Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.3-Tangent/105-4.3.4.2-a+b-tan-^m-c+d-tan-^n-A+B-tan+C-tan^2-

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [171]. This is test number [105].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (171)	0.00 (0)
Mathematica	98.83 (169)	1.17 (2)
Maple	71.35 (122)	28.65 (49)
Mupad	60.23 (103)	39.77 (68)
Fricas	49.12 (84)	50.88 (87)
Giac	49.12 (84)	50.88 (87)
Maxima	49.12 (84)	50.88 (87)
Sympy	36.84 (63)	63.16 (108)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

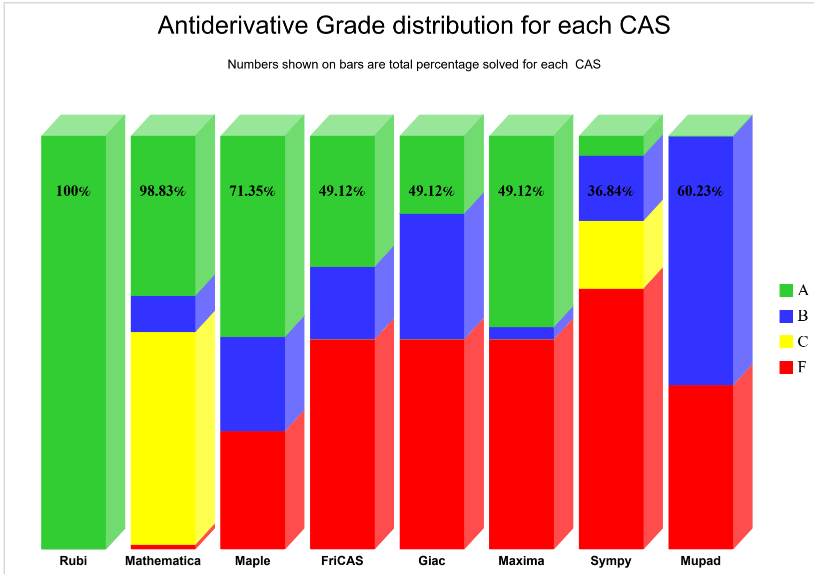
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

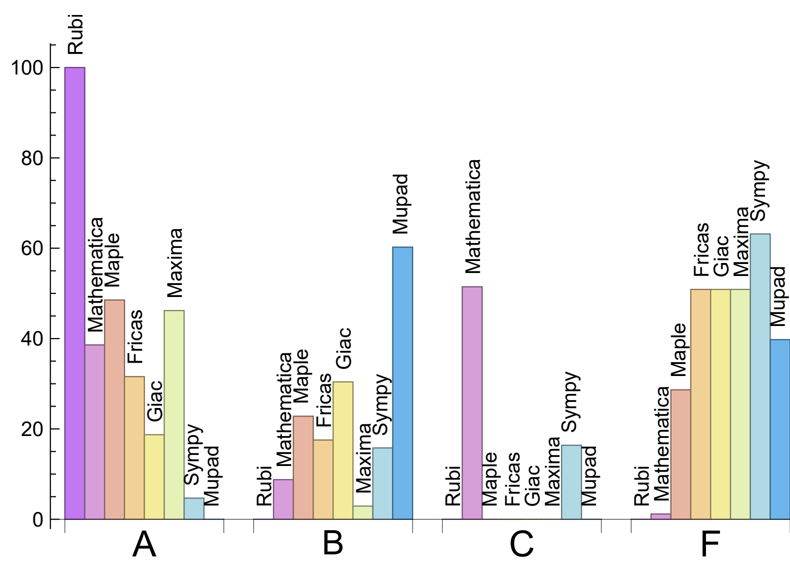
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	48.54	22.81	0.00	28.65
Maxima	46.20	2.92	0.00	50.88
Mathematica	38.60	8.77	51.46	1.17
Fricas	31.58	17.54	0.00	50.88
Giac	18.71	30.41	0.00	50.88
Sympy	4.68	15.79	16.37	63.16
Mupad	N/A	60.23	0.00	39.77

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	49	93.88 %	6.12 %	0.00 %
Fricas	87	14.94 %	85.06 %	0.00 %
Giac	87	12.64 %	26.44 %	60.92 %
Maxima	87	36.78 %	41.38 %	21.84 %
Sympy	108	71.30 %	6.48 %	22.22 %
Mupad	68	38.24 %	61.76 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

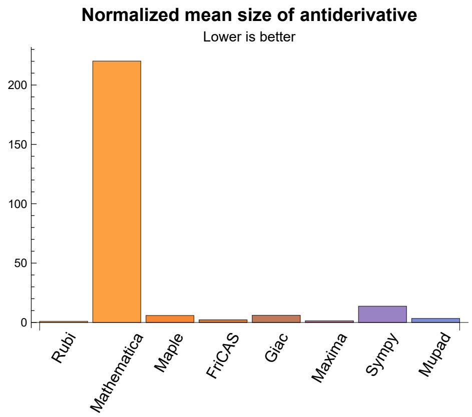
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

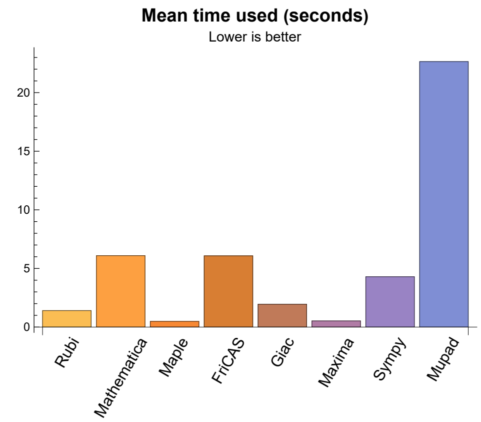
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.41	323.47	1.00	287.00	1.00
Mathematica	6.09	110159.63	220.14	314.00	1.25
Maple	0.49	2019.01	5.83	386.00	1.33
Maxima	0.53	378.46	1.36	217.50	1.21
Fricas	6.08	807.71	2.24	272.00	1.59
Sympy	4.30	3297.83	13.67	711.00	2.75
Giac	1.95	1924.63	6.03	492.00	2.34
Mupad	22.65	866.03	3.34	307.00	1.38

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
50
100
150
200
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {69, 84, 132, 138, 139, 143, 144, 145, 146, 153, 154, 155, 159, 160}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

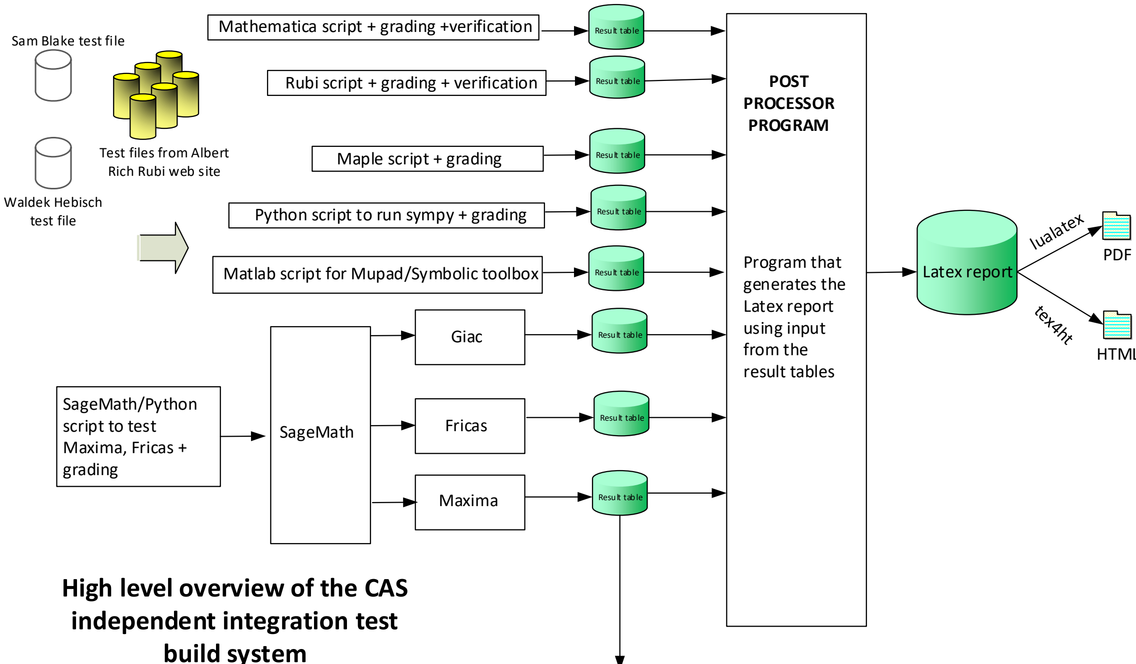
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 28, 45, 46, 47, 48, 53, 69, 74, 75, 76, 81, 82, 84, 88, 91, 92, 93, 94, 95, 98, 99, 100, 101, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 120, 128, 129, 130, 131, 133, 134, 135, 136, 137, 141, 142, 147, 148, 149, 150, 151, 152, 156, 157, 158, 161, 162, 163, 166, 167, 168, 169, 170 }

B grade: { 83, 89, 90, 96, 97, 102, 103, 108, 109, 121, 126, 127, 140, 165, 171 }

C grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 77, 78, 79, 80, 85, 86, 87, 116, 117, 118, 119, 122, 123, 124, 125, 132, 138, 139, 143, 144, 145, 146, 153, 154, 155, 159, 160 }

F grade: { 49, 164 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

B grade: { 64, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 84, 85, 86, 87 }

B grade: { 76, 82, 83, 88, 89 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 64, 65, 66, 67, 70, 71, 72, 73, 74, 79, 80 }

B grade: { 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 55, 56, 62, 63, 68, 69, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.6 Sympy

A grade: { 1, 2, 9, 10, 11, 18, 19, 20 }

B grade: { 3, 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 50, 51, 52, 53, 57, 58, 59, 60, 64, 65, 66 }

C grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 54, 55, 61, 62, 67, 68, 70, 71, 72, 73, 74, 77, 78, 79, 80 }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 63, 69, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.7 Giac

A grade: { 3, 4, 11, 12, 13, 18, 19, 20, 21, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 44, 54, 55, 61, 67, 70, 71, 72, 73, 74, 79 }

B grade: { 1, 2, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 22, 23, 24, 34, 35, 36, 40, 41, 42, 43, 50, 51, 52, 53, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 68, 69, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 100, 101, 106, 110, 111, 112, 113, 114, 115, 117, 118, 119, 123, 124, 125 }

C grade: { }

F grade: { 45, 46, 47, 48, 49, 90, 91, 96, 97, 98, 99, 102, 103, 104, 105, 107, 108, 109, 116, 120, 121, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	99	86	85	139	1017	84
N.S.	1	1.00	0.99	1.14	0.99	0.98	1.60	11.69	0.97
time (sec)	N/A	0.096	0.410	0.083	0.501	4.634	0.115	1.045	8.825

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	74	66	66	105	616	63
N.S.	1	1.00	1.02	1.12	1.00	1.00	1.59	9.33	0.95
time (sec)	N/A	0.035	0.225	0.070	0.516	4.076	0.096	0.818	8.838

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	59	53	50	50	82	50	58
N.S.	1	1.00	1.40	1.26	1.19	1.19	1.95	1.19	1.38
time (sec)	N/A	0.050	0.046	0.237	0.501	4.456	0.348	0.803	8.791

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	43	52	59	85	53	69
N.S.	1	1.00	1.19	1.16	1.41	1.59	2.30	1.43	1.86
time (sec)	N/A	0.080	0.056	0.280	0.495	3.687	0.523	0.967	8.956

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	78	53	68	73	122	119	87
N.S.	1	1.00	1.81	1.23	1.58	1.70	2.84	2.77	2.02
time (sec)	N/A	0.090	0.114	0.216	0.515	3.563	0.925	1.134	8.875

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	77	77	86	95	150	179	108
N.S.	1	1.00	1.17	1.17	1.30	1.44	2.27	2.71	1.64
time (sec)	N/A	0.118	0.328	0.302	0.502	4.566	1.375	1.395	8.944

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	101	95	104	121	180	237	127
N.S.	1	1.00	1.16	1.09	1.20	1.39	2.07	2.72	1.46
time (sec)	N/A	0.146	0.723	0.277	0.518	3.568	2.231	1.634	8.888

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	100	108	122	138	211	299	145
N.S.	1	1.00	0.93	1.00	1.13	1.28	1.95	2.77	1.34
time (sec)	N/A	0.170	0.785	0.277	0.496	3.605	3.031	1.436	8.821

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	221	176	147	146	250	2228	151
N.S.	1	1.00	1.49	1.19	0.99	0.99	1.69	15.05	1.02
time (sec)	N/A	0.211	6.162	0.100	0.507	2.305	0.171	1.916	8.844

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	172	135	120	119	194	1509	121
N.S.	1	1.00	1.54	1.21	1.07	1.06	1.73	13.47	1.08
time (sec)	N/A	0.083	1.246	0.071	0.513	1.375	0.132	1.254	8.795

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	96	102	91	91	151	95	91
N.S.	1	1.00	1.10	1.17	1.05	1.05	1.74	1.09	1.05
time (sec)	N/A	0.100	0.320	0.309	0.492	1.418	0.564	1.151	8.848

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	91	82	85	92	136	86	90
N.S.	1	1.00	1.30	1.17	1.21	1.31	1.94	1.23	1.29
time (sec)	N/A	0.135	0.192	0.270	0.490	2.608	0.846	1.410	8.853

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	100	84	93	112	165	118	100
N.S.	1	1.00	1.39	1.17	1.29	1.56	2.29	1.64	1.39
time (sec)	N/A	0.149	0.172	0.262	0.492	2.404	1.345	1.695	8.999

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	123	107	120	122	212	237	127
N.S.	1	1.00	1.40	1.22	1.36	1.39	2.41	2.69	1.44
time (sec)	N/A	0.184	0.235	0.296	0.537	3.354	2.213	0.998	8.979

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	152	136	149	157	258	334	156
N.S.	1	1.00	1.29	1.15	1.26	1.33	2.19	2.83	1.32
time (sec)	N/A	0.221	0.781	0.271	0.511	4.867	3.052	1.063	9.078

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	180	162	175	191	311	435	182
N.S.	1	1.00	1.19	1.07	1.16	1.26	2.06	2.88	1.21
time (sec)	N/A	0.260	1.901	0.296	0.527	3.472	5.024	1.098	8.860

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	209	213	179	178	313	2870	181
N.S.	1	1.00	1.27	1.29	1.08	1.08	1.90	17.39	1.10
time (sec)	N/A	0.132	1.095	0.093	0.499	4.127	0.165	2.470	8.835

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	130	161	143	142	248	158	142
N.S.	1	1.00	0.93	1.15	1.02	1.01	1.77	1.13	1.01
time (sec)	N/A	0.144	0.709	0.287	0.507	6.586	0.894	1.653	8.963

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	113	131	124	133	211	129	118
N.S.	1	1.00	0.97	1.12	1.06	1.14	1.80	1.10	1.01
time (sec)	N/A	0.233	0.324	0.296	0.501	3.194	1.243	1.790	8.963

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	113	123	125	145	221	152	114
N.S.	1	1.00	0.95	1.03	1.05	1.22	1.86	1.28	0.96
time (sec)	N/A	0.234	0.327	0.272	0.500	5.318	2.228	1.296	8.860

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	126	138	142	162	260	193	135
N.S.	1	1.00	0.99	1.09	1.12	1.28	2.05	1.52	1.06
time (sec)	N/A	0.248	0.308	0.355	0.494	3.151	2.983	1.416	8.967

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	164	166	180	181	330	390	169
N.S.	1	1.00	1.06	1.08	1.17	1.18	2.14	2.53	1.10
time (sec)	N/A	0.300	0.839	0.316	0.507	2.886	4.910	1.537	8.999

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	199	203	215	225	398	528	204
N.S.	1	1.00	1.04	1.06	1.13	1.18	2.08	2.76	1.07
time (sec)	N/A	0.362	0.505	0.310	0.493	4.021	6.512	1.596	8.941

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	237	244	250	266	469	670	238
N.S.	1	1.00	1.02	1.05	1.07	1.14	2.01	2.88	1.02
time (sec)	N/A	0.389	0.780	0.345	0.502	3.010	10.142	1.663	9.120

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	138	127	130	190	1306	135	144
N.S.	1	1.00	1.09	1.00	1.02	1.50	10.28	1.06	1.13
time (sec)	N/A	0.316	0.986	0.194	0.512	4.860	0.823	0.787	9.071

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	118	101	109	149	1020	110	117
N.S.	1	1.00	1.17	1.00	1.08	1.48	10.10	1.09	1.16
time (sec)	N/A	0.174	0.492	0.177	0.511	4.217	0.641	0.686	8.768

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	98	87	94	110	711	95	100
N.S.	1	1.00	1.15	1.02	1.11	1.29	8.36	1.12	1.18
time (sec)	N/A	0.120	0.135	0.179	0.492	2.356	0.519	0.647	9.065

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	67	82	88	76	541	94	93
N.S.	1	1.00	1.16	1.41	1.52	1.31	9.33	1.62	1.60
time (sec)	N/A	0.095	0.089	0.369	0.518	1.651	1.415	0.801	9.125

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	113	101	107	118	966	113	115
N.S.	1	1.00	1.41	1.26	1.34	1.48	12.08	1.41	1.44
time (sec)	N/A	0.138	0.250	0.493	0.520	1.145	2.497	0.930	9.457

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	138	123	131	177	2064	157	140
N.S.	1	1.00	1.34	1.19	1.27	1.72	20.04	1.52	1.36
time (sec)	N/A	0.232	0.610	0.477	0.513	1.192	5.022	1.137	10.339

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	163	152	158	234	2592	214	175
N.S.	1	1.00	1.19	1.11	1.15	1.71	18.92	1.56	1.28
time (sec)	N/A	0.434	0.950	0.482	0.507	3.995	8.454	1.469	10.929

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	444	172	220	434	4541	290	210
N.S.	1	1.00	2.13	0.83	1.06	2.09	21.83	1.39	1.01
time (sec)	N/A	0.363	2.817	0.229	0.510	3.062	1.198	0.863	9.648

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	324	155	197	311	3497	244	165
N.S.	1	1.00	2.06	0.99	1.25	1.98	22.27	1.55	1.05
time (sec)	N/A	0.223	1.486	0.258	0.493	3.889	1.050	0.708	9.107

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	140	145	185	221	2995	241	163
N.S.	1	1.00	1.22	1.26	1.61	1.92	26.04	2.10	1.42
time (sec)	N/A	0.103	1.515	0.165	0.517	3.609	0.857	0.705	9.011

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	190	141	177	222	2895	234	153
N.S.	1	1.00	1.71	1.27	1.59	2.00	26.08	2.11	1.38
time (sec)	N/A	0.146	1.516	0.429	0.503	4.127	2.228	0.953	9.094

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	159	163	208	323	4461	279	180
N.S.	1	1.00	1.16	1.19	1.52	2.36	32.56	2.04	1.31
time (sec)	N/A	0.280	1.653	0.504	0.506	8.037	4.024	1.297	10.693

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	193	196	262	465	8097	362	230
N.S.	1	1.00	1.01	1.02	1.36	2.42	42.17	1.89	1.20
time (sec)	N/A	0.427	2.355	0.564	0.509	9.190	7.367	1.324	12.148

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	1146	263	389	890	0	505	335
N.S.	1	1.00	3.46	0.79	1.18	2.69	0.00	1.53	1.01
time (sec)	N/A	0.606	6.615	0.386	0.516	8.004	0.000	1.105	10.428

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	462	242	366	666	0	458	307
N.S.	1	1.00	1.85	0.97	1.46	2.66	0.00	1.83	1.23
time (sec)	N/A	0.374	3.225	0.302	0.515	5.069	0.000	0.967	9.316

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	288	223	333	478	0	410	280
N.S.	1	1.00	1.52	1.18	1.76	2.53	0.00	2.17	1.48
time (sec)	N/A	0.280	3.578	0.235	0.519	3.728	0.000	0.837	9.181

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	188	213	330	488	0	410	282
N.S.	1	1.00	1.05	1.19	1.84	2.73	0.00	2.29	1.58
time (sec)	N/A	0.179	2.735	0.224	0.499	4.126	0.000	0.839	9.280

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	243	208	321	482	0	409	279
N.S.	1	1.00	1.39	1.19	1.83	2.75	0.00	2.34	1.59
time (sec)	N/A	0.224	2.793	0.509	0.506	2.766	0.000	1.295	8.941

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	223	243	372	683	0	479	315
N.S.	1	1.00	1.04	1.13	1.73	3.18	0.00	2.23	1.47
time (sec)	N/A	0.474	2.081	0.651	0.518	2.346	0.000	1.281	10.976

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	288	289	454	917	0	560	380
N.S.	1	1.00	1.00	1.01	1.58	3.20	0.00	1.95	1.32
time (sec)	N/A	0.653	6.281	0.684	0.510	3.219	0.000	1.621	13.986

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.307	0.423	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	115	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.274	0.504	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	133	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.378	0.650	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	133	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.376	0.625	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.135	45.839	0.787	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	280	639	423	421	1001	11805	477
N.S.	1	1.00	0.79	1.81	1.20	1.19	2.84	33.44	1.35
time (sec)	N/A	0.554	5.855	0.206	0.511	1.481	0.351	8.909	8.997

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	243	386	280	278	617	6502	300
N.S.	1	1.00	0.98	1.56	1.13	1.12	2.49	26.22	1.21
time (sec)	N/A	0.309	2.268	0.118	0.497	3.354	0.229	3.760	8.982

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	161	187	156	154	326	2918	153
N.S.	1	1.00	1.00	1.16	0.97	0.96	2.02	18.12	0.95
time (sec)	N/A	0.168	1.066	0.118	0.507	2.700	0.143	1.698	8.842

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	76	80	78	77	131	918	75
N.S.	1	1.00	1.04	1.10	1.07	1.05	1.79	12.58	1.03
time (sec)	N/A	0.045	0.329	0.071	0.494	3.153	0.100	0.856	8.679

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	155	148	173	187	231	2387	186	186
N.S.	1	0.99	0.95	1.11	1.20	1.48	15.30	1.19	1.19
time (sec)	N/A	0.237	0.787	0.209	0.505	5.584	1.022	0.669	10.127

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	589	321	342	564	9721	531	1875
N.S.	1	1.00	2.22	1.21	1.29	2.13	36.68	2.00	7.08
time (sec)	N/A	0.324	4.486	0.323	0.516	6.443	1.653	0.764	21.136

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	331	494	580	996	0	1037	502
N.S.	1	1.00	1.03	1.54	1.81	3.11	0.00	3.24	1.57
time (sec)	N/A	0.497	4.619	0.303	0.528	7.206	0.000	0.954	15.885

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	573	1239	699	697	1819	24014	891
N.S.	1	1.00	0.87	1.87	1.06	1.05	2.75	36.33	1.35
time (sec)	N/A	1.434	6.452	0.238	0.513	7.551	0.500	18.531	9.287

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	352	770	470	468	1134	13549	561
N.S.	1	1.00	0.79	1.74	1.06	1.06	2.56	30.58	1.27
time (sec)	N/A	0.826	5.179	0.181	0.512	5.084	0.347	9.574	9.119

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	264	241	386	266	264	617	6502	300
N.S.	1	0.99	0.91	1.45	1.00	0.99	2.32	24.44	1.13
time (sec)	N/A	0.317	1.768	0.123	0.520	5.501	0.230	3.877	9.006

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	176	162	140	138	241	2128	141
N.S.	1	1.00	1.34	1.24	1.07	1.05	1.84	16.24	1.08
time (sec)	N/A	0.105	0.726	0.077	0.515	5.627	0.135	1.411	8.808

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	252	190	317	295	403	4444	338	325
N.S.	1	0.99	0.75	1.25	1.16	1.59	17.50	1.33	1.28
time (sec)	N/A	0.549	1.960	0.253	0.510	4.500	2.639	0.849	11.275

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	2640	552	501	973	16225	912	2500
N.S.	1	1.00	6.36	1.33	1.21	2.34	39.10	2.20	6.02
time (sec)	N/A	0.733	7.394	0.372	0.550	5.958	3.496	0.916	34.031

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	2499	865	845	1711	0	1714	807
N.S.	1	1.00	4.19	1.45	1.42	2.87	0.00	2.87	1.35
time (sec)	N/A	0.883	7.513	0.584	0.549	3.946	0.000	1.203	29.277

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	603	419	1239	688	686	1819	24014	891
N.S.	1	1.00	0.69	2.05	1.14	1.14	3.02	39.82	1.48
time (sec)	N/A	1.079	6.412	0.355	0.516	2.448	0.491	20.705	9.310

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	387	297	639	394	392	1001	11805	478
N.S.	1	0.99	0.76	1.64	1.01	1.01	2.57	30.35	1.23
time (sec)	N/A	0.521	6.258	0.161	0.517	1.589	0.349	8.913	9.038

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	212	265	208	206	410	4300	221
N.S.	1	1.00	1.11	1.39	1.09	1.08	2.15	22.51	1.16
time (sec)	N/A	0.174	1.613	0.103	0.523	0.994	0.180	2.887	8.789

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	255	542	442	630	7096	573	508
N.S.	1	1.00	0.70	1.49	1.22	1.74	19.55	1.58	1.40
time (sec)	N/A	1.040	3.035	0.342	0.520	2.580	25.439	1.097	13.004

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	2467	829	691	1522	24300	1357	701
N.S.	1	1.00	4.30	1.44	1.20	2.65	42.33	2.36	1.22
time (sec)	N/A	1.524	7.750	0.623	0.542	4.128	33.116	1.238	15.699

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	798	798	1451	1271	1126	2562	0	2505	1172
N.S.	1	1.00	1.82	1.59	1.41	3.21	0.00	3.14	1.47
time (sec)	N/A	1.846	13.872	0.895	0.569	4.554	0.000	1.495	19.238

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	258	542	451	634	7096	573	508
N.S.	1	1.00	0.77	1.61	1.34	1.88	21.06	1.70	1.51
time (sec)	N/A	1.011	2.857	0.430	0.558	4.203	25.846	1.119	13.392

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	190	317	299	396	4444	338	325
N.S.	1	1.00	0.81	1.34	1.27	1.68	18.83	1.43	1.38
time (sec)	N/A	0.519	1.917	0.249	0.544	6.800	2.681	0.825	11.200

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	148	173	182	217	2387	186	186
N.S.	1	1.00	0.95	1.11	1.17	1.39	15.30	1.19	1.19
time (sec)	N/A	0.226	0.720	0.219	0.549	3.878	1.024	0.709	10.246

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	117	100	109	122	966	109	109
N.S.	1	1.00	1.18	1.01	1.10	1.23	9.76	1.10	1.10
time (sec)	N/A	0.069	0.140	0.186	0.591	4.741	0.609	0.635	9.898

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	164	313	197	247	307	24052	272	196
N.S.	1	0.99	1.90	1.19	1.50	1.86	145.77	1.65	1.19
time (sec)	N/A	0.177	0.974	0.335	0.597	10.941	43.033	0.765	21.398

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	543	364	525	1355	0	846	393
N.S.	1	1.00	1.93	1.30	1.87	4.82	0.00	3.01	1.40
time (sec)	N/A	0.524	4.377	0.840	0.538	11.323	0.000	0.893	63.656

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	898	647	1103	3657	0	2127	2500
N.S.	1	1.00	1.88	1.36	2.31	7.67	0.00	4.46	5.24
time (sec)	N/A	1.166	7.952	1.596	0.632	12.067	0.000	1.224	24.034

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	2463	829	690	1487	24300	1355	701
N.S.	1	1.00	4.25	1.43	1.19	2.57	41.97	2.34	1.21
time (sec)	N/A	1.403	7.740	0.464	0.535	12.852	33.483	1.235	16.677

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	2636	552	498	948	16225	912	2500
N.S.	1	1.00	6.32	1.32	1.19	2.27	38.91	2.19	6.00
time (sec)	N/A	0.729	7.352	0.351	0.558	4.403	3.737	0.940	35.258

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	288	606	321	323	513	9721	528	1875
N.S.	1	0.99	2.08	1.10	1.11	1.76	33.29	1.81	6.42
time (sec)	N/A	0.360	4.457	0.359	0.531	3.104	1.691	0.774	22.014

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	207	173	209	262	4396	299	184
N.S.	1	1.00	1.48	1.24	1.49	1.87	31.40	2.14	1.31
time (sec)	N/A	0.143	1.776	0.206	0.514	3.288	0.986	0.713	11.345

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	561	365	518	1285	0	846	430
N.S.	1	1.00	1.91	1.25	1.77	4.39	0.00	2.89	1.47
time (sec)	N/A	0.543	4.920	0.758	0.531	10.131	0.000	0.938	85.865

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	508	984	577	1192	4188	0	2893	2500
N.S.	1	1.00	1.93	1.13	2.34	8.23	0.00	5.68	4.91
time (sec)	N/A	1.402	7.841	1.601	0.616	13.104	0.000	1.227	31.511

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	841	841	1758	951	2528	9612	0	3176	2500
N.S.	1	1.00	2.09	1.13	3.01	11.43	0.00	3.78	2.97
time (sec)	N/A	2.715	7.698	3.594	0.717	20.927	0.000	1.233	58.468

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	804	804	1445	1271	1117	2503	0	2505	1172
N.S.	1	1.00	1.80	1.58	1.39	3.11	0.00	3.12	1.46
time (sec)	N/A	1.790	13.708	1.010	0.603	4.638	0.000	1.487	20.600

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	2499	865	833	1630	0	1709	807
N.S.	1	1.00	4.19	1.45	1.40	2.73	0.00	2.86	1.35
time (sec)	N/A	0.947	7.493	0.587	0.548	3.365	0.000	1.144	30.686

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	349	331	493	549	906	0	1037	502
N.S.	1	0.99	0.94	1.40	1.56	2.57	0.00	2.95	1.43
time (sec)	N/A	0.509	4.617	0.319	0.551	4.610	0.000	0.950	16.535

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	261	262	373	575	0	548	327
N.S.	1	1.00	1.25	1.25	1.78	2.75	0.00	2.62	1.56
time (sec)	N/A	0.250	3.485	0.281	0.547	5.260	0.000	0.893	11.877

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	912	649	1085	3510	0	2125	2500
N.S.	1	1.00	1.87	1.33	2.23	7.21	0.00	4.36	5.13
time (sec)	N/A	1.230	8.125	1.829	0.597	48.335	0.000	1.190	24.606

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	861	860	1732	949	2546	9585	0	3176	2500
N.S.	1	1.00	2.01	1.10	2.96	11.13	0.00	3.69	2.90
time (sec)	N/A	2.888	7.754	3.471	0.707	71.528	0.000	1.161	47.926

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	1232	3066	0	0	0	0	-1
N.S.	1	1.00	2.66	6.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.415	6.286	0.684	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	314	2208	0	0	0	0	-1
N.S.	1	1.00	0.97	6.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.863	3.185	0.491	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	220	1398	0	0	0	0	2500
N.S.	1	1.00	0.98	6.24	0.00	0.00	0.00	0.00	11.16
time (sec)	N/A	0.416	1.316	0.479	0.000	0.000	0.000	0.000	60.113

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	150	846	0	0	0	0	1199
N.S.	1	1.00	0.97	5.46	0.00	0.00	0.00	0.00	7.74
time (sec)	N/A	0.206	0.388	0.425	0.000	0.000	0.000	0.000	17.403

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	233	1394	0	0	0	0	2500
N.S.	1	1.00	1.00	5.96	0.00	0.00	0.00	0.00	10.68
time (sec)	N/A	0.720	0.458	0.618	0.000	0.000	0.000	0.000	36.224

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	362	2143	0	0	0	0	2500
N.S.	1	1.00	1.14	6.76	0.00	0.00	0.00	0.00	7.89
time (sec)	N/A	0.981	4.112	0.580	0.000	0.000	0.000	0.000	45.420

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	2819	3102	0	0	0	0	-1
N.S.	1	1.00	5.19	5.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.869	6.278	0.629	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	1290	5095	0	0	0	0	-1
N.S.	1	1.00	2.35	9.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.841	6.286	0.559	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	350	3689	0	0	0	0	-1
N.S.	1	1.00	0.88	9.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.128	3.955	0.523	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	260	2315	0	0	0	0	-1
N.S.	1	1.00	0.95	8.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	3.029	0.489	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	202	1293	0	0	0	0	2500
N.S.	1	1.00	1.08	6.91	0.00	0.00	0.00	0.00	13.37
time (sec)	N/A	0.290	0.870	0.498	0.000	0.000	0.000	0.000	44.865

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	266	2291	0	0	0	0	2500
N.S.	1	1.00	0.98	8.45	0.00	0.00	0.00	0.00	9.23
time (sec)	N/A	1.205	1.650	0.635	0.000	0.000	0.000	0.000	58.881

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	1732	3605	0	0	0	0	-1
N.S.	1	1.00	4.66	9.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.676	4.137	0.680	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	7678	4969	0	0	0	0	-1
N.S.	1	1.00	14.43	9.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.850	6.349	0.654	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	564	5353	0	0	0	0	-1
N.S.	1	1.00	1.12	10.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.546	6.321	0.577	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	351	324	3368	0	0	0	0	-1
N.S.	1	0.99	0.92	9.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.833	3.498	0.526	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	262	1787	0	0	0	0	2500
N.S.	1	1.00	1.14	7.80	0.00	0.00	0.00	0.00	10.92
time (sec)	N/A	0.416	1.391	0.482	0.000	0.000	0.000	0.000	117.306

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	322	3356	0	0	0	0	-1
N.S.	1	1.00	0.96	9.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.852	3.444	0.642	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	6112	5214	0	0	0	0	-1
N.S.	1	1.00	12.92	11.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.642	6.349	0.686	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	18214	7283	0	0	0	0	-1
N.S.	1	1.00	28.33	11.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.322	6.604	0.784	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	392	8228	0	0	0	0	2500
N.S.	1	1.00	0.96	20.22	0.00	0.00	0.00	0.00	6.14
time (sec)	N/A	1.105	5.772	0.456	0.000	0.000	0.000	0.000	122.079

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	275	5860	0	0	0	0	2500
N.S.	1	1.00	0.96	20.42	0.00	0.00	0.00	0.00	8.71
time (sec)	N/A	0.647	3.920	0.458	0.000	0.000	0.000	0.000	47.981

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	192	1349	0	0	0	0	2500
N.S.	1	1.00	0.99	6.95	0.00	0.00	0.00	0.00	12.89
time (sec)	N/A	0.326	0.985	0.437	0.000	0.000	0.000	0.000	23.482

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	129	1894	0	0	0	0	2500
N.S.	1	1.00	0.97	14.24	0.00	0.00	0.00	0.00	18.80
time (sec)	N/A	0.151	0.150	0.432	0.000	0.000	0.000	0.000	14.206

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	194	3761	0	0	0	0	2500
N.S.	1	1.00	0.92	17.91	0.00	0.00	0.00	0.00	11.90
time (sec)	N/A	0.414	0.290	0.531	0.000	0.000	0.000	0.000	69.145

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	338	5887	0	0	0	0	2500
N.S.	1	1.00	1.03	18.00	0.00	0.00	0.00	0.00	7.65
time (sec)	N/A	0.943	4.717	0.580	0.000	0.000	0.000	0.000	57.653

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	920	13578	0	0	0	0	-1
N.S.	1	1.00	1.80	26.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.656	6.503	0.655	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	476	9979	0	0	0	0	2500
N.S.	1	1.00	1.39	29.09	0.00	0.00	0.00	0.00	7.29
time (sec)	N/A	0.901	6.354	0.527	0.000	0.000	0.000	0.000	66.251

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	290	6237	0	0	0	0	2500
N.S.	1	1.00	1.44	31.03	0.00	0.00	0.00	0.00	12.44
time (sec)	N/A	0.372	1.710	0.481	0.000	0.000	0.000	0.000	41.071

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	218	3124	0	0	0	0	2500
N.S.	1	1.00	1.39	19.90	0.00	0.00	0.00	0.00	15.92
time (sec)	N/A	0.201	0.697	0.454	0.000	0.000	0.000	0.000	19.614

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	296	6344	0	0	0	0	-1
N.S.	1	1.00	1.13	24.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.872	3.275	0.634	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	446	2078	10088	0	0	0	0	-1
N.S.	1	1.00	4.65	22.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.923	6.216	0.650	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	670	21768	0	0	0	0	-1
N.S.	1	1.00	1.15	37.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.006	6.590	0.649	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	414	15609	0	0	0	0	2500
N.S.	1	1.00	1.16	43.60	0.00	0.00	0.00	0.00	6.98
time (sec)	N/A	1.038	4.681	0.704	0.000	0.000	0.000	0.000	116.899

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	271	300	9672	0	0	0	0	2500
N.S.	1	0.99	1.10	35.43	0.00	0.00	0.00	0.00	9.16
time (sec)	N/A	0.540	1.965	0.554	0.000	0.000	0.000	0.000	88.469

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	223	4918	0	0	0	0	2500
N.S.	1	1.00	1.07	23.53	0.00	0.00	0.00	0.00	11.96
time (sec)	N/A	0.323	0.650	0.464	0.000	0.000	0.000	0.000	37.590

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	1948	10189	0	0	0	0	-1
N.S.	1	1.00	5.34	27.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.691	6.189	0.614	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	678	6052	15715	0	0	0	0	-1
N.S.	1	1.00	8.91	23.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.471	6.283	0.747	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	1202	0	0	0	0	0	-1
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.307	8.579	180.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	835	0	0	0	0	0	-1
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.431	7.887	180.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	383	619	0	0	0	0	0	-1
N.S.	1	1.01	1.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.560	7.189	180.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	441	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.924	2.738	180.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	621058	0	0	0	0	0	-1
N.S.	1	1.00	2070.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.876	33.566	180.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	424	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.341	6.146	180.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	1109	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.381	6.886	180.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	682	682	1304	0	0	0	0	0	-1
N.S.	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.725	8.081	180.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	867	0	0	0	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.339	7.978	180.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	580	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.104	6.546	180.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	1073629	0	0	0	0	0	-1
N.S.	1	1.00	2810.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.280	36.188	180.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	1347065	0	0	0	0	0	-1
N.S.	1	1.00	3350.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.144	36.897	180.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	586	3134	0	0	0	0	0	-1
N.S.	1	1.00	5.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.465	7.957	180.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	697	697	1261	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.539	8.415	180.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	780	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.480	7.969	180.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	1654245	0	0	0	0	0	-1
N.S.	1	1.00	3092.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.184	39.156	180.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	2018669	0	0	0	0	0	-1
N.S.	1	1.00	3703.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.262	40.815	180.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	2345519	0	0	0	0	0	-1
N.S.	1	1.00	3975.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	10.903	41.820	180.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F(-1)	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	946	946	2719441	0	0	0	0	0	-1
N.S.	1	1.00	2874.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.317	46.370	180.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	785	0	0	0	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.374	7.643	180.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	582	0	0	0	0	0	-1
N.S.	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.034	4.582	180.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	450	0	0	0	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.945	4.340	180.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	362	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.094	1.534	180.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	264	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.652	1.716	180.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	388	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.209	4.107	180.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	1653959	0	0	0	0	0	-1
N.S.	1	1.00	3132.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.902	39.127	180.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	1073499	0	0	0	0	0	-1
N.S.	1	1.00	2825.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.118	36.186	180.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	621084	0	0	0	0	0	-1
N.S.	1	1.00	2077.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.476	33.565	180.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	275	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	2.198	180.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	382	484	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.277	6.498	180.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	902	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.357	6.618	180.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	2018643	0	0	0	0	0	-1
N.S.	1	1.00	3676.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.959	40.927	180.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	1347117	0	0	0	0	0	-1
N.S.	1	1.00	3309.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.473	37.116	180.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	434	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.296	6.105	180.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	403	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.236	3.720	180.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	650	903	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.397	6.648	180.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.616	16.614	0.370	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	551	1390	0	0	0	0	0	-1
N.S.	1	0.98	2.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.550	6.298	0.618	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	360	308	0	0	0	0	0	-1
N.S.	1	0.99	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.771	4.836	0.324	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	202	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.355	1.995	0.250	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	135	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.180	0.160	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	204	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.336	0.753	0.342	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	402	360	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.842	5.702	0.398	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	702	702	2238	0	0	0	0	0	-1
N.S.	1	1.00	3.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.106	6.169	0.601	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [128] had the largest ratio of [49]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	36	0.139
2	A	3	3	1.00	30	0.100
3	A	3	3	1.00	36	0.083
4	A	5	4	1.00	38	0.105
5	A	4	4	1.00	38	0.105
6	A	5	5	1.00	38	0.132
7	A	6	5	1.00	38	0.132
8	A	7	5	1.00	38	0.132
9	A	6	6	1.00	38	0.158
10	A	4	4	1.00	32	0.125
11	A	4	4	1.00	38	0.105
12	A	5	4	1.00	40	0.100
13	A	5	4	1.00	40	0.100
14	A	5	5	1.00	40	0.125
15	A	6	6	1.00	40	0.150
16	A	7	6	1.00	40	0.150
17	A	5	4	1.00	32	0.125
18	A	5	4	1.00	38	0.105
19	A	6	5	1.00	40	0.125
20	A	6	5	1.00	40	0.125
21	A	6	5	1.00	40	0.125
22	A	6	6	1.00	40	0.150
23	A	7	7	1.00	40	0.175
24	A	8	7	1.00	40	0.175
25	A	7	7	1.00	40	0.175

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	6	1.00	38	0.158
27	A	6	4	1.00	32	0.125
28	A	3	3	1.00	38	0.079
29	A	4	4	1.00	40	0.100
30	A	5	5	1.00	40	0.125
31	A	6	6	1.00	40	0.150
32	A	7	7	1.00	40	0.175
33	A	6	6	1.00	38	0.158
34	A	3	3	1.00	32	0.094
35	A	4	4	1.00	38	0.105
36	A	5	5	1.00	40	0.125
37	A	6	6	1.00	40	0.150
38	A	8	8	1.00	40	0.200
39	A	7	7	1.00	40	0.175
40	A	5	5	1.00	38	0.132
41	A	4	4	1.00	32	0.125
42	A	5	4	1.00	38	0.105
43	A	6	6	1.00	40	0.150
44	A	7	6	1.00	40	0.150
45	A	7	5	1.00	39	0.128
46	A	7	5	1.00	39	0.128
47	A	7	5	1.00	41	0.122
48	A	7	5	1.00	41	0.122
49	A	13	7	1.00	43	0.163
50	A	6	5	1.00	43	0.116
51	A	5	5	1.00	43	0.116
52	A	4	4	1.00	41	0.098
53	A	3	3	1.00	31	0.097
54	A	5	5	0.99	43	0.116
55	A	5	5	1.00	43	0.116
56	A	4	4	1.00	43	0.093
57	A	7	6	1.00	45	0.133
58	A	6	6	1.00	45	0.133
59	A	5	5	0.99	43	0.116
60	A	4	4	1.00	33	0.121

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	6	6	0.99	45	0.133
62	A	6	6	1.00	45	0.133
63	A	6	6	1.00	45	0.133
64	A	7	6	1.00	45	0.133
65	A	6	5	0.99	43	0.116
66	A	5	4	1.00	33	0.121
67	A	7	6	1.00	45	0.133
68	A	7	7	1.00	45	0.156
69	A	7	6	1.00	45	0.133
70	A	7	6	1.00	45	0.133
71	A	6	6	1.00	45	0.133
72	A	5	5	1.00	43	0.116
73	A	4	4	1.00	33	0.121
74	A	3	2	0.99	45	0.044
75	A	4	3	1.00	45	0.067
76	A	5	3	1.00	45	0.067
77	A	7	7	1.00	45	0.156
78	A	6	6	1.00	45	0.133
79	A	5	5	0.99	43	0.116
80	A	3	3	1.00	33	0.091
81	A	4	3	1.00	45	0.067
82	A	5	3	1.00	45	0.067
83	A	6	3	1.00	45	0.067
84	A	7	6	1.00	45	0.133
85	A	6	6	1.00	45	0.133
86	A	4	4	0.99	43	0.093
87	A	4	4	1.00	33	0.121
88	A	5	3	1.00	45	0.067
89	A	6	3	1.00	45	0.067
90	A	12	8	1.00	47	0.170
91	A	11	8	1.00	47	0.170
92	A	10	7	1.00	45	0.156
93	A	9	6	1.00	35	0.171
94	A	12	7	1.00	47	0.149
95	A	12	7	1.00	47	0.149

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	13	8	1.00	47	0.170
97	A	13	8	1.00	47	0.170
98	A	12	8	1.00	47	0.170
99	A	11	7	1.00	45	0.156
100	A	10	6	1.00	35	0.171
101	A	13	7	1.00	47	0.149
102	A	13	8	1.00	47	0.170
103	A	13	7	1.00	47	0.149
104	A	13	8	1.00	47	0.170
105	A	12	7	0.99	45	0.156
106	A	11	6	1.00	35	0.171
107	A	14	7	1.00	47	0.149
108	A	14	8	1.00	47	0.170
109	A	14	8	1.00	47	0.170
110	A	11	7	1.00	47	0.149
111	A	10	7	1.00	47	0.149
112	A	9	6	1.00	45	0.133
113	A	8	5	1.00	35	0.143
114	A	11	6	1.00	47	0.128
115	A	12	7	1.00	47	0.149
116	A	11	8	1.00	47	0.170
117	A	10	7	1.00	47	0.149
118	A	9	6	1.00	45	0.133
119	A	8	5	1.00	35	0.143
120	A	12	7	1.00	47	0.149
121	A	13	7	1.00	47	0.149
122	A	11	7	1.00	47	0.149
123	A	10	7	1.00	47	0.149
124	A	9	6	0.99	45	0.133
125	A	9	6	1.00	35	0.171
126	A	13	7	1.00	47	0.149
127	A	14	7	1.00	47	0.149
128	A	16	8	1.00	49	0.163
129	A	15	8	1.00	49	0.163
130	A	14	8	1.01	49	0.163

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	13	8	1.00	49	0.163
132	A	13	8	1.00	49	0.163
133	A	9	6	1.00	49	0.122
134	A	10	6	1.00	49	0.122
135	A	16	8	1.00	49	0.163
136	A	15	8	1.00	49	0.163
137	A	14	8	1.00	49	0.163
138	A	14	9	1.00	49	0.184
139	A	14	8	1.00	49	0.163
140	A	10	6	1.00	49	0.122
141	A	16	8	1.00	49	0.163
142	A	15	8	1.00	49	0.163
143	A	15	9	1.00	49	0.184
144	A	15	9	1.00	49	0.184
145	A	15	8	1.00	49	0.163
146	A	11	6	1.00	49	0.122
147	A	15	8	1.00	49	0.163
148	A	14	8	1.00	49	0.163
149	A	13	8	1.00	49	0.163
150	A	12	7	1.00	49	0.143
151	A	8	5	1.00	49	0.102
152	A	9	5	1.00	49	0.102
153	A	15	9	1.00	49	0.184
154	A	14	9	1.00	49	0.184
155	A	13	8	1.00	49	0.163
156	A	8	5	1.00	49	0.102
157	A	9	5	1.00	49	0.102
158	A	10	5	1.00	49	0.102
159	A	15	9	1.00	49	0.184
160	A	14	8	1.00	49	0.163
161	A	9	6	1.00	49	0.122
162	A	9	5	1.00	49	0.102
163	A	10	5	1.00	49	0.102
164	A	9	6	1.00	45	0.133
165	A	9	6	0.98	45	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	8	6	0.99	45	0.133
167	A	7	5	1.00	43	0.116
168	A	6	4	1.00	33	0.121
169	A	8	5	1.00	45	0.111
170	A	9	6	1.00	45	0.133
171	A	10	6	1.00	45	0.133

Chapter 3

Listing of integrals

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3.59 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$ 379
3.60 $\int (c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$ . . . 386
3.61 $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$ . . . 391

3.62 $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$ 398
3.63 $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$ 408
3.64 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 416
3.65 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 425
3.66 $\int (c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 432
3.67 $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$ 438
3.68 $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$ 445
3.69 $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$ 455
3.70 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$ 464
3.71 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$ 471
3.72 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$ 478
3.73 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx$ 484
3.74 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx$ 489
3.75 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$ 495
3.76 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$ 501
3.77 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$ 510
3.78 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$ 520
3.79 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$ 530
3.80 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx$ 538
3.81 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$ 544
3.82 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$ 550
3.83 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$ 559
3.84 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$ 570
3.85 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$ 579
3.86 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$ 587
3.87 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx$ 593
3.88 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$ 598
3.89 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$ 607
3.90 $\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 618
3.91 $\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 626
3.92 $\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 632
3.93 $\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 639
3.94 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$ 645

3.95 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$ 652
3.96 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$ 660
3.97 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 669
3.98 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 676
3.99 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 683
3.100 $\int (c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 689
3.101 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$ 696
3.102 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$ 704
3.103 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$ 712
3.104 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 719
3.105 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 725
3.106 $\int (c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 731
3.107 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$ 738
3.108 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$ 745
3.109 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$ 751
3.110 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$ 757
3.111 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$ 764
3.112 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$ 771
3.113 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$ 778
3.114 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$ 785
3.115 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$ 793
3.116 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$ 800
3.117 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$ 806
3.118 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$ 813
3.119 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$ 819
3.120 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$ 826
3.121 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$ 831
3.122 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$ 837
3.123 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$ 843
3.124 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$ 850
3.125 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx$ 856

3.126 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$ . . . 864
3.127 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$ . . . 870
3.128 $\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 876
3.129 $\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 883
3.130 $\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 889
3.131 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$ . . . 894
3.132 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$ . . . 899
3.133 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$ . . . 904
3.134 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$ . . . 909
3.135 $\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 914
3.136 $\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 921
3.137 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$ . . . 927
3.138 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$ . . . 933
3.139 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$ . . . 939
3.140 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$ . . . 945
3.141 $\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$ 952
3.142 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$ . . . 959
3.143 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$ . . . 965
3.144 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$ . . . 971
3.145 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$ . . . 977
3.146 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{9/2}}\,dx$ . . . 983
3.147 $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$ . . . 989
3.148 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$ . . . 995
3.149 $\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$ . . . 1001
3.150 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx$ . . . 1006
3.151 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx$ . . . 1011
3.152 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$ . . . 1016
3.153 $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$ . . . 1021

3.154 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$ 1027

3.155 $\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$ 1033

3.156 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx$ 1038

3.157 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$ 1043

3.158 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$ 1048

3.159 $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$ 1053

3.160 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$ 1059

3.161 $\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$ 1065

3.162 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx$ 1070

3.163 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$ 1075

3.164 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 1080

3.165 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 1085

3.166 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 1091

3.167 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 1096

3.168 $\int (a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$ 1100

3.169 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$ 1104

3.170 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$ 1108

3.171 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$ 1113

3.1 $\int \tan(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(\right.$

Optimal. Leaf size=87

$$-((aB-bC)x)+\frac{(bB+aC)\log(\cos(c+dx))}{d}+\frac{(aB-bC)\tan(c+dx)}{d}+\frac{(bB+aC)\tan^2(c+dx)}{2d}+\frac{bC\tan^3(c}{3d}$$

```
[Out] -(B*a-C*b)*x+(B*b+C*a)*ln(cos(d*x+c))/d+(B*a-C*b)*tan(d*x+c)/d+1/2*(B*b+C*a
)*tan(d*x+c)^2/d+1/3*b*C*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3713, 3673, 3609, 3606, 3556}

$$\frac{(aC+bB)\tan^2(c+dx)}{2d}+\frac{(aB-bC)\tan(c+dx)}{d}+\frac{(aC+bB)\log(\cos(c+dx))}{d}-x(aB-bC)+\frac{bC\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x
]
```

```
[Out] -((a*B - b*C)*x) + ((b*B + a*C)*Log[Cos[c + d*x]])/d + ((a*B - b*C)*Tan[c +
 d*x])/d + ((b*B + a*C)*Tan[c + d*x]^2)/(2*d) + (b*C*Tan[c + d*x]^3)/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3673

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \tan^2(c+dx)(a+b\tan(c+dx))$$

$$= \frac{bC\tan^3(c+dx)}{3d} + \int \tan^2(c+dx$$

$$= \frac{(bB+aC)\tan^2(c+dx)}{2d} + \frac{bC\tan}{}$$

$$= -(aB-bC)x + \frac{(aB-bC)\tan(c}{d}$$

$$= -(aB-bC)x + \frac{(bB+aC)\log(co}{d}$$

Mathematica [**A**]
time = 0.41, size = 86, normalized size = 0.99

$$\frac{(-6aB+6bC)\mathrm{ArcTan}(\tan(c+dx))+6(bB+aC)\log(\cos(c+dx))+6(aB-bC)\tan(c+dx)+3(bB+aC)\tan^2(c+dx)+2bC\tan^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x
]^2),x]
```

```
[Out] ((-6*a*B + 6*b*C)*ArcTan[Tan[c + d*x]] + 6*(b*B + a*C)*Log[Cos[c + d*x]] +
6*(a*B - b*C)*Tan[c + d*x] + 3*(b*B + a*C)*Tan[c + d*x]^2 + 2*b*C*Tan[c + d
*x]^3)/(6*d)
```

Maple [**A**]
time = 0.08, size = 99, normalized size = 1.14

method	result
norman	$(-aB+Cb)\,x+\frac{(aB-Cb)\tan(dx+c)}{d}+\frac{(Bb+Ca)\left(\tan^{2}(dx+c)\right)}{2d}+\frac{bC\left(\tan^{3}(dx+c)\right)}{3d}-\frac{(Bb+Ca)\ln\left(1+\tan^{2}\right.}{2d}$
derivativedivides	$\frac{\frac{bC\left(\tan^{3}(dx+c)\right)}{3}+\frac{bB\left(\tan^{2}(dx+c)\right)}{2}+\frac{Ca\left(\tan^{2}(dx+c)\right)}{2}+B\tan(dx+c)a-Cb\tan(dx+c)+\frac{(-Bb-Ca)\ln\left(1+\tan^{2}(dx+c)\right)}{2}+(-a}{d}$
default	$\frac{\frac{bC\left(\tan^{3}(dx+c)\right)}{3}+\frac{bB\left(\tan^{2}(dx+c)\right)}{2}+\frac{Ca\left(\tan^{2}(dx+c)\right)}{2}+B\tan(dx+c)a-Cb\tan(dx+c)+\frac{(-Bb-Ca)\ln\left(1+\tan^{2}(dx+c)\right)}{2}+(-a}{d}$
risch	$-iBbx-iCax-Bax+Cbx-\frac{2iBbc}{d}-\frac{2iCac}{d}+\frac{2i\left(-3iBb\,e^{4i(dx+c)}-3iCa\,e^{4i(dx+c)}+3aB\,e^{4i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/d*(1/3*C*b*tan(d*x+c)^3+1/2*B*b*tan(d*x+c)^2+1/2*C*a*tan(d*x+c)^2+B*tan(d
*x+c)*a-C*b*tan(d*x+c)+1/2*(-B*b-C*a)*ln(1+tan(d*x+c)^2)+(-B*a+C*b)*arctan(
tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 86, normalized size = 0.99

$$\frac{2\,Cb\tan\left(dx+c\right)^{3}+3\,(Ca+Bb)\tan\left(dx+c\right)^{2}-6\,(Ba-Cb)(dx+c)-3\,(Ca+Bb)\log\left(\tan\left(dx+c\right)^{2}+1\right)+6\,(Ba-Cb)\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/6*(2*C*b*tan(d*x + c)^3 + 3*(C*a + B*b)*tan(d*x + c)^2 - 6*(B*a - C*b)*(d
*x + c) - 3*(C*a + B*b)*log(tan(d*x + c)^2 + 1) + 6*(B*a - C*b)*tan(d*x + c
))/d
```

Fricas [A]
time = 4.63, size = 85, normalized size = 0.98

$$\frac{2\,Cb\tan\left(dx+c\right)^{3}-6\,(Ba-Cb)dx+3\,(Ca+Bb)\tan\left(dx+c\right)^{2}+3\,(Ca+Bb)\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)+6\,(Ba-Cb)\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/6*(2*C*b*tan(d*x + c)^3 - 6*(B*a - C*b)*d*x + 3*(C*a + B*b)*tan(d*x + c)^
2 + 3*(C*a + B*b)*log(1/(tan(d*x + c)^2 + 1)) + 6*(B*a - C*b)*tan(d*x + c))
/d
```

Sympy [A]
time = 0.12, size = 139, normalized size = 1.60

$$\begin{cases} -Bax + \frac{Ba\tan(c+dx)}{d} - \frac{Bb\log(\tan^2(c+dx)+1)}{2d} + \frac{Bb\tan^2(c+dx)}{2d} - \frac{Ca\log(\tan^2(c+dx)+1)}{2d} + \frac{Ca\tan^2(c+dx)}{2d} + Cbx + \frac{Cb\tan^3(c+dx)}{3d} - \frac{Cb\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\tan(c))(B\tan(c)+C\tan^2(c))\tan(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((-B*a*x + B*a*tan(c + d*x)/d - B*b*log(tan(c + d*x)**2 + 1)/(2*d)
 + B*b*tan(c + d*x)**2/(2*d) - C*a*log(tan(c + d*x)**2 + 1)/(2*d) + C*a*tan
(c + d*x)**2/(2*d) + C*b*x + C*b*tan(c + d*x)**3/(3*d) - C*b*tan(c + d*x)/d
, Ne(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(c)**2)*tan(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. 2(83) = 166.
time = 1.05, size = 1017, normalized size = 11.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] -1/6*(6*B*a*d*x*tan(d*x)^3*tan(c)^3 - 6*C*b*d*x*tan(d*x)^3*tan(c)^3 - 3*C*a
*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 3*
B*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 -
 18*B*a*d*x*tan(d*x)^2*tan(c)^2 + 18*C*b*d*x*tan(d*x)^2*tan(c)^2 - 3*C*a*ta
n(d*x)^3*tan(c)^3 - 3*B*b*tan(d*x)^3*tan(c)^3 + 9*C*a*log(4*(tan(d*x)^4*tan
(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 9*B*b*log(4*(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 6*B*a*tan(d*x)^3*tan(
c)^2 - 6*C*b*tan(d*x)^3*tan(c)^2 + 6*B*a*tan(d*x)^2*tan(c)^3 - 6*C*b*tan(d*
x)^2*tan(c)^3 + 18*B*a*d*x*tan(d*x)*tan(c) - 18*C*b*d*x*tan(d*x)*tan(c) - 3
*C*a*tan(d*x)^3*tan(c) - 3*B*b*tan(d*x)^3*tan(c) + 3*C*a*tan(d*x)^2*tan(c)^
2 + 3*B*b*tan(d*x)^2*tan(c)^2 - 3*C*a*tan(d*x)*tan(c)^3 - 3*B*b*tan(d*x)*ta
n(c)^3 + 2*C*b*tan(d*x)^3 - 9*C*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3
*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)
^2 + 1))*tan(d*x)*tan(c) - 9*B*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(d*x)*tan(c) - 12*B*a*tan(d*x)^2*tan(c) + 18*C*b*tan(d*x)^2*tan(
c) - 12*B*a*tan(d*x)*tan(c)^2 + 18*C*b*tan(d*x)*tan(c)^2 + 2*C*b*tan(c)^3 -
```

```
 6*B*a*d*x + 6*C*b*d*x + 3*C*a*tan(d*x)^2 + 3*B*b*tan(d*x)^2 - 3*C*a*tan(d*
x)*tan(c) - 3*B*b*tan(d*x)*tan(c) + 3*C*a*tan(c)^2 + 3*B*b*tan(c)^2 + 3*C*a
*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 3*B*b*log(4*(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 6*B*a*tan(d*x) - 6*C*b*tan(d*x) + 6*B*a
*tan(c) - 6*C*b*tan(c) + 3*C*a + 3*B*b)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*
x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 8.83, size = 84, normalized size = 0.97

$$\frac{\tan(c+d\,x)\,\left(B\,a-C\,b\right)-\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,b}{2}+\frac{C\,a}{2}\right)+\tan(c+d\,x)^2\,\left(\frac{B\,b}{2}+\frac{C\,a}{2}\right)-d\,x\,\left(B\,a-C\,b\right)+\frac{C\,b\,\tan(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)),x
)
```

```
[Out] (tan(c + d*x)*(B*a - C*b) - log(tan(c + d*x)^2 + 1)*((B*b)/2 + (C*a)/2) + t
an(c + d*x)^2*((B*b)/2 + (C*a)/2) - d*x*(B*a - C*b) + (C*b*tan(c + d*x)^3)/
3)/d
```

3.2 $\int (a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$

Optimal. Leaf size=66

$$-((bB+aC)x)-\frac{(aB-bC)\log(\cos(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2bd}$$

```
[Out] -(B*b+C*a)*x-(B*a-C*b)*ln(cos(d*x+c))/d+b*B*tan(d*x+c)/d+1/2*C*(a+b*tan(d*x
+c))^2/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3711, 3606, 3556}

$$-\frac{(aB-bC)\log(\cos(c+dx))}{d}-x(aC+bB)+\frac{C(a+b\tan(c+dx))^2}{2bd}+\frac{bB\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] -((b*B + a*C)*x) - ((a*B - b*C)*Log[Cos[c + d*x]])/d + (b*B*Tan[c + d*x])/d
 + (C*(a + b*Tan[c + d*x])^2)/(2*b*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{C(a+b\tan(c+dx))^2}{2bd}+\int (a+b\tan(c+dx$$
$$= -(bB+aC)x+\frac{bB\tan(c+dx)}{d}+\frac{C(a+b\,\mathrm{t}}{}$$
$$= -(bB+aC)x-\frac{(aB-bC)\log(\cos(c+dx))}{d}$$

Mathematica [A]
time = 0.22, size = 67, normalized size = 1.02

$$\frac{-2(bB+aC)\mathrm{ArcTan}(\tan(c+dx))+2(-aB+bC)\log(\cos(c+dx))+2(bB+aC)\tan(c+dx)+bC\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
[Out] (-2*(b*B + a*C)*ArcTan[Tan[c + d*x]] + 2*(-(a*B) + b*C)*Log[Cos[c + d*x]] +
 2*(b*B + a*C)*Tan[c + d*x] + b*C*Tan[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.07, size = 74, normalized size = 1.12

method	result
norman	$(-Bb-Ca)\,x+\frac{(Bb+Ca)\tan(dx+c)}{d}+\frac{Cb\left(\tan^2(dx+c)\right)}{2d}+\frac{(aB-Cb)\ln\left(1+\tan^2(dx+c)\right)}{2d}$
derivativedivides	$\frac{\frac{Cb\left(\tan^2(dx+c)\right)}{2}+bB\tan(dx+c)+C\tan(dx+c)a+\frac{(aB-Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-Bb-Ca)\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{Cb\left(\tan^2(dx+c)\right)}{2}+bB\tan(dx+c)+C\tan(dx+c)a+\frac{(aB-Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-Bb-Ca)\arctan(\tan(dx+c))}{d}$
risch	$-Bbx-Cax+iBax-iCbx+\frac{2iaBc}{d}-\frac{2iCbc}{d}+\frac{2i\left(-iCb\,e^{2i(dx+c)}+Bb\,e^{2i(dx+c)}+Ca\,e^{2i(dx+c)}+Bb-\right.}{d\left(e^{2i(dx+c)}+1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*C*tan(d*x+c)^2*b+b*B*tan(d*x+c)+C*tan(d*x+c)*a+1/2*(B*a-C*b)*ln(1+
tan(d*x+c)^2)+(-B*b-C*a)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 66, normalized size = 1.00

$$\frac{Cb\tan(dx+c)^2-2(Ca+Bb)(dx+c)+(Ba-Cb)\log\left(\tan(dx+c)^2+1\right)+2(Ca+Bb)\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(C*b*tan(d*x + c)^2 - 2*(C*a + B*b)*(d*x + c) + (B*a - C*b)*log(tan(d*x
 + c)^2 + 1) + 2*(C*a + B*b)*tan(d*x + c))/d
```

Fricas [A]
time = 4.08, size = 66, normalized size = 1.00

$$\frac{Cb\tan\left(dx+c\right)^{2}-2\left(Ca+Bb\right)dx-\left(Ba-Cb\right)\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)+2\left(Ca+Bb\right)\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] 1/2*(C*b*tan(d*x + c)^2 - 2*(C*a + B*b)*d*x - (B*a - C*b)*log(1/(tan(d*x +
c)^2 + 1)) + 2*(C*a + B*b)*tan(d*x + c))/d
```

Sympy [A]
time = 0.10, size = 105, normalized size = 1.59

$$\begin{cases} \frac{Ba\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}-Bbx+\frac{Bb\tan\left(c+dx\right)}{d}-Cax+\frac{Ca\tan\left(c+dx\right)}{d}-\frac{Cb\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{Cb\tan^{2}\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))\left(B\tan\left(c\right)+C\tan^{2}\left(c\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a*log(tan(c + d*x)**2 + 1)/(2*d) - B*b*x + B*b*tan(c + d*x)/d
- C*a*x + C*a*tan(c + d*x)/d - C*b*log(tan(c + d*x)**2 + 1)/(2*d) + C*b*tan
(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(c)**2),
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(64) = 128.
time = 0.82, size = 616, normalized size = 9.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] -1/2*(2*C*a*d*x*tan(d*x)^2*tan(c)^2 + 2*B*b*d*x*tan(d*x)^2*tan(c)^2 + B*a*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
```

```
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - C*b*
log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 4*C
*a*d*x*tan(d*x)*tan(c) - 4*B*b*d*x*tan(d*x)*tan(c) - C*b*tan(d*x)^2*tan(c)^
2 - 2*B*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan
(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)
 + 2*C*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)*tan(c)
+ 2*C*a*tan(d*x)^2*tan(c) + 2*B*b*tan(d*x)^2*tan(c) + 2*C*a*tan(d*x)*tan(c)
^2 + 2*B*b*tan(d*x)*tan(c)^2 + 2*C*a*d*x + 2*B*b*d*x - C*b*tan(d*x)^2 - C*b
*tan(c)^2 + B*a*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - C*b*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 2*C*a*tan(d*x) - 2*B*b*tan
(d*x) - 2*C*a*tan(c) - 2*B*b*tan(c) - C*b)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan
(d*x)*tan(c) + d)
```

Mupad [B]
time = 8.84, size = 63, normalized size = 0.95

$$\frac{\tan(c+d\,x)\,\left(B\,b+C\,a\right)+\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,a}{2}-\frac{C\,b}{2}\right)-d\,x\,\left(B\,b+C\,a\right)+\frac{C\,b\tan(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)),x)
```

```
[Out] (tan(c + d*x)*(B*b + C*a) + log(tan(c + d*x)^2 + 1)*((B*a)/2 - (C*b)/2) - d
*x*(B*b + C*a) + (C*b*tan(c + d*x)^2)/2)/d
```

3.3 $\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c\right.$

Optimal. Leaf size=42

$$(aB-bC)x-\frac{(bB+aC)\log(\cos(c+dx))}{d}+\frac{bC\tan(c+dx)}{d}$$

```
[Out] (B*a-C*b)*x-(B*b+C*a)*ln(cos(d*x+c))/d+b*C*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3713, 3606, 3556}

$$-\frac{(aC+bB)\log(\cos(c+dx))}{d}+x(aB-bC)+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x
]
```

```
[Out] (a*B - b*C)*x - ((b*B + a*C)*Log[Cos[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int (a+b\tan(c+dx))(B+C\tan$$
$$= (aB-bC)x+\frac{bC\tan(c+dx)}{d}+$$
$$= (aB-bC)x-\frac{(bB+aC)\log(\cos}{d}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 1.40

$$aBx-\frac{bC\text{ArcTan}(\tan(c+dx))}{d}-\frac{bB\log(\cos(c+dx))}{d}-\frac{aC\log(\cos(c+dx))}{d}+\frac{bC\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x
]^2),x]
```

```
[Out] a*B*x - (b*C*ArcTan[Tan[c + d*x]])/d - (b*B*Log[Cos[c + d*x]])/d - (a*C*Log
[Cos[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

Maple [A]
time = 0.24, size = 53, normalized size = 1.26

method	result
norman	$(aB-Cb)\,x+\frac{bC\tan(dx+c)}{d}+\frac{(Bb+Ca)\ln\left(1+\tan^2(dx+c)\right)}{2d}$
derivativedivides	$\frac{-Bb\ln(\cos(dx+c))+Cb(\tan(dx+c)-dx-c)+aB(dx+c)-Ca\ln(\cos(dx+c))}{d}$
default	$\frac{-Bb\ln(\cos(dx+c))+Cb(\tan(dx+c)-dx-c)+aB(dx+c)-Ca\ln(\cos(dx+c))}{d}$
risch	$iBbx+iCax+Bax-Cbx+\frac{2iBbc}{d}+\frac{2iCac}{d}+\frac{2iCb}{d\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{2i(dx+c)}+1\right)Bb}{d}-\frac{\ln\left(e^{2i(dx+c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/d*(-B*b*ln(cos(d*x+c))+C*b*(tan(d*x+c)-d*x-c)+a*B*(d*x+c)-C*a*ln(cos(d*x+
c)))
```

Maxima [A]
time = 0.50, size = 50, normalized size = 1.19

$$\frac{2\,Cb\tan\left(dx+c\right)+2\,(Ba-Cb)(dx+c)+(Ca+Bb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] 1/2*(2*C*b*tan(d*x + c) + 2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*x
 + c)^2 + 1))/d
```

Fricas [A]
time = 4.46, size = 50, normalized size = 1.19

$$\frac{2\,(Ba-Cb)dx+2\,Cb\tan\left(dx+c\right)-(Ca+Bb)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] 1/2*(2*(B*a - C*b)*d*x + 2*C*b*tan(d*x + c) - (C*a + B*b)*log(1/(tan(d*x +
c)^2 + 1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.
time = 0.35, size = 82, normalized size = 1.95

$$\begin{cases} Bax+\frac{Bb\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ca\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-Cbx+\frac{Cb\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a*x + B*b*log(tan(c + d*x)**2 + 1)/(2*d) + C*a*log(tan(c + d*x
)**2 + 1)/(2*d) - C*b*x + C*b*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))*
(B*tan(c) + C*tan(c)**2)*cot(c), True))
```

Giac [A]
time = 0.80, size = 50, normalized size = 1.19

$$\frac{2\,Cb\tan\left(dx+c\right)+2\,(Ba-Cb)(dx+c)+(Ca+Bb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] 1/2*(2*C*b*tan(d*x + c) + 2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*x
 + c)^2 + 1))/d
```

Mupad [B]
time = 8.79, size = 58, normalized size = 1.38

$$B\,a\,x-C\,b\,x+\frac{C\,b\,\tan(c+d\,x)}{d}+\frac{B\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}+\frac{C\,a\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)),x
)
```

```
[Out] B*a*x - C*b*x + (C*b*tan(c + d*x))/d + (B*b*log(tan(c + d*x)^2 + 1))/(2*d)
+ (C*a*log(tan(c + d*x)^2 + 1))/(2*d)
```

3.4 $\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(\right.$

Optimal. Leaf size=37

$$(bB+aC)x-\frac{bC\log(\cos(c+dx))}{d}+\frac{aB\log(\sin(c+dx))}{d}$$

```
[Out] (B*b+C*a)*x-b*C*ln(cos(d*x+c))/d+a*B*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3713, 3670, 3556, 3612}

$$x(aC+bB)+\frac{aB\log(\sin(c+dx))}{d}-\frac{bC\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (b*B + a*C)*x - (b*C*Log[Cos[c + d*x]])/d + (a*B*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3670

```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B*(d/
b), Int[Tan[e + f*x], x], x] + Dist[1/b, Int[Simp[A*b*c + (A*b*d + B*(b*c -
 a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d,
e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot(c+dx)(a+b\tan(c+dx)$$

$$= (bC)\int \tan(c+dx)\,dx + \int \cot($$

$$= (bB+aC)x - \frac{bC\log(\cos(c+d}{d}$$

$$= (bB+aC)x - \frac{bC\log(\cos(c+d}{d}$$

Mathematica [A]
time = 0.06, size = 44, normalized size = 1.19

$$bBx + aCx - \frac{bC\log(\cos(c+dx))}{d} + \frac{aB(\log(\cos(c+dx)) + \log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] b*B*x + a*C*x - (b*C*Log[Cos[c + d*x]])/d + (a*B*(Log[Cos[c + d*x]] + Log[T
an[c + d*x]]))/d
```

Maple [A]
time = 0.28, size = 43, normalized size = 1.16

method	result	size
derivativedivides	$\frac{Bb(dx+c)-Cb\ln(\cos(dx+c))+aB\ln(\sin(dx+c))+Ca(dx+c)}{d}$	43
default	$\frac{Bb(dx+c)-Cb\ln(\cos(dx+c))+aB\ln(\sin(dx+c))+Ca(dx+c)}{d}$	43
norman	$(Bb+Ca)\,x+\frac{aB\ln(\tan(dx+c))}{d}-\frac{(aB-Cb)\ln(1+\tan^2(dx+c))}{2d}$	48
risch	$Bbx+Cax-iBax+iCbx+\frac{2iCbc}{d}-\frac{2iaBc}{d}-\frac{\ln(e^{2i(dx+c)}+1)Cb}{d}+\frac{a\ln(e^{2i(dx+c)}-1)B}{d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(B*b*(d*x+c)-C*b*ln(cos(d*x+c))+a*B*ln(sin(d*x+c))+C*a*(d*x+c))
```

Maxima [A]
time = 0.50, size = 52, normalized size = 1.41

$$\frac{2\,Ba\log\left(\tan\left(dx+c\right)\right)+2\,(Ca+Bb)(dx+c)-(Ba-Cb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*B*a*log(tan(d*x + c)) + 2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*log(ta
n(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 3.69, size = 59, normalized size = 1.59

$$\frac{2\,(Ca+Bb)dx+Ba\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)-Cb\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(C*a + B*b)*d*x + B*a*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - C*b
*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(34) = 68$.
time = 0.52, size = 85, normalized size = 2.30

$$\begin{cases} -\frac{Ba\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ba\log\left(\tan\left(c+dx\right)\right)}{d}+Bbx+Cax+\frac{Cb\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} & \text{for } d\neq 0 \\ x(a+b\tan\left(c\right))\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^2\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((-B*a*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*x))/d +
B*b*x + C*a*x + C*b*log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*(a + b*ta
n(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

Giac [A]
time = 0.97, size = 53, normalized size = 1.43

$$\frac{2\,Ba\log\left(\left|\tan\left(dx+c\right)\right|\right)+2\,(Ca+Bb)(dx+c)-(Ba-Cb)\log\left(\tan\left(dx+c\right)^{2}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*B*a*log(abs(tan(d*x + c))) + 2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*l
og(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 8.96, size = 69, normalized size = 1.86

$$\frac{B\,a\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)*1i)/(2*d) - (log(tan(c + d*x)
 - 1i)*(B + C*1i)*(a + b*1i))/(2*d) + (B*a*log(tan(c + d*x)))/d
```

3.5 $\int \cot^3(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(\right.$

Optimal. Leaf size=43

$$-((aB-bC)x)-\frac{aB\cot(c+dx)}{d}+\frac{(bB+aC)\log(\sin(c+dx))}{d}$$

```
[Out] -(B*a-C*b)*x-a*B*cot(d*x+c)/d+(B*b+C*a)*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3713, 3672, 3612, 3556}

$$\frac{(aC+bB)\log(\sin(c+dx))}{d}-(x(aB-bC))-\frac{aB\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] -((a*B - b*C)*x) - (a*B*Cot[c + d*x])/d + ((b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^2(c+dx)(a+b\tan(c+dx$$
$$= -\frac{aB\cot(c+dx)}{d} + \int \cot(c+d$$
$$= -(aB-bC)x - \frac{aB\cot(c+dx)}{d}$$
$$= -(aB-bC)x - \frac{aB\cot(c+dx)}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 78, normalized size = 1.81

$$bCx - \frac{aB\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} + \frac{bB(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d} + \frac{aC(\log(\cos(c+dx))+\log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] b*C*x - (a*B*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])
/d + (b*B*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d + (a*C*(Log[Cos[c + d*
x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.22, size = 53, normalized size = 1.23

method	result
derivativedivides	$\frac{Bb\ln(\sin(dx+c))+Cb(dx+c)+aB(-\cot(dx+c)-dx-c)+Ca\ln(\sin(dx+c))}{d}$
default	$\frac{Bb\ln(\sin(dx+c))+Cb(dx+c)+aB(-\cot(dx+c)-dx-c)+Ca\ln(\sin(dx+c))}{d}$
norman	$\frac{(-aB+Cb)x(\tan^2(dx+c))-\frac{aB\tan(dx+c)}{d}}{\tan(dx+c)^2} + \frac{(Bb+Ca)\ln(\tan(dx+c))}{d} - \frac{(Bb+Ca)\ln(1+\tan^2(dx+c))}{2d}$

risch	$-iBbx - iCax - Bax + Cbx - \frac{2iBbc}{d} - \frac{2iCac}{d} - \frac{2iaB}{d(e^{2i(dx+c)}-1)} + \frac{\ln(e^{2i(dx+c)}-1)Bb}{d} + \frac{\ln(e^{2i(dx+}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(B*b*ln(sin(d*x+c))+C*b*(d*x+c)+a*B*(-cot(d*x+c)-d*x-c)+C*a*ln(sin(d*x+
c)))
```

Maxima [A]
time = 0.52, size = 68, normalized size = 1.58

$$-\frac{2(Ba-Cb)(dx+c)+(Ca+Bb)\log\left(\tan(dx+c)^2+1\right)-2(Ca+Bb)\log\left(\tan(dx+c)\right)+\frac{2Ba}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*x + c)^2 + 1) - 2*(C*
a + B*b)*log(tan(d*x + c)) + 2*B*a/tan(d*x + c))/d
```

Fricas [A]
time = 3.56, size = 73, normalized size = 1.70

$$-\frac{2(Ba-Cb)dx\tan(dx+c)-(Ca+Bb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)+2Ba}{2d\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] -1/2*(2*(B*a - C*b)*d*x*tan(d*x + c) - (C*a + B*b)*log(tan(d*x + c)^2/(tan(
d*x + c)^2 + 1))*tan(d*x + c) + 2*B*a)/(d*tan(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(36) = 72$.
time = 0.93, size = 122, normalized size = 2.84

$$\begin{cases} \text{NaN} & \text{for}\ (c=0\vee c=-dx)\wedge(c=-dx\vee d=0) \\ x(a+b\tan(c))\left(B\tan(c)+C\tan^2(c)\right)\cot^3(c) & \text{for}\ d=0 \\ -Bax-\frac{Ba}{d\tan(c+dx)}-\frac{Bb\log(\tan^2(c+dx)+1)}{2d}+\frac{Bb\log(\tan(c+dx))}{d}-\frac{Ca\log(\tan^2(c+dx)+1)}{2d}+\frac{Ca\log(\tan(c+dx))}{d}+Cbx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**3, Eq(d, 0)), (-B*a*x - B*a/(
d*tan(c + d*x)) - B*b*log(tan(c + d*x)**2 + 1)/(2*d) + B*b*log(tan(c + d*x)
)/d - C*a*log(tan(c + d*x)**2 + 1)/(2*d) + C*a*log(tan(c + d*x))/d + C*b*x,
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(43) = 86.
time = 1.13, size = 119, normalized size = 2.77

$$\frac{Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,(Ba-Cb)(dx+c)-2\,(Ca+Bb)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)+2\,(Ca+Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Ba}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*(B*a*tan(1/2*d*x + 1/2*c) - 2*(B*a - C*b)*(d*x + c) - 2*(C*a + B*b)*log
(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c)))
 - (2*C*a*tan(1/2*d*x + 1/2*c) + 2*B*b*tan(1/2*d*x + 1/2*c) + B*a)/tan(1/2*
d*x + 1/2*c))/d
```

Mupad [B]
time = 8.87, size = 87, normalized size = 2.02

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,b+C\,a\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(B-C\,1{}\mathrm{i}\right)\,\left(b+a\,1{}\mathrm{i}\right)}{2\,d}-\frac{B\,a\cot(c+d\,x)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1{}\mathrm{i}\right)\,\left(a+b\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
,x)
```

```
[Out] (log(tan(c + d*x))*(B*b + C*a))/d + (log(tan(c + d*x) - 1i)*(B + C*1i)*(a +
 b*1i)*1i)/(2*d) - (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b))/(2*d) - (
B*a*cot(c + d*x))/d
```

3.6 $\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(\right.$

Optimal. Leaf size=66

$$-((bB+aC)x)-\frac{(bB+aC)\cot(c+dx)}{d}-\frac{aB\cot^2(c+dx)}{2d}-\frac{(aB-bC)\log(\sin(c+dx))}{d}$$

```
[Out] -(B*b+C*a)*x-(B*b+C*a)*cot(d*x+c)/d-1/2*a*B*cot(d*x+c)^2/d-(B*a-C*b)*ln(sin
(d*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3713, 3672, 3610, 3612, 3556}

$$-\frac{(aC+bB)\cot(c+dx)}{d}-\frac{(aB-bC)\log(\sin(c+dx))}{d}-x(aC+bB)-\frac{aB\cot^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] -((b*B + a*C)*x) - ((b*B + a*C)*Cot[c + d*x])/d - (a*B*Cot[c + d*x]^2)/(2*d
) - ((a*B - b*C)*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^3(c+dx)(a+b\tan(c+dx$$

$$= -\frac{aB\cot^2(c+dx)}{2d} + \int \cot^2(c+$$

$$= -\frac{(bB+aC)\cot(c+dx)}{d} - \frac{aB}{}$$

$$= -(bB+aC)x - \frac{(bB+aC)\cot(}{d}$$

$$= -(bB+aC)x - \frac{(bB+aC)\cot(}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.33, size = 77, normalized size = 1.17

$$-\frac{aB\cot^2(c+dx)+2(bB+aC)\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)+2(aB-bC)(\log(\cos(c+dx))+\log(\tan(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] -1/2*(a*B*Cot[c + d*x]^2 + 2*(b*B + a*C)*Cot[c + d*x]*Hypergeometric2F1[-1/
2, 1, 1/2, -Tan[c + d*x]^2] + 2*(a*B - b*C)*(Log[Cos[c + d*x]] + Log[Tan[c
+ d*x]]))/d
```

Maple [A]
time = 0.30, size = 77, normalized size = 1.17

method	result
derivativedivides	$\frac{Bb(-\cot(dx+c)-dx-c)+Cb\ln(\sin(dx+c))+aB\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+Ca(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{Bb(-\cot(dx+c)-dx-c)+Cb\ln(\sin(dx+c))+aB\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+Ca(-\cot(dx+c)-dx-c)}{d}$
norman	$\frac{(-Bb-Ca)x\left(\tan^3(dx+c)\right)-\frac{(Bb+Ca)\left(\tan^2(dx+c)\right)}{d}-\frac{aB\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{(aB-Cb)\ln(\tan(dx+c))}{d}+\frac{(aB-Cb)\ln\left(1+\tan^2(\right.}{2d}$
risch	$-Bbx-Cax+iBax-iCbx+\frac{2iaBc}{d}-\frac{2iCbc}{d}-\frac{2i\left(iBa\,e^{2i(dx+c)}+Bb\,e^{2i(dx+c)}+Ca\,e^{2i(dx+c)}-Bb-Ca\right.}{d\left(e^{2i(dx+c)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(B*b*(-cot(d*x+c)-d*x-c)+C*b*ln(sin(d*x+c))+a*B*(-1/2*cot(d*x+c)^2-ln(s
in(d*x+c)))+C*a*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 86, normalized size = 1.30

$$-\frac{2\,(Ca+Bb)(dx+c)-(Ba-Cb)\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(Ba-Cb)\log\left(\tan\left(dx+c\right)\right)+\frac{Ba+2\,(Ca+Bb)\tan(dx+c)}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*log(tan(d*x + c)^2 + 1) + 2*(B*
a - C*b)*log(tan(d*x + c)) + (B*a + 2*(C*a + B*b)*tan(d*x + c))/tan(d*x + c
)^2)/d
```

Fricas [A]
time = 4.57, size = 95, normalized size = 1.44

$$-\frac{(Ba-Cb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+\left(2\,(Ca+Bb)dx+Ba\right)\tan\left(dx+c\right)^2+Ba+2\,(Ca+Bb)\tan\left(dx+c\right)}{2\,d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] -1/2*((B*a - C*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 +
 (2*(C*a + B*b)*d*x + B*a)*tan(d*x + c)^2 + B*a + 2*(C*a + B*b)*tan(d*x + c
))/(d*tan(d*x + c)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(56) = 112$.
time = 1.37, size = 150, normalized size = 2.27

$$\begin{cases} \text{NaN} & \text{for}\ (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))\left(B\tan(c) + C\tan^2(c)\right)\cot^4(c) & \text{for}\ d = 0 \\ \frac{Ba\log(\tan^2(c+dx)+1)}{2d} - \frac{Ba\log(\tan(c+dx))}{d} - \frac{Ba}{2d\tan^2(c+dx)} - Bbx - \frac{Bb}{d\tan(c+dx)} - Cax - \frac{Ca}{d\tan(c+dx)} - \frac{Cb\log(\tan^2(c+dx)+1)}{2d} + \frac{Cb\log(\tan(c+dx))}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**4, Eq(d, 0)), (B*a*log(tan(c
+ d*x)**2 + 1)/(2*d) - B*a*log(tan(c + d*x))/d - B*a/(2*d*tan(c + d*x)**2)
- B*b*x - B*b/(d*tan(c + d*x)) - C*a*x - C*a/(d*tan(c + d*x)) - C*b*log(tan
(c + d*x)**2 + 1)/(2*d) + C*b*log(tan(c + d*x))/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(64) = 128$.
time = 1.40, size = 179, normalized size = 2.71

$$-\frac{Ba\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 4Ca\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 4Bb\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 8(Ca + Bb)(dx + c) - 8(Ba - Cb)\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right) + 8(Ba - Cb)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - \frac{12Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 12Cb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 4Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 4Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - Ba}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] -1/8*(B*a*tan(1/2*d*x + 1/2*c)^2 - 4*C*a*tan(1/2*d*x + 1/2*c) - 4*B*b*tan(1
/2*d*x + 1/2*c) + 8*(C*a + B*b)*(d*x + c) - 8*(B*a - C*b)*log(tan(1/2*d*x +
 1/2*c)^2 + 1) + 8*(B*a - C*b)*log(abs(tan(1/2*d*x + 1/2*c))) - (12*B*a*tan
(1/2*d*x + 1/2*c)^2 - 12*C*b*tan(1/2*d*x + 1/2*c)^2 - 4*C*a*tan(1/2*d*x + 1
/2*c) - 4*B*b*tan(1/2*d*x + 1/2*c) - B*a)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 8.94, size = 108, normalized size = 1.64

$$-\frac{\ln\left(\tan(c + d\,x)\right)\,\left(B\,a - C\,b\right)}{d} - \frac{\cot(c + d\,x)^2\,\left(\frac{B\,a}{2} + \tan(c + d\,x)\,\left(B\,b + C\,a\right)\right)}{d} + \frac{\ln\left(\tan(c + d\,x) - \mathrm{i}\right)\,\left(B + C\,1\mathrm{i}\right)\,\left(a + b\,1\mathrm{i}\right)}{2\,d} - \frac{\ln\left(\tan(c + d\,x) + 1\mathrm{i}\right)\,\left(B - C\,1\mathrm{i}\right)\,\left(b + a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(B + C*1i)*(a + b*1i))/(2*d) - (cot(c + d*x)^2*((B*
a)/2 + tan(c + d*x)*(B*b + C*a)))/d - (log(tan(c + d*x))*(B*a - C*b))/d - (
log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)*1i)/(2*d)
```

3.7 $\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(\right.$

Optimal. Leaf size=87

$$(aB-bC)x+\frac{(aB-bC)\cot(c+dx)}{d}-\frac{(bB+aC)\cot^2(c+dx)}{2d}-\frac{aB\cot^3(c+dx)}{3d}-\frac{(bB+aC)\log(\sin(c+dx)}{d}$$

```
[Out] (B*a-C*b)*x+(B*a-C*b)*cot(d*x+c)/d-1/2*(B*b+C*a)*cot(d*x+c)^2/d-1/3*a*B*cot
(d*x+c)^3/d-(B*b+C*a)*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3713, 3672, 3610, 3612, 3556}

$$-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}-\frac{(aC+bB)\log(\sin(c+dx))}{d}+x(aB-bC)-\frac{aB\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (a*B - b*C)*x + ((a*B - b*C)*Cot[c + d*x])/d - ((b*B + a*C)*Cot[c + d*x]^2)
/(2*d) - (a*B*Cot[c + d*x]^3)/(3*d) - ((b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^4(c+dx)(a+b\tan(c+dx$$

$$= -\frac{aB\cot^3(c+dx)}{3d} + \int \cot^3(c+$$

$$= -\frac{(bB+aC)\cot^2(c+dx)}{2d} - \frac{aB$$

$$= \frac{(aB-bC)\cot(c+dx)}{d} - \frac{(bB+$$

$$= (aB-bC)x + \frac{(aB-bC)\cot(c}{d}$$

$$= (aB-bC)x + \frac{(aB-bC)\cot(c}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.72, size = 101, normalized size = 1.16

$$-\frac{2aB\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)+6bC\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)+3(bB+aC)\left(\cot^2(c+dx)+2(\log(\cos(c+dx))+\log(\tan(c+dx)))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] -1/6*(2*a*B*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2
] + 6*b*C*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2] + 3
*(b*B + a*C)*(Cot[c + d*x]^2 + 2*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/
d
```

Maple [A]
time = 0.28, size = 95, normalized size = 1.09

method	result
derivativedivides	$\frac{Bb\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+Cb(-\cot(dx+c)-dx-c)+aB\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+Ca\left(-\frac{\left(\cot^2\right.}{}\right.}{d}$
default	$\frac{Bb\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+Cb(-\cot(dx+c)-dx-c)+aB\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+Ca\left(-\frac{\left(\cot^2\right.}{}\right.}{d}$
norman	$\frac{\frac{(aB-Cb)\left(\tan^3(dx+c)\right)}{d}+(aB-Cb)x\left(\tan^4(dx+c)\right)-\frac{(Bb+Ca)\left(\tan^2(dx+c)\right)}{2d}-\frac{aB\tan(dx+c)}{3d}}{\tan(dx+c)^4}-\frac{(Bb+Ca)\ln(\tan(dx+c))}{d}+$
risch	$iBbx+iCax+Bax-Cbx+\frac{2iBbc}{d}+\frac{2iCac}{d}-\frac{2i\left(3iBb\,e^{4i(dx+c)}+3iCa\,e^{4i(dx+c)}-6aB\,e^{4i(dx+c)}+3Cb\,e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(B*b*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+C*b*(-cot(d*x+c)-d*x-c)+a*B*(-1
/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+C*a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.52, size = 104, normalized size = 1.20

$$\frac{6\,(Ba-Cb)(dx+c)+3\,(Ca+Bb)\log\left(\tan\left(dx+c\right)^2+1\right)-6\,(Ca+Bb)\log\left(\tan\left(dx+c\right)\right)+\frac{6\,(Ba-Cb)\tan(dx+c)^2-2\,Ba-3\,(Ca+Bb)\tan(dx+c)}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/6*(6*(B*a - C*b)*(d*x + c) + 3*(C*a + B*b)*log(tan(d*x + c)^2 + 1) - 6*(C
*a + B*b)*log(tan(d*x + c)) + (6*(B*a - C*b)*tan(d*x + c)^2 - 2*B*a - 3*(C*
a + B*b)*tan(d*x + c))/tan(d*x + c)^3)/d
```

Fricas [A]
time = 3.57, size = 121, normalized size = 1.39

$$-\frac{3\,(Ca+Bb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3-3\,(2\,(Ba-Cb)dx-Ca-Bb)\tan\left(dx+c\right)^3-6\,(Ba-Cb)\tan\left(dx+c\right)^2+2\,Ba+3\,(Ca+Bb)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] -1/6*(3*(C*a + B*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^3
 - 3*(2*(B*a - C*b)*d*x - C*a - B*b)*tan(d*x + c)^3 - 6*(B*a - C*b)*tan(d*x
 + c)^2 + 2*B*a + 3*(C*a + B*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(75) = 150$.
time = 2.23, size = 180, normalized size = 2.07

$$\begin{cases} \text{NaN} & \text{for}\ (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan{(c)}) \left(B\tan{(c)} + C\tan^{2}{(c)}\right)\cot^{5}{(c)} & \text{for}\ d = 0 \\ Bax + \frac{Ba}{d\tan{(c+dx)}} - \frac{Ba}{3d\tan^{3}{(c+dx)}} + \frac{Bb\log{(\tan^{2}{(c+dx)}+1)}}{2d} - \frac{Bb\log{(\tan{(c+dx)})}}{d} - \frac{Bb}{2d\tan^{2}{(c+dx)}} + \frac{Ca\log{(\tan^{2}{(c+dx)}+1)}}{2d} - \frac{Ca\log{(\tan{(c+dx)})}}{d} - \frac{Ca}{2d\tan^{2}{(c+dx)}} - Cbx - \frac{Cb}{d\tan{(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**5, Eq(d, 0)), (B*a*x + B*a/(d
*tan(c + d*x)) - B*a/(3*d*tan(c + d*x)**3) + B*b*log(tan(c + d*x)**2 + 1)/(
2*d) - B*b*log(tan(c + d*x))/d - B*b/(2*d*tan(c + d*x)**2) + C*a*log(tan(c
+ d*x)**2 + 1)/(2*d) - C*a*log(tan(c + d*x))/d - C*a/(2*d*tan(c + d*x)**2)
- C*b*x - C*b/(d*tan(c + d*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(83) = 166$.
time = 1.63, size = 237, normalized size = 2.72

$$\frac{Ba\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{3} - 3Ca\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2} - 3Bb\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2} - 15Ba\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 12Cb\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24(Ba - Cb)(dx + c) + 24(Ca + Bb)\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{2} + 1\right) - 24(Ca + Bb)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + \frac{44Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3} + 44Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3} + 15Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2} - 12Cb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2} - 3Ca\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 3Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - Ba}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/24*(B*a*tan(1/2*d*x + 1/2*c)^3 - 3*C*a*tan(1/2*d*x + 1/2*c)^2 - 3*B*b*tan
(1/2*d*x + 1/2*c)^2 - 15*B*a*tan(1/2*d*x + 1/2*c) + 12*C*b*tan(1/2*d*x + 1/
2*c) + 24*(B*a - C*b)*(d*x + c) + 24*(C*a + B*b)*log(tan(1/2*d*x + 1/2*c)^2
 + 1) - 24*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C*a*tan(1/2*d*x
 + 1/2*c)^3 + 44*B*b*tan(1/2*d*x + 1/2*c)^3 + 15*B*a*tan(1/2*d*x + 1/2*c)^2
 - 12*C*b*tan(1/2*d*x + 1/2*c)^2 - 3*C*a*tan(1/2*d*x + 1/2*c) - 3*B*b*tan(1
/2*d*x + 1/2*c) - B*a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 8.89, size = 127, normalized size = 1.46

$$-\frac{\cot(c + d\,x)^{3}\,\left(\left(C\,b - B\,a\right)\,\tan(c + d\,x)^{2} + \left(\frac{B\,b}{2} + \frac{C\,a}{2}\right)\,\tan(c + d\,x) + \frac{B\,a}{3}\right)}{d} - \frac{\ln\left(\tan(c + d\,x)\right)\,\left(B\,b + C\,a\right)}{d} - \frac{\ln\left(\tan(c + d\,x) - \mathrm{i}\right)\,\left(B + C\,1\mathrm{i}\right)\,\left(a + b\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d} + \frac{\ln\left(\tan(c + d\,x) + 1\mathrm{i}\right)\,\left(B - C\,1\mathrm{i}\right)\,\left(b + a\,1\mathrm{i}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b))/(2*d) - (log(tan(c + d*x))*(
B*b + C*a))/d - (log(tan(c + d*x) - 1i)*(B + C*1i)*(a + b*1i)*1i)/(2*d) - (
cot(c + d*x)^3*((B*a)/3 + tan(c + d*x)*((B*b)/2 + (C*a)/2) - tan(c + d*x)^2
*(B*a - C*b)))/d
```

3.8 $\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2\right.$

Optimal. Leaf size=108

$$(bB+aC)x+\frac{(bB+aC)\cot(c+dx)}{d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}-\frac{(bB+aC)\cot^3(c+dx)}{3d}-\frac{aB\cot^4(c+dx)}{4d}+$$

```
[Out] (B*b+C*a)*x+(B*b+C*a)*cot(d*x+c)/d+1/2*(B*a-C*b)*cot(d*x+c)^2/d-1/3*(B*b+C*
a)*cot(d*x+c)^3/d-1/4*a*B*cot(d*x+c)^4/d+(B*a-C*b)*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.17, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3713, 3672, 3610, 3612, 3556}

$$-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}+\frac{(aB-bC)\log(\sin(c+dx))}{d}+x(aC+bB)-\frac{aB\cot^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (b*B + a*C)*x + ((b*B + a*C)*Cot[c + d*x])/d + ((a*B - b*C)*Cot[c + d*x]^2)
/(2*d) - ((b*B + a*C)*Cot[c + d*x]^3)/(3*d) - (a*B*Cot[c + d*x]^4)/(4*d) +
((a*B - b*C)*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3672

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c +
 b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m,
 -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^5(c+dx)(a+b\tan(c+dx)$$

$$= -\frac{aB\cot^4(c+dx)}{4d} + \int \cot^4(c+$$

$$= -\frac{(bB+aC)\cot^3(c+dx)}{3d} - \frac{aB\,c}{}$$

$$= \frac{(aB-bC)\cot^2(c+dx)}{2d} - \frac{(bB+}{}$$

$$= \frac{(bB+aC)\cot(c+dx)}{d} + \frac{(aB-}{}$$

$$= (bB+aC)x + \frac{(bB+aC)\cot(c+}{d}$$

$$= (bB+aC)x + \frac{(bB+aC)\cot(c+}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.78, size = 100, normalized size = 0.93

$$-\frac{4(bB+aC)\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)+3\left((-2aB+2bC)\cot^2(c+dx)+aB\cot^4(c+dx)-4(aB-bC)(\log(\cos(c+dx))+\log(\tan(c+dx)))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] -1/12*(4*(b*B + a*C)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c
 + d*x]^2] + 3*((-2*a*B + 2*b*C)*Cot[c + d*x]^2 + a*B*Cot[c + d*x]^4 - 4*(a
*B - b*C)*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/d
```

Maple [A]
time = 0.28, size = 108, normalized size = 1.00

method	result
derivativedivides	$\frac{Bb\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+Cb\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+aB\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\right.}{d}$
default	$\frac{Bb\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+Cb\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+aB\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\right.}{d}$
norman	$\frac{\frac{(Bb+Ca)\left(\tan^4(dx+c)\right)}{d}+(Bb+Ca)x\left(\tan^5(dx+c)\right)-\frac{(Bb+Ca)\left(\tan^2(dx+c)\right)}{3d}+\frac{(aB-Cb)\left(\tan^3(dx+c)\right)}{2d}-\frac{aB\tan(dx+c)}{4d}}{\tan(dx+c)^5}+\frac{(aB}{}$
risch	$Bbx+Cax-iBax+iCbx-\frac{2iaBc}{d}+\frac{2iCbc}{d}-\frac{2\left(-6iBb\,e^{6i(dx+c)}-6iCa\,e^{6i(dx+c)}+6Ba\,e^{6i(dx+c)}-3C\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(B*b*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+C*b*(-1/2*cot(d*x+c)^2-ln(sin
(d*x+c)))+a*B*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+C*a*(-1/3
*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 122, normalized size = 1.13

$$\frac{12\,(Ca+Bb)(dx+c)-6\,(Ba-Cb)\log\left(\tan\left(dx+c\right)^2+1\right)+12\,(Ba-Cb)\log\left(\tan\left(dx+c\right)\right)+\frac{12\,(Ca+Bb)\tan(dx+c)^3+6\,(Ba-Cb)\tan(dx+c)^2-3\,Ba-4\,(Ca+Bb)\tan(dx+c)}{\tan(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/12*(12*(C*a + B*b)*(d*x + c) - 6*(B*a - C*b)*log(tan(d*x + c)^2 + 1) + 12
*(B*a - C*b)*log(tan(d*x + c)) + (12*(C*a + B*b)*tan(d*x + c)^3 + 6*(B*a -
C*b)*tan(d*x + c)^2 - 3*B*a - 4*(C*a + B*b)*tan(d*x + c))/tan(d*x + c)^4)/d
```

Fricas [A]
time = 3.60, size = 138, normalized size = 1.28

$$\frac{6\,(Ba-Cb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\,(4\,(Ca+Bb)dx+3\,Ba-2\,Cb)\tan\left(dx+c\right)^4+12\,(Ca+Bb)\tan\left(dx+c\right)^3+6\,(Ba-Cb)\tan\left(dx+c\right)^2-3\,Ba-4\,(Ca+Bb)\tan\left(dx+c\right)}{12\,d\tan\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/12*(6*(B*a - C*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4
 + 3*(4*(C*a + B*b)*d*x + 3*B*a - 2*C*b)*tan(d*x + c)^4 + 12*(C*a + B*b)*ta
n(d*x + c)^3 + 6*(B*a - C*b)*tan(d*x + c)^2 - 3*B*a - 4*(C*a + B*b)*tan(d*x
 + c))/(d*tan(d*x + c)^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(95) = 190$.
time = 3.03, size = 211, normalized size = 1.95

$$\begin{cases} \text{NaN} & \text{for}\: \left(c = 0 \vee c = - d x\right) \wedge \left(c = - d x \vee d = 0\right) \\ x \left(a + b \tan{\left(c \right)}\right) \left(B \tan{\left(c \right)} + C \tan^{2}{\left(c \right)}\right) \cot^{6}{\left(c \right)} & \text{for}\: d = 0 \\ - \frac{B a \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{B a \log{\left(\tan{\left(c + d x \right)} \right)}}{d} + \frac{B a}{2 d \tan^{2}{\left(c + d x \right)}} - \frac{B a}{4 d \tan^{4}{\left(c + d x \right)}} + B b x + \frac{B b}{d \tan{\left(c + d x \right)}} - \frac{B b}{3 d \tan^{3}{\left(c + d x \right)}} + C a x + \frac{C a}{d \tan{\left(c + d x \right)}} - \frac{C a}{3 d \tan^{3}{\left(c + d x \right)}} + \frac{C b \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - \frac{C b \log{\left(\tan{\left(c + d x \right)} \right)}}{d} - \frac{C b}{2 d \tan^{2}{\left(c + d x \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**6, Eq(d, 0)), (-B*a*log(tan(c
 + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*x))/d + B*a/(2*d*tan(c + d*x)**2)
 - B*a/(4*d*tan(c + d*x)**4) + B*b*x + B*b/(d*tan(c + d*x)) - B*b/(3*d*tan(
c + d*x)**3) + C*a*x + C*a/(d*tan(c + d*x)) - C*a/(3*d*tan(c + d*x)**3) + C
*b*log(tan(c + d*x)**2 + 1)/(2*d) - C*b*log(tan(c + d*x))/d - C*b/(2*d*tan(
c + d*x)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(102) = 204$.
time = 1.44, size = 299, normalized size = 2.77

$$-\frac{3 \, B a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 8 \, C a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 8 \, B b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 36 \, B a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 24 \, C b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 120 \, C a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 120 \, B b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 192 \, {\left(C a + B b\right)} {\left(d x + c\right)} + 192 \, {\left(B a - C b\right)} \log\left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 1\right) - 192 \, {\left(B a - C b\right)} \log\left({\left| \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) \right|}\right) + \frac{400 \, B a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 400 \, C b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 120 \, C a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 120 \, B b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 36 \, B a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 24 \, C b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 8 \, C a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 8 \, B b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 3 \, B a}{\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4}}}{192 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] -1/192*(3*B*a*tan(1/2*d*x + 1/2*c)^4 - 8*C*a*tan(1/2*d*x + 1/2*c)^3 - 8*B*b
*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*tan(1/2*d*x + 1/2*c)^2 + 24*C*b*tan(1/2*d*
x + 1/2*c)^2 + 120*C*a*tan(1/2*d*x + 1/2*c) + 120*B*b*tan(1/2*d*x + 1/2*c)
- 192*(C*a + B*b)*(d*x + c) + 192*(B*a - C*b)*log(tan(1/2*d*x + 1/2*c)^2 +
1) - 192*(B*a - C*b)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*B*a*tan(1/2*d*x
+ 1/2*c)^4 - 400*C*b*tan(1/2*d*x + 1/2*c)^4 - 120*C*a*tan(1/2*d*x + 1/2*c)^
3 - 120*B*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*tan(1/2*d*x + 1/2*c)^2 + 24*C*b
*tan(1/2*d*x + 1/2*c)^2 + 8*C*a*tan(1/2*d*x + 1/2*c) + 8*B*b*tan(1/2*d*x +
1/2*c) + 3*B*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.82, size = 145, normalized size = 1.34

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,a-C\,b\right)}{d}-\frac{\cot(c+d\,x)^4\,\left(\left(-B\,b-C\,a\right)\,\tan(c+d\,x)^3+\left(\frac{C\,b}{2}-\frac{B\,a}{2}\right)\,\tan(c+d\,x)^2+\left(\frac{B\,b}{3}+\frac{C\,a}{3}\right)\,\tan(c+d\,x)+\frac{B\,a}{4}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
,x)
```

```
[Out] (log(tan(c + d*x))*(B*a - C*b))/d - (cot(c + d*x)^4*((B*a)/4 + tan(c + d*x)
*((B*b)/3 + (C*a)/3) - tan(c + d*x)^3*(B*b + C*a) - tan(c + d*x)^2*((B*a)/2
 - (C*b)/2)))/d - (log(tan(c + d*x) - 1i)*(B + C*1i)*(a + b*1i))/(2*d) + (l
og(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)*1i)/(2*d)
```

3.9 $\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(\right.$

Optimal. Leaf size=148

$$-\left(\left(a^2B-b^2B-2abC\right)x\right)+\frac{\left(2abB+a^2C-b^2C\right)\log(\cos(c+dx))}{d}-\frac{b(bB+aC)\tan(c+dx)}{d}-\frac{C(a+b\tan(c}{2d}$$

```
[Out] -(B*a^2-B*b^2-2*C*a*b)*x+(2*B*a*b+C*a^2-C*b^2)*ln(cos(d*x+c))/d-b*(B*b+C*a)
*tan(d*x+c)/d-1/2*C*(a+b*tan(d*x+c))^2/d+1/12*(4*B*b-C*a)*(a+b*tan(d*x+c))^
3/b^2/d+1/4*C*tan(d*x+c)*(a+b*tan(d*x+c))^3/b/d
```

Rubi [A]
time = 0.21, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3713, 3688, 3711, 3609, 3606, 3556}

$$\frac{\left(a^2C+2abB-b^2C\right)\log(\cos(c+dx))}{d}-x\left(a^2B-2abC-b^2B\right)+\frac{(4bB-aC)(a+b\tan(c+dx))^3}{12b^2d}-\frac{b(aC+bB)\tan(c+dx)}{d}+\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{C(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] -((a^2*B - b^2*B - 2*a*b*C)*x) + ((2*a*b*B + a^2*C - b^2*C)*Log[Cos[c + d*x
]])/d - (b*(b*B + a*C)*Tan[c + d*x])/d - (C*(a + b*Tan[c + d*x])^2)/(2*d) +
 ((4*b*B - a*C)*(a + b*Tan[c + d*x])^3)/(12*b^2*d) + (C*Tan[c + d*x]*(a + b
*Tan[c + d*x])^3)/(4*b*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1] &
& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \tan(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \tan^2(c+dx)(a+b\tan(c+d$$

$$= \frac{C\tan(c+dx)(a+b\tan(c+dx}{4bd}$$

$$= \frac{(4bB-aC)(a+b\tan(c+dx))^3}{12b^2d}$$

$$= -\frac{C(a+b\tan(c+dx))^2}{2d} + \frac{(4bE}{}$$

$$= -\left(a^2B-b^2B-2abC\right)x - \frac{b(bI}{}$$

$$= -\left(a^2B-b^2B-2abC\right)x + \frac{(2a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.16, size = 221, normalized size = 1.49

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}+\frac{\frac{(4bB-aC)(a+b\tan(c+dx))^3}{3bd}+\frac{2((bB-aC)(i(a+ib)^2\log(i-\tan(c+dx))-i(a-ib)^2\log(i+\tan(c+dx))-2b^2\tan(c+dx))-C((ia-b)^3\log(i-\tan(c+dx))-(ia+b)^3\log(i+\tan(c+dx))+6ab^2\tan(c+dx)+b^3\tan^2(c+dx)))}{d}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] (C*Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*d) + (((4*b*B - a*C)*(a + b*Ta
n[c + d*x])^3)/(3*b*d) + (2*((b*B - a*C)*(I*(a + I*b)^2*Log[I - Tan[c + d*x
]] - I*(a - I*b)^2*Log[I + Tan[c + d*x]] - 2*b^2*Tan[c + d*x]) - C*((I*a -
b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*a*b^2*Ta
n[c + d*x] + b^3*Tan[c + d*x]^2)))/d)/(4*b)
```

Maple [A]
time = 0.10, size = 176, normalized size = 1.19

method	result
norman	$\left(-a^2B+b^2B+2Cab\right)x+\frac{\left(a^2B-b^2B-2Cab\right)\tan(dx+c)}{d}+\frac{\left(2Bab+C\,a^2-b^2C\right)\left(\tan^2(dx+c)\right)}{2d}+\frac{b(Bb+2C$
derivativedivides	$\frac{\frac{b^2C\left(\tan^4(dx+c)\right)}{4}+\frac{B\,b^2\left(\tan^3(dx+c)\right)}{3}+\frac{2Cab\left(\tan^3(dx+c)\right)}{3}+Bab\left(\tan^2(dx+c)\right)+\frac{C\,a^2\left(\tan^2(dx+c)\right)}{2}-\frac{b^2C\left(\tan^2(dx+c)\right)}{2}+a^2B$
default	$\frac{\frac{b^2C\left(\tan^4(dx+c)\right)}{4}+\frac{B\,b^2\left(\tan^3(dx+c)\right)}{3}+\frac{2Cab\left(\tan^3(dx+c)\right)}{3}+Bab\left(\tan^2(dx+c)\right)+\frac{C\,a^2\left(\tan^2(dx+c)\right)}{2}-\frac{b^2C\left(\tan^2(dx+c)\right)}{2}+a^2B$
risch	$-B\,a^2x+B\,b^2x+2Cabx-\frac{2iC\,a^2c}{d}+\frac{2iC\,b^2c}{d}+\frac{2i\left(6iC\,b^2e^{2i(dx+c)}-12iBab\,e^{4i(dx+c)}-6iC\,a^2e^{4i(dx+c)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^2*C*tan(d*x+c)^4+1/3*B*b^2*tan(d*x+c)^3+2/3*C*a*b*tan(d*x+c)^3+B
*a*b*tan(d*x+c)^2+1/2*C*a^2*tan(d*x+c)^2-1/2*b^2*C*tan(d*x+c)^2+a^2*B*tan(d
*x+c)-b^2*B*tan(d*x+c)-2*C*a*b*tan(d*x+c)+1/2*(-2*B*a*b-C*a^2+C*b^2)*ln(1+t
an(d*x+c)^2)+(-B*a^2+B*b^2+2*C*a*b)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 147, normalized size = 0.99

$$\frac{3\,Cb^2\tan(dx+c)^4+4\,(2\,Cab+Bb^2)\tan(dx+c)^3+6\,(Ca^2+2\,Bab-Cb^2)\tan(dx+c)^2-12\,(Ba^2-2\,Cab-Bb^2)(dx+c)-6\,(Ca^2+2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)+12\,(Ba^2-2\,Cab-Bb^2)\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/12*(3*C*b^2*tan(d*x + c)^4 + 4*(2*C*a*b + B*b^2)*tan(d*x + c)^3 + 6*(C*a^
2 + 2*B*a*b - C*b^2)*tan(d*x + c)^2 - 12*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c
) - 6*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1) + 12*(B*a^2 - 2*C*a
*b - B*b^2)*tan(d*x + c))/d
```

Fricas [**A**]
time = 2.30, size = 146, normalized size = 0.99

$$\frac{3\,Cb^2\tan\left(dx+c\right)^4+4\,\left(2\,Cab+Bb^2\right)\tan\left(dx+c\right)^3-12\,\left(Ba^2-2\,Cab-Bb^2\right)dx+6\,\left(Ca^2+2\,Bab-Cb^2\right)\tan\left(dx+c\right)^2+6\,\left(Ca^2+2\,Bab-Cb^2\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+12\,\left(Ba^2-2\,Cab-Bb^2\right)\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/12*(3*C*b^2*tan(d*x + c)^4 + 4*(2*C*a*b + B*b^2)*tan(d*x + c)^3 - 12*(B*a
^2 - 2*C*a*b - B*b^2)*d*x + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^2 + 6*
(C*a^2 + 2*B*a*b - C*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 12*(B*a^2 - 2*C*a*b
 - B*b^2)*tan(d*x + c))/d
```

Sympy [**A**]
time = 0.17, size = 250, normalized size = 1.69

$$\begin{cases} -Ba^2x+\frac{Ba^2\tan\left(c+dx\right)}{d}-\frac{Bab\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{Bab\tan^2\left(c+dx\right)}{d}+Bb^2x+\frac{Bb^2\tan^3\left(c+dx\right)}{3d}-\frac{Bb^2\tan\left(c+dx\right)}{d}-\frac{Ca^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ca^2\tan^2\left(c+dx\right)}{2d}+2Cabx+\frac{2Cab\tan^3\left(c+dx\right)}{3d}-\frac{2Cab\tan\left(c+dx\right)}{d}+\frac{Cb^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Cb^2\tan^4\left(c+dx\right)}{4d}-\frac{Cb^2\tan^2\left(c+dx\right)}{2d} & \text{for}\ d\neq 0 \\ x(a+b\tan\left(c\right))^2\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\tan\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((-B*a**2*x + B*a**2*tan(c + d*x)/d - B*a*b*log(tan(c + d*x)**2 +
1)/d + B*a*b*tan(c + d*x)**2/d + B*b**2*x + B*b**2*tan(c + d*x)**3/(3*d) -
B*b**2*tan(c + d*x)/d - C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*tan(
c + d*x)**2/(2*d) + 2*C*a*b*x + 2*C*a*b*tan(c + d*x)**3/(3*d) - 2*C*a*b*tan
(c + d*x)/d + C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*tan(c + d*x)**
4/(4*d) - C*b**2*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**2*(B*
tan(c) + C*tan(c)**2)*tan(c), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2228 vs. $2(141) = 282$.
time = 1.92, size = 2228, normalized size = 15.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] -1/12*(12*B*a^2*d*x*tan(d*x)^4*tan(c)^4 - 24*C*a*b*d*x*tan(d*x)^4*tan(c)^4
- 12*B*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*C*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2
```

```
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 12*B*a*b*log(4*(tan(d*x)^4*tan(c)
^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 6*C*b^2*log(4*(tan(d*x)^4*t
an(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*
x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 48*B*a^2*d*x*tan(d*x)^
3*tan(c)^3 + 96*C*a*b*d*x*tan(d*x)^3*tan(c)^3 + 48*B*b^2*d*x*tan(d*x)^3*tan
(c)^3 - 6*C*a^2*tan(d*x)^4*tan(c)^4 - 12*B*a*b*tan(d*x)^4*tan(c)^4 + 9*C*b^
2*tan(d*x)^4*tan(c)^4 + 24*C*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(d*x)^3*tan(c)^3 + 48*B*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 24*C*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2
*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 12*B*a^2*tan(d*x)^4*tan(c)^3 - 24
*C*a*b*tan(d*x)^4*tan(c)^3 - 12*B*b^2*tan(d*x)^4*tan(c)^3 + 12*B*a^2*tan(d*
x)^3*tan(c)^4 - 24*C*a*b*tan(d*x)^3*tan(c)^4 - 12*B*b^2*tan(d*x)^3*tan(c)^4
 + 72*B*a^2*d*x*tan(d*x)^2*tan(c)^2 - 144*C*a*b*d*x*tan(d*x)^2*tan(c)^2 - 7
2*B*b^2*d*x*tan(d*x)^2*tan(c)^2 - 6*C*a^2*tan(d*x)^4*tan(c)^2 - 12*B*a*b*ta
n(d*x)^4*tan(c)^2 + 6*C*b^2*tan(d*x)^4*tan(c)^2 + 12*C*a^2*tan(d*x)^3*tan(c
)^3 + 24*B*a*b*tan(d*x)^3*tan(c)^3 - 24*C*b^2*tan(d*x)^3*tan(c)^3 - 6*C*a^2
*tan(d*x)^2*tan(c)^4 - 12*B*a*b*tan(d*x)^2*tan(c)^4 + 6*C*b^2*tan(d*x)^2*ta
n(c)^4 + 8*C*a*b*tan(d*x)^4*tan(c) + 4*B*b^2*tan(d*x)^4*tan(c) - 36*C*a^2*l
og(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan
(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 72*B
*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2
+ 36*C*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan
(c)^2 - 36*B*a^2*tan(d*x)^3*tan(c)^2 + 96*C*a*b*tan(d*x)^3*tan(c)^2 + 48*B*
b^2*tan(d*x)^3*tan(c)^2 - 36*B*a^2*tan(d*x)^2*tan(c)^3 + 96*C*a*b*tan(d*x)^
2*tan(c)^3 + 48*B*b^2*tan(d*x)^2*tan(c)^3 + 8*C*a*b*tan(d*x)*tan(c)^4 + 4*B
*b^2*tan(d*x)*tan(c)^4 - 3*C*b^2*tan(d*x)^4 - 48*B*a^2*d*x*tan(d*x)*tan(c)
+ 96*C*a*b*d*x*tan(d*x)*tan(c) + 48*B*b^2*d*x*tan(d*x)*tan(c) + 12*C*a^2*ta
n(d*x)^3*tan(c) + 24*B*a*b*tan(d*x)^3*tan(c) - 24*C*b^2*tan(d*x)^3*tan(c) -
 12*C*a^2*tan(d*x)^2*tan(c)^2 - 24*B*a*b*tan(d*x)^2*tan(c)^2 + 12*C*b^2*tan
(d*x)^2*tan(c)^2 + 12*C*a^2*tan(d*x)*tan(c)^3 + 24*B*a*b*tan(d*x)*tan(c)^3
- 24*C*b^2*tan(d*x)*tan(c)^3 - 3*C*b^2*tan(c)^4 - 8*C*a*b*tan(d*x)^3 - 4*B*
b^2*tan(d*x)^3 + 24*C*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))
*tan(d*x)*tan(c) + 48*B*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)*tan(c) - 24*C*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
 1))*tan(d*x)*tan(c) + 36*B*a^2*tan(d*x)^2*tan(c) - 96*C*a*b*tan(d*x)^2*tan
```

```
(c) - 48*B*b^2*tan(d*x)^2*tan(c) + 36*B*a^2*tan(d*x)*tan(c)^2 - 96*C*a*b*ta
n(d*x)*tan(c)^2 - 48*B*b^2*tan(d*x)*tan(c)^2 - 8*C*a*b*tan(c)^3 - 4*B*b^2*t
an(c)^3 + 12*B*a^2*d*x - 24*C*a*b*d*x - 12*B*b^2*d*x - 6*C*a^2*tan(d*x)^2 -
 12*B*a*b*tan(d*x)^2 + 6*C*b^2*tan(d*x)^2 + 12*C*a^2*tan(d*x)*tan(c) + 24*B
*a*b*tan(d*x)*tan(c) - 24*C*b^2*tan(d*x)*tan(c) - 6*C*a^2*tan(c)^2 - 12*B*a
*b*tan(c)^2 + 6*C*b^2*tan(c)^2 - 6*C*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan
(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(c)^2 + 1)) - 12*B*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
 + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)
) + 6*C*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) - 12*B*a^2*ta
n(d*x) + 24*C*a*b*tan(d*x) + 12*B*b^2*tan(d*x) - 12*B*a^2*tan(c) + 24*C*a*b
*tan(c) + 12*B*b^2*tan(c) - 6*C*a^2 - 12*B*a*b + 9*C*b^2)/(d*tan(d*x)^4*tan
(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*ta
n(c) + d)
```

Mupad [B]

time = 8.84, size = 151, normalized size = 1.02

$$x\left(-B\,a^2+2\,C\,a\,b+B\,b^2\right)+\frac{\tan(c+d\,x)^3\,\left(\frac{B\,b^2}{3}+\frac{2\,C\,a\,b}{3}\right)}{d}-\frac{\tan(c+d\,x)\,\left(-B\,a^2+2\,C\,a\,b+B\,b^2\right)}{d}-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{C\,a^2}{2}+B\,a\,b-\frac{C\,b^2}{2}\right)}{d}+\frac{\tan(c+d\,x)^2\,\left(\frac{C\,a^2}{2}+B\,a\,b-\frac{C\,b^2}{2}\right)}{d}+\frac{C\,b^2\,\tan(c+d\,x)^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^2
,x)
```

```
[Out] x*(B*b^2 - B*a^2 + 2*C*a*b) + (tan(c + d*x)^3*((B*b^2)/3 + (2*C*a*b)/3))/d
- (tan(c + d*x)*(B*b^2 - B*a^2 + 2*C*a*b))/d - (log(tan(c + d*x)^2 + 1)*((C
*a^2)/2 - (C*b^2)/2 + B*a*b))/d + (tan(c + d*x)^2*((C*a^2)/2 - (C*b^2)/2 +
B*a*b))/d + (C*b^2*tan(c + d*x)^4)/(4*d)
```

3.10 $\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)$

Optimal. Leaf size=112

$$-\left(\left(2abB+a^2C-b^2C\right)x\right)-\frac{\left(a^2B-b^2B-2abC\right)\log(\cos(c+dx))}{d}+\frac{b(aB-bC)\tan(c+dx)}{d}+\frac{B(a+b\tan(c}{2d}$$

```
[Out] -(2*B*a*b+C*a^2-C*b^2)*x-(B*a^2-B*b^2-2*C*a*b)*ln(cos(d*x+c))/d+b*(B*a-C*b)
*tan(d*x+c)/d+1/2*B*(a+b*tan(d*x+c))^2/d+1/3*C*(a+b*tan(d*x+c))^3/b/d
```

Rubi [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3711, 3609, 3606, 3556}

$$-\frac{\left(a^2B-2abC-b^2B\right)\log(\cos(c+dx))}{d}-x\left(a^2C+2abB-b^2C\right)+\frac{b(aB-bC)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] -((2*a*b*B + a^2*C - b^2*C)*x) - ((a^2*B - b^2*B - 2*a*b*C)*Log[Cos[c + d*x
]])/d + (b*(a*B - b*C)*Tan[c + d*x])/d + (B*(a + b*Tan[c + d*x])^2)/(2*d) +
 (C*(a + b*Tan[c + d*x])^3)/(3*b*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
```

```
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{C(a+b\tan(c+dx))^3}{3bd} + \int (a+b\tan(c+d$$
$$= \frac{B(a+b\tan(c+dx))^2}{2d} + \frac{C(a+b\tan(c+d}{3bd}$$
$$= -\left(2abB+a^2C-b^2C\right)x + \frac{b(aB-bC)\tan}{d}$$
$$= -\left(2abB+a^2C-b^2C\right)x - \frac{(a^2B-b^2B-2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.25, size = 172, normalized size = 1.54

$$\frac{2C(a+b\tan(c+dx))^3+3(aB+bC)\left(i((a+ib)^2\log(i-\tan(c+dx))-(a-ib)^2\log(i+\tan(c+dx)))-2b^2\tan(c+dx)\right)+3B((ia-b)^3\log(i-\tan(c+dx))-(ia+b)^3\log(i+\tan(c+dx))+6ab^2\tan(c+dx)+b^3\tan^2(c+dx))}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] (2*C*(a + b*Tan[c + d*x])^3 + 3*(a*B + b*C)*(I*((a + I*b)^2*Log[I - Tan[c +
 d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]]) - 2*b^2*Tan[c + d*x]) + 3*B*((I
*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*a*b
^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(6*b*d)
```

Maple [A]
time = 0.07, size = 135, normalized size = 1.21

method	result
norman	$(-2Bab-C\,a^2+b^2C)\,x+\frac{(2Bab+C\,a^2-b^2C)\tan(dx+c)}{d}+\frac{b(Bb+2Ca)\left(\tan^2(dx+c)\right)}{2d}+\frac{b^2C\left(\tan^3(dx+\right.}{3d}$
derivativedivides	$\frac{\frac{b^2C\left(\tan^3(dx+c)\right)}{3}+\frac{B\,b^2\left(\tan^2(dx+c)\right)}{2}+Cab\left(\tan^2(dx+c)\right)+2Bab\tan(dx+c)+C\,a^2\tan(dx+c)-b^2C\tan(dx+c)+\frac{\left(a^2B-b^2\right.}{}}{d}$
default	$\frac{\frac{b^2C\left(\tan^3(dx+c)\right)}{3}+\frac{B\,b^2\left(\tan^2(dx+c)\right)}{2}+Cab\left(\tan^2(dx+c)\right)+2Bab\tan(dx+c)+C\,a^2\tan(dx+c)-b^2C\tan(dx+c)+\frac{\left(a^2B-b^2\right.}{}}{d}$
risch	$-\frac{4iCabc}{d}-iB\,b^2x+\frac{2iB\,a^2c}{d}-2Babx-C\,a^2x+C\,b^2x+iB\,a^2x+\frac{2i\left(-3iB\,b^2e^{4i(dx+c)}-6iCab\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*b^2*C*tan(d*x+c)^3+1/2*B*b^2*tan(d*x+c)^2+C*a*b*tan(d*x+c)^2+2*B*a
*b*tan(d*x+c)+C*a^2*tan(d*x+c)-b^2*C*tan(d*x+c)+1/2*(B*a^2-B*b^2-2*C*a*b)*l
n(1+tan(d*x+c)^2)+(-2*B*a*b-C*a^2+C*b^2)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 120, normalized size = 1.07

$$\frac{2\,Cb^2\tan\left(dx+c\right)^3+3\,(2\,Cab+Bb^2)\tan\left(dx+c\right)^2-6\,(Ca^2+2\,Bab-Cb^2)(dx+c)+3\,(Ba^2-2\,Cab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)+6\,(Ca^2+2\,Bab-Cb^2)\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/6*(2*C*b^2*tan(d*x + c)^3 + 3*(2*C*a*b + B*b^2)*tan(d*x + c)^2 - 6*(C*a^2
 + 2*B*a*b - C*b^2)*(d*x + c) + 3*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c
)^2 + 1) + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c))/d
```

Fricas [A]
time = 1.37, size = 119, normalized size = 1.06

$$\frac{2\,Cb^2\tan\left(dx+c\right)^3-6\,(Ca^2+2\,Bab-Cb^2)dx+3\,(2\,Cab+Bb^2)\tan\left(dx+c\right)^2-3\,(Ba^2-2\,Cab-Bb^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+6\,(Ca^2+2\,Bab-Cb^2)\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(2*C*b^2*tan(d*x + c)^3 - 6*(C*a^2 + 2*B*a*b - C*b^2)*d*x + 3*(2*C*a*b
+ B*b^2)*tan(d*x + c)^2 - 3*(B*a^2 - 2*C*a*b - B*b^2)*log(1/(tan(d*x + c)^2
 + 1)) + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c))/d
```

Sympy [A]
time = 0.13, size = 194, normalized size = 1.73

$$\begin{cases} \frac{Ba^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} - 2Babx + \frac{2Bab\tan\left(c+dx\right)}{d} - \frac{Bb^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{Bb^2\tan^2\left(c+dx\right)}{2d} - Ca^2x + \frac{Ca^2\tan\left(c+dx\right)}{d} - \frac{Cab\log\left(\tan^2\left(c+dx\right)+1\right)}{d} + \frac{Cab\tan^2\left(c+dx\right)}{d} + Cb^2x + \frac{Cb^2\tan^3\left(c+dx\right)}{3d} - \frac{Cb^2\tan\left(c+dx\right)}{d} & \text{for}\: d \neq 0 \\ x\left(a+b\tan\left(c\right)\right)^2\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*B*a*b*x + 2*B*a*b*tan(
c + d*x)/d - B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**2*tan(c + d*x)**2
/(2*d) - C*a**2*x + C*a**2*tan(c + d*x)/d - C*a*b*log(tan(c + d*x)**2 + 1)/
d + C*a*b*tan(c + d*x)**2/d + C*b**2*x + C*b**2*tan(c + d*x)**3/(3*d) - C*b
```

```
**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*tan(c)**2
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(108) = 216$.
time = 1.25, size = 1509, normalized size = 13.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*C*a^2*d*x*tan(d*x)^3*tan(c)^3 + 12*B*a*b*d*x*tan(d*x)^3*tan(c)^3 -
6*C*b^2*d*x*tan(d*x)^3*tan(c)^3 + 3*B*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 6*C*a*b*log(4*(tan(d*x)^4*tan(c)^2 -
 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 3*B*b^2*log(4*(tan(d*x)^4*tan(c
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 18*C*a^2*d*x*tan(d*x)^2*ta
n(c)^2 - 36*B*a*b*d*x*tan(d*x)^2*tan(c)^2 + 18*C*b^2*d*x*tan(d*x)^2*tan(c)^
2 - 6*C*a*b*tan(d*x)^3*tan(c)^3 - 3*B*b^2*tan(d*x)^3*tan(c)^3 - 9*B*a^2*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 18*C*a
*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 +
9*B*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)
^2 + 6*C*a^2*tan(d*x)^3*tan(c)^2 + 12*B*a*b*tan(d*x)^3*tan(c)^2 - 6*C*b^2*t
an(d*x)^3*tan(c)^2 + 6*C*a^2*tan(d*x)^2*tan(c)^3 + 12*B*a*b*tan(d*x)^2*tan(
c)^3 - 6*C*b^2*tan(d*x)^2*tan(c)^3 + 18*C*a^2*d*x*tan(d*x)*tan(c) + 36*B*a*
b*d*x*tan(d*x)*tan(c) - 18*C*b^2*d*x*tan(d*x)*tan(c) - 6*C*a*b*tan(d*x)^3*t
an(c) - 3*B*b^2*tan(d*x)^3*tan(c) + 6*C*a*b*tan(d*x)^2*tan(c)^2 + 3*B*b^2*t
an(d*x)^2*tan(c)^2 - 6*C*a*b*tan(d*x)*tan(c)^3 - 3*B*b^2*tan(d*x)*tan(c)^3
+ 2*C*b^2*tan(d*x)^3 + 9*B*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)*tan(c) - 18*C*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(d*x)*tan(c) - 9*B*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3
*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)
^2 + 1))*tan(d*x)*tan(c) - 12*C*a^2*tan(d*x)^2*tan(c) - 24*B*a*b*tan(d*x)^2
*tan(c) + 18*C*b^2*tan(d*x)^2*tan(c) - 12*C*a^2*tan(d*x)*tan(c)^2 - 24*B*a*
b*tan(d*x)*tan(c)^2 + 18*C*b^2*tan(d*x)*tan(c)^2 + 2*C*b^2*tan(c)^3 - 6*C*a
^2*d*x - 12*B*a*b*d*x + 6*C*b^2*d*x + 6*C*a*b*tan(d*x)^2 + 3*B*b^2*tan(d*x)
```

```
^2 - 6*C*a*b*tan(d*x)*tan(c) - 3*B*b^2*tan(d*x)*tan(c) + 6*C*a*b*tan(c)^2 +
 3*B*b^2*tan(c)^2 - 3*B*a^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
)) + 6*C*a*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 3*B*b^2*lo
g(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 6*C*a^2*tan(d*x) + 12*B*a
*b*tan(d*x) - 6*C*b^2*tan(d*x) + 6*C*a^2*tan(c) + 12*B*a*b*tan(c) - 6*C*b^2
*tan(c) + 6*C*a*b + 3*B*b^2)/(d*tan(d*x)^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)
^2 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 8.80, size = 121, normalized size = 1.08

$$\frac{\tan(c+dx)^2\left(\frac{Bb^2}{2}+Cab\right)}{d}-x\left(Ca^2+2Bab-Cb^2\right)+\frac{\tan(c+dx)\,\left(Ca^2+2Bab-Cb^2\right)}{d}-\frac{\ln\left(\tan(c+dx)^2+1\right)\,\left(-\frac{Ba^2}{2}+Cab+\frac{Bb^2}{2}\right)}{d}+\frac{Cb^2\tan(c+dx)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (tan(c + d*x)^2*((B*b^2)/2 + C*a*b))/d - x*(C*a^2 - C*b^2 + 2*B*a*b) + (tan
(c + d*x)*(C*a^2 - C*b^2 + 2*B*a*b))/d - (log(tan(c + d*x)^2 + 1)*((B*b^2)/
2 - (B*a^2)/2 + C*a*b))/d + (C*b^2*tan(c + d*x)^3)/(3*d)
```

3.11 $\int \cot(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=87

$$\left(a^2B-b^2B-2abC\right)x-\frac{\left(2abB+a^2C-b^2C\right)\log(\cos(c+dx))}{d}+\frac{b(bB+aC)\tan(c+dx)}{d}+\frac{C(a+b\tan(c+}{2d}$$

```
[Out] (B*a^2-B*b^2-2*C*a*b)*x-(2*B*a*b+C*a^2-C*b^2)*ln(cos(d*x+c))/d+b*(B*b+C*a)*
tan(d*x+c)/d+1/2*C*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3713, 3609, 3606, 3556}

$$-\frac{\left(a^2C+2abB-b^2C\right)\log(\cos(c+dx))}{d}+x\left(a^2B-2abC-b^2B\right)+\frac{b(aC+bB)\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (a^2*B - b^2*B - 2*a*b*C)*x - ((2*a*b*B + a^2*C - b^2*C)*Log[Cos[c + d*x]])
/d + (b*(b*B + a*C)*Tan[c + d*x])/d + (C*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx=\int(a+b\tan(c+dx))^2(B+C\tan$$

$$=\frac{C(a+b\tan(c+dx))^2}{2d}+\int(a+b$$

$$=\left(a^2B-b^2B-2abC\right)x+\frac{b(bB+}{}$$

$$=\left(a^2B-b^2B-2abC\right)x-\frac{(2abB}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 96, normalized size = 1.10

$$\frac{(a+ib)^2(-iB+C)\log(i-\tan(c+dx))+(a-ib)^2(iB+C)\log(i+\tan(c+dx))+2b(bB+2aC)\tan(c+dx)+b^2C\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] ((a + I*b)^2*((-I)*B + C)*Log[I - Tan[c + d*x]] + (a - I*b)^2*(I*B + C)*Log
[I + Tan[c + d*x]] + 2*b*(b*B + 2*a*C)*Tan[c + d*x] + b^2*C*Tan[c + d*x]^2)
/(2*d)
```

Maple [A]
time = 0.31, size = 102, normalized size = 1.17

method	result
norman	$\left(a^2B-b^2B-2Cab\right)x+\frac{b(Bb+2Ca)\tan(dx+c)}{d}+\frac{b^2C\left(\tan^2(dx+c)\right)}{2d}+\frac{\left(2Bab+C\,a^2-b^2C\right)\ln\left(1+\tan^2(dx+\right.}{2d}$
derivativedivides	$\frac{b^2B(\tan(dx+c)-dx-c)+b^2C\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)-2Bab\ln(\cos(dx+c))+2Cab(\tan(dx+c)-dx-c)+a^2B(dx}{d}$
default	$\frac{b^2B(\tan(dx+c)-dx-c)+b^2C\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)-2Bab\ln(\cos(dx+c))+2Cab(\tan(dx+c)-dx-c)+a^2B(dx}{d}$

risch	$B\,a^2x - B\,b^2x - 2Cabx + \frac{2ib\left(-iCb\,e^{2i(dx+c)}+Bb\,e^{2i(dx+c)}+2Ca\,e^{2i(dx+c)}+Bb+2Ca\right)}{d\left(e^{2i(dx+c)}+1\right)^2} - \frac{2iC\,b^2c}{d} - iC$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(b^2*B*(tan(d*x+c)-d*x-c)+b^2*C*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))-2*B*a
*b*ln(cos(d*x+c))+2*C*a*b*(tan(d*x+c)-d*x-c)+a^2*B*(d*x+c)-C*a^2*ln(cos(d*x
+c)))
```

Maxima [A]
time = 0.49, size = 91, normalized size = 1.05

$$\frac{Cb^2\tan\left(dx+c\right)^2 + 2\left(Ba^2 - 2\,Cab - Bb^2\right)(dx+c) + \left(Ca^2 + 2\,Bab - Cb^2\right)\log\left(\tan\left(dx+c\right)^2 + 1\right) + 2\left(2\,Cab + Bb^2\right)\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/2*(C*b^2*tan(d*x + c)^2 + 2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2
+ 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1) + 2*(2*C*a*b + B*b^2)*tan(d*x +
c))/d
```

Fricas [A]
time = 1.42, size = 91, normalized size = 1.05

$$\frac{Cb^2\tan\left(dx+c\right)^2 + 2\left(Ba^2 - 2\,Cab - Bb^2\right)dx - \left(Ca^2 + 2\,Bab - Cb^2\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 2\left(2\,Cab + Bb^2\right)\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/2*(C*b^2*tan(d*x + c)^2 + 2*(B*a^2 - 2*C*a*b - B*b^2)*d*x - (C*a^2 + 2*B*
a*b - C*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 2*(2*C*a*b + B*b^2)*tan(d*x + c)
)/d
```

Sympy [A]
time = 0.56, size = 151, normalized size = 1.74

$$\begin{cases} Ba^2x + \frac{Bab\log\left(\tan^2\left(c+dx\right)+1\right)}{d} - Bb^2x + \frac{Bb^2\tan\left(c+dx\right)}{d} + \frac{Ca^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} - 2Cabx + \frac{2Cab\tan\left(c+dx\right)}{d} - \frac{Cb^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{Cb^2\tan^2\left(c+dx\right)}{2d} & \text{for}\ d \neq 0 \\ x\left(a+b\tan\left(c\right)\right)^2\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**2*x + B*a*b*log(tan(c + d*x)**2 + 1)/d - B*b**2*x + B*b**2*
tan(c + d*x)/d + C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*C*a*b*x + 2*C*a*
b*tan(c + d*x)/d - C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*tan(c + d
*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(
c), True))
```

Giac [A]
time = 1.15, size = 95, normalized size = 1.09

$$\frac{Cb^2\tan\left(dx+c\right)^2+4\,Cab\tan\left(dx+c\right)+2\,Bb^2\tan\left(dx+c\right)+2\left(Ba^2-2\,Cab-Bb^2\right)\left(dx+c\right)+\left(Ca^2+2\,Bab-Cb^2\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/2*(C*b^2*tan(d*x + c)^2 + 4*C*a*b*tan(d*x + c) + 2*B*b^2*tan(d*x + c) + 2
*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*
x + c)^2 + 1))/d
```

Mupad [B]
time = 8.85, size = 91, normalized size = 1.05

$$\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{C\,a^2}{2}+B\,a\,b-\frac{C\,b^2}{2}\right)}{d}-x\,\left(-B\,a^2+2\,C\,a\,b+B\,b^2\right)+\frac{\tan(c+d\,x)\,\left(B\,b^2+2\,C\,a\,b\right)}{d}+\frac{C\,b^2\,\tan(c+d\,x)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^2
,x)
```

```
[Out] (log(tan(c + d*x)^2 + 1)*((C*a^2)/2 - (C*b^2)/2 + B*a*b))/d - x*(B*b^2 - B*
a^2 + 2*C*a*b) + (tan(c + d*x)*(B*b^2 + 2*C*a*b))/d + (C*b^2*tan(c + d*x)^2
)/(2*d)
```

3.12 $\int \cot^2(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan$

Optimal. Leaf size=70

$$(2abB+a^2C-b^2C)\,x-\frac{b(bB+2aC)\log(\cos(c+dx))}{d}+\frac{a^2B\log(\sin(c+dx))}{d}+\frac{b^2C\tan(c+dx)}{d}$$

```
[Out] (2*B*a*b+C*a^2-C*b^2)*x-b*(B*b+2*C*a)*ln(cos(d*x+c))/d+a^2*B*ln(sin(d*x+c))
/d+b^2*C*tan(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3713, 3687, 3705, 3556}

$$x\left(a^2C+2abB-b^2C\right)+\frac{a^2B\log(\sin(c+dx))}{d}-\frac{b(2aC+bB)\log(\cos(c+dx))}{d}+\frac{b^2C\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (2*a*b*B + a^2*C - b^2*C)*x - (b*(b*B + 2*a*C)*Log[Cos[c + d*x]])/d + (a^2*
B*Log[Sin[c + d*x]])/d + (b^2*C*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3687

```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (
f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b^2
*B*(Tan[e + f*x]/(d*f)), x] + Dist[1/d, Int[(a^2*A*d - b^2*B*c + (2*a*A*b +
 B*(a^2 - b^2))*d*Tan[e + f*x] + (A*b^2*d - b*B*(b*c - 2*a*d))*Tan[e + f*x]
^2)/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx=\int \cot(c+dx)(a+b\tan(c+dx)$$

$$=\frac{b^2C\tan(c+dx)}{d}+\int \cot(c+dx$$

$$=\left(2abB+a^2C-b^2C\right)x+\frac{b^2C\,\mathrm{ta}}{}$$

$$=\left(2abB+a^2C-b^2C\right)x-\frac{b(bB+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 91, normalized size = 1.30

$$-\frac{(a+ib)^2(B+iC)\log(i-\tan(c+dx))-2a^2B\log(\tan(c+dx))+(a-ib)^2(B-iC)\log(i+\tan(c+dx))-2b^2C\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] -1/2*((a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] - 2*a^2*B*Log[Tan[c + d*x
]] + (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]] - 2*b^2*C*Tan[c + d*x])/d
```

Maple [A]
time = 0.27, size = 82, normalized size = 1.17

method	result
derivativedivides	$\frac{-b^2B\ln(\cos(dx+c))+b^2C(\tan(dx+c)-dx-c)+2Bab(dx+c)-2Cab\ln(\cos(dx+c))+a^2B\ln(\sin(dx+c))+C\,a^2(dx+c)}{d}$
default	$\frac{-b^2B\ln(\cos(dx+c))+b^2C(\tan(dx+c)-dx-c)+2Bab(dx+c)-2Cab\ln(\cos(dx+c))+a^2B\ln(\sin(dx+c))+C\,a^2(dx+c)}{d}$
norman	$\frac{(2Bab+C\,a^2-b^2C)x\tan(dx+c)+\frac{b^2C\left(\tan^2(dx+c)\right)}{d}}{\tan(dx+c)}+\frac{a^2B\ln(\tan(dx+c))}{d}-\frac{(a^2B-b^2B-2Cab)\ln(1+\tan^2(dx+c))}{2d}$
risch	$iB\,b^2x-\frac{2iB\,a^2c}{d}-iB\,a^2x+2Babx+C\,a^2x-C\,b^2x+\frac{4iCabc}{d}+\frac{2iB\,b^2c}{d}+\frac{2ib^2C}{d\left(e^{2i(dx+c)}+1\right)}+2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^2*B*ln(cos(d*x+c))+b^2*C*(tan(d*x+c)-d*x-c)+2*B*a*b*(d*x+c)-2*C*a*b
*ln(cos(d*x+c))+a^2*B*ln(sin(d*x+c))+C*a^2*(d*x+c))
```

Maxima [A]
time = 0.49, size = 85, normalized size = 1.21

$$\frac{2\,Ba^2\log\left(\tan\left(dx+c\right)\right)+2\,Cb^2\tan\left(dx+c\right)+2\,(Ca^2+2\,Bab-Cb^2)(dx+c)-(Ba^2-2\,Cab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/2*(2*B*a^2*log(tan(d*x + c)) + 2*C*b^2*tan(d*x + c) + 2*(C*a^2 + 2*B*a*b
- C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 2.61, size = 92, normalized size = 1.31

$$\frac{Ba^2\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+2\,Cb^2\tan\left(dx+c\right)+2\,(Ca^2+2\,Bab-Cb^2)dx-(2\,Cab+Bb^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + 2*C*b^2*tan(d*x + c)
+ 2*(C*a^2 + 2*B*a*b - C*b^2)*d*x - (2*C*a*b + B*b^2)*log(1/(tan(d*x + c)^2
 + 1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(66) = 132$.
time = 0.85, size = 136, normalized size = 1.94

$$\begin{cases} -\frac{Ba^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ba^2\log\left(\tan\left(c+dx\right)\right)}{d}+2Babx+\frac{Bb^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+Ca^2x+\frac{Cab\log\left(\tan^2\left(c+dx\right)+1\right)}{d}-Cb^2x+\frac{Cb^2\tan\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x(a+b\tan\left(c\right))^2\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^2\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((-B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*log(tan(c + d*x)
)/d + 2*B*a*b*x + B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*x + C*a*b*
log(tan(c + d*x)**2 + 1)/d - C*b**2*x + C*b**2*tan(c + d*x)/d, Ne(d, 0)), (
x*(a + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

Giac [A]

time = 1.41, size = 86, normalized size = 1.23

$$\frac{2\,Ba^2\log\left(|\tan\left(dx+c\right)|\right)+2\,Cb^2\tan\left(dx+c\right)+2\,(Ca^2+2\,Bab-Cb^2)(dx+c)-(Ba^2-2\,Cab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(2*B*a^2*log(abs(tan(d*x + c))) + 2*C*b^2*tan(d*x + c) + 2*(C*a^2 + 2*B
*a*b - C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)
)/d
```

Mupad [B]

time = 8.85, size = 90, normalized size = 1.29

$$\frac{B\,a^2\,\ln\left(\tan(c+d\,x)\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(B-C\,1{}\mathrm{i}\right)\,{\left(b+a\,1{}\mathrm{i}\right)}^2}{2\,d}+\frac{C\,b^2\tan(c+d\,x)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1{}\mathrm{i}\right)\,{\left(-b+a\,1{}\mathrm{i}\right)}^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^2,x)
```

```
[Out] (B*a^2*log(tan(c + d*x)))/d + (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)
^2)/(2*d) + (C*b^2*tan(c + d*x))/d + (log(tan(c + d*x) - 1i)*(B + C*1i)*(a*
1i - b)^2)/(2*d)
```

3.13 $\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=72

$$-\left(\left(a^2B-b^2B-2abC\right)x\right)-\frac{a^2B\cot(c+dx)}{d}-\frac{b^2C\log(\cos(c+dx))}{d}+\frac{a(2bB+aC)\log(\sin(c+dx))}{d}$$

```
[Out] -(B*a^2-B*b^2-2*C*a*b)*x-a^2*B*cot(d*x+c)/d-b^2*C*ln(cos(d*x+c))/d+a*(2*B*b
+C*a)*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {3713, 3685, 3705, 3556}

$$-x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot(c+dx)}{d}+\frac{a(aC+2bB)\log(\sin(c+dx))}{d}-\frac{b^2C\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] -((a^2*B - b^2*B - 2*a*b*C)*x) - (a^2*B*Cot[c + d*x])/d - (b^2*C*Log[Cos[c
+ d*x]])/d + (a*(2*b*B + a*C)*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
 + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*
c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^2(c+dx)(a+b\tan(c+dx$$

$$= -\frac{a^2B\cot(c+dx)}{d} + \int \cot(c+$$

$$= -\left(a^2B-b^2B-2abC\right)x - \frac{a^2B}{}$$

$$= -\left(a^2B-b^2B-2abC\right)x - \frac{a^2B}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 100, normalized size = 1.39

$$\frac{-2a^2B\cot(c+dx)+i(a+ib)^2(B+iC)\log(i-\tan(c+dx))+2a(2bB+aC)\log(\tan(c+dx))-(a-ib)^2(iB+C)\log(i+\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-2*a^2*B*Cot[c + d*x] + I*(a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] + 2*
a*(2*b*B + a*C)*Log[Tan[c + d*x]] - (a - I*b)^2*(I*B + C)*Log[I + Tan[c + d
*x]])/(2*d)
```

Maple [A]
time = 0.26, size = 84, normalized size = 1.17

method	result
derivativedivides	$\frac{b^2B(dx+c)-b^2C\ln(\cos(dx+c))+2Bab\ln(\sin(dx+c))+2Cab(dx+c)+a^2B(-\cot(dx+c)-dx-c)+Ca^2\ln(\sin(dx+c))}{d}$
default	$\frac{b^2B(dx+c)-b^2C\ln(\cos(dx+c))+2Bab\ln(\sin(dx+c))+2Cab(dx+c)+a^2B(-\cot(dx+c)-dx-c)+Ca^2\ln(\sin(dx+c))}{d}$
norman	$\frac{(-a^2B+b^2B+2Cab)x(\tan^2(dx+c))-\frac{a^2B\tan(dx+c)}{d}}{\tan(dx+c)^2}+\frac{a(2Bb+Ca)\ln(\tan(dx+c))}{d}-\frac{(2Bab+Ca^2-b^2C)\ln(1+\tan^2(}{2d}$

risch	$-B\,a^2x+B\,b^2x+2Cabx-\frac{2ia^2B}{d\left(e^{2i(dx+c)}-1\right)}+\frac{2iC\,b^2c}{d}-\frac{4iBabc}{d}+iC\,b^2x-2iBabx-\frac{2iC\,a^2c}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*B*(d*x+c)-b^2*C*ln(cos(d*x+c))+2*B*a*b*ln(sin(d*x+c))+2*C*a*b*(d*x
+c)+a^2*B*(-cot(d*x+c)-d*x-c)+C*a^2*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.49, size = 93, normalized size = 1.29

$$-\frac{2\,(Ba^2-2\,Cab-Bb^2)(dx+c)+(Ca^2+2\,Bab-Cb^2)\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(Ca^2+2\,Bab)\log\left(\tan\left(dx+c\right)\right)+\frac{2\,Ba^2}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*log
(tan(d*x + c)^2 + 1) - 2*(C*a^2 + 2*B*a*b)*log(tan(d*x + c)) + 2*B*a^2/tan(
d*x + c))/d
```

Fricas [A]
time = 2.40, size = 112, normalized size = 1.56

$$-\frac{Cb^2\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)+2\,(Ba^2-2\,Cab-Bb^2)dx\tan\left(dx+c\right)+2\,Ba^2-(Ca^2+2\,Bab)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/2*(C*b^2*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*(B*a^2 - 2*C*a*b -
 B*b^2)*d*x*tan(d*x + c) + 2*B*a^2 - (C*a^2 + 2*B*a*b)*log(tan(d*x + c)^2/(
tan(d*x + c)^2 + 1))*tan(d*x + c))/(d*tan(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.
time = 1.35, size = 165, normalized size = 2.29

$$\begin{cases} \text{NaN} & \text{for}\: (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ x(a+b\tan(c))^2\,(B\tan(c)+C\tan^2(c))\cot^3(c) & \text{for}\: d=0 \\ -Ba^2x-\frac{Ba^2}{d\tan(c+dx)}-\frac{Bab\log(\tan^2(c+dx)+1)}{d}+\frac{2Bab\log(\tan(c+dx))}{d}+Bb^2x-\frac{Ca^2\log(\tan^2(c+dx)+1)}{2d}+\frac{Ca^2\log(\tan(c+dx))}{d}+2Cabx+\frac{Cb^2\log(\tan^2(c+dx)+1)}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**3, Eq(d, 0)), (-B*a**2*x -
 B*a**2/(d*tan(c + d*x)) - B*a*b*log(tan(c + d*x)**2 + 1)/d + 2*B*a*b*log(t
an(c + d*x))/d + B*b**2*x - C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*
log(tan(c + d*x))/d + 2*C*a*b*x + C*b**2*log(tan(c + d*x)**2 + 1)/(2*d), Tr
ue))
```

Giac [A]
time = 1.70, size = 118, normalized size = 1.64

$$-\frac{2\,(Ba^2-2\,Cab-Bb^2)(dx+c)+(Ca^2+2\,Bab-Cb^2)\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(Ca^2+2\,Bab)\log\left(\left|\tan\left(dx+c\right)\right|\right)+\frac{2\,(Ca^2\tan(dx+c)+2\,Bab\tan(dx+c)+Ba^2)}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/2*(2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*log
(tan(d*x + c)^2 + 1) - 2*(C*a^2 + 2*B*a*b)*log(abs(tan(d*x + c))) + 2*(C*a^
2*tan(d*x + c) + 2*B*a*b*tan(d*x + c) + B*a^2)/tan(d*x + c))/d
```

Mupad [B]
time = 9.00, size = 100, normalized size = 1.39

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(C\,a^2+2\,B\,b\,a\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)\,{\left(-b+a\,1\mathrm{i}\right)}^2}{2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^2}{2\,d}-\frac{B\,a^2\cot(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^2,x)
```

```
[Out] (log(tan(c + d*x))*(C*a^2 + 2*B*a*b))/d - (log(tan(c + d*x) - 1i)*(B*1i - C
)*(a*1i - b)^2)/(2*d) + (log(tan(c + d*x) + 1i)*(B*1i + C)*(a*1i + b)^2)/(2
*d) - (B*a^2*cot(c + d*x))/d
```

3.14 $\int \cot^4(c+dx)(a+b\tan(c+dx))^2 (B\tan(c+dx)+C\tan$

Optimal. Leaf size=88

$$\left(b^2C-a(2bB+aC)\right)x-\frac{a(2bB+aC)\cot(c+dx)}{d}-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{(a^2B-b^2B-2abC)\log(\sin(c+d$$

```
[Out] (b^2*C-a*(2*B*b+C*a))*x-a*(2*B*b+C*a)*cot(d*x+c)/d-1/2*a^2*B*cot(d*x+c)^2/d
-(B*a^2-B*b^2-2*C*a*b)*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.18, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3713, 3685, 3709, 3612, 3556}

$$-\frac{(a^2B-2abC-b^2B)\log(\sin(c+dx))}{d}-\frac{a^2B\cot^2(c+dx)}{2d}+x\left(b^2C-a(aC+2bB)\right)-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (b^2*C - a*(2*b*B + a*C))*x - (a*(2*b*B + a*C)*Cot[c + d*x])/d - (a^2*B*Cot
[c + d*x]^2)/(2*d) - ((a^2*B - b^2*B - 2*a*b*C)*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
 + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*
```

```
c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^3(c+dx)(a+b\tan(c+dx$$

$$= -\frac{a^2B\cot^2(c+dx)}{2d} + \int \cot^2(c+$$

$$= -\frac{a(2bB+aC)\cot(c+dx)}{d} - \frac{a^2}{}$$

$$= \left(b^2C-a(2bB+aC)\right)x - \frac{a(2bB}{}$$

$$= \left(b^2C-a(2bB+aC)\right)x - \frac{a(2bB}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 123, normalized size = 1.40

$$\frac{-2a(2bB+aC)\cot(c+dx)-a^2B\cot^2(c+dx)+(a+ib)^2(B+iC)\log(i-\tan(c+dx))-2(a^2B-b^2B-2abC)\log(\tan(c+dx))+(a-ib)^2(B-iC)\log(i+\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

[Out] (-2*a*(2*b*B + a*C)*Cot[c + d*x] - a^2*B*Cot[c + d*x]^2 + (a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] - 2*(a^2*B - b^2*B - 2*a*b*C)*Log[Tan[c + d*x]] + (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]])/(2*d)

Maple [A]
time = 0.30, size = 107, normalized size = 1.22

method	result
derivativedivides	$\frac{b^2 B \ln(\sin(dx+c))+b^2 C(dx+c)+2Bab(-\cot(dx+c)-dx-c)+2Cab\ln(\sin(dx+c))+a^2 B\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c)\right.}{d}$
default	$\frac{b^2 B \ln(\sin(dx+c))+b^2 C(dx+c)+2Bab(-\cot(dx+c)-dx-c)+2Cab\ln(\sin(dx+c))+a^2 B\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c)\right.}{d}$
norman	$\frac{(-2Bab-C a^2+b^2C)x\left(\tan^3(dx+c)\right)-\frac{a(2Bb+Ca)\left(\tan^2(dx+c)\right)}{d}-\frac{a^2 B\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{(a^2B-b^2B-2Cab)\ln(\tan(dx+c))}{d}$
risch	$-\frac{2iB b^2 c}{d}-\frac{2ia\left(2Bb\,e^{2i(dx+c)}+Ca\,e^{2i(dx+c)}+iBa\,e^{2i(dx+c)}-2Bb-Ca\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-iB b^2 x-2Babx-C a^2 x+C$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(b^2*B*ln(sin(d*x+c))+b^2*C*(d*x+c)+2*B*a*b*(-cot(d*x+c)-d*x-c)+2*C*a*b*ln(sin(d*x+c))+a^2*B*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+C*a^2*(-cot(d*x+c)-d*x-c))

Maxima [A]
time = 0.54, size = 120, normalized size = 1.36

$$-\frac{2\,(Ca^2+2\,Bab-Cb^2)(dx+c)-(Ba^2-2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)+2\,(Ba^2-2\,Cab-Bb^2)\log\left(\tan(dx+c)\right)+\frac{Ba^2+2\,(Ca^2+2\,Bab)\tan(dx+c)}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="maxima")

[Out] -1/2*(2*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1) + 2*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)) + (B*a^2 + 2*(C*a^2 + 2*B*a*b)*tan(d*x + c))/tan(d*x + c)^2)/d

Fricas [A]
time = 3.35, size = 122, normalized size = 1.39

$$-\frac{(Ba^2-2\,Cab-Bb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+Ba^2+(Ba^2+2\,(Ca^2+2\,Bab-Cb^2)dx)\tan(dx+c)^2+2\,(Ca^2+2\,Bab)\tan(dx+c)}{2\,d\tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/2*((B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*ta
n(d*x + c)^2 + B*a^2 + (B*a^2 + 2*(C*a^2 + 2*B*a*b - C*b^2)*d*x)*tan(d*x +
c)^2 + 2*(C*a^2 + 2*B*a*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(78) = 156$.
time = 2.21, size = 212, normalized size = 2.41

$$\begin{cases} \text{NaN} & \text{for}\: (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))^2 (B\tan(c) + C\tan^2(c))\cot^4(c) & \text{for}\: d = 0 \\ \frac{Ba^2\log(\tan^2(c+dx)+1)}{2d} - \frac{Ba^2\log(\tan(c+dx))}{d} - \frac{Ba^2}{2d\tan^2(c+dx)} - 2Babx - \frac{2Bab}{d\tan(c+dx)} - \frac{Bb^2\log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^2\log(\tan(c+dx))}{d} - Ca^2x - \frac{Ca^2}{d\tan(c+dx)} - \frac{Cab\log(\tan^2(c+dx)+1)}{d} + \frac{2Cab\log(\tan(c+dx))}{d} + Cb^2x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**4, Eq(d, 0)), (B*a**2*log(
tan(c + d*x)**2 + 1)/(2*d) - B*a**2*log(tan(c + d*x))/d - B*a**2/(2*d*tan(c
 + d*x)**2) - 2*B*a*b*x - 2*B*a*b/(d*tan(c + d*x)) - B*b**2*log(tan(c + d*x
)**2 + 1)/(2*d) + B*b**2*log(tan(c + d*x))/d - C*a**2*x - C*a**2/(d*tan(c +
 d*x)) - C*a*b*log(tan(c + d*x)**2 + 1)/d + 2*C*a*b*log(tan(c + d*x))/d + C
*b**2*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(86) = 172$.
time = 1.00, size = 237, normalized size = 2.69

$$-\frac{Ba^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 4Ca^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 8Bab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 8(Ca^2 + 2Bab - Cb^2)(dx + c) - 8(Ba^2 - 2Cab - Bb^2)\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right) + 8(Ba^2 - 2Cab - Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - \frac{12Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 24Cab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 12Bb^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 4Ca^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 8Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - Ba^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/8*(B*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*C*a^2*tan(1/2*d*x + 1/2*c) - 8*B*a*b
*tan(1/2*d*x + 1/2*c) + 8*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - 8*(B*a^2 -
2*C*a*b - B*b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 8*(B*a^2 - 2*C*a*b - B*b
^2)*log(abs(tan(1/2*d*x + 1/2*c))) - (12*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*
C*a*b*tan(1/2*d*x + 1/2*c)^2 - 12*B*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*C*a^2*ta
n(1/2*d*x + 1/2*c) - 8*B*a*b*tan(1/2*d*x + 1/2*c) - B*a^2)/tan(1/2*d*x + 1/
2*c)^2)/d
```

Mupad [B]
time = 8.98, size = 127, normalized size = 1.44

$$\frac{\ln(\tan(c + d\,x))\,(-B\,a^2 + 2\,C\,a\,b + B\,b^2)}{d} - \frac{\cot(c + d\,x)^2\,\left(\frac{B\,a^2}{2} + \tan(c + d\,x)\,(C\,a^2 + 2\,B\,b\,a)\right)}{d} - \frac{\ln(\tan(c + d\,x) + 1\mathrm{i})\,(B - C\,1\mathrm{i})\,(b + a\,1\mathrm{i})^2}{2\,d} - \frac{\ln(\tan(c + d\,x) - \mathrm{i})\,(B + C\,1\mathrm{i})\,(-b + a\,1\mathrm{i})^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^2,x)
```

```
[Out] (log(tan(c + d*x))*(B*b^2 - B*a^2 + 2*C*a*b))/d - (cot(c + d*x)^2*((B*a^2)/
2 + tan(c + d*x)*(C*a^2 + 2*B*a*b)))/d - (log(tan(c + d*x) + 1i)*(B - C*1i)
*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x) - 1i)*(B + C*1i)*(a*1i - b)^2)/(2*
d)
```

3.15 $\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=118

$$\left(a^2B-b^2B-2abC\right)x+\frac{\left(a^2B-b^2B-2abC\right)\cot(c+dx)}{d}-\frac{a(2bB+aC)\cot^2(c+dx)}{2d}-\frac{a^2B\cot^3(c+dx)}{3d}+\frac{(}{}$$

```
[Out] (B*a^2-B*b^2-2*C*a*b)*x+(B*a^2-B*b^2-2*C*a*b)*cot(d*x+c)/d-1/2*a*(2*B*b+C*a
)*cot(d*x+c)^2/d-1/3*a^2*B*cot(d*x+c)^3/d+(b^2*C-a*(2*B*b+C*a))*ln(sin(d*x+
c))/d
```

Rubi [A]
time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {3713, 3685, 3709, 3610, 3612, 3556}

$$\frac{\left(a^2B-2abC-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot^3(c+dx)}{3d}+\frac{\left(b^2C-a(aC+2bB)\right)\log(\sin(c+dx))}{d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (a^2*B - b^2*B - 2*a*b*C)*x + ((a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x])/d -
(a*(2*b*B + a*C)*Cot[c + d*x]^2)/(2*d) - (a^2*B*Cot[c + d*x]^3)/(3*d) + ((b
^2*C - a*(2*b*B + a*C))*Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
 + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*
c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^4(c+dx)(a+b\tan(c+d$$

$$= -\frac{a^2B\cot^3(c+dx)}{3d} + \int \cot^3(c$$

$$= -\frac{a(2bB+aC)\cot^2(c+dx)}{2d} -$$

$$= \frac{(a^2B-b^2B-2abC)\cot(c+d}{d}$$

$$= \left(a^2B-b^2B-2abC\right)x + \frac{(a^2B}{}$$

$$= \left(a^2B-b^2B-2abC\right)x + \frac{(a^2B}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.78, size = 152, normalized size = 1.29

$$\frac{6(a^2B-b^2B-2abC)\cot(c+dx)-3a(2bB+aC)\cot^2(c+dx)-2a^2B\cot^3(c+dx)+3(a+ib)^2(-iB+C)\log(i-\tan(c+dx))-6(2abB+a^2C-b^2C)\log(\tan(c+dx))+3(a-ib)^2(iB+C)\log(i+\tan(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (6*(a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x] - 3*a*(2*b*B + a*C)*Cot[c + d*x]^
2 - 2*a^2*B*Cot[c + d*x]^3 + 3*(a + I*b)^2*((-I)*B + C)*Log[I - Tan[c + d*x
]] - 6*(2*a*b*B + a^2*C - b^2*C)*Log[Tan[c + d*x]] + 3*(a - I*b)^2*(I*B + C
)*Log[I + Tan[c + d*x]])/(6*d)
```

Maple [A]
time = 0.27, size = 136, normalized size = 1.15

method	result
derivativedivides	$\frac{b^2B(-\cot(dx+c)-dx-c)+b^2C\ln(\sin(dx+c))+2Bab\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2Cab(-\cot(dx+c)-dx-c)+a^2\cdots}{d}$
default	$\frac{b^2B(-\cot(dx+c)-dx-c)+b^2C\ln(\sin(dx+c))+2Bab\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2Cab(-\cot(dx+c)-dx-c)+a^2\cdots}{d}$
norman	$\frac{\frac{\left(a^2B-b^2B-2Cab\right)\left(\tan^3(dx+c)\right)}{d}+\left(a^2B-b^2B-2Cab\right)x\left(\tan^4(dx+c)\right)-\frac{a(2Bb+Ca)\left(\tan^2(dx+c)\right)}{2d}-\frac{a^2B\tan(dx+c)}{3d}}{\tan(dx+c)^4}-\frac{(2Bab\cdots}{}$
risch	$B\,a^2x-B\,b^2x-2Cabx-\frac{2iC\,b^2c}{d}+\frac{2iC\,a^2c}{d}-iC\,b^2x+\frac{4iBabc}{d}+2iBabx-\frac{2i\left(6iBab\,e^{4i(dx+c)}+\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*B*(-cot(d*x+c)-d*x-c)+b^2*C*ln(sin(d*x+c))+2*B*a*b*(-1/2*cot(d*x+c
)^2-ln(sin(d*x+c)))+2*C*a*b*(-cot(d*x+c)-d*x-c)+a^2*B*(-1/3*cot(d*x+c)^3+co
t(d*x+c)+d*x+c)+C*a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.51, size = 149, normalized size = 1.26

$$\frac{6\,(Ba^2-2\,Cab-Bb^2)(dx+c)+3\,(Ca^2+2\,Bab-Cb^2)\log\left(\tan\left(dx+c\right)^2+1\right)-6\,(Ca^2+2\,Bab-Cb^2)\log\left(\tan\left(dx+c\right)\right)-\frac{2\,Ba^2-6\,(Ba^2-2\,Cab-Bb^2)\tan(dx+c)^2+3\,(Ca^2+2\,Bab)\tan(dx+c)}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/6*(6*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + 3*(C*a^2 + 2*B*a*b - C*b^2)*lo
g(tan(d*x + c)^2 + 1) - 6*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)) - (2*
B*a^2 - 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 + 3*(C*a^2 + 2*B*a*b)*ta
n(d*x + c))/tan(d*x + c)^3)/d
```

Fricas [A]
time = 4.87, size = 157, normalized size = 1.33

$$-\frac{3\,(Ca^2+2\,Bab-Cb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3+3\,(Ca^2+2\,Bab-2\,(Ba^2-2\,Cab-Bb^2)dx)\tan\left(dx+c\right)^3+2\,Ba^2-6\,(Ba^2-2\,Cab-Bb^2)\tan\left(dx+c\right)^2+3\,(Ca^2+2\,Bab)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/6*(3*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*
tan(d*x + c)^3 + 3*(C*a^2 + 2*B*a*b - 2*(B*a^2 - 2*C*a*b - B*b^2)*d*x)*tan(
d*x + c)^3 + 2*B*a^2 - 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 + 3*(C*a^
2 + 2*B*a*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(107) = 214$.
time = 3.05, size = 258, normalized size = 2.19

$$\begin{cases} \text{NaN} & \text{for}\: \left(c = 0 \vee c = - dx\right) \wedge \left(c = - dx \vee d = 0\right) \\ x \left(a + b \tan{\left(c \right)}\right)^{2} \left(B \tan{\left(c \right)} + C \tan^{2}{\left(c \right)}\right) \cot^{5}{\left(c \right)} & \text{for}\: d = 0 \\ Ba^{2} x + \frac{Ba^{2}}{d \tan{\left(c + dx \right)}} - \frac{Ba^{2}}{3 d \tan^{3}{\left(c + dx \right)}} + \frac{Bab \log{\left(\tan^{2}{\left(c + dx \right)} + 1 \right)}}{d} - \frac{2 Bab \log{\left(\tan{\left(c + dx \right)} \right)}}{d} - \frac{Bab}{d \tan^{2}{\left(c + dx \right)}} - Bb^{2} x - \frac{Bb^{2}}{d \tan{\left(c + dx \right)}} + \frac{Ca^{2} \log{\left(\tan^{2}{\left(c + dx \right)} + 1 \right)}}{2 d} - \frac{Ca^{2} \log{\left(\tan{\left(c + dx \right)} \right)}}{d} - \frac{Ca^{2}}{2 d \tan^{2}{\left(c + dx \right)}} - 2 Cabx - \frac{2 Cab}{d \tan{\left(c + dx \right)}} - \frac{Cb^{2} \log{\left(\tan^{2}{\left(c + dx \right)} + 1 \right)}}{2 d} + \frac{Cb^{2} \log{\left(\tan{\left(c + dx \right)} \right)}}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**5, Eq(d, 0)), (B*a**2*x +
B*a**2/(d*tan(c + d*x)) - B*a**2/(3*d*tan(c + d*x)**3) + B*a*b*log(tan(c +
d*x)**2 + 1)/d - 2*B*a*b*log(tan(c + d*x))/d - B*a*b/(d*tan(c + d*x)**2) -
B*b**2*x - B*b**2/(d*tan(c + d*x)) + C*a**2*log(tan(c + d*x)**2 + 1)/(2*d)
- C*a**2*log(tan(c + d*x))/d - C*a**2/(2*d*tan(c + d*x)**2) - 2*C*a*b*x - 2
*C*a*b/(d*tan(c + d*x)) - C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*lo
g(tan(c + d*x))/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(114) = 228$.
time = 1.06, size = 334, normalized size = 2.83

$$\frac{Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Cab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,(Ba^2-2\,Cab-Bb^2)(dx+c)+24\,(Ca^2+2\,Bab-Cb^2)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-24\,(Ca^2+2\,Bab-Cb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{44\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+88\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-44\,Cb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,Cab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ba^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(B*a^2*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*B*a
*b*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^2*tan(1/2*d*x + 1/2*c) + 24*C*a*b*tan(1/
2*d*x + 1/2*c) + 12*B*b^2*tan(1/2*d*x + 1/2*c) + 24*(B*a^2 - 2*C*a*b - B*b^
2)*(d*x + c) + 24*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1)
 - 24*(C*a^2 + 2*B*a*b - C*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C*a^2*
tan(1/2*d*x + 1/2*c)^3 + 88*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 44*C*b^2*tan(1/2
*d*x + 1/2*c)^3 + 15*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*C*a*b*tan(1/2*d*x +
1/2*c)^2 - 12*B*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*C*a^2*tan(1/2*d*x + 1/2*c) -
 6*B*a*b*tan(1/2*d*x + 1/2*c) - B*a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 9.08, size = 156, normalized size = 1.32

$$-\frac{\cot(c+d\,x)^3\,\left(\frac{B\,a^2}{3}+\tan(c+d\,x)^2\,\left(-B\,a^2+2\,C\,a\,b+B\,b^2\right)+\tan(c+d\,x)\,\left(\frac{C\,a^2}{2}+B\,b\,a\right)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(C\,a^2+2\,B\,a\,b-C\,b^2\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2}{2\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^2,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(B*1i - C)*(a*1i - b)^2)/(2*d) - (log(tan(c + d*x))
*(C*a^2 - C*b^2 + 2*B*a*b))/d - (cot(c + d*x)^3*((B*a^2)/3 + tan(c + d*x)^2
*(B*b^2 - B*a^2 + 2*C*a*b) + tan(c + d*x)*((C*a^2)/2 + B*a*b)))/d - (log(ta
n(c + d*x) + 1i)*(B*1i + C)*(a*1i + b)^2)/(2*d)
```

3.16 $\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=151

$$\left(2abB+a^2C-b^2C\right)x-\frac{\left(b^2C-a(2bB+aC)\right)\cot(c+dx)}{d}+\frac{\left(a^2B-b^2B-2abC\right)\cot^2(c+dx)}{2d}-\frac{a(2bB+a}{}$$

```
[Out] (2*B*a*b+C*a^2-C*b^2)*x-(b^2*C-a*(2*B*b+C*a))*cot(d*x+c)/d+1/2*(B*a^2-B*b^2
-2*C*a*b)*cot(d*x+c)^2/d-1/3*a*(2*B*b+C*a)*cot(d*x+c)^3/d-1/4*a^2*B*cot(d*x
+c)^4/d+(B*a^2-B*b^2-2*C*a*b)*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.26, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3713, 3685, 3709, 3610, 3612, 3556}

$$\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B-2abC-b^2B\right)\log(\sin(c+dx))}{d}+x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{\left(b^2C-a(aC+2bB)\right)\cot(c+dx)}{d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (2*a*b*B + a^2*C - b^2*C)*x - ((b^2*C - a*(2*b*B + a*C))*Cot[c + d*x])/d +
((a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x]^2)/(2*d) - (a*(2*b*B + a*C)*Cot[c +
 d*x]^3)/(3*d) - (a^2*B*Cot[c + d*x]^4)/(4*d) + ((a^2*B - b^2*B - 2*a*b*C)*
Log[Sin[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
 + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*
c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^5(c+dx)(a+b\tan(c+d$$
$$= -\frac{a^2 B\cot^4(c+dx)}{4d} + \int \cot^4(c$$
$$= -\frac{a(2bB+aC)\cot^3(c+dx)}{3d} -$$
$$= \frac{(a^2B-b^2B-2abC)\cot^2(c+$$
$$= -\frac{(b^2C-a(2bB+aC))\cot(c+}{d}$$
$$= \left(2abB+a^2C-b^2C\right)x - \frac{(b^2C}{}$$
$$= \left(2abB+a^2C-b^2C\right)x - \frac{(b^2C}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.90, size = 180, normalized size = 1.19

$$\frac{12(2abB+a^2C-b^2C)\cot(c+dx)+6(a^2B-b^2B-2abC)\cot^2(c+dx)-4a(2bB+aC)\cot^3(c+dx)-3a^2B\cot^4(c+dx)-6((a+ib)^2(B+iC)\log(i-\tan(c+dx))+(-2a^2B+2b^2B+4abC)\log(\tan(c+dx))+(a-ib)^2(B-iC)\log(i+\tan(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (12*(2*a*b*B + a^2*C - b^2*C)*Cot[c + d*x] + 6*(a^2*B - b^2*B - 2*a*b*C)*Co
t[c + d*x]^2 - 4*a*(2*b*B + a*C)*Cot[c + d*x]^3 - 3*a^2*B*Cot[c + d*x]^4 -
6*((a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] + (-2*a^2*B + 2*b^2*B + 4*a*
b*C)*Log[Tan[c + d*x]] + (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]]))/(12*
d)
```

Maple [A]
time = 0.30, size = 162, normalized size = 1.07

method	result
derivativedivides	$\frac{b^2B\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b^2C(-\cot(dx+c)-dx-c)+2Bab\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2Cab}{}$
default	$\frac{b^2B\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b^2C(-\cot(dx+c)-dx-c)+2Bab\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2Cab}{}$

norman	$\frac{\frac{\left(2Bab+C\,a^{2}-b^{2}C\right)\left(\tan^{4}(dx+c)\right)}{d}+\left(2Bab+C\,a^{2}-b^{2}C\right)x\left(\tan^{5}(dx+c)\right)+\frac{\left(a^{2}B-b^{2}B-2Cab\right)\left(\tan^{3}(dx+c)\right)}{2d}-\frac{a(2Bb+Ca)\left(\tan^{2}(d\right.}{3d}}{\tan(dx+c)^{5}}$
risch	$\frac{2iB\,b^{2}c}{d}+iB\,b^{2}x-\frac{2iB\,a^{2}c}{d}+2Babx+C\,a^{2}x-C\,b^{2}x-iB\,a^{2}x+\frac{4iCabc}{d}+2iCabx+\frac{-8iC\,a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*B*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+b^2*C*(-cot(d*x+c)-d*x-c)+2*B
*a*b*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*C*a*b*(-1/2*cot(d*x+c)^2-ln(sin
(d*x+c)))+a^2*B*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+C*a^2*(
-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.53, size = 175, normalized size = 1.16

$$\frac{12\,(Ca^2+2\,Bab-Cb^2)(dx+c)-6\,(Ba^2-2\,Cab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)+12\,(Ba^2-2\,Cab-Bb^2)\log\left(\tan\left(dx+c\right)\right)+\frac{12\,(Ca^2+2\,Bab-Cb^2)\tan(dx+c)^3-3\,Ba^2+6\,(Ba^2-2\,Cab-Bb^2)\tan(dx+c)^2-4\,(Ca^2+2\,Bab)\tan(dx+c)}{\tan(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/12*(12*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - 6*(B*a^2 - 2*C*a*b - B*b^2)*
log(tan(d*x + c)^2 + 1) + 12*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)) +
(12*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^3 - 3*B*a^2 + 6*(B*a^2 - 2*C*a*b
 - B*b^2)*tan(d*x + c)^2 - 4*(C*a^2 + 2*B*a*b)*tan(d*x + c))/tan(d*x + c)^4
)/d
```

Fricas [A]
time = 3.47, size = 191, normalized size = 1.26

$$\frac{6\,(Ba^2-2\,Cab-Bb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\,(3\,Ba^2-4\,Cab-2\,Bb^2+4\,(Ca^2+2\,Bab-Cb^2)dx)\tan\left(dx+c\right)^4+12\,(Ca^2+2\,Bab-Cb^2)\tan\left(dx+c\right)^3-3\,Ba^2+6\,(Ba^2-2\,Cab-Bb^2)\tan\left(dx+c\right)^2-4\,(Ca^2+2\,Bab)\tan\left(dx+c\right)}{12\,d\tan\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/12*(6*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*
tan(d*x + c)^4 + 3*(3*B*a^2 - 4*C*a*b - 2*B*b^2 + 4*(C*a^2 + 2*B*a*b - C*b^
2)*d*x)*tan(d*x + c)^4 + 12*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^3 - 3*B*
a^2 + 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 - 4*(C*a^2 + 2*B*a*b)*tan(
d*x + c))/(d*tan(d*x + c)^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(136) = 272$.
time = 5.02, size = 311, normalized size = 2.06

$$\begin{cases} \mathrm{NaN} & \text{for}\ (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a + b\tan(c))^2 (B\tan(c) + C\tan^2(c))\cot^6(c) & \text{for}\ d = 0 \\ -\frac{Ba^2\log(\tan^2(c+dx)+1)}{2d} + \frac{Ba^2\log(\tan(c+dx))}{d} + \frac{Ba^2}{2d\tan^2(c+dx)} - \frac{Ba^2}{4d\tan^4(c+dx)} + 2Babx + \frac{2Bab}{d\tan(c+dx)} - \frac{2Bab}{3d\tan^3(c+dx)} + \frac{Bb^2\log(\tan^2(c+dx)+1)}{2d} - \frac{Bb^2\log(\tan(c+dx))}{d} - \frac{Bb^2}{2d\tan^2(c+dx)} + Ca^2x + \frac{Ca^2}{d\tan(c+dx)} - \frac{Ca^2}{3d\tan^3(c+dx)} + \frac{Cab\log(\tan^2(c+dx)+1)}{d} - \frac{2Cab\log(\tan(c+dx))}{d} - \frac{Cab}{d\tan^2(c+dx)} - Cb^2x - \frac{Cb^2}{d\tan(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**6, Eq(d, 0)), (-B*a**2*log
(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*log(tan(c + d*x))/d + B*a**2/(2*d*tan(
c + d*x)**2) - B*a**2/(4*d*tan(c + d*x)**4) + 2*B*a*b*x + 2*B*a*b/(d*tan(c
+ d*x)) - 2*B*a*b/(3*d*tan(c + d*x)**3) + B*b**2*log(tan(c + d*x)**2 + 1)/(
2*d) - B*b**2*log(tan(c + d*x))/d - B*b**2/(2*d*tan(c + d*x)**2) + C*a**2*x
 + C*a**2/(d*tan(c + d*x)) - C*a**2/(3*d*tan(c + d*x)**3) + C*a*b*log(tan(c
 + d*x)**2 + 1)/d - 2*C*a*b*log(tan(c + d*x))/d - C*a*b/(d*tan(c + d*x)**2)
 - C*b**2*x - C*b**2/(d*tan(c + d*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(145) = 290$.
time = 1.10, size = 435, normalized size = 2.88

$$-\frac{3Ba^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 8Ca^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 16Bab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 36Ba^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 48Cab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 24Bb^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 120Ca^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 240Bab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 96Cb^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 192(Ca^2 + 2Bab - Cb^2)(dx + c) + 192(Ba^2 - 2Cab - Bb^2)\log\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right) - 192(Ba^2 - 2Cab - Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + \frac{400Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 800Cab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 400Bb^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 120Ca^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 240Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 96Cb^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 36Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 48Cab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 24Bb^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 8Ca^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 16Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3Ba^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/192*(3*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 8*C*a^2*tan(1/2*d*x + 1/2*c)^3 - 1
6*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*tan(1/2*d*x + 1/2*c)^2 + 48*C*a*b
*tan(1/2*d*x + 1/2*c)^2 + 24*B*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*C*a^2*tan(1
/2*d*x + 1/2*c) + 240*B*a*b*tan(1/2*d*x + 1/2*c) - 96*C*b^2*tan(1/2*d*x + 1
/2*c) - 192*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) + 192*(B*a^2 - 2*C*a*b - B*
b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(B*a^2 - 2*C*a*b - B*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c))) + (400*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 800*C*a*b*ta
n(1/2*d*x + 1/2*c)^4 - 400*B*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*C*a^2*tan(1/2
*d*x + 1/2*c)^3 - 240*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 96*C*b^2*tan(1/2*d*x +
 1/2*c)^3 - 36*B*a^2*tan(1/2*d*x + 1/2*c)^2 + 48*C*a*b*tan(1/2*d*x + 1/2*c)
^2 + 24*B*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*C*a^2*tan(1/2*d*x + 1/2*c) + 16*B*
a*b*tan(1/2*d*x + 1/2*c) + 3*B*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.86, size = 182, normalized size = 1.21

$$-\frac{\cot(c+d\,x)^4\left(\frac{B\,a^2}{4} + \tan(c+d\,x)^2\left(-\frac{B\,a^2}{2} + C\,a\,b + \frac{B\,b^2}{2}\right) - \tan(c+d\,x)^3\,(C\,a^2 + 2\,B\,a\,b - C\,b^2) + \tan(c+d\,x)\,\left(\frac{C\,a^2}{3} + \frac{2\,B\,b\,a}{3}\right)\right)}{d} - \frac{\ln(\tan(c+d\,x))\,(-B\,a^2 + 2\,C\,a\,b + B\,b^2)}{d} + \frac{\ln(\tan(c+d\,x) + 1\mathrm{i})\,(B - C\,1\mathrm{i})\,(b + a\,1\mathrm{i})^2}{2\,d} + \frac{\ln(\tan(c+d\,x) - \mathrm{i})\,(B + C\,1\mathrm{i})\,(-b + a\,1\mathrm{i})^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x))
*(B*b^2 - B*a^2 + 2*C*a*b))/d - (cot(c + d*x)^4*((B*a^2)/4 + tan(c + d*x)^2
*((B*b^2)/2 - (B*a^2)/2 + C*a*b) - tan(c + d*x)^3*(C*a^2 - C*b^2 + 2*B*a*b)
 + tan(c + d*x)*((C*a^2)/3 + (2*B*a*b)/3)))/d + (log(tan(c + d*x) - 1i)*(B
+ C*1i)*(a*1i - b)^2)/(2*d)
```

3.17 $\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)$

Optimal. Leaf size=165

$$-\left(\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x\right)-\frac{\left(a^3B-3ab^2B-3a^2bC+b^3C\right)\log(\cos(c+dx))}{d}+\frac{b(a^2B-b^2B-2a}{d}$$

```
[Out] -(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*x-(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*ln(c
os(d*x+c))/d+b*(B*a^2-B*b^2-2*C*a*b)*tan(d*x+c)/d+1/2*(B*a-C*b)*(a+b*tan(d*
x+c))^2/d+1/3*B*(a+b*tan(d*x+c))^3/d+1/4*C*(a+b*tan(d*x+c))^4/b/d
```

Rubi [A]
time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3711, 3609, 3606, 3556}

$$\frac{b(a^2B-2abC-b^2B)\tan(c+dx)}{d}-\frac{(a^3B-3a^2bC-3ab^2B+b^3C)\log(\cos(c+dx))}{d}-x(a^3C+3a^2bB-3ab^2C-b^3B)+\frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] -((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x) - ((a^3*B - 3*a*b^2*B - 3*a^2*
b*C + b^3*C)*Log[Cos[c + d*x]])/d + (b*(a^2*B - b^2*B - 2*a*b*C)*Tan[c + d*
x])/d + ((a*B - b*C)*(a + b*Tan[c + d*x])^2)/(2*d) + (B*(a + b*Tan[c + d*x]
)^3)/(3*d) + (C*(a + b*Tan[c + d*x])^4)/(4*b*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx &= \frac{C(a+b\tan(c+dx))^4}{4bd}+\int (a+b\tan(c+dx) \\
&= \frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx)}{4bd} \\
&= \frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan}{3} \\
&= -\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x+\frac{b(a^2B}{} \\
&= -\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x-\frac{(a^3B-}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.10, size = 209, normalized size = 1.27

$$\frac{-6i(a+ib)^4B\log(i-\tan(c+dx))+6i(a-ib)^4B\log(i+\tan(c+dx))-12b^2(-6a^2+b^2)\,B\tan(c+dx)+24ab^3B\tan^2(c+dx)+4b^4B\tan^3(c+dx)+3C(a+b\tan(c+dx))^4-6(aB+bC)\left((ia-b)^3\log(i-\tan(c+dx))-(ia+b)^3\log(i+\tan(c+dx))+6ab^2\tan(c+dx)+b^3\tan^2(c+dx)\right)}{12bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

```
[Out] ((-6*I)*(a + I*b)^4*B*Log[I - Tan[c + d*x]] + (6*I)*(a - I*b)^4*B*Log[I + T
an[c + d*x]] - 12*b^2*(-6*a^2 + b^2)*B*Tan[c + d*x] + 24*a*b^3*B*Tan[c + d*
x]^2 + 4*b^4*B*Tan[c + d*x]^3 + 3*C*(a + b*Tan[c + d*x])^4 - 6*(a*B + b*C)*
((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*
a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(12*b*d)
```

Maple [A]
time = 0.09, size = 213, normalized size = 1.29

method	result
norman	$\left(-3B\,a^2b+B\,b^3-C\,a^3+3Ca\,b^2\right)x+\frac{\left(3B\,a^2b-B\,b^3+C\,a^3-3Ca\,b^2\right)\tan(dx+c)}{d}+\frac{C\,b^3\left(\tan^4(dx+c)\right)}{4d}+$
derivativedivides	$\frac{C\,b^3\left(\tan^4(dx+c)\right)}{4}+\frac{B\,b^3\left(\tan^3(dx+c)\right)}{3}+Ca\,b^2\left(\tan^3(dx+c)\right)+\frac{3Ba\,b^2\left(\tan^2(dx+c)\right)}{2}+\frac{3C\,a^2b\left(\tan^2(dx+c)\right)}{2}-\frac{C\,b^3\left(\tan^2(dx+c)\right)}{2}+$

default	$\frac{\frac{C\,b^{3}\left(\tan^{4}(dx+c)\right)}{4}+\frac{B\,b^{3}\left(\tan^{3}(dx+c)\right)}{3}+C a\,b^{2}\left(\tan^{3}(dx+c)\right)+\frac{3B a\,b^{2}\left(\tan^{2}(dx+c)\right)}{2}+\frac{3C\,a^{2}b\left(\tan^{2}(dx+c)\right)}{2}-\frac{C\,b^{3}\left(\tan^{2}(dx+c)\right)}{2}}{}$
risch	$\frac{2iB\,a^{3}c}{d}-3iBa\,b^{2}x+iB\,a^{3}x+\frac{2iC\,b^{3}c}{d}-3B\,a^{2}bx+B\,b^{3}x-C\,a^{3}x+3Ca\,b^{2}x+\frac{2i(-12Ca}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/4*C*b^3*tan(d*x+c)^4+1/3*B*b^3*tan(d*x+c)^3+C*a*b^2*tan(d*x+c)^3+3/2
*B*a*b^2*tan(d*x+c)^2+3/2*C*a^2*b*tan(d*x+c)^2-1/2*C*b^3*tan(d*x+c)^2+3*B*a
^2*b*tan(d*x+c)-B*b^3*tan(d*x+c)+C*a^3*tan(d*x+c)-3*C*a*b^2*tan(d*x+c)+1/2*
(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*ln(1+tan(d*x+c)^2)+(-3*B*a^2*b+B*b^3-C*a^
3+3*C*a*b^2)*arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 179, normalized size = 1.08

$$\frac{3\,Cb^{3}\tan\left(dx+c\right)^{4}+4\,\left(3\,Cab^{2}+Bb^{3}\right)\tan\left(dx+c\right)^{3}+6\,\left(3\,Ca^{2}b+3\,Bab^{2}-Cb^{3}\right)\tan\left(dx+c\right)^{2}-12\,\left(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3}\right)\left(dx+c\right)+6\,\left(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2}+Cb^{3}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)+12\,\left(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3}\right)\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] 1/12*(3*C*b^3*tan(d*x + c)^4 + 4*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^3 + 6*(3*
C*a^2*b + 3*B*a*b^2 - C*b^3)*tan(d*x + c)^2 - 12*(C*a^3 + 3*B*a^2*b - 3*C*a
*b^2 - B*b^3)*(d*x + c) + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan
(d*x + c)^2 + 1) + 12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c))
/d
```

Fricas [A]
time = 4.13, size = 178, normalized size = 1.08

$$\frac{3\,Cb^{3}\tan\left(dx+c\right)^{4}+4\,\left(3\,Cab^{2}+Bb^{3}\right)\tan\left(dx+c\right)^{3}-12\,\left(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3}\right)dx+6\,\left(3\,Ca^{2}b+3\,Bab^{2}-Cb^{3}\right)\tan\left(dx+c\right)^{2}-6\,\left(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2}+Cb^{3}\right)\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)+12\,\left(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3}\right)\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*C*b^3*tan(d*x + c)^4 + 4*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^3 - 12*(C
*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x + 6*(3*C*a^2*b + 3*B*a*b^2 - C*b^
3)*tan(d*x + c)^2 - 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(1/(tan(d*
x + c)^2 + 1)) + 12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(151) = 302$.
time = 0.16, size = 313, normalized size = 1.90

$$\begin{cases} \frac{Ba^3\log(\tan^2(c+dx)+1)}{2d} - 3Ba^2bx + \frac{3Ba^2b\tan(c+dx)}{d} - \frac{3Bab^2\log(\tan^2(c+dx)+1)}{2d} + \frac{3Bab^2\tan^2(c+dx)}{2d} + Bb^3x + \frac{Bb^3\tan^3(c+dx)}{3d} - \frac{Bb^3\tan(c+dx)}{d} - Ca^3x + \frac{Ca^3\tan(c+dx)}{d} - \frac{3Ca^2b\log(\tan^2(c+dx)+1)}{2d} + \frac{3Ca^2b\tan^2(c+dx)}{2d} + 3Cab^2x + \frac{Cab^2\tan^3(c+dx)}{d} - \frac{3Cab^2\tan(c+dx)}{d} + \frac{Cb^3\log(\tan^2(c+dx)+1)}{2d} + \frac{Cb^3\tan^4(c+dx)}{4d} - \frac{Cb^3\tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a+b\tan(c))^3(B\tan(c)+C\tan^2(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*B*a**2*b*x + 3*B*a**2*
b*tan(c + d*x)/d - 3*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*a*b**2*t
an(c + d*x)**2/(2*d) + B*b**3*x + B*b**3*tan(c + d*x)**3/(3*d) - B*b**3*tan
(c + d*x)/d - C*a**3*x + C*a**3*tan(c + d*x)/d - 3*C*a**2*b*log(tan(c + d*x
)**2 + 1)/(2*d) + 3*C*a**2*b*tan(c + d*x)**2/(2*d) + 3*C*a*b**2*x + C*a*b**
2*tan(c + d*x)**3/d - 3*C*a*b**2*tan(c + d*x)/d + C*b**3*log(tan(c + d*x)**
2 + 1)/(2*d) + C*b**3*tan(c + d*x)**4/(4*d) - C*b**3*tan(c + d*x)**2/(2*d),
 Ne(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*tan(c)**2), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2870 vs. $2(159) = 318$.
time = 2.47, size = 2870, normalized size = 17.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] -1/12*(12*C*a^3*d*x*tan(d*x)^4*tan(c)^4 + 36*B*a^2*b*d*x*tan(d*x)^4*tan(c)^
4 - 36*C*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 12*B*b^3*d*x*tan(d*x)^4*tan(c)^4 +
 6*B*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c
)^4 - 18*C*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x
)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)
^4*tan(c)^4 - 18*B*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
 tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*
tan(d*x)^4*tan(c)^4 + 6*C*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
 1))*tan(d*x)^4*tan(c)^4 - 48*C*a^3*d*x*tan(d*x)^3*tan(c)^3 - 144*B*a^2*b*d
*x*tan(d*x)^3*tan(c)^3 + 144*C*a*b^2*d*x*tan(d*x)^3*tan(c)^3 + 48*B*b^3*d*x
*tan(d*x)^3*tan(c)^3 - 18*C*a^2*b*tan(d*x)^4*tan(c)^4 - 18*B*a*b^2*tan(d*x)
^4*tan(c)^4 + 9*C*b^3*tan(d*x)^4*tan(c)^4 - 24*B*a^3*log(4*(tan(d*x)^4*tan(
c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*
tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 72*C*a^2*b*log(4*(tan(d*x
)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*t
```

```
an(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 72*B*a*b^2*log(4*
(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)
^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 24*C*b^3*
log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 12*
C*a^3*tan(d*x)^4*tan(c)^3 + 36*B*a^2*b*tan(d*x)^4*tan(c)^3 - 36*C*a*b^2*tan
(d*x)^4*tan(c)^3 - 12*B*b^3*tan(d*x)^4*tan(c)^3 + 12*C*a^3*tan(d*x)^3*tan(c
)^4 + 36*B*a^2*b*tan(d*x)^3*tan(c)^4 - 36*C*a*b^2*tan(d*x)^3*tan(c)^4 - 12*
B*b^3*tan(d*x)^3*tan(c)^4 + 72*C*a^3*d*x*tan(d*x)^2*tan(c)^2 + 216*B*a^2*b*
d*x*tan(d*x)^2*tan(c)^2 - 216*C*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 72*B*b^3*d*
x*tan(d*x)^2*tan(c)^2 - 18*C*a^2*b*tan(d*x)^4*tan(c)^2 - 18*B*a*b^2*tan(d*x
)^4*tan(c)^2 + 6*C*b^3*tan(d*x)^4*tan(c)^2 + 36*C*a^2*b*tan(d*x)^3*tan(c)^3
 + 36*B*a*b^2*tan(d*x)^3*tan(c)^3 - 24*C*b^3*tan(d*x)^3*tan(c)^3 - 18*C*a^2
*b*tan(d*x)^2*tan(c)^4 - 18*B*a*b^2*tan(d*x)^2*tan(c)^4 + 6*C*b^3*tan(d*x)^
2*tan(c)^4 + 12*C*a*b^2*tan(d*x)^4*tan(c) + 4*B*b^3*tan(d*x)^4*tan(c) + 36*
B*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2
 - 108*C*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2
*tan(c)^2 - 108*B*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*t
an(d*x)^2*tan(c)^2 + 36*C*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan
(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 +
 1))*tan(d*x)^2*tan(c)^2 - 36*C*a^3*tan(d*x)^3*tan(c)^2 - 108*B*a^2*b*tan(d
*x)^3*tan(c)^2 + 144*C*a*b^2*tan(d*x)^3*tan(c)^2 + 48*B*b^3*tan(d*x)^3*tan(
c)^2 - 36*C*a^3*tan(d*x)^2*tan(c)^3 - 108*B*a^2*b*tan(d*x)^2*tan(c)^3 + 144
*C*a*b^2*tan(d*x)^2*tan(c)^3 + 48*B*b^3*tan(d*x)^2*tan(c)^3 + 12*C*a*b^2*ta
n(d*x)*tan(c)^4 + 4*B*b^3*tan(d*x)*tan(c)^4 - 3*C*b^3*tan(d*x)^4 - 48*C*a^3
*d*x*tan(d*x)*tan(c) - 144*B*a^2*b*d*x*tan(d*x)*tan(c) + 144*C*a*b^2*d*x*ta
n(d*x)*tan(c) + 48*B*b^3*d*x*tan(d*x)*tan(c) + 36*C*a^2*b*tan(d*x)^3*tan(c)
 + 36*B*a*b^2*tan(d*x)^3*tan(c) - 24*C*b^3*tan(d*x)^3*tan(c) - 36*C*a^2*b*t
an(d*x)^2*tan(c)^2 - 36*B*a*b^2*tan(d*x)^2*tan(c)^2 + 12*C*b^3*tan(d*x)^2*t
an(c)^2 + 36*C*a^2*b*tan(d*x)*tan(c)^3 + 36*B*a*b^2*tan(d*x)*tan(c)^3 - 24*
C*b^3*tan(d*x)*tan(c)^3 - 3*C*b^3*tan(c)^4 - 12*C*a*b^2*tan(d*x)^3 - 4*B*b^
3*tan(d*x)^3 - 24*B*a^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) +
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*t
an(d*x)*tan(c) + 72*C*a^2*b*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)*tan(c) + 72*B*a*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
 + 1))*tan(d*x)*tan(c) - 24*C*b^3*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3
*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)
^2 + 1))*tan(d*x)*tan(c) + 36*C*a^3*tan(d*x)^2*tan(c) + 108*B*a^2*b*tan(d*x
)^2*tan(c) - 144*C*a*b^2*tan(d*x)^2*tan(c) - 48*B*b^3*tan(d*x)^2*tan(c) + 3
6*C*a^3*tan(d*x)*tan(c)^2 + 108*B*a^2*b*tan(d*x)*tan(c)^2 - 144*C*a*b^2*tan
```

```
(d*x)*tan(c)^2 - 48*B*b^3*tan(d*x)*tan(c)^2 - 12*C*a*b^2*tan(c)^3 - 4*B*b^3
*tan(c)^3 + 12*C*a^3*d*x + 36*B*a^2*b*d*x - 36*C*a*b^2*d*x - 12*B*b^3*d*x -
 18*C*a^2*b*tan(d*x)^2 - 18*B*a*b^2*tan(d*x)^2 + 6*C*b^3*tan(d*x)^2 + 36*C*
a^2*b*tan(d*x)*tan(c) + 36*B*a*b^2*tan(d*x)*tan(c) - 24*C*b^3*tan(d*x)*tan(
c) - 18*C*a^2*b*tan(c)^2 - 18*B*a*b^2*tan(c)^2 ...
```

Mupad [B]
time = 8.83, size = 181, normalized size = 1.10

$$x\left(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3\right)-\frac{\tan(c+d\,x)^2\left(\frac{C\,b^3}{2}-\frac{3\,a\,b\,(B\,b+C\,a)}{2}\right)}{d}-\frac{\tan(c+d\,x)\left(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3\right)}{d}+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\left(\frac{B\,a^3}{2}-\frac{3\,C\,a^2\,b}{2}-\frac{3\,B\,a\,b^2}{2}+\frac{C\,b^3}{2}\right)}{d}+\frac{\tan(c+d\,x)^3\left(\frac{B\,b^3}{3}+C\,a\,b^2\right)}{d}+\frac{C\,b^3\tan(c+d\,x)^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] x*(B*b^3 - C*a^3 - 3*B*a^2*b + 3*C*a*b^2) - (tan(c + d*x)^2*((C*b^3)/2 - (3
*a*b*(B*b + C*a))/2))/d - (tan(c + d*x)*(B*b^3 - C*a^3 - 3*B*a^2*b + 3*C*a*
b^2))/d + (log(tan(c + d*x)^2 + 1)*((B*a^3)/2 + (C*b^3)/2 - (3*B*a*b^2)/2 -
 (3*C*a^2*b)/2))/d + (tan(c + d*x)^3*((B*b^3)/3 + C*a*b^2))/d + (C*b^3*tan(
c + d*x)^4)/(4*d)
```

3.18 $\int \cot(c+dx)(a+b\tan(c+dx))^3\,(B\tan(c+dx)+C\tan$

Optimal. Leaf size=140

$$\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x-\frac{(3a^2bB-b^3B+a^3C-3ab^2C)\log(\cos(c+dx))}{d}+\frac{b(2abB+a^2C-b^2C)}{d}$$

```
[Out] (B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*x-(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*ln(co
s(d*x+c))/d+b*(2*B*a*b+C*a^2-C*b^2)*tan(d*x+c)/d+1/2*(B*b+C*a)*(a+b*tan(d*x
+c))^2/d+1/3*C*(a+b*tan(d*x+c))^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3713, 3609, 3606, 3556}

$$\frac{b(a^2C+2abB-b^2C)\tan(c+dx)}{d}-\frac{(a^3C+3a^2bB-3ab^2C-b^3B)\log(\cos(c+dx))}{d}+x(a^3B-3a^2bC-3ab^2B+b^3C)+\frac{(aC+bB)(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2)
,x]
```

```
[Out] (a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x - ((3*a^2*b*B - b^3*B + a^3*C - 3
*a*b^2*C)*Log[Cos[c + d*x]])/d + (b*(2*a*b*B + a^2*C - b^2*C)*Tan[c + d*x])
/d + ((b*B + a*C)*(a + b*Tan[c + d*x])^2)/(2*d) + (C*(a + b*Tan[c + d*x])^3
)/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx = \int (a+b\tan(c+dx))^3(B+C\tan$$

$$= \frac{C(a+b\tan(c+dx))^3}{3d} + \int (a+b$$

$$= \frac{(bB+aC)(a+b\tan(c+dx))^2}{2d} +$$

$$= \left(a^3B-3ab^2B-3a^2bC+b^3C\right)x$$

$$= \left(a^3B-3ab^2B-3a^2bC+b^3C\right)x$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.71, size = 130, normalized size = 0.93

$$\frac{3(a+ib)^3(-iB+C)\log(i-\tan(c+dx))+3(a-ib)^3(iB+C)\log(i+\tan(c+dx))+6b(3abB+3a^2C-b^2C)\tan(c+dx)+3b^2(bB+3aC)\tan^2(c+dx)+2b^3C\tan^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d
*x]^2),x]
```

```
[Out] (3*(a + I*b)^3*((-I)*B + C)*Log[I - Tan[c + d*x]] + 3*(a - I*b)^3*(I*B + C)
*Log[I + Tan[c + d*x]] + 6*b*(3*a*b*B + 3*a^2*C - b^2*C)*Tan[c + d*x] + 3*b
^2*(b*B + 3*a*C)*Tan[c + d*x]^2 + 2*b^3*C*Tan[c + d*x]^3)/(6*d)
```

Maple [A]
time = 0.29, size = 161, normalized size = 1.15

method	result
norman	$\left(B\,a^3-3Ba\,b^2-3C\,a^2b+C\,b^3\right)x+\frac{b\left(3Bab+3C\,a^2-b^2C\right)\tan(dx+c)}{d}+\frac{C\,b^3\left(\tan^3(dx+c)\right)}{3d}+\frac{b^2(Bb+3}{}$

derivativedivides	$\frac{B\,b^{3}\left(\frac{\left(\tan^{2}(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+C\,b^{3}\left(\frac{\left(\tan^{3}(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3Ba\,b^{2}(\tan(dx+c)-dx-c)+3Ca\,b^{2}\left(\right.}{d}$
default	$\frac{B\,b^{3}\left(\frac{\left(\tan^{2}(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+C\,b^{3}\left(\frac{\left(\tan^{3}(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3Ba\,b^{2}(\tan(dx+c)-dx-c)+3Ca\,b^{2}\left(\right.}{d}$
risch	$B\,a^{3}x-3Ba\,b^{2}x-3C\,a^{2}bx+C\,b^{3}x-3iCa\,b^{2}x+\frac{6iB\,a^{2}bc}{d}+iC\,a^{3}x+\frac{2iC\,a^{3}c}{d}+\frac{2ib(-3iB}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(B*b^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+C*b^3*(1/3*tan(d*x+c)^3-tan(d*
x+c)+d*x+c)+3*B*a*b^2*(tan(d*x+c)-d*x-c)+3*C*a*b^2*(1/2*tan(d*x+c)^2+ln(cos
(d*x+c)))-3*B*a^2*b*ln(cos(d*x+c))+3*C*a^2*b*(tan(d*x+c)-d*x-c)+B*a^3*(d*x+
c)-C*a^3*ln(cos(d*x+c)))
```

Maxima [A]

time = 0.51, size = 143, normalized size = 1.02

$$\frac{2\,Cb^{3}\tan\left(dx+c\right)^{3}+3\left(3\,Cab^{2}+Bb^{3}\right)\tan\left(dx+c\right)^{2}+6\left(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2}+Cb^{3}\right)(dx+c)+3\left(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)+6\left(3\,Ca^{2}b+3\,Bab^{2}-Cb^{3}\right)\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] 1/6*(2*C*b^3*tan(d*x + c)^3 + 3*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^2 + 6*(B*a
^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 + 3*B*a^2*b - 3*C*
a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) + 6*(3*C*a^2*b + 3*B*a*b^2 - C*b^3)*
tan(d*x + c))/d
```

Fricas [A]

time = 6.59, size = 142, normalized size = 1.01

$$\frac{2\,Cb^{3}\tan\left(dx+c\right)^{3}+6\left(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2}+Cb^{3}\right)dx+3\left(3\,Cab^{2}+Bb^{3}\right)\tan\left(dx+c\right)^{2}-3\left(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3}\right)\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)+6\left(3\,Ca^{2}b+3\,Bab^{2}-Cb^{3}\right)\tan\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] 1/6*(2*C*b^3*tan(d*x + c)^3 + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*x
 + 3*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^2 - 3*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2
- B*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 6*(3*C*a^2*b + 3*B*a*b^2 - C*b^3)*ta
n(d*x + c))/d
```

Sympy [A]
time = 0.89, size = 248, normalized size = 1.77

$$\begin{cases} Ba^3x + \frac{3Ba^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} - 3Bab^2x + \frac{3Bab^2\tan\left(c+dx\right)}{d} - \frac{Bb^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{Bb^3\tan^2\left(c+dx\right)}{2d} + \frac{Ca^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} - 3Ca^2bx + \frac{3Ca^2b\tan\left(c+dx\right)}{d} - \frac{3Cab^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{3Cab^2\tan^2\left(c+dx\right)}{2d} + Cb^3x + \frac{Cb^3\tan^3\left(c+dx\right)}{3d} - \frac{Cb^3\tan\left(c+dx\right)}{d} & \text{for}\: d \neq 0 \\ x(a+b\tan\left(c\right))^3\left(B\tan\left(c\right) + C\tan^2\left(c\right)\right)\cot\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

```
[Out] Piecewise((B*a**3*x + 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) - 3*B*a*b**
2*x + 3*B*a*b**2*tan(c + d*x)/d - B*b**3*log(tan(c + d*x)**2 + 1)/(2*d) + B
*b**3*tan(c + d*x)**2/(2*d) + C*a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*C*a
**2*b*x + 3*C*a**2*b*tan(c + d*x)/d - 3*C*a*b**2*log(tan(c + d*x)**2 + 1)/(
2*d) + 3*C*a*b**2*tan(c + d*x)**2/(2*d) + C*b**3*x + C*b**3*tan(c + d*x)**3
/(3*d) - C*b**3*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) +
 C*tan(c)**2)*cot(c), True))
```

Giac [A]
time = 1.65, size = 158, normalized size = 1.13

$$\frac{2\,Cb^3\tan\left(dx+c\right)^3 + 9\,Cab^2\tan\left(dx+c\right)^2 + 3\,Bb^3\tan\left(dx+c\right)^2 + 18\,Ca^2b\tan\left(dx+c\right) + 18\,Bab^2\tan\left(dx+c\right) - 6\,Cb^3\tan\left(dx+c\right) + 6\left(Ba^3 - 3\,Ca^2b - 3\,Bab^2 + Cb^3\right)(dx+c) + 3\left(Ca^3 + 3\,Ba^2b - 3\,Cab^2 - Bb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] 1/6*(2*C*b^3*tan(d*x + c)^3 + 9*C*a*b^2*tan(d*x + c)^2 + 3*B*b^3*tan(d*x +
c)^2 + 18*C*a^2*b*tan(d*x + c) + 18*B*a*b^2*tan(d*x + c) - 6*C*b^3*tan(d*x
+ c) + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 + 3*B
*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 8.96, size = 142, normalized size = 1.01

$$x\left(B\,a^3 - 3\,C\,a^2\,b - 3\,B\,a\,b^2 + C\,b^3\right) - \frac{\ln\left(\tan(c+d\,x)^2+1\right)\left(-\frac{C\,a^3}{2} - \frac{3\,B\,a^2\,b}{2} + \frac{3\,C\,a\,b^2}{2} + \frac{B\,b^3}{2}\right)}{d} + \frac{\tan(c+d\,x)^2\left(\frac{B\,b^3}{2} + \frac{3\,C\,a\,b^2}{2}\right)}{d} - \frac{\tan(c+d\,x)\,\left(C\,b^3 - 3\,a\,b\,(B\,b + C\,a)\right)}{d} + \frac{C\,b^3\tan(c+d\,x)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^3
,x)
```

```
[Out] x*(B*a^3 + C*b^3 - 3*B*a*b^2 - 3*C*a^2*b) - (log(tan(c + d*x)^2 + 1)*((B*b^
3)/2 - (C*a^3)/2 - (3*B*a^2*b)/2 + (3*C*a*b^2)/2))/d + (tan(c + d*x)^2*((B*
b^3)/2 + (3*C*a*b^2)/2))/d - (tan(c + d*x)*(C*b^3 - 3*a*b*(B*b + C*a)))/d +
 (C*b^3*tan(c + d*x)^3)/(3*d)
```

3.19 $\int \cot^2(c+dx)(a+b\tan(c+dx))^3 (B\tan(c+dx)+C\tan$

Optimal. Leaf size=117

$$\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x-\frac{b(3abB+3a^2C-b^2C)\log(\cos(c+dx))}{d}+\frac{a^3B\log(\sin(c+dx))}{d}+\frac{b^2(bB-$$

```
[Out] (3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*x-b*(3*B*a*b+3*C*a^2-C*b^2)*ln(cos(d*x+c)
)/d+a^3*B*ln(sin(d*x+c))/d+b^2*(B*b+2*C*a)*tan(d*x+c)/d+1/2*b*C*(a+b*tan(d*
x+c))^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3713, 3688, 3718, 3705, 3556}

$$\frac{a^3B\log(\sin(c+dx))}{d}-\frac{b(3a^2C+3abB-b^2C)\log(\cos(c+dx))}{d}+x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)+\frac{b^2(2aC+bB)\tan(c+dx)}{d}+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x - (b*(3*a*b*B + 3*a^2*C - b^2*C)*
Log[Cos[c + d*x]])/d + (a^3*B*Log[Sin[c + d*x]])/d + (b^2*(b*B + 2*a*C)*Tan
[c + d*x])/d + (b*C*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1] &
& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
```

```
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx=\int \cot(c+dx)(a+b\tan(c+dx)$$

$$=\frac{bC(a+b\tan(c+dx))^2}{2d}+\frac{1}{2}\int c$$

$$=\frac{b^2(bB+2aC)\tan(c+dx)}{d}+\frac{bC}{}$$

$$=\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x$$

$$=\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 113, normalized size = 0.97

$$\frac{-(a+ib)^3(B+iC)\log(i-\tan(c+dx))+2a^3B\log(\tan(c+dx))-(a-ib)^3(B-iC)\log(i+\tan(c+dx))+2b^2(bB+3aC)\tan(c+dx)+b^3C\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-((a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]]) + 2*a^3*B*Log[Tan[c + d*x]]
 - (a - I*b)^3*(B - I*C)*Log[I + Tan[c + d*x]] + 2*b^2*(b*B + 3*a*C)*Tan[c
+ d*x] + b^3*C*Tan[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.30, size = 131, normalized size = 1.12

method	result
derivativedivides	$\frac{B\,b^3(\tan(dx+c)-dx-c)+C\,b^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)-3Ba\,b^2\ln(\cos(dx+c))+3Ca\,b^2(\tan(dx+c)-dx-c)+3B}{d}$
default	$\frac{B\,b^3(\tan(dx+c)-dx-c)+C\,b^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)-3Ba\,b^2\ln(\cos(dx+c))+3Ca\,b^2(\tan(dx+c)-dx-c)+3B}{d}$
norman	$\frac{(3B\,a^2b-B\,b^3+C\,a^3-3Ca\,b^2)x\tan(dx+c)+\frac{b^2(Bb+3Ca)\left(\tan^2(dx+c)\right)}{d}+\frac{C\,b^3\left(\tan^3(dx+c)\right)}{2d}}{\tan(dx+c)}+\frac{B\,a^3\ln(\tan(dx+c))}{d}-($
risch	$-\frac{2iB\,a^3c}{d}+3iBa\,b^2x+\frac{6iBa\,b^2c}{d}+\frac{6ibC\,a^2c}{d}+3B\,a^2bx-B\,b^3x+C\,a^3x-3Ca\,b^2x-\frac{2iC\,b^3c}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(B*b^3*(tan(d*x+c)-d*x-c)+C*b^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))-3*B*a
*b^2*ln(cos(d*x+c))+3*C*a*b^2*(tan(d*x+c)-d*x-c)+3*B*a^2*b*(d*x+c)-3*C*a^2*
b*ln(cos(d*x+c))+B*a^3*ln(sin(d*x+c))+C*a^3*(d*x+c))
```

Maxima [A]
time = 0.50, size = 124, normalized size = 1.06

$$\frac{Cb^3\tan(dx+c)^2+2\,Ba^3\log(\tan(dx+c))+2\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)(dx+c)-(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)\log\left(\tan(dx+c)^2+1\right)+2\,(3\,Cab^2+Bb^3)\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/2*(C*b^3*tan(d*x + c)^2 + 2*B*a^3*log(tan(d*x + c)) + 2*(C*a^3 + 3*B*a^2*
b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*
log(tan(d*x + c)^2 + 1) + 2*(3*C*a*b^2 + B*b^3)*tan(d*x + c))/d
```

Fricas [A]
time = 3.19, size = 133, normalized size = 1.14

$$\frac{Cb^3\tan(dx+c)^2+Ba^3\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+2\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)dx-(3\,Ca^2b+3\,Bab^2-Cb^3)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+2\,(3\,Cab^2+Bb^3)\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(C*b^3*tan(d*x + c)^2 + B*a^3*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
+ 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x - (3*C*a^2*b + 3*B*a*b^2 -
C*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 2*(3*C*a*b^2 + B*b^3)*tan(d*x + c))/d
```

Sympy [A]
time = 1.24, size = 211, normalized size = 1.80

$$\begin{cases} -\frac{Ba^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ba^3\log\left(\tan\left(c+dx\right)\right)}{d}+3Ba^2bx+\frac{3Bab^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-Bb^3x+\frac{Bb^3\tan\left(c+dx\right)}{d}+Ca^3x+\frac{3Ca^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-3Cab^2x+\frac{3Cab^2\tan\left(c+dx\right)}{d}-\frac{Cb^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Cb^3\tan^2\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x(a+b\tan\left(c\right))^3\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^2\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((-B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**3*log(tan(c + d*x)
)/d + 3*B*a**2*b*x + 3*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - B*b**3*x +
 B*b**3*tan(c + d*x)/d + C*a**3*x + 3*C*a**2*b*log(tan(c + d*x)**2 + 1)/(2*
d) - 3*C*a*b**2*x + 3*C*a*b**2*tan(c + d*x)/d - C*b**3*log(tan(c + d*x)**2
+ 1)/(2*d) + C*b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**3*
(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

Giac [A]
time = 1.79, size = 129, normalized size = 1.10

$$\frac{Cb^3\tan\left(dx+c\right)^2+2\,Ba^3\log\left(\left|\tan\left(dx+c\right)\right|\right)+6\,Cab^2\tan\left(dx+c\right)+2\,Bb^3\tan\left(dx+c\right)+2\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)(dx+c)-(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(C*b^3*tan(d*x + c)^2 + 2*B*a^3*log(abs(tan(d*x + c))) + 6*C*a*b^2*tan(
d*x + c) + 2*B*b^3*tan(d*x + c) + 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)
*(d*x + c) - (B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1
))/d
```

Mupad [B]
time = 8.96, size = 118, normalized size = 1.01

$$\frac{\tan(c+d\,x)\,\left(B\,b^3+3\,C\,a\,b^2\right)}{d}+\frac{B\,a^3\,\ln\left(\tan(c+d\,x)\right)}{d}+\frac{C\,b^3\,\tan(c+d\,x)^2}{2\,d}-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(B-C\,1{}\mathrm{i}\right)\,{\left(b+a\,1{}\mathrm{i}\right)}^3\,1{}\mathrm{i}}{2\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1{}\mathrm{i}\right)\,{\left(-b+a\,1{}\mathrm{i}\right)}^3\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^3,x)
```

```
[Out] (tan(c + d*x)*(B*b^3 + 3*C*a*b^2))/d + (B*a^3*log(tan(c + d*x)))/d - (log(t
an(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)^3*1i)/(2*d) - (log(tan(c + d*x) - 1
i)*(B + C*1i)*(a*1i - b)^3*1i)/(2*d) + (C*b^3*tan(c + d*x)^2)/(2*d)
```

3.20 $\int \cot^3(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=119

$$-\left(\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x\right)-\frac{b^2(bB+3aC)\log(\cos(c+dx))}{d}+\frac{a^2(3bB+aC)\log(\sin(c+dx))}{d}+\frac{b^2(a}{}$$

```
[Out] -(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*x-b^2*(B*b+3*C*a)*ln(cos(d*x+c))/d+a^2*(
3*B*b+C*a)*ln(sin(d*x+c))/d+b^2*(B*a+C*b)*tan(d*x+c)/d-a*B*cot(d*x+c)*(a+b*
tan(d*x+c))^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3713, 3686, 3718, 3705, 3556}

$$\frac{a^2(aC+3bB)\log(\sin(c+dx))}{d}-x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{b^2(3aC+bB)\log(\cos(c+dx))}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] -((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x) - (b^2*(b*B + 3*a*C)*Log[Cos[c
 + d*x]])/d + (a^2*(3*b*B + a*C)*Log[Sin[c + d*x]])/d + (b^2*(a*B + b*C)*Ta
n[c + d*x])/d - (a*B*Cot[c + d*x]*(a + b*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^2(c+dx)(a+b\tan(c+d$$

$$= -\frac{aB\cot(c+dx)(a+b\tan(c+}{d}$$

$$= \frac{b^2(aB+bC)\tan(c+dx)}{d} - \frac{aE}{}$$

$$= -\left(a^3B-3ab^2B-3a^2bC+b^3C\right.$$

$$= -\left(a^3B-3ab^2B-3a^2bC+b^3C\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 113, normalized size = 0.95

$$\frac{-2a^3B\cot(c+dx)+i(a+ib)^3(B+iC)\log(i-\tan(c+dx))+2a^2(3bB+aC)\log(\tan(c+dx))+(ia+b)^3(B-iC)\log(i+\tan(c+dx))+2b^3C\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-2*a^3*B*Cot[c + d*x] + I*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 2*
a^2*(3*b*B + a*C)*Log[Tan[c + d*x]] + (I*a + b)^3*(B - I*C)*Log[I + Tan[c +
 d*x]] + 2*b^3*C*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.27, size = 123, normalized size = 1.03

method	result
derivativedivides	$\frac{-B\,b^3\ln(\cos(dx+c))+C\,b^3(\tan(dx+c)-dx-c)+3Ba\,b^2(dx+c)-3Ca\,b^2\ln(\cos(dx+c))+3B\,a^2b\ln(\sin(dx+c))+3C\,a^2b(dx}{d}$
default	$\frac{-B\,b^3\ln(\cos(dx+c))+C\,b^3(\tan(dx+c)-dx-c)+3Ba\,b^2(dx+c)-3Ca\,b^2\ln(\cos(dx+c))+3B\,a^2b\ln(\sin(dx+c))+3C\,a^2b(dx}{d}$
norman	$\frac{(-B\,a^3+3Ba\,b^2+3C\,a^2b-C\,b^3)x\left(\tan^2(dx+c)\right)+\frac{C\,b^3\left(\tan^3(dx+c)\right)}{d}-\frac{B\,a^3\tan(dx+c)}{d}}{\tan(dx+c)^2}+\frac{a^2(3Bb+Ca)\ln(\tan(dx+c))}{d}-$
risch	$-B\,a^3x+3Ba\,b^2x+3C\,a^2bx-C\,b^3x+3iCa\,b^2x-\frac{6iB\,a^2bc}{d}-iC\,a^3x-\frac{2iC\,a^3c}{d}+\frac{2iB\,b^3c}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(-B*b^3*ln(cos(d*x+c))+C*b^3*(tan(d*x+c)-d*x-c)+3*B*a*b^2*(d*x+c)-3*C*a
*b^2*ln(cos(d*x+c))+3*B*a^2*b*ln(sin(d*x+c))+3*C*a^2*b*(d*x+c)+B*a^3*(-cot(
d*x+c)-d*x-c)+C*a^3*ln(sin(d*x+c)))
```

Maxima [A]
time = 0.50, size = 125, normalized size = 1.05

$$\frac{2\,Cb^3\tan\left(dx+c\right)-\frac{2\,Ba^3}{\tan(dx+c)}-2\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)(dx+c)-(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(Ca^3+3\,Ba^2b)\log\left(\tan\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/2*(2*C*b^3*tan(d*x + c) - 2*B*a^3/tan(d*x + c) - 2*(B*a^3 - 3*C*a^2*b - 3
*B*a*b^2 + C*b^3)*(d*x + c) - (C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(t
an(d*x + c)^2 + 1) + 2*(C*a^3 + 3*B*a^2*b)*log(tan(d*x + c)))/d
```

Fricas [A]
time = 5.32, size = 145, normalized size = 1.22

$$\frac{2\,Cb^3\tan\left(dx+c\right)^2-2\,Ba^3-2\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)dx\tan\left(dx+c\right)+(Ca^3+3\,Ba^2b)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)-(3\,Cab^2+Bb^3)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/2*(2*C*b^3*tan(d*x + c)^2 - 2*B*a^3 - 2*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 +
C*b^3)*d*x*tan(d*x + c) + (C*a^3 + 3*B*a^2*b)*log(tan(d*x + c)^2/(tan(d*x +
 c)^2 + 1))*tan(d*x + c) - (3*C*a*b^2 + B*b^3)*log(1/(tan(d*x + c)^2 + 1))*
tan(d*x + c))/(d*tan(d*x + c))
```

Sympy [A]
time = 2.23, size = 221, normalized size = 1.86

$$\begin{cases} \text{NaN} & \text{for}\: \left(c = 0 \vee c = - d x\right) \wedge \left(c = - d x \vee d = 0\right) \\ x \left(a + b \tan{\left(c \right)}\right)^{3} \left(B \tan{\left(c \right)} + C \tan^{2}{\left(c \right)}\right) \cot^{3}{\left(c \right)} & \text{for}\: d = 0 \\ - B a^{3} x - \frac{B a^{3}}{d \tan{\left(c + d x \right)}} - \frac{3 B a^{2} b \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{3 B a^{2} b \log{\left(\tan{\left(c + d x \right)} \right)}}{d} + 3 B a b^{2} x + \frac{B b^{3} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - \frac{C a^{3} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{C a^{3} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} + 3 C a^{2} b x + \frac{3 C a b^{2} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - C b^{3} x + \frac{C b^{3} \tan{\left(c + d x \right)}}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**3*(B*tan(c) + C*tan(c)**2)*cot(c)**3, Eq(d, 0)), (-B*a**3*x -
 B*a**3/(d*tan(c + d*x)) - 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*
a**2*b*log(tan(c + d*x))/d + 3*B*a*b**2*x + B*b**3*log(tan(c + d*x)**2 + 1)
/(2*d) - C*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**3*log(tan(c + d*x))/d
 + 3*C*a**2*b*x + 3*C*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - C*b**3*x + C*
b**3*tan(c + d*x)/d, True))
```

Giac [A]
time = 1.30, size = 152, normalized size = 1.28

$$\frac{2 \, C b^{3} \tan\left(d x + c\right) - 2 \, {\left(B a^{3} - 3 \, C a^{2} b - 3 \, B a b^{2} + C b^{3}\right)} {\left(d x + c\right)} - {\left(C a^{3} + 3 \, B a^{2} b - 3 \, C a b^{2} - B b^{3}\right)} \log\left(\tan\left(d x + c\right)^{2} + 1\right) + 2 \, {\left(C a^{3} + 3 \, B a^{2} b\right)} \log\left({\left| \tan\left(d x + c\right) \right|}\right) - \frac{2 \, {\left(C a^{3} \tan\left(d x + c\right) + 3 \, B a^{2} b \tan\left(d x + c\right) + B a^{3}\right)}}{\tan\left(d x + c\right)}}{2 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/2*(2*C*b^3*tan(d*x + c) - 2*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x
+ c) - (C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) + 2*
(C*a^3 + 3*B*a^2*b)*log(abs(tan(d*x + c))) - 2*(C*a^3*tan(d*x + c) + 3*B*a^
2*b*tan(d*x + c) + B*a^3)/tan(d*x + c))/d
```

Mupad [B]
time = 8.86, size = 114, normalized size = 0.96

$$\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)\right)\,\left(C\,a^3+3\,B\,b\,a^2\right)}{d}-\frac{B\,a^3\,\mathrm{cot}\left(c+d\,x\right)}{d}+\frac{C\,b^3\,\mathrm{tan}\left(c+d\,x\right)}{d}+\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)\,\left(B+C\,1{}\mathrm{i}\right)\,{\left(a+b\,1{}\mathrm{i}\right)}^3\,1{}\mathrm{i}}{2\,d}-\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)\,\left(B-C\,1{}\mathrm{i}\right)\,{\left(a-b\,1{}\mathrm{i}\right)}^3\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^3,x)
```

```
[Out] (log(tan(c + d*x))*(C*a^3 + 3*B*a^2*b))/d + (log(tan(c + d*x) - 1i)*(B + C*
1i)*(a + b*1i)^3*1i)/(2*d) - (log(tan(c + d*x) + 1i)*(B - C*1i)*(a - b*1i)^
3*1i)/(2*d) - (B*a^3*cot(c + d*x))/d + (C*b^3*tan(c + d*x))/d
```

3.21 $\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=127

$$-\left(\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x\right)-\frac{a^2(2bB+aC)\cot(c+dx)}{d}-\frac{b^3C\log(\cos(c+dx))}{d}-\frac{a(a^2B-3b^2B-$$

```
[Out] -(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*x-a^2*(2*B*b+C*a)*cot(d*x+c)/d-b^3*C*ln(
cos(d*x+c))/d-a*(B*a^2-3*B*b^2-3*C*a*b)*ln(sin(d*x+c))/d-1/2*a*B*cot(d*x+c)
^2*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3713, 3686, 3716, 3705, 3556}

$$-\frac{a(a^2B-3abC-3b^2B)\log(\sin(c+dx))}{d}-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{b^3C\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] -((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x) - (a^2*(2*b*B + a*C)*Cot[c + d
*x])/d - (b^3*C*Log[Cos[c + d*x]])/d - (a*(a^2*B - 3*b^2*B - 3*a*b*C)*Log[S
in[c + d*x]])/d - (a*B*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3705

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Dist[A, Int[1/Tan[e
 + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B, C
}, x] && NeQ[A, C]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx &= \int \cot^3(c+dx)(a+b\tan(c+dx \\
&= -\frac{aB\cot^2(c+dx)(a+b\tan(c+}{2d} \\
&= -\frac{a^2(2bB+aC)\cot(c+dx)}{d} - \frac{a}{} \\
&= -\left(3a^2bB-b^3B+a^3C-3ab^2C\right. \\
&= -\left(3a^2bB-b^3B+a^3C-3ab^2C\right.
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.31, size = 126, normalized size = 0.99

$$
\frac{-2a^2(3bB+aC)\cot(c+dx)-a^3B\cot^2(c+dx)+(a+ib)^3(B+iC)\log(i-\tan(c+dx))-2a(a^2B-3b^2B-3abC)\log(\tan(c+dx))+(a-ib)^3(B-iC)\log(i+\tan(c+dx))}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-2*a^2*(3*b*B + a*C)*Cot[c + d*x] - a^3*B*Cot[c + d*x]^2 + (a + I*b)^3*(B
+ I*C)*Log[I - Tan[c + d*x]] - 2*a*(a^2*B - 3*b^2*B - 3*a*b*C)*Log[Tan[c +
d*x]] + (a - I*b)^3*(B - I*C)*Log[I + Tan[c + d*x]])/(2*d)
```

Maple [A]
time = 0.36, size = 138, normalized size = 1.09

method	result
derivativedivides	$\frac{B\,b^3(dx+c)-C\,b^3\ln(\cos(dx+c))+3Ba\,b^2\ln(\sin(dx+c))+3Ca\,b^2(dx+c)+3B\,a^2b(-\cot(dx+c)-dx-c)+3C\,a^2b\ln(\sin(d}{d}$
default	$\frac{B\,b^3(dx+c)-C\,b^3\ln(\cos(dx+c))+3Ba\,b^2\ln(\sin(dx+c))+3Ca\,b^2(dx+c)+3B\,a^2b(-\cot(dx+c)-dx-c)+3C\,a^2b\ln(\sin(d}{d}$
norman	$\frac{(-3B\,a^2b+B\,b^3-C\,a^3+3Ca\,b^2)x(\tan^3(dx+c))-\frac{B\,a^3\tan(dx+c)}{2d}-\frac{a^2(3Bb+Ca)(\tan^2(dx+c))}{d}}{\tan(dx+c)^3}+\frac{(B\,a^3-3Ba\,b^2-3C\,a^2b}{}$
risch	$\frac{2iB\,a^3c}{d}-3iBa\,b^2x+iB\,a^3x-3iC\,a^2bx-3B\,a^2bx+B\,b^3x-C\,a^3x+3Ca\,b^2x-\frac{2ia^2(3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(B*b^3*(d*x+c)-C*b^3*ln(cos(d*x+c))+3*B*a*b^2*ln(sin(d*x+c))+3*C*a*b^2*
(d*x+c)+3*B*a^2*b*(-cot(d*x+c)-d*x-c)+3*C*a^2*b*ln(sin(d*x+c))+B*a^3*(-1/2*
cot(d*x+c)^2-ln(sin(d*x+c)))+C*a^3*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 142, normalized size = 1.12

$$-\frac{2\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)(dx+c)-(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(Ba^3-3\,Ca^2b-3\,Bab^2)\log\left(\tan\left(dx+c\right)\right)+\frac{Ba^3+2\,(Ca^3+3\,Ba^2b)\tan(dx+c)}{\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a^
2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 2*(B*a^3 - 3*C*a^2*b - 3
*B*a*b^2)*log(tan(d*x + c)) + (B*a^3 + 2*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/
tan(d*x + c)^2)/d
```

Fricas [A]
time = 3.15, size = 162, normalized size = 1.28

$$-\frac{Cb^3\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+Ba^3+(Ba^3-3Ca^2b-3Bab^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+(Ba^3+2(Ca^3+3Ba^2b-3Cab^2-Bb^3)dx)\tan(dx+c)^2+2(Ca^3+3Ba^2b)\tan(dx+c)}{2d\tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/2*(C*b^3*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 + B*a^3 + (B*a^3 - 3
*C*a^2*b - 3*B*a*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)
^2 + (B*a^3 + 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c)^2
 + 2*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.
time = 2.98, size = 260, normalized size = 2.05

$$\begin{cases} \text{NaN} & \text{for}\: (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ x(a+b\tan(c))^3(B\tan(c)+C\tan^2(c))\cot^4(c) & \text{for}\: d = 0 \\ \frac{Ba^3\log(\tan^2(c+dx)+1)}{2d} - \frac{Ba^3\log(\tan(c+dx))}{d} - \frac{Ba^3}{2d\tan^2(c+dx)} - 3Ba^2bx - \frac{3Ba^2b}{d\tan(c+dx)} - \frac{3Bab^2\log(\tan^2(c+dx)+1)}{2d} + \frac{3Bab^2\log(\tan(c+dx))}{d} + Bb^3x - Ca^3x - \frac{Ca^3}{d\tan(c+dx)} - \frac{3Ca^2b\log(\tan^2(c+dx)+1)}{2d} + \frac{3Ca^2b\log(\tan(c+dx))}{d} + 3Cab^2x + \frac{Cb^3\log(\tan^2(c+dx)+1)}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**3*(B*tan(c) + C*tan(c)**2)*cot(c)**4, Eq(d, 0)), (B*a**3*log(
tan(c + d*x)**2 + 1)/(2*d) - B*a**3*log(tan(c + d*x))/d - B*a**3/(2*d*tan(c
 + d*x)**2) - 3*B*a**2*b*x - 3*B*a**2*b/(d*tan(c + d*x)) - 3*B*a*b**2*log(t
an(c + d*x)**2 + 1)/(2*d) + 3*B*a*b**2*log(tan(c + d*x))/d + B*b**3*x - C*a
**3*x - C*a**3/(d*tan(c + d*x)) - 3*C*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d)
 + 3*C*a**2*b*log(tan(c + d*x))/d + 3*C*a*b**2*x + C*b**3*log(tan(c + d*x)*
*2 + 1)/(2*d), True))
```

Giac [A]
time = 1.42, size = 193, normalized size = 1.52

$$-\frac{2(Ca^3+3Ba^2b-3Cab^2-Bb^3)(dx+c)-(Ba^3-3Ca^2b-3Bab^2+Cb^3)\log\left(\tan(dx+c)^2+1\right)+2(Ba^3-3Ca^2b-3Bab^2)\log\left(|\tan(dx+c)|\right)-\frac{3Ba^3\tan(dx+c)^2-9Ca^2b\tan(dx+c)^2-9Bab^2\tan(dx+c)^2-2Ca^3\tan(dx+c)-6Ba^2b\tan(dx+c)-Ba^3}{\tan(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/2*(2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a^
2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 2*(B*a^3 - 3*C*a^2*b - 3
*B*a*b^2)*log(abs(tan(d*x + c))) - (3*B*a^3*tan(d*x + c)^2 - 9*C*a^2*b*tan(
```

```
d*x + c)^2 - 9*B*a*b^2*tan(d*x + c)^2 - 2*C*a^3*tan(d*x + c) - 6*B*a^2*b*ta
n(d*x + c) - B*a^3)/tan(d*x + c)^2)/d
```

Mupad [B]
time = 8.97, size = 135, normalized size = 1.06

$$\frac{\ln\left(\tan(c+dx)\right)\,\left(-B\,a^3+3\,C\,a^2\,b+3\,B\,a\,b^2\right)}{d}-\frac{\cot(c+dx)^2\,\left(\tan(c+dx)\,\left(C\,a^3+3\,B\,b\,a^2\right)+\frac{B\,a^3}{2}\right)}{d}+\frac{\ln\left(\tan(c+dx)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{2\,d}+\frac{\ln\left(\tan(c+dx)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,{\left(-b+a\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^3,x)
```

```
[Out] (log(tan(c + d*x))*(3*B*a*b^2 - B*a^3 + 3*C*a^2*b))/d - (cot(c + d*x)^2*(ta
n(c + d*x)*(C*a^3 + 3*B*a^2*b) + (B*a^3)/2))/d + (log(tan(c + d*x) + 1i)*(B
 - C*1i)*(a*1i + b)^3*1i)/(2*d) + (log(tan(c + d*x) - 1i)*(B + C*1i)*(a*1i
- b)^3*1i)/(2*d)
```

3.22 $\int \cot^5(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=154

$$\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x+\frac{a(3a^2B-8b^2B-9abC)\cot(c+dx)}{3d}-\frac{a^2(5bB+3aC)\cot^2(c+dx)}{6d}-\frac{(3a^2}{}$$

```
[Out] (B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*x+1/3*a*(3*B*a^2-8*B*b^2-9*C*a*b)*cot(d*x
+c)/d-1/6*a^2*(5*B*b+3*C*a)*cot(d*x+c)^2/d-(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2
)*ln(sin(d*x+c))/d-1/3*a*B*cot(d*x+c)^3*(a+b*tan(d*x+c))^2/d
```

Rubi [A]

time = 0.30, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3713, 3686, 3716, 3709, 3612, 3556}

$$\frac{a(3a^2B-9abC-8b^2B)\cot(c+dx)}{3d}-\frac{a^2(3aC+5bB)\cot^2(c+dx)}{6d}-\frac{(a^3C+3a^2bB-3ab^2C-b^3B)\log(\sin(c+dx))}{d}+x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)-\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x + (a*(3*a^2*B - 8*b^2*B - 9*a*b*C
)*Cot[c + d*x])/(3*d) - (a^2*(5*b*B + 3*a*C)*Cot[c + d*x]^2)/(6*d) - ((3*a^
2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]
^3*(a + b*Tan[c + d*x])^2)/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
```

```
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^4(c+dx)(a+b\tan(c+dx$$

$$= -\frac{aB\cot^3(c+dx)(a+b\tan(c+}{3d}$$

$$= -\frac{a^2(5bB+3aC)\cot^2(c+dx)}{6d} -$$

$$= \frac{a(3a^2B-8b^2B-9abC)\cot(c+}{3d}$$

$$= \left(a^3B-3ab^2B-3a^2bC+b^3C\right)x$$

$$= \left(a^3B-3ab^2B-3a^2bC+b^3C\right)x$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.84, size = 164, normalized size = 1.06

$$\frac{6a(a^2B-3b^2B-3abC)\cot(c+dx)-3a^2(3bB+aC)\cot^2(c+dx)-2a^3B\cot^3(c+dx)+3(a+ib)^3(-iB+C)\log(i-\tan(c+dx))-6(3a^2bB-b^3B+a^3C-3ab^2C)\log(\tan(c+dx))+3(a-ib)^3(iB+C)\log(i+\tan(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (6*a*(a^2*B - 3*b^2*B - 3*a*b*C)*Cot[c + d*x] - 3*a^2*(3*b*B + a*C)*Cot[c +
 d*x]^2 - 2*a^3*B*Cot[c + d*x]^3 + 3*(a + I*b)^3*((-I)*B + C)*Log[I - Tan[c
 + d*x]] - 6*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Log[Tan[c + d*x]] + 3*
(a - I*b)^3*(I*B + C)*Log[I + Tan[c + d*x]])/(6*d)
```

Maple [A]
time = 0.32, size = 166, normalized size = 1.08

method	result
derivativedivides	$\frac{B\,b^3\ln(\sin(dx+c))+C\,b^3(dx+c)+3Ba\,b^2(-\cot(dx+c)-dx-c)+3Ca\,b^2\ln(\sin(dx+c))+3B\,a^2b\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(d\right.}{}$
default	$\frac{B\,b^3\ln(\sin(dx+c))+C\,b^3(dx+c)+3Ba\,b^2(-\cot(dx+c)-dx-c)+3Ca\,b^2\ln(\sin(dx+c))+3B\,a^2b\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(d\right.}{}$
norman	$\frac{\left(B\,a^3-3Ba\,b^2-3C\,a^2b+C\,b^3\right)x\left(\tan^4(dx+c)\right)+\frac{a\left(a^2B-3b^2B-3Cab\right)\left(\tan^3(dx+c)\right)}{d}-\frac{B\,a^3\tan(dx+c)}{3d}-\frac{a^2(3Bb+Ca)\left(\tan^2(dx+\right.}{2d}}{\tan(dx+c)^4}$
risch	$B\,a^3x-3Ba\,b^2x-3C\,a^2bx+C\,b^3x-3iCa\,b^2x+\frac{6iB\,a^2bc}{d}-\frac{2ia\left(9iBab\,\mathrm{e}^{4i(dx+c)}+3iC\,a^2\mathrm{e}^{4i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(B*b^3*ln(sin(d*x+c))+C*b^3*(d*x+c)+3*B*a*b^2*(-cot(d*x+c)-d*x-c)+3*C*a
*b^2*ln(sin(d*x+c))+3*B*a^2*b*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+3*C*a^2*b*
(-cot(d*x+c)-d*x-c)+B*a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+C*a^3*(-1/2*
cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.51, size = 180, normalized size = 1.17

$$\frac{6\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)(dx+c)+3\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right)-6\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)\log\left(\tan\left(dx+c\right)\right)-\frac{2\,Ba^3-6\,(Ba^3-3\,Ca^2b-3\,Bab^2)\tan(dx+c)^2+3\,(Ca^3+3\,Ba^2b)\tan(dx+c)}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/6*(6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 + 3*B*a
^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) - 6*(C*a^3 + 3*B*a^2*b -
3*C*a*b^2 - B*b^3)*log(tan(d*x + c)) - (2*B*a^3 - 6*(B*a^3 - 3*C*a^2*b - 3*
B*a*b^2)*tan(d*x + c)^2 + 3*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x + c)^
3)/d
```

Fricas [A]
time = 2.89, size = 181, normalized size = 1.18

$$-\frac{3\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3+2\,Ba^3+3\,(Ca^3+3\,Ba^2b-2\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)dx)\tan\left(dx+c\right)^3-6\,(Ba^3-3\,Ca^2b-3\,Bab^2)\tan\left(dx+c\right)^2+3\,(Ca^3+3\,Ba^2b)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] -1/6*(3*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d*x
 + c)^2 + 1))*tan(d*x + c)^3 + 2*B*a^3 + 3*(C*a^3 + 3*B*a^2*b - 2*(B*a^3 -
3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*x)*tan(d*x + c)^3 - 6*(B*a^3 - 3*C*a^2*b -
 3*B*a*b^2)*tan(d*x + c)^2 + 3*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x
 + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(150) = 300$.
time = 4.91, size = 330, normalized size = 2.14

$$\begin{cases} \text{NaN} & \text{for}\: \left(c = 0 \vee c = - d x\right) \wedge \left(c = - d x \vee d = 0\right) \\ x \left(a + b \tan{\left(c \right)}\right)^{3} \left(B \tan{\left(c \right)} + C \tan^{2}{\left(c \right)}\right) \cot^{5}{\left(c \right)} & \text{for}\: d = 0 \\ B a^{3} x + \frac{B a^{3}}{d \tan{\left(c + d x \right)}} - \frac{B a^{3}}{3 d \tan^{3}{\left(c + d x \right)}} + \frac{3 B a^{2} b \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - \frac{3 B a^{2} b \log{\left(\tan{\left(c + d x \right)} \right)}}{d} - \frac{3 B a^{2} b}{2 d \tan^{2}{\left(c + d x \right)}} - 3 B a b^{2} x - \frac{3 B a b^{2}}{d \tan{\left(c + d x \right)}} - \frac{B b^{3} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{B b^{3} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} + \frac{C a^{3} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - \frac{C a^{3} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} - \frac{C a^{3}}{2 d \tan^{2}{\left(c + d x \right)}} - 3 C a^{2} b x - \frac{3 C a^{2} b}{d \tan{\left(c + d x \right)}} - \frac{3 C a b^{2} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{3 C a b^{2} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} + C b^{3} x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**3*(B*tan(c) + C*tan(c)**2)*cot(c)**5, Eq(d, 0)), (B*a**3*x +
B*a**3/(d*tan(c + d*x)) - B*a**3/(3*d*tan(c + d*x)**3) + 3*B*a**2*b*log(tan
(c + d*x)**2 + 1)/(2*d) - 3*B*a**2*b*log(tan(c + d*x))/d - 3*B*a**2*b/(2*d*
tan(c + d*x)**2) - 3*B*a*b**2*x - 3*B*a*b**2/(d*tan(c + d*x)) - B*b**3*log(
tan(c + d*x)**2 + 1)/(2*d) + B*b**3*log(tan(c + d*x))/d + C*a**3*log(tan(c
+ d*x)**2 + 1)/(2*d) - C*a**3*log(tan(c + d*x))/d - C*a**3/(2*d*tan(c + d*x
)**2) - 3*C*a**2*b*x - 3*C*a**2*b/(d*tan(c + d*x)) - 3*C*a*b**2*log(tan(c +
 d*x)**2 + 1)/(2*d) + 3*C*a*b**2*log(tan(c + d*x))/d + C*b**3*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(148) = 296.
time = 1.54, size = 390, normalized size = 2.53

$$\frac{Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+36Ca^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+36Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24\left(Ba^3-3Ca^2b-3Bab^2+Cb^3\right)(dx+c)+24\left(Ca^3+3Ba^2b-3Cab^2-Bb^3\right)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)-24\left(Ca^3+3Ba^2b-3Cab^2-Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+\frac{44Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+132Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-132Cab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-44Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-36Ca^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-36Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3Ca^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ba^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/24*(B*a^3*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*B*a
^2*b*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^3*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b*ta
n(1/2*d*x + 1/2*c) + 36*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*(B*a^3 - 3*C*a^2*
b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 24*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^
3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 24*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*
b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C*a^3*tan(1/2*d*x + 1/2*c)^3 + 13
2*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 132*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 44*
B*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*C*a^2*b
*tan(1/2*d*x + 1/2*c)^2 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*C*a^3*tan(1
/2*d*x + 1/2*c) - 9*B*a^2*b*tan(1/2*d*x + 1/2*c) - B*a^3)/tan(1/2*d*x + 1/2
*c)^3)/d
```

Mupad [B]
time = 9.00, size = 169, normalized size = 1.10

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3\right)}{d}-\frac{\cot(c+d\,x)^3\,\left(\tan(c+d\,x)\,\left(\frac{C\,a^3}{2}+\frac{3\,B\,b\,a^2}{2}\right)+\frac{B\,a^3}{3}+\tan(c+d\,x)^2\,\left(-B\,a^3+3\,C\,a^2\,b+3\,B\,a\,b^2\right)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,{\left(a+b\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,{\left(a-b\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^3,x)
```

```
[Out] (log(tan(c + d*x))*(B*b^3 - C*a^3 - 3*B*a^2*b + 3*C*a*b^2))/d - (cot(c + d*
x)^3*(tan(c + d*x)*((C*a^3)/2 + (3*B*a^2*b)/2) + (B*a^3)/3 + tan(c + d*x)^2
*(3*B*a*b^2 - B*a^3 + 3*C*a^2*b)))/d - (log(tan(c + d*x) - 1i)*(B + C*1i)*(
a + b*1i)^3*1i)/(2*d) + (log(tan(c + d*x) + 1i)*(B - C*1i)*(a - b*1i)^3*1i)
/(2*d)
```

3.23 $\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=191

$$\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x+\frac{\left(3a^2bB-b^3B+a^3C-3ab^2C\right)\cot(c+dx)}{d}+\frac{a\left(2a^2B-5b^2B-6abC\right)\cot}{4d}$$

```
[Out] (3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*x+(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*cot(d
*x+c)/d+1/4*a*(2*B*a^2-5*B*b^2-6*C*a*b)*cot(d*x+c)^2/d-1/6*a^2*(3*B*b+2*C*a
)*cot(d*x+c)^3/d+(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*ln(sin(d*x+c))/d-1/4*a*B
*cot(d*x+c)^4*(a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.36, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {3713, 3686, 3716, 3709, 3610, 3612, 3556}

$$\frac{a\left(2a^2B-6abC-5b^2B\right)\cot^2(c+dx)}{4d}-\frac{a^2(2aC+3bB)\cot^3(c+dx)}{6d}+\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\cot(c+dx)}{d}+\frac{\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\log(\sin(c+dx))}{d}+x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)-\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x + ((3*a^2*b*B - b^3*B + a^3*C - 3
*a*b^2*C)*Cot[c + d*x])/d + (a*(2*a^2*B - 5*b^2*B - 6*a*b*C)*Cot[c + d*x]^2
)/(4*d) - (a^2*(3*b*B + 2*a*C)*Cot[c + d*x]^3)/(6*d) + ((a^3*B - 3*a*b^2*B
- 3*a^2*b*C + b^3*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]^4*(a + b*Tan[
c + d*x])^2)/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
```

-1]

Rubi steps

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \int \cot^5(c+dx)(a+b\tan(c+dx$$

$$= -\frac{aB\cot^4(c+dx)(a+b\tan(c+}{4d}$$

$$= -\frac{a^2(3bB+2aC)\cot^3(c+dx)}{6d} -$$

$$= \frac{a(2a^2B-5b^2B-6abC)\cot^2(c-}{4d}$$

$$= \frac{(3a^2bB-b^3B+a^3C-3ab^2C)\,c}{d}$$

$$= \left(3a^2bB-b^3B+a^3C-3ab^2C\right)x$$

$$= \left(3a^2bB-b^3B+a^3C-3ab^2C\right)x$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.51, size = 199, normalized size = 1.04

$$\frac{12(3a^2bB-b^3B+a^3C-3ab^2C)\cot(c+dx)+6a(a^2B-3b^2B-3abC)\cot^2(c+dx)-4a^2(3bB+aC)\cot^3(c+dx)-3a^3B\cot^4(c+dx)-6(a+ib)^3(B+iC)\log(i-\tan(c+dx))+12(a^3B-3ab^2B-3a^2bC+b^3C)\log(\tan(c+dx))-6(a-ib)^3(B-iC)\log(i+\tan(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (12*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Cot[c + d*x] + 6*a*(a^2*B - 3*b
^2*B - 3*a*b*C)*Cot[c + d*x]^2 - 4*a^2*(3*b*B + a*C)*Cot[c + d*x]^3 - 3*a^3
*B*Cot[c + d*x]^4 - 6*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 12*(a^3
*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*Log[Tan[c + d*x]] - 6*(a - I*b)^3*(B -
I*C)*Log[I + Tan[c + d*x]])/(12*d)
```

Maple [A]
time = 0.31, size = 203, normalized size = 1.06

method	result
derivativedivides	$\frac{B\,b^3(-\cot(dx+c)-dx-c)+C\,b^3\ln(\sin(dx+c))+3Ba\,b^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3Ca\,b^2(-\cot(dx+c)-dx-c)+}{}$

default	$\frac{B\,b^{3}(-\cot(dx+c)-dx-c)+C\,b^{3}\ln(\sin(dx+c))+3Ba\,b^{2}\left(-\frac{\left(\cot^{2}(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3Ca\,b^{2}(-\cot(dx+c)-dx-c}{}$
norman	$\frac{\frac{\left(3B\,a^{2}b-B\,b^{3}+C\,a^{3}-3Ca\,b^{2}\right)\left(\tan^{4}(dx+c)\right)}{d}+(3B\,a^{2}b-B\,b^{3}+C\,a^{3}-3Ca\,b^{2})x\left(\tan^{5}(dx+c)\right)-\frac{B\,a^{3}\tan(dx+c)}{4d}+\frac{a\left(a^{2}B-3b^{2}B\right.}{}}{\tan(dx+c)^{5}}$
risch	$-\frac{2iB\,a^{3}c}{d}+3iBa\,b^{2}x-iB\,a^{3}x+3iC\,a^{2}bx+3B\,a^{2}bx-B\,b^{3}x+C\,a^{3}x-3Ca\,b^{2}x+\frac{6ibC}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(B*b^3*(-cot(d*x+c)-d*x-c)+C*b^3*ln(sin(d*x+c))+3*B*a*b^2*(-1/2*cot(d*x
+c)^2-ln(sin(d*x+c)))+3*C*a*b^2*(-cot(d*x+c)-d*x-c)+3*B*a^2*b*(-1/3*cot(d*x
+c)^3+cot(d*x+c)+d*x+c)+3*C*a^2*b*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+B*a^3*
(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+C*a^3*(-1/3*cot(d*x+c)^
3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 215, normalized size = 1.13

$$\frac{12\,(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3})(dx+c)-6\,(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2}+Cb^{3})\log\left(\tan\left(dx+c\right)^{2}+1\right)+12\,(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2}+Cb^{3})\log\left(\tan\left(dx+c\right)\right)-\frac{3\,Ba^{3}-12\,(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3})\tan(dx+c)^{3}-6\,(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2})\tan(dx+c)^{2}+4\,(Ca^{3}+3\,Ba^{2}b)\tan(dx+c)}{\tan(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] 1/12*(12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - 6*(B*a^3 - 3*C
*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 12*(B*a^3 - 3*C*a^2*b
 - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)) - (3*B*a^3 - 12*(C*a^3 + 3*B*a^2*b
- 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 - 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2)*tan
(d*x + c)^2 + 4*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x + c)^4)/d
```

Fricas [A]
time = 4.02, size = 225, normalized size = 1.18

$$\frac{6\,(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2}+Cb^{3})\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)\tan\left(dx+c\right)^{4}+3\,(3\,Ba^{3}-6\,Ca^{2}b-6\,Bab^{2}+4\,(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3})dx)\tan\left(dx+c\right)^{4}-3\,Ba^{3}+12\,(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3})\tan\left(dx+c\right)^{3}+6\,(Ba^{3}-3\,Ca^{2}b-3\,Bab^{2})\tan\left(dx+c\right)^{2}-4\,(Ca^{3}+3\,Ba^{2}b)\tan\left(dx+c\right)}{12\,d\tan\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/12*(6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2/(tan(d*x
 + c)^2 + 1))*tan(d*x + c)^4 + 3*(3*B*a^3 - 6*C*a^2*b - 6*B*a*b^2 + 4*(C*a^
3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c)^4 - 3*B*a^3 + 12*(C*a^
```

```
3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 + 6*(B*a^3 - 3*C*a^2*b -
3*B*a*b^2)*tan(d*x + c)^2 - 4*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x
+ c)^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(187) = 374$.
time = 6.51, size = 398, normalized size = 2.08

$$\begin{cases} \text{NaN} & \text{for}\: \left(c = 0 \vee c = - d x\right) \wedge \left(c = - d x \vee d = 0\right) \\ x \left(a + b \tan{\left(c \right)}\right)^{3} \left(B \tan{\left(c \right)} + C \tan^{2}{\left(c \right)}\right) \cot^{6}{\left(c \right)} & \text{for}\: d = 0 \\ - \frac{B a^{3} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{B a^{3} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} + \frac{B a^{3}}{2 d \tan^{2}{\left(c + d x \right)}} - \frac{B a^{3}}{4 d \tan^{4}{\left(c + d x \right)}} + 3 B a^{2} b x + \frac{3 B a^{2} b}{d \tan{\left(c + d x \right)}} - \frac{B a^{2} b}{d \tan^{3}{\left(c + d x \right)}} + \frac{3 B a b^{2} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - \frac{3 B a b^{2} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} - \frac{3 B a b^{2}}{2 d \tan^{2}{\left(c + d x \right)}} - B b^{3} x - \frac{B b^{3}}{d \tan{\left(c + d x \right)}} + C a^{3} x + \frac{C a^{3}}{d \tan{\left(c + d x \right)}} - \frac{C a^{3}}{3 d \tan^{3}{\left(c + d x \right)}} + \frac{3 C a^{2} b \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - \frac{3 C a^{2} b \log{\left(\tan{\left(c + d x \right)} \right)}}{d} - \frac{3 C a^{2} b}{2 d \tan^{2}{\left(c + d x \right)}} - 3 C a b^{2} x - \frac{3 C a b^{2}}{d \tan{\left(c + d x \right)}} - \frac{C b^{3} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{C b^{3} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**3*(B*tan(c) + C*tan(c)**2)*cot(c)**6, Eq(d, 0)), (-B*a**3*log
(tan(c + d*x)**2 + 1)/(2*d) + B*a**3*log(tan(c + d*x))/d + B*a**3/(2*d*tan(
c + d*x)**2) - B*a**3/(4*d*tan(c + d*x)**4) + 3*B*a**2*b*x + 3*B*a**2*b/(d*
tan(c + d*x)) - B*a**2*b/(d*tan(c + d*x)**3) + 3*B*a*b**2*log(tan(c + d*x)*
*2 + 1)/(2*d) - 3*B*a*b**2*log(tan(c + d*x))/d - 3*B*a*b**2/(2*d*tan(c + d*
x)**2) - B*b**3*x - B*b**3/(d*tan(c + d*x)) + C*a**3*x + C*a**3/(d*tan(c +
d*x)) - C*a**3/(3*d*tan(c + d*x)**3) + 3*C*a**2*b*log(tan(c + d*x)**2 + 1)/
(2*d) - 3*C*a**2*b*log(tan(c + d*x))/d - 3*C*a**2*b/(2*d*tan(c + d*x)**2) -
 3*C*a*b**2*x - 3*C*a*b**2/(d*tan(c + d*x)) - C*b**3*log(tan(c + d*x)**2 +
1)/(2*d) + C*b**3*log(tan(c + d*x))/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(185) = 370$.
time = 1.60, size = 528, normalized size = 2.76

$$-\frac{3 \, B a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 8 \, C a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 24 \, B a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 36 \, B a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 72 \, C a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 72 \, B a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 120 \, C a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 360 \, B a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 288 \, C a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 96 \, B b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 192 \, {\left(C a^{3} + 3 \, B a^{2} b - 3 \, C a b^{2} - B b^{3}\right)} {\left(d x + c\right)} + 192 \, {\left(B a^{3} - 3 \, C a^{2} b - 3 \, B a b^{2} + C b^{3}\right)} \log\left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 1\right) - 192 \, {\left(B a^{3} - 3 \, C a^{2} b - 3 \, B a b^{2} + C b^{3}\right)} \log\left({\left| \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) \right|}\right) + \frac{400 \, B a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 1200 \, C a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 1200 \, B a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} + 400 \, C b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 120 \, C a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 360 \, B a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 288 \, C a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 96 \, B b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 36 \, B a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 72 \, C a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 72 \, B a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 8 \, C a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 24 \, B a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 3 \, B a^{3}}{\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4}}}{192 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] -1/192*(3*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 2
4*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*C*a
^2*b*tan(1/2*d*x + 1/2*c)^2 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*C*a^3
*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) - 288*C*a*b^2*tan(
1/2*d*x + 1/2*c) - 96*B*b^3*tan(1/2*d*x + 1/2*c) - 192*(C*a^3 + 3*B*a^2*b -
 3*C*a*b^2 - B*b^3)*(d*x + c) + 192*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)
*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b
^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 12
00*C*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 1200*B*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 4
```

```
00*C*b^3*tan(1/2*d*x + 1/2*c)^4 - 120*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 360*B*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 288*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 96*B*b^
3*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*C*a^2*b*tan
(1/2*d*x + 1/2*c)^2 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*C*a^3*tan(1/2*d
*x + 1/2*c) + 24*B*a^2*b*tan(1/2*d*x + 1/2*c) + 3*B*a^3)/tan(1/2*d*x + 1/2*
c)^4)/d
```

Mupad [B]
time = 8.94, size = 204, normalized size = 1.07

$$\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,a^3-3\,C\,a^2\,b-3\,B\,a\,b^2+C\,b^3\right)}{d}-\frac{\cot(c+d\,x)^4\,\left(\tan(c+d\,x)\,\left(\frac{C\,a^3}{3}+B\,b\,a^2\right)+\frac{B\,a^3}{4}+\tan(c+d\,x)^2\,\left(-\frac{B\,a^3}{2}+\frac{3\,C\,a^2\,b}{2}+\frac{3\,B\,a\,b^2}{2}\right)+\tan(c+d\,x)^3\,\left(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3\right)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{2\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,{\left(-b+a\,1\mathrm{i}\right)}^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^3,x)
```

```
[Out] (log(tan(c + d*x))*(B*a^3 + C*b^3 - 3*B*a*b^2 - 3*C*a^2*b))/d - (cot(c + d*
x)^4*(tan(c + d*x)*((C*a^3)/3 + B*a^2*b) + (B*a^3)/4 + tan(c + d*x)^2*((3*B
*a*b^2)/2 - (B*a^3)/2 + (3*C*a^2*b)/2) + tan(c + d*x)^3*(B*b^3 - C*a^3 - 3*
B*a^2*b + 3*C*a*b^2)))/d - (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)^3*
1i)/(2*d) - (log(tan(c + d*x) - 1i)*(B + C*1i)*(a*1i - b)^3*1i)/(2*d)
```

3.24 $\int \cot^7(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan\right.$

Optimal. Leaf size=233

$$-\left(\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x\right)-\frac{\left(a^3B-3ab^2B-3a^2bC+b^3C\right)\cot(c+dx)}{d}+\frac{\left(3a^2bB-b^3B+a^3C-\right.}{2d}$$

```
[Out] -(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*x-(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*cot(
d*x+c)/d+1/2*(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*cot(d*x+c)^2/d+1/15*a*(5*B*a
^2-12*B*b^2-15*C*a*b)*cot(d*x+c)^3/d-1/20*a^2*(7*B*b+5*C*a)*cot(d*x+c)^4/d+
(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*ln(sin(d*x+c))/d-1/5*a*B*cot(d*x+c)^5*(a+
b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.39, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {3713, 3686, 3716, 3709, 3610, 3612, 3556}

$$\frac{a\left(5a^2B-15abC-12b^2B\right)\cot^3(c+dx)}{15d}-\frac{a^2(5aC+7bB)\cot^4(c+dx)}{20d}+\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\cot^2(c+dx)}{2d}-\frac{\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\cot(c+dx)}{d}+\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\log(\sin(c+dx))}{d}-x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)-\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] -((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x) - ((a^3*B - 3*a*b^2*B - 3*a^2*
b*C + b^3*C)*Cot[c + d*x])/d + ((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Cot
[c + d*x]^2)/(2*d) + (a*(5*a^2*B - 12*b^2*B - 15*a*b*C)*Cot[c + d*x]^3)/(15
*d) - (a^2*(7*b*B + 5*a*C)*Cot[c + d*x]^4)/(20*d) + ((3*a^2*b*B - b^3*B + a
^3*C - 3*a*b^2*C)*Log[Sin[c + d*x]])/d - (a*B*Cot[c + d*x]^5*(a + b*Tan[c +
 d*x])^2)/(5*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
```

```
-1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^7(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx &= \int \cot^6(c+dx)(a+b\tan(c+dx \\
&= -\frac{aB\cot^5(c+dx)(a+b\tan(c+}{5d} \\
&= -\frac{a^2(7bB+5aC)\cot^4(c+dx)}{20d} - \\
&= \frac{a(5a^2B-12b^2B-15abC)\cot^3(}{15d} \\
&= \frac{(3a^2bB-b^3B+a^3C-3ab^2C)\,c}{2d} \\
&= -\frac{(a^3B-3ab^2B-3a^2bC+b^3C}{d} \\
&= -\left(a^3B-3ab^2B-3a^2bC+b^3C\right. \\
&= -\left(a^3B-3ab^2B-3a^2bC+b^3C\right.
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.78, size = 237, normalized size = 1.02

$$\frac{-60(a^3B-3ab^2B-3a^2bC+b^3C)\cot(c+dx)+30(3a^2bB-b^3B+a^3C-3ab^2C)\cot^2(c+dx)+20a(a^2B-3b^2B-3abC)\cot^3(c+dx)-15a^2(3bB+aC)\cot^4(c+dx)-12a^3B\cot^5(c+dx)+30i(a+ib)^3(B+iC)\log(i-\tan(c+dx))+60(3a^2bB-b^3B+a^3C-3ab^2C)\log(\tan(c+dx))+30(ia+b)^3(B-iC)\log(i+\tan(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (-60*(a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*Cot[c + d*x] + 30*(3*a^2*b*B -
 b^3*B + a^3*C - 3*a*b^2*C)*Cot[c + d*x]^2 + 20*a*(a^2*B - 3*b^2*B - 3*a*b*
C)*Cot[c + d*x]^3 - 15*a^2*(3*b*B + a*C)*Cot[c + d*x]^4 - 12*a^3*B*Cot[c +
d*x]^5 + (30*I)*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 60*(3*a^2*b*B
 - b^3*B + a^3*C - 3*a*b^2*C)*Log[Tan[c + d*x]] + 30*(I*a + b)^3*(B - I*C)*
Log[I + Tan[c + d*x]])/(60*d)
```

Maple [A]
time = 0.34, size = 244, normalized size = 1.05 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(B*b^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+C*b^3*(-cot(d*x+c)-d*x-c)+3*B
*a*b^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*C*a*b^2*(-1/2*cot(d*x+c)^2-ln
(sin(d*x+c)))+3*B*a^2*b*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))
+3*C*a^2*b*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+B*a^3*(-1/5*cot(d*x+c)^5+1/
3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+C*a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+
ln(sin(d*x+c))))
```

Maxima [**A**]
time = 0.50, size = 250, normalized size = 1.07

$$-\frac{60\left(Ba^3-3Ca^2b-3Bab^2+Cb^3\right)(dx+c)+30\left(Ca^3+3Ba^2b-3Cab^2-Bb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)-60\left(Ca^3+3Ba^2b-3Cab^2-Bb^3\right)\log\left(\tan\left(dx+c\right)\right)+\frac{60\left(Ba^3-3Ca^2b-3Bab^2+Cb^3\right)\tan(dx+c)^4+12Ba^3-30\left(Ca^3+3Ba^2b-3Cab^2-Bb^3\right)\tan(dx+c)^3-20\left(Ba^3-3Ca^2b-3Bab^2\right)\tan(dx+c)^2+15\left(Ca^3+3Ba^2b\right)\tan(dx+c)}{\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] -1/60*(60*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 30*(C*a^3 + 3
*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) - 60*(C*a^3 + 3*B*a^2
*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)) + (60*(B*a^3 - 3*C*a^2*b - 3*B*a*
b^2 + C*b^3)*tan(d*x + c)^4 + 12*B*a^3 - 30*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2
- B*b^3)*tan(d*x + c)^3 - 20*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2)*tan(d*x + c)^2
 + 15*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x + c)^5)/d
```

Fricas [**A**]
time = 3.01, size = 266, normalized size = 1.14

$$\frac{30\left(Ca^3+3Ba^2b-3Cab^2-Bb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5+15\left(3Ca^3+9Ba^2b-6Cab^2-2Bb^3-4\left(Ba^3-3Ca^2b-3Bab^2+Cb^3\right)dx\right)\tan\left(dx+c\right)^5-60\left(Ba^3-3Ca^2b-3Bab^2+Cb^3\right)\tan\left(dx+c\right)^4-12Ba^3+30\left(Ca^3+3Ba^2b-3Cab^2-Bb^3\right)\tan\left(dx+c\right)^3+20\left(Ba^3-3Ca^2b-3Bab^2\right)\tan\left(dx+c\right)^2-15\left(Ca^3+3Ba^2b\right)\tan\left(dx+c\right)}{60\,d\tan\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] 1/60*(30*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d*
x + c)^2 + 1))*tan(d*x + c)^5 + 15*(3*C*a^3 + 9*B*a^2*b - 6*C*a*b^2 - 2*B*b
^3 - 4*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*x)*tan(d*x + c)^5 - 60*(B*
a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*tan(d*x + c)^4 - 12*B*a^3 + 30*(C*a^3
+ 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 + 20*(B*a^3 - 3*C*a^2*b - 3
*B*a*b^2)*tan(d*x + c)^2 - 15*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x
+ c)^5)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(231) = 462$.
time = 10.14, size = 469, normalized size = 2.01

$$\begin{cases} \text{NaN} & \text{for}\: \left(c = 0 \vee c = - d x\right) \wedge \left(c = - d x \vee d = 0\right) \\ x \left(a + b \tan{\left(c \right)}\right)^{3} \left(B \tan{\left(c \right)} + C \tan^{2}{\left(c \right)}\right) \cot^{7}{\left(c \right)} & \text{for}\: d = 0 \\ - B a^{3} x - \frac{B a^{3}}{d \tan{\left(c + d x \right)}} + \frac{B a^{3}}{3 d \tan^{3}{\left(c + d x \right)}} - \frac{B a^{3}}{5 d \tan^{5}{\left(c + d x \right)}} - \frac{3 B a^{2} b \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{3 B a^{2} b \log{\left(\tan{\left(c + d x \right)} \right)}}{d} + \frac{3 B a^{2} b}{2 d \tan^{2}{\left(c + d x \right)}} - \frac{3 B a^{2} b}{4 d \tan^{4}{\left(c + d x \right)}} + 3 B a b^{2} x + \frac{3 B a b^{2}}{d \tan{\left(c + d x \right)}} - \frac{B a b^{2}}{d \tan^{3}{\left(c + d x \right)}} + \frac{B b^{3} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - \frac{B b^{3} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} - \frac{B b^{3}}{2 d \tan^{2}{\left(c + d x \right)}} - \frac{C a^{3} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} + \frac{C a^{3} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} + \frac{C a^{3}}{2 d \tan^{2}{\left(c + d x \right)}} - \frac{C a^{3}}{4 d \tan^{4}{\left(c + d x \right)}} + 3 C a^{2} b x + \frac{3 C a^{2} b}{d \tan{\left(c + d x \right)}} - \frac{C a^{2} b}{d \tan^{3}{\left(c + d x \right)}} + \frac{3 C a b^{2} \log{\left(\tan^{2}{\left(c + d x \right)} + 1 \right)}}{2 d} - \frac{3 C a b^{2} \log{\left(\tan{\left(c + d x \right)} \right)}}{d} - \frac{3 C a b^{2}}{2 d \tan^{2}{\left(c + d x \right)}} - C b^{3} x - \frac{C b^{3}}{d \tan{\left(c + d x \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Piecewise((nan, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x*(a
 + b*tan(c))**3*(B*tan(c) + C*tan(c)**2)*cot(c)**7, Eq(d, 0)), (-B*a**3*x -
 B*a**3/(d*tan(c + d*x)) + B*a**3/(3*d*tan(c + d*x)**3) - B*a**3/(5*d*tan(c
 + d*x)**5) - 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*a**2*b*log(ta
n(c + d*x))/d + 3*B*a**2*b/(2*d*tan(c + d*x)**2) - 3*B*a**2*b/(4*d*tan(c +
d*x)**4) + 3*B*a*b**2*x + 3*B*a*b**2/(d*tan(c + d*x)) - B*a*b**2/(d*tan(c +
 d*x)**3) + B*b**3*log(tan(c + d*x)**2 + 1)/(2*d) - B*b**3*log(tan(c + d*x)
)/d - B*b**3/(2*d*tan(c + d*x)**2) - C*a**3*log(tan(c + d*x)**2 + 1)/(2*d)
+ C*a**3*log(tan(c + d*x))/d + C*a**3/(2*d*tan(c + d*x)**2) - C*a**3/(4*d*t
an(c + d*x)**4) + 3*C*a**2*b*x + 3*C*a**2*b/(d*tan(c + d*x)) - C*a**2*b/(d*
tan(c + d*x)**3) + 3*C*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - 3*C*a*b**2*l
og(tan(c + d*x))/d - 3*C*a*b**2/(2*d*tan(c + d*x)**2) - C*b**3*x - C*b**3/(
d*tan(c + d*x)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(225) = 450.
time = 1.66, size = 670, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] 1/960*(6*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^3*tan(1/2*d*x + 1/2*c)^4 - 4
5*B*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 70*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 120*C*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 120*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 180*C*a
^3*tan(1/2*d*x + 1/2*c)^2 + 540*B*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 360*C*a*b^
2*tan(1/2*d*x + 1/2*c)^2 - 120*B*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*B*a^3*tan
(1/2*d*x + 1/2*c) - 1800*C*a^2*b*tan(1/2*d*x + 1/2*c) - 1800*B*a*b^2*tan(1/
2*d*x + 1/2*c) + 480*C*b^3*tan(1/2*d*x + 1/2*c) - 960*(B*a^3 - 3*C*a^2*b -
3*B*a*b^2 + C*b^3)*(d*x + c) - 960*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*
log(tan(1/2*d*x + 1/2*c)^2 + 1) + 960*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^
3)*log(abs(tan(1/2*d*x + 1/2*c))) - (2192*C*a^3*tan(1/2*d*x + 1/2*c)^5 + 65
76*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 6576*C*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
192*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 660*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 1800*
C*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 1800*B*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 480*
C*b^3*tan(1/2*d*x + 1/2*c)^4 - 180*C*a^3*tan(1/2*d*x + 1/2*c)^3 - 540*B*a^2
*b*tan(1/2*d*x + 1/2*c)^3 + 360*C*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*B*b^3*
tan(1/2*d*x + 1/2*c)^3 - 70*B*a^3*tan(1/2*d*x + 1/2*c)^2 + 120*C*a^2*b*tan(
1/2*d*x + 1/2*c)^2 + 120*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*C*a^3*tan(1/2*
```

```
d*x + 1/2*c) + 45*B*a^2*b*tan(1/2*d*x + 1/2*c) + 6*B*a^3)/tan(1/2*d*x + 1/2
*c)^5)/d
```

Mupad [B]
time = 9.12, size = 238, normalized size = 1.02

$$-\frac{\cot(c+d\,x)^5\,\left(\tan(c+d\,x)\,\left(\frac{C\,a^3}{4}+\frac{3\,B\,b\,a^2}{4}\right)+\frac{B\,a^3}{5}+\tan(c+d\,x)^2\,\left(-\frac{B\,a^3}{3}+C\,a^2\,b+B\,a\,b^2\right)+\tan(c+d\,x)^4\,(B\,a^3-3\,C\,a^2\,b-3\,B\,a\,b^2+C\,b^3)+\tan(c+d\,x)^3\,\left(-\frac{C\,a^3}{2}-\frac{3\,B\,a^2\,b}{2}+\frac{3\,C\,a\,b^2}{2}+\frac{B\,b^3}{2}\right)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)\right)\,(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,(B+C\,1\mathrm{i})\,(a+b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,(B-C\,1\mathrm{i})\,(a-b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))
^3,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(B + C*1i)*(a + b*1i)^3*1i)/(2*d) - (log(tan(c + d*
x))*(B*b^3 - C*a^3 - 3*B*a^2*b + 3*C*a*b^2))/d - (cot(c + d*x)^5*(tan(c + d
*x)*((C*a^3)/4 + (3*B*a^2*b)/4) + (B*a^3)/5 + tan(c + d*x)^2*(B*a*b^2 - (B*
a^3)/3 + C*a^2*b) + tan(c + d*x)^4*(B*a^3 + C*b^3 - 3*B*a*b^2 - 3*C*a^2*b)
+ tan(c + d*x)^3*((B*b^3)/2 - (C*a^3)/2 - (3*B*a^2*b)/2 + (3*C*a*b^2)/2)))/
d - (log(tan(c + d*x) + 1i)*(B - C*1i)*(a - b*1i)^3*1i)/(2*d)
```

3.25 $\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=127

$$-\frac{(bB-aC)x}{a^2+b^2}+\frac{(aB+bC)\log(\cos(c+dx))}{(a^2+b^2)\,d}-\frac{a^3(bB-aC)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)\,d}+\frac{(bB-aC)\tan(c+dx)}{b^2d}+$$

```
[Out] -(B*b-C*a)*x/(a^2+b^2)+(B*a+C*b)*ln(cos(d*x+c))/(a^2+b^2)/d-a^3*(B*b-C*a)*l
n(a+b*tan(d*x+c))/b^3/(a^2+b^2)/d+(B*b-C*a)*tan(d*x+c)/b^2/d+1/2*C*tan(d*x+
c)^2/b/d
```

Rubi [A]
time = 0.32, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {3713, 3688, 3728, 3707, 3698, 31, 3556}

$$\frac{(aB+bC)\log(\cos(c+dx))}{d\,(a^2+b^2)}-\frac{x(bB-aC)}{a^2+b^2}-\frac{a^3(bB-aC)\log(a+b\tan(c+dx))}{b^3d\,(a^2+b^2)}+\frac{(bB-aC)\tan(c+dx)}{b^2d}+\frac{C\tan^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
]),x]
```

```
[Out] -(((b*B - a*C)*x)/(a^2 + b^2)) + ((a*B + b*C)*Log[Cos[c + d*x]])/((a^2 + b^
2)*d) - (a^3*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)*d) + ((b
*B - a*C)*Tan[c + d*x])/(b^2*d) + (C*Tan[c + d*x]^2)/(2*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3688

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan
[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
+ n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b*
(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1] &
& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$= \frac{C\tan^2(c+dx)}{2bd} + \frac{\int \frac{\tan(c+dx)(-2aC-2bC\tan(c+dx)+2(bB-}{a+b\tan(c+dx)}}{2b}$$

$$= \frac{(bB-aC)\tan(c+dx)}{b^2d} + \frac{C\tan^2(c+dx)}{2bd} + \frac{\int \frac{-2a(bB}{}}{}$$

$$= -\frac{(bB-aC)x}{a^2+b^2} + \frac{(bB-aC)\tan(c+dx)}{b^2d} + \frac{C\tan^2(c}{2bd}$$

$$= -\frac{(bB-aC)x}{a^2+b^2} + \frac{(aB+bC)\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{(bB}{}$$

$$= -\frac{(bB-aC)x}{a^2+b^2} + \frac{(aB+bC)\log(\cos(c+dx))}{(a^2+b^2)\,d} - \frac{a^3(b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.99, size = 138, normalized size = 1.09

$$\frac{-\frac{b(B+iC)\log(i-\tan(c+dx))}{a+ib} - \frac{b(B-iC)\log(i+\tan(c+dx))}{a-ib} + \frac{2a^3(-bB+aC)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)} + \frac{2(bB-aC)\tan(c+dx)}{b} + C\tan^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x]),x]
```

```
[Out] (-((b*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)) - (b*(B - I*C)*Log[I + Ta
n[c + d*x]])/(a - I*b) + (2*a^3*(-(b*B) + a*C)*Log[a + b*Tan[c + d*x]])/(b^
2*(a^2 + b^2)) + (2*(b*B - a*C)*Tan[c + d*x])/b + C*Tan[c + d*x]^2)/(2*b*d)
```

Maple [A]
time = 0.19, size = 127, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{Cb\left(\tan^2(dx+c)\right)}{2}+bB\tan(dx+c)-C\tan(dx+c)a}{b^2}+\frac{\frac{(-aB-Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-Bb+Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3(Bb-Ca)\ln(a+}{b^3\left(a^2+\right.}}{d}$
default	$\frac{\frac{Cb\left(\tan^2(dx+c)\right)}{2}+bB\tan(dx+c)-C\tan(dx+c)a}{b^2}+\frac{\frac{(-aB-Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-Bb+Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3(Bb-Ca)\ln(a+}{b^3\left(a^2+\right.}}{d}$
norman	$\frac{(Bb-Ca)\tan(dx+c)}{b^2d}-\frac{(Bb-Ca)x}{a^2+b^2}+\frac{C\left(\tan^2(dx+c)\right)}{2bd}-\frac{(aB+Cb)\ln\left(1+\tan^2(dx+c)\right)}{2d(a^2+b^2)}-\frac{a^3(Bb-Ca)\ln(a+b\tan(d}{b^3(a^2+b^2)d}$

risch	$-\frac{2iaBc}{b^2d}-\frac{xC}{ib-a}+\frac{2iC\,a^2x}{b^3}-\frac{2ia^4Cc}{(a^2+b^2)\,b^3d}+\frac{2ia^3Bx}{b^2(a^2+b^2)}-\frac{2iCc}{bd}+\frac{2i\left(-iCb\,\mathrm{e}^{2i(dx+c)}+Bb\,\mathrm{e}^{2i(dx+c)}-Ca\,\mathrm{e}^{2i(d}\right.}{b^2d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*(1/2*C*tan(d*x+c)^2*b+b*B*tan(d*x+c)-C*tan(d*x+c)*a)+1/(a^2+b^2)
*(1/2*(-B*a-C*b)*ln(1+tan(d*x+c)^2)+(-B*b+C*a)*arctan(tan(d*x+c)))-1/b^3*a^
3*(B*b-C*a)/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 130, normalized size = 1.02

$$\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}+\frac{2\,(Ca^4-Ba^3b)\log(b\tan(dx+c)+a)}{a^2b^3+b^5}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{Cb\tan(dx+c)^2-2\,(Ca-Bb)\tan(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a^4 - B*a^3*b)*log(b*tan(d*
x + c) + a)/(a^2*b^3 + b^5) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^
2) + (C*b*tan(d*x + c)^2 - 2*(C*a - B*b)*tan(d*x + c))/b^2)/d
```

Fricas [A]
time = 4.86, size = 190, normalized size = 1.50

$$\frac{2\,(Cab^3-Bb^4)dx+(Ca^2b^2+Cb^4)\tan\left(dx+c\right)^2+(Ca^4-Ba^3b)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(Ca^4-Ba^3b-Bab^3-Cb^4)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\,(Ca^3b-Ba^2b^2+Cab^3-Bb^4)\tan\left(dx+c\right)}{2\,(a^2b^3+b^5)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(C*a*b^3 - B*b^4)*d*x + (C*a^2*b^2 + C*b^4)*tan(d*x + c)^2 + (C*a^4
- B*a^3*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c
)^2 + 1)) - (C*a^4 - B*a^3*b - B*a*b^3 - C*b^4)*log(1/(tan(d*x + c)^2 + 1))
 - 2*(C*a^3*b - B*a^2*b^2 + C*a*b^3 - B*b^4)*tan(d*x + c))/((a^2*b^3 + b^5)
*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.82, size = 1306, normalized size = 10.28

$$\begin{cases} \tilde{\infty}x(B\tan(c)+C\tan^2(c))\tan(c) & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{-\frac{B\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{B\tan^2(c+dx)}{2d}+Cx+\frac{C\tan^3(c+dx)}{3d}-\frac{C\tan(c+dx)}{d}}{a} & \text{for } b=0 \\ -\frac{3Bdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{3iBdx}{2bd\tan(c+dx)-2ibd}+\frac{iB\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{B\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)-2ibd}+\frac{2B\tan^2(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{3B}{2bd\tan(c+dx)-2ibd}-\frac{3iCdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{3Cdx}{2bd\tan(c+dx)-2ibd}-\frac{2C\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{2iC\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)-2ibd}+\frac{C\tan^3(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{iC\tan^2(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{3iC}{2bd\tan(c+dx)-2ibd} & \text{for } a=-ib \\ -\frac{3Bdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{3iBdx}{2bd\tan(c+dx)+2ibd}-\frac{iB\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{B\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)+2ibd}+\frac{2B\tan^2(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{3B}{2bd\tan(c+dx)+2ibd}+\frac{3iCdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{3Cdx}{2bd\tan(c+dx)+2ibd}-\frac{2C\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{2iC\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)+2ibd}+\frac{C\tan^3(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{iC\tan^2(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{3iC}{2bd\tan(c+dx)+2ibd} & \text{for } a=ib \\ \frac{x(B\tan(c)+C\tan^2(c))\tan^2(c)}{a+b\tan(c)} & \text{for } d=0 \\ -\frac{2Ba^3b\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2b^3d+2b^5d}+\frac{2Ba^2b^2\tan(c+dx)}{2a^2b^3d+2b^5d}-\frac{Bab^3\log\left(\tan^2(c+dx)+1\right)}{2a^2b^3d+2b^5d}-\frac{2Bb^4dx}{2a^2b^3d+2b^5d}+\frac{2Bb^4\tan(c+dx)}{2a^2b^3d+2b^5d}+\frac{2Ca^4\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2b^3d+2b^5d}-\frac{2Ca^3b\tan(c+dx)}{2a^2b^3d+2b^5d}+\frac{Ca^2b^2\tan^2(c+dx)}{2a^2b^3d+2b^5d}+\frac{2Cab^3dx}{2a^2b^3d+2b^5d}-\frac{2Cab^3\tan(c+dx)}{2a^2b^3d+2b^5d}-\frac{Cb^4\log\left(\tan^2(c+dx)+1\right)}{2a^2b^3d+2b^5d}+\frac{Cb^4\tan^2(c+dx)}{2a^2b^3d+2b^5d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(
d, 0)), ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d*x)**2/(2*d) + C*x
 + C*tan(c + d*x)**3/(3*d) - C*tan(c + d*x)/d)/a, Eq(b, 0)), (-3*B*d*x*tan(
c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*B*d*x/(2*b*d*tan(c + d*x) - 2
*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2
*I*b*d) + B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*B*t
an(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*B/(2*b*d*tan(c + d*x) - 2
*I*b*d) - 3*I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - 3*C*d*x/(
2*b*d*tan(c + d*x) - 2*I*b*d) - 2*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(
2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c
 + d*x) - 2*I*b*d) + C*tan(c + d*x)**3/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*C
*tan(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*C/(2*b*d*tan(c + d*x)
 - 2*I*b*d), Eq(a, -I*b)), (-3*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I
*b*d) - 3*I*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*B*log(tan(c + d*x)**2
+ 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*log(tan(c + d*x)**2 +
1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x)
 + 2*I*b*d) + 3*B/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3*I*C*d*x*tan(c + d*x)/(
2*b*d*tan(c + d*x) + 2*I*b*d) - 3*C*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*
C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*
I*C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*tan(c + d*x
)**3/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*C*tan(c + d*x)**2/(2*b*d*tan(c + d*
x) + 2*I*b*d) - 3*I*C/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(B*ta
n(c) + C*tan(c)**2)*tan(c)**2/(a + b*tan(c)), Eq(d, 0)), (-2*B*a**3*b*log(a
/b + tan(c + d*x))/(2*a**2*b**3*d + 2*b**5*d) + 2*B*a**2*b**2*tan(c + d*x)/
(2*a**2*b**3*d + 2*b**5*d) - B*a*b**3*log(tan(c + d*x)**2 + 1)/(2*a**2*b**3
*d + 2*b**5*d) - 2*B*b**4*d*x/(2*a**2*b**3*d + 2*b**5*d) + 2*B*b**4*tan(c +
 d*x)/(2*a**2*b**3*d + 2*b**5*d) + 2*C*a**4*log(a/b + tan(c + d*x))/(2*a**2
*b**3*d + 2*b**5*d) - 2*C*a**3*b*tan(c + d*x)/(2*a**2*b**3*d + 2*b**5*d) +
C*a**2*b**2*tan(c + d*x)**2/(2*a**2*b**3*d + 2*b**5*d) + 2*C*a*b**3*d*x/(2*
a**2*b**3*d + 2*b**5*d) - 2*C*a*b**3*tan(c + d*x)/(2*a**2*b**3*d + 2*b**5*d
) - C*b**4*log(tan(c + d*x)**2 + 1)/(2*a**2*b**3*d + 2*b**5*d) + C*b**4*tan
(c + d*x)**2/(2*a**2*b**3*d + 2*b**5*d), True))
```

Giac [A]
time = 0.79, size = 135, normalized size = 1.06

$$\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Ca^4-Ba^3b)\log(|b\tan(dx+c)+a|)}{a^2b^3+b^5}+\frac{Cb\tan(dx+c)^2-2\,Ca\tan(dx+c)+2\,Bb\tan(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2 +
 1)/(a^2 + b^2) + 2*(C*a^4 - B*a^3*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b^3
 + b^5) + (C*b*tan(d*x + c)^2 - 2*C*a*tan(d*x + c) + 2*B*b*tan(d*x + c))/b^
2)/d
```

Mupad [B]
time = 9.07, size = 144, normalized size = 1.13

$$\frac{\tan(c+d\,x)\,\left(\frac{B}{b}-\frac{C\,a}{b^2}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^4-B\,a^3\,b\right)}{d\,\left(a^2\,b^3+b^5\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)}+\frac{C\tan(c+d\,x)^2}{2\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
)),x)
```

```
[Out] (tan(c + d*x)*(B/b - (C*a)/b^2))/d - (log(tan(c + d*x) - 1i)*(B*1i - C))/(2
*d*(a*1i - b)) + (log(a + b*tan(c + d*x))*(C*a^4 - B*a^3*b))/(d*(b^5 + a^2*
b^3)) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a - b*1i)) + (C*tan(c + d
*x)^2)/(2*b*d)
```

3.26 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=101

$$-\frac{(aB+bC)x}{a^2+b^2}-\frac{(bB-aC)\log(\cos(c+dx))}{(a^2+b^2)\,d}+\frac{a^2(bB-aC)\log(a+b\tan(c+dx))}{b^2\,(a^2+b^2)\,d}+\frac{C\tan(c+dx)}{bd}$$

```
[Out] -(B*a+C*b)*x/(a^2+b^2)-(B*b-C*a)*ln(cos(d*x+c))/(a^2+b^2)/d+a^2*(B*b-C*a)*l
n(a+b*tan(d*x+c))/b^2/(a^2+b^2)/d+C*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.17, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3713, 3687, 3707, 3698, 31, 3556}

$$\frac{a^2(bB-aC)\log(a+b\tan(c+dx))}{b^2d\,(a^2+b^2)}-\frac{(bB-aC)\log(\cos(c+dx))}{d\,(a^2+b^2)}-\frac{x(aB+bC)}{a^2+b^2}+\frac{C\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
,x]
```

```
[Out] -(((a*B + b*C)*x)/(a^2 + b^2)) - ((b*B - a*C)*Log[Cos[c + d*x]])/((a^2 + b^
2)*d) + (a^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)*d) + (C*
Tan[c + d*x])/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3687

```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (
f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b^2
*B*(Tan[e + f*x]/(d*f)), x] + Dist[1/d, Int[(a^2*A*d - b^2*B*c + (2*a*A*b +
 B*(a^2 - b^2))*d*Tan[e + f*x] + (A*b^2*d - b*B*(b*c - 2*a*d))*Tan[e + f*x]
^2)/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx=\int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{C\tan(c+dx)}{bd}+\frac{\int \frac{-aC-bC\tan(c+dx)+(bB-aC)\tan^2(c+dx)}{a+b\tan(c+dx)}}{b}$$
$$=-\frac{(aB+bC)x}{a^2+b^2}+\frac{C\tan(c+dx)}{bd}+\frac{(bB-aC)\int\tan($$
$$=-\frac{(aB+bC)x}{a^2+b^2}-\frac{(bB-aC)\log(\cos(c+dx))}{(a^2+b^2)\,d}+\frac{C\,\mathrm{t}}{}$$
$$=-\frac{(aB+bC)x}{a^2+b^2}-\frac{(bB-aC)\log(\cos(c+dx))}{(a^2+b^2)\,d}+\frac{a^2(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.49, size = 118, normalized size = 1.17

$$\frac{\frac{i(B+iC)\log(i-\tan(c+dx))}{a+ib}-\frac{(iB+C)\log(i+\tan(c+dx))}{a-ib}+\frac{2a^2(bB-aC)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)}+\frac{2C\tan(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x]),x]

[Out] ((I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) - ((I*B + C)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*a^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)) + (2*C*Tan[c + d*x])/b)/(2*d)

Maple [A]
time = 0.18, size = 101, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{C\tan(dx+c)}{b}+\frac{a^2(Bb-Ca)\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}+\frac{\frac{(Bb-Ca)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-aB-Cb)\arctan(\tan(dx+c))}{a^2+b^2}}{d}$
default	$\frac{\frac{C\tan(dx+c)}{b}+\frac{a^2(Bb-Ca)\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}+\frac{\frac{(Bb-Ca)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-aB-Cb)\arctan(\tan(dx+c))}{a^2+b^2}}{d}$
norman	$\frac{C\tan(dx+c)}{bd}-\frac{(aB+Cb)x}{a^2+b^2}+\frac{a^2(Bb-Ca)\ln(a+b\tan(dx+c))}{b^2(a^2+b^2)d}+\frac{(Bb-Ca)\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}$
risch	$\frac{xB}{ib-a}-\frac{ixC}{ib-a}-\frac{2ia^2Bx}{b(a^2+b^2)}-\frac{2ia^2Bc}{bd(a^2+b^2)}+\frac{2ia^3Cx}{b^2(a^2+b^2)}+\frac{2ia^3Cc}{b^2d(a^2+b^2)}+\frac{2iBx}{b}+\frac{2iBc}{bd}-\frac{2iCax}{b^2}-\frac{2iCac}{b^2d}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(C/b*tan(d*x+c)+1/b^2*a^2*(B*b-C*a)/(a^2+b^2)*ln(a+b*tan(d*x+c))+1/(a^2+b^2)*(1/2*(B*b-C*a)*ln(1+tan(d*x+c)^2)+(-B*a-C*b)*arctan(tan(d*x+c))))

Maxima [A]
time = 0.51, size = 109, normalized size = 1.08

$$-\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{2\,(Ca^3-Ba^2b)\,\log(b\tan(dx+c)+a)}{a^2b^2+b^4}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,C\tan(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a^3 - B*a^2*b)*log(b*tan(d*x + c) + a)/(a^2*b^2 + b^4) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*C*tan(d*x + c)/b)/d

Fricas [A]
time = 4.22, size = 149, normalized size = 1.48

$$-\frac{2\left(Bab^2+Cb^3\right)dx+\left(Ca^3-Ba^2b\right)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-\left(Ca^3-Ba^2b+Cab^2-Bb^3\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\left(Ca^2b+Cb^3\right)\tan\left(dx+c\right)}{2\left(a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] -1/2*(2*(B*a*b^2 + C*b^3)*d*x + (C*a^3 - B*a^2*b)*log((b^2*tan(d*x + c)^2 +
 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (C*a^3 - B*a^2*b + C*a*b
^2 - B*b^3)*log(1/(tan(d*x + c)^2 + 1)) - 2*(C*a^2*b + C*b^3)*tan(d*x + c))
/((a^2*b^2 + b^4)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.64, size = 1020, normalized size = 10.10

$$\begin{cases} \tilde{\infty}x\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right) & \text{for}\ a=0\wedge b=0\wedge d=0\\ \frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{Bdx}{2bd\tan(c+dx)-2ibd}+\frac{B\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{iB\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)-2ibd}-\frac{iB}{2bd\tan(c+dx)-2ibd}-\frac{3Cdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{3iCdx}{2bd\tan(c+dx)-2ibd}+\frac{iC\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{C\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)-2ibd}+\frac{2C\tan^2(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{3C}{2bd\tan(c+dx)-2ibd} & \text{for}\ a=-ib\\ -\frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{Bdx}{2bd\tan(c+dx)+2ibd}+\frac{B\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{iB\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)+2ibd}+\frac{iB}{2bd\tan(c+dx)+2ibd}-\frac{3Cdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{3iCdx}{2bd\tan(c+dx)+2ibd}-\frac{iC\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{C\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)+2ibd}+\frac{2C\tan^2(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{3C}{2bd\tan(c+dx)+2ibd} & \text{for}\ a=ib\\ \frac{-Bx+\frac{B\tan(c+dx)}{d}-\frac{C\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{C\tan^2(c+dx)}{2d}}{a} & \text{for}\ b=0\\ \frac{x\left(B\tan(c)+C\tan^2(c)\right)\tan(c)}{a+b\tan(c)} & \text{for}\ d=0\\ \frac{2Ba^2b\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2b^2d+2b^4d}-\frac{2Bab^2dx}{2a^2b^2d+2b^4d}+\frac{Bb^3\log\left(\tan^2(c+dx)+1\right)}{2a^2b^2d+2b^4d}-\frac{2Ca^3\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2b^2d+2b^4d}+\frac{2Ca^2b\tan(c+dx)}{2a^2b^2d+2b^4d}-\frac{Cab^2\log\left(\tan^2(c+dx)+1\right)}{2a^2b^2d+2b^4d}-\frac{2Cb^3dx}{2a^2b^2d+2b^4d}+\frac{2Cb^3\tan(c+dx)}{2a^2b^2d+2b^4d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)),
 (I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*d*x/(2*b*d*tan(c
+ d*x) - 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c +
d*x) - 2*I*b*d) - I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*
d) - I*B/(2*b*d*tan(c + d*x) - 2*I*b*d) - 3*C*d*x*tan(c + d*x)/(2*b*d*tan(c
 + d*x) - 2*I*b*d) + 3*I*C*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*C*log(tan
(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + C*log(tan(c
 + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*C*tan(c + d*x)**2/(2*b*d
*tan(c + d*x) - 2*I*b*d) + 3*C/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b))
, (-I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(
c + d*x) + 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c
+ d*x) + 2*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*
b*d) + I*B/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*C*d*x*tan(c + d*x)/(2*b*d*tan
(c + d*x) + 2*I*b*d) - 3*I*C*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*C*log(t
an(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*log(tan
(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*C*tan(c + d*x)**2/(2*b
*d*tan(c + d*x) + 2*I*b*d) + 3*C/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)
), ((-B*x + B*tan(c + d*x)/d - C*log(tan(c + d*x)**2 + 1)/(2*d) + C*tan(c +
 d*x)**2/(2*d))/a, Eq(b, 0)), (x*(B*tan(c) + C*tan(c)**2)*tan(c)/(a + b*tan
(c)), Eq(d, 0)), (2*B*a**2*b*log(a/b + tan(c + d*x))/(2*a**2*b**2*d + 2*b**
```

```
4*d) - 2*B*a*b**2*d*x/(2*a**2*b**2*d + 2*b**4*d) + B*b**3*log(tan(c + d*x)*
*2 + 1)/(2*a**2*b**2*d + 2*b**4*d) - 2*C*a**3*log(a/b + tan(c + d*x))/(2*a*
*2*b**2*d + 2*b**4*d) + 2*C*a**2*b*tan(c + d*x)/(2*a**2*b**2*d + 2*b**4*d)
- C*a*b**2*log(tan(c + d*x)**2 + 1)/(2*a**2*b**2*d + 2*b**4*d) - 2*C*b**3*d
*x/(2*a**2*b**2*d + 2*b**4*d) + 2*C*b**3*tan(c + d*x)/(2*a**2*b**2*d + 2*b*
*4*d), True))
```

Giac [A]
time = 0.69, size = 110, normalized size = 1.09

$$-\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Ca^3-Ba^2b)\log(|b\tan(dx+c)+a|)}{a^2b^2+b^4}-\frac{2\,C\tan(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="giac")
```

```
[Out] -1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) + 2*(C*a^3 - B*a^2*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b^
2 + b^4) - 2*C*tan(d*x + c)/b)/d
```

Mupad [B]
time = 8.77, size = 117, normalized size = 1.16

$$\frac{C\tan(c+d\,x)}{b\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^3-B\,a^2\,b\right)}{d\,\left(a^2\,b^2+b^4\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x))
,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*1i + b)) - (log(a + b*tan(c + d
*x))*(C*a^3 - B*a^2*b))/(d*(b^4 + a^2*b^2)) + (C*tan(c + d*x))/(b*d) + (log
(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a + b*1i))
```

3.27 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{(bB-aC)x}{a^2+b^2}-\frac{(aB+bC)\log(\cos(c+dx))}{(a^2+b^2)\,d}-\frac{a(bB-aC)\log(a+b\tan(c+dx))}{b\,(a^2+b^2)\,d}$$

```
[Out] (B*b-C*a)*x/(a^2+b^2)-(B*a+C*b)*ln(cos(d*x+c))/(a^2+b^2)/d-a*(B*b-C*a)*ln(a
+b*tan(d*x+c))/b/(a^2+b^2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1643, 649, 209, 266}

$$-\frac{a(bB-aC)\log(a+b\tan(c+dx))}{bd\,(a^2+b^2)}-\frac{(aB+bC)\log(\cos(c+dx))}{d\,(a^2+b^2)}+\frac{x(bB-aC)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((b*B - a*C)*x)/(a^2 + b^2) - ((a*B + b*C)*Log[Cos[c + d*x]])/((a^2 + b^2)*
d) - (a*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x(B+Cx)}{(a+bx)(1+x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a(-bB+aC)}{(a^2+b^2)(a+bx)} + \frac{bB-aC+(aB+bC)x}{(a^2+b^2)(1+x^2)}\right)\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{a(bB-aC)\log(a+b\tan(c+dx))}{b(a^2+b^2)d} + \frac{\text{Subst}\left(\int \frac{bB-aC+(aB+bC)x}{1+x^2}\,dx\right.}{(a^2+b^2)d}$$

$$= -\frac{a(bB-aC)\log(a+b\tan(c+dx))}{b(a^2+b^2)d} + \frac{(bB-aC)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.}{(a^2+b^2)d}$$

$$= \frac{(bB-aC)x}{a^2+b^2} - \frac{(aB+bC)\log(\cos(c+dx))}{(a^2+b^2)d} - \frac{a(bB-aC)\log(a+}{b(a^2+b^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 98, normalized size = 1.15

$$\frac{(a-ib)b(B+iC)\log(i-\tan(c+dx))+(a+ib)b(B-iC)\log(i+\tan(c+dx))+2a(-bB+aC)\log(a+b\tan(c+dx))}{2b(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a - I*b)*b*(B + I*C)*Log[I - Tan[c + d*x]] + (a + I*b)*b*(B - I*C)*Log[I
+ Tan[c + d*x]] + 2*a*(-(b*B) + a*C)*Log[a + b*Tan[c + d*x]])/(2*b*(a^2 + b
^2)*d)
```

Maple [A]
time = 0.18, size = 87, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{(aB+Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(Bb-Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a(Bb-Ca)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}}{d}$
default	$\frac{\frac{\frac{(aB+Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(Bb-Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a(Bb-Ca)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}}{d}$
norman	$\frac{(Bb-Ca)x}{a^2+b^2}+\frac{(aB+Cb)\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)}-\frac{a(Bb-Ca)\ln(a+b\tan(dx+c))}{b(a^2+b^2)d}$
risch	$\frac{ixB}{ib-a}+\frac{xC}{ib-a}+\frac{2iCx}{b}+\frac{2iCc}{bd}+\frac{2iBax}{a^2+b^2}+\frac{2iBac}{d(a^2+b^2)}-\frac{2ia^2Cx}{(a^2+b^2)b}-\frac{2ia^2Cc}{(a^2+b^2)bd}-\frac{\ln(e^{2i(dx+c)}+1)C}{bd}-\frac{\ln\left(e^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)*(1/2*(B*a+C*b)*ln(1+tan(d*x+c)^2)+(B*b-C*a)*arctan(tan(d*x
+c)))-a*(B*b-C*a)/(a^2+b^2)/b*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 94, normalized size = 1.11

$$-\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{2\,(Ca^2-Bab)\log(b\tan(dx+c)+a)}{a^2b+b^3}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a^2 - B*a*b)*log(b*tan(d*x
 + c) + a)/(a^2*b + b^3) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))
/d
```

Fricas [A]
time = 2.36, size = 110, normalized size = 1.29

$$-\frac{2\,(Cab-Bb^2)dx-(Ca^2-Bab)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+(Ca^2+Cb^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,(a^2b+b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*(2*(C*a*b - B*b^2)*d*x - (C*a^2 - B*a*b)*log((b^2*tan(d*x + c)^2 + 2*a
*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + (C*a^2 + C*b^2)*log(1/(tan(d
*x + c)^2 + 1)))/((a^2*b + b^3)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 711, normalized size = 8.36

$$\begin{cases} \frac{\tilde{\infty}x\left(B\tan(c)+C\tan^2(c)\right)}{\tan(c)} & \text{for}\: a=0\wedge b=0\wedge d=0 \\ \frac{Bdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{iBdx}{2bd\tan(c+dx)-2ibd}-\frac{B}{2bd\tan(c+dx)-2ibd}+\frac{iCdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{Cdx}{2bd\tan(c+dx)-2ibd}+\frac{C\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{iC\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)-2ibd}-\frac{iC}{2bd\tan(c+dx)-2ibd} & \text{for}\: a=-ib \\ \frac{Bdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{iBdx}{2bd\tan(c+dx)+2ibd}-\frac{B}{2bd\tan(c+dx)+2ibd}-\frac{iCdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{Cdx}{2bd\tan(c+dx)+2ibd}+\frac{C\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{iC\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)+2ibd}+\frac{iC}{2bd\tan(c+dx)+2ibd} & \text{for}\: a=ib \\ \frac{\frac{B\log\left(\tan^2(c+dx)+1\right)}{2d}-Cx+\frac{C\tan(c+dx)}{d}}{a} & \text{for}\: b=0 \\ \frac{x\left(B\tan(c)+C\tan^2(c)\right)}{a+b\tan(c)} & \text{for}\: d=0 \\ -\frac{2Bab\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2bd+2b^3d}+\frac{Bab\log\left(\tan^2(c+dx)+1\right)}{2a^2bd+2b^3d}+\frac{2Bb^2dx}{2a^2bd+2b^3d}+\frac{2Ca^2\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2bd+2b^3d}-\frac{2Cabdx}{2a^2bd+2b^3d}+\frac{Cb^2\log\left(\tan^2(c+dx)+1\right)}{2a^2bd+2b^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(
d, 0)), (B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B*d*x/(2*b*d
*tan(c + d*x) - 2*I*b*d) - B/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*C*d*x*tan(c
 + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + C*d*x/(2*b*d*tan(c + d*x) - 2*I*b*
d) + C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d)
 - I*C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*C/(2*b*d
*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (B*d*x*tan(c + d*x)/(2*b*d*tan(c +
d*x) + 2*I*b*d) + I*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - B/(2*b*d*tan(c +
 d*x) + 2*I*b*d) - I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*
d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*log(tan(c + d*x)**2 + 1)*tan(c + d*x
)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(
c + d*x) + 2*I*b*d) + I*C/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), ((B*
log(tan(c + d*x)**2 + 1)/(2*d) - C*x + C*tan(c + d*x)/d)/a, Eq(b, 0)), (x*(
B*tan(c) + C*tan(c)**2)/(a + b*tan(c)), Eq(d, 0)), (-2*B*a*b*log(a/b + tan(
c + d*x))/(2*a**2*b*d + 2*b**3*d) + B*a*b*log(tan(c + d*x)**2 + 1)/(2*a**2*
b*d + 2*b**3*d) + 2*B*b**2*d*x/(2*a**2*b*d + 2*b**3*d) + 2*C*a**2*log(a/b +
 tan(c + d*x))/(2*a**2*b*d + 2*b**3*d) - 2*C*a*b*d*x/(2*a**2*b*d + 2*b**3*d
) + C*b**2*log(tan(c + d*x)**2 + 1)/(2*a**2*b*d + 2*b**3*d), True))
```

Giac [A]
time = 0.65, size = 95, normalized size = 1.12

$$-\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,(Ca^2-Bab)\log(|b\tan(dx+c)+a|)}{a^2b+b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) - 2*(C*a^2 - B*a*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b +
b^3))/d
```

Mupad [B]
time = 9.07, size = 100, normalized size = 1.18

$$\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)}-\frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,b-C\,a\right)}{b\,d\,\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(c + d*x) + C*tan(c + d*x)^2)/(a + b*tan(c + d*x)),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*1i - b)) + (log(tan(c + d*x) +
1i)*(B - C*1i))/(2*d*(a - b*1i)) - (a*log(a + b*tan(c + d*x))*(B*b - C*a))/
(b*d*(a^2 + b^2))
```

3.28 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{(aB+bC)x}{a^2+b^2}+\frac{(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] (B*a+C*b)*x/(a^2+b^2)+(B*b-C*a)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {3713, 3612, 3611}

$$\frac{(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)}+\frac{x(aB+bC)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
,x]
```

```
[Out] ((a*B + b*C)*x)/(a^2 + b^2) + ((b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d
*x]])/((a^2 + b^2)*d)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
```

```
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \int \frac{B+C\tan(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{(aB+bC)x}{a^2+b^2} + \frac{(bB-aC)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{(aB+bC)x}{a^2+b^2} + \frac{(bB-aC)\log(a\cos(c+dx)+b\sin(c}{(a^2+b^2)\,d}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 1.16

$$\frac{-2(aB+bC)\mathrm{ArcTan}(\cot(c+dx))+(bB-aC)\left(2\log(b+a\cot(c+dx))-\log\left(\csc^2(c+dx)\right)\right)}{2\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x]),x]
```

```
[Out] (-2*(a*B + b*C)*ArcTan[Cot[c + d*x]] + (b*B - a*C)*(2*Log[b + a*Cot[c + d*x
]] - Log[Csc[c + d*x]^2]))/(2*(a^2 + b^2)*d)
```

Maple [A]
time = 0.37, size = 82, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{(Bb-Ca)\ln(a+b\tan(dx+c))}{a^2+b^2}+\frac{\frac{(-Bb+Ca)\ln\left(1+\tan^2(dx+c)\right)}{2}+(aB+Cb)\arctan(\tan(dx+c))}{a^2+b^2}}{d}$
default	$\frac{\frac{(Bb-Ca)\ln(a+b\tan(dx+c))}{a^2+b^2}+\frac{\frac{(-Bb+Ca)\ln\left(1+\tan^2(dx+c)\right)}{2}+(aB+Cb)\arctan(\tan(dx+c))}{a^2+b^2}}{d}$
norman	$\frac{(aB+Cb)x}{a^2+b^2}+\frac{(Bb-Ca)\ln(a+b\tan(dx+c))}{d(a^2+b^2)}-\frac{(Bb-Ca)\ln\left(1+\tan^2(dx+c)\right)}{2d(a^2+b^2)}$
risch	$-\frac{xB}{ib-a}+\frac{ixC}{ib-a}-\frac{2ibBx}{a^2+b^2}+\frac{2iCax}{a^2+b^2}-\frac{2ibBc}{d(a^2+b^2)}+\frac{2iCac}{d(a^2+b^2)}+\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)B}{d(a^2+b^2)}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right.}{d(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/d*((B*b-C*a)/(a^2+b^2)*ln(a+b*tan(d*x+c))+1/(a^2+b^2)*(1/2*(-B*b+C*a)*ln(
1+tan(d*x+c)^2)+(B*a+C*b)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.52, size = 88, normalized size = 1.52

$$\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}-\frac{2\,(Ca-Bb)\log(b\tan(dx+c)+a)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] 1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a - B*b)*log(b*tan(d*x + c)
 + a)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Fricas [A]
time = 1.65, size = 76, normalized size = 1.31

$$\frac{2\,(Ba+Cb)dx-(Ca-Bb)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\,(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] 1/2*(2*(B*a + C*b)*d*x - (C*a - B*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*
x + c) + a^2)/(tan(d*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.41, size = 541, normalized size = 9.33

$$\begin{cases} \frac{\tilde{\infty}x(B\tan(c)+C\tan^2(c))\cot(c)}{\tan(c)} & \text{for}\ a=0\wedge b=0\wedge d=0 \\ \frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{Bdx}{2bd\tan(c+dx)-2ibd}+\frac{iB}{2bd\tan(c+dx)-2ibd}+\frac{Cdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{iCdx}{2bd\tan(c+dx)-2ibd}-\frac{C}{2bd\tan(c+dx)-2ibd} & \text{for}\ a=-ib \\ -\frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{Bdx}{2bd\tan(c+dx)+2ibd}-\frac{iB}{2bd\tan(c+dx)+2ibd}+\frac{Cdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{iCdx}{2bd\tan(c+dx)+2ibd}-\frac{C}{2bd\tan(c+dx)+2ibd} & \text{for}\ a=ib \\ \frac{x(B\tan(c)+C\tan^2(c))\cot(c)}{a+b\tan(c)} & \text{for}\ d=0 \\ \frac{Bx+\frac{C\log\left(\tan^2(c+dx)+1\right)}{2d}}{a} & \text{for}\ b=0 \\ \frac{2Badx}{2a^2d+2b^2d}+\frac{2Bb\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2d+2b^2d}-\frac{Bb\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}-\frac{2Ca\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2d+2b^2d}+\frac{Ca\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}+\frac{2Cbdx}{2a^2d+2b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)/tan(c), Eq(a, 0) & Eq(b, 0
) & Eq(d, 0)), (I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*d*x
```

```
/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*B/(2*b*d*tan(c + d*x) - 2*I*b*d) + C*d*
x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*C*d*x/(2*b*d*tan(c + d*x)
 - 2*I*b*d) - C/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (-I*B*d*x*tan
(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) + 2*I*
b*d) - I*B/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*d*x*tan(c + d*x)/(2*b*d*tan(c
 + d*x) + 2*I*b*d) + I*C*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - C/(2*b*d*tan(
c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2)*cot(c)/(a + b
*tan(c)), Eq(d, 0)), ((B*x + C*log(tan(c + d*x)**2 + 1)/(2*d))/a, Eq(b, 0))
, (2*B*a*d*x/(2*a**2*d + 2*b**2*d) + 2*B*b*log(a/b + tan(c + d*x))/(2*a**2*
d + 2*b**2*d) - B*b*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d) - 2*C*a*
log(a/b + tan(c + d*x))/(2*a**2*d + 2*b**2*d) + C*a*log(tan(c + d*x)**2 + 1
)/(2*a**2*d + 2*b**2*d) + 2*C*b*d*x/(2*a**2*d + 2*b**2*d), True))
```

Giac [A]

time = 0.80, size = 94, normalized size = 1.62

$$\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,(Cab-Bb^2)\log(|b\tan(dx+c)+a|)}{a^2b+b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algo
rithm="giac")
```

```
[Out] 1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2 +
 1)/(a^2 + b^2) - 2*(C*a*b - B*b^2)*log(abs(b*tan(d*x + c) + a))/(a^2*b + b
^3))/d
```

Mupad [B]

time = 9.12, size = 93, normalized size = 1.60

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,b-C\,a\right)}{d\,\left(a^2+b^2\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x))
,x)
```

```
[Out] (log(a + b*tan(c + d*x))*(B*b - C*a))/(d*(a^2 + b^2)) - (log(tan(c + d*x) +
 1i)*(B - C*1i))/(2*d*(a*1i + b)) - (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*
d*(a + b*1i))
```

3.29 $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{(bB-aC)x}{a^2+b^2}+\frac{B\log(\sin(c+dx))}{ad}-\frac{b(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a\left(a^2+b^2\right)d}$$

```
[Out] -(B*b-C*a)*x/(a^2+b^2)+B*ln(sin(d*x+c))/a/d-b*(B*b-C*a)*ln(a*cos(d*x+c)+b*s
in(d*x+c))/a/(a^2+b^2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3713, 3692, 3611, 3556}

$$-\frac{b(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}-\frac{x(bB-aC)}{a^2+b^2}+\frac{B\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
]),x]
```

```
[Out] -(((b*B - a*C)*x)/(a^2 + b^2)) + (B*Log[Sin[c + d*x]])/(a*d) - (b*(b*B - a*
C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3692

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(B*(b*
c + a*d) + A*(a*c - b*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[b*((A*b
 - a*B)/((b*c - a*d)*(a^2 + b^2))), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e +
 f*x]), x], x] + Dist[d*((B*c - A*d)/((b*c - a*d)*(c^2 + d^2))), Int[(d - c
*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A,
 B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx=\int \frac{\cot(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=-\frac{(bB-aC)x}{a^2+b^2}+\frac{B\int\cot(c+dx)\,dx}{a}-\frac{(b(bB-aC)}{a\,(}$$
$$=-\frac{(bB-aC)x}{a^2+b^2}+\frac{B\log(\sin(c+dx))}{ad}-\frac{b(bB-aC)\,\mathrm{l}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 113, normalized size = 1.41

$$-\frac{\frac{(B+iC)\log(i-\tan(c+dx))}{a+ib}-\frac{2B\log(\tan(c+dx))}{a}+\frac{(B-iC)\log(i+\tan(c+dx))}{a-ib}+\frac{2b(bB-aC)\log(a+b\tan(c+dx))}{a(a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x]),x]
```

```
[Out] -1/2*(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) - (2*B*Log[Tan[c + d*x]])
/a + ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b*(b*B - a*C)*Log[a +
 b*Tan[c + d*x]])/(a*(a^2 + b^2)))/d
```

Maple [A]
time = 0.49, size = 101, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{\frac{(-aB-Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-Bb+Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{(Bb-Ca)b\ln(a+b\tan(dx+c))}{a\left(a^2+b^2\right)}+\frac{B\ln(\tan(dx+c))}{a}}{d}$
default	$\frac{\frac{\frac{(-aB-Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-Bb+Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{(Bb-Ca)b\ln(a+b\tan(dx+c))}{a\left(a^2+b^2\right)}+\frac{B\ln(\tan(dx+c))}{a}}{d}$

norman	$-\frac{(Bb-Ca)x}{a^2+b^2}+\frac{B\ln(\tan(dx+c))}{ad}-\frac{(aB+Cb)\ln\left(1+\tan^2(dx+c)\right)}{2d(a^2+b^2)}-\frac{(Bb-Ca)b\ln(a+b\tan(dx+c))}{(a^2+b^2)ad}$
risch	$-\frac{ixB}{ib-a}-\frac{xC}{ib-a}+\frac{2ib^2Bx}{a(a^2+b^2)}+\frac{2ib^2Bc}{ad(a^2+b^2)}-\frac{2ibCx}{a^2+b^2}-\frac{2ibCc}{(a^2+b^2)d}-\frac{2ixB}{a}-\frac{2iBc}{ad}-\frac{b^2\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{ad(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)*(1/2*(-B*a-C*b)*ln(1+tan(d*x+c)^2)+(-B*b+C*a)*arctan(tan(d
*x+c)))-(B*b-C*a)*b/a/(a^2+b^2)*ln(a+b*tan(d*x+c))+B/a*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 107, normalized size = 1.34

$$\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}+\frac{2\,(Cab-Bb^2)\log(b\tan(dx+c)+a)}{a^3+ab^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,B\log(\tan(dx+c))}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a*b - B*b^2)*log(b*tan(d*x
+ c) + a)/(a^3 + a*b^2) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) +
 2*B*log(tan(d*x + c))/a)/d
```

Fricas [A]
time = 1.14, size = 118, normalized size = 1.48

$$\frac{2\,(Ca^2-Bab)dx+(Ba^2+Bb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+(Cab-Bb^2)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\,(a^3+ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^2 - B*a*b)*d*x + (B*a^2 + B*b^2)*log(tan(d*x + c)^2/(tan(d*x +
c)^2 + 1)) + (C*a*b - B*b^2)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) +
 a^2)/(tan(d*x + c)^2 + 1)))/((a^3 + a*b^2)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.50, size = 966, normalized size = 12.08

$$\begin{cases} \frac{\tilde{\infty}x\left(B\tan(c)+C\tan^2(c)\right)\cot^2(c)}{\tan(c)} & \text{for}\: a=0\wedge b=0\wedge d=0 \\ \frac{-\frac{B\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{B\log(\tan(c+dx))}{d}+Cx}{a} & \text{for}\: b=0 \\ \frac{-Bx-\frac{B}{d\tan(c+dx)}-\frac{C\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{C\log(\tan(c+dx))}{d}}{b} & \text{for}\: a=0 \\ \frac{Bdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{iBdx}{2bd\tan(c+dx)-2ibd}-\frac{iB\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{B\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)-2ibd}+\frac{2iB\log(\tan(c+dx))\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{2B\log(\tan(c+dx))}{2bd\tan(c+dx)-2ibd}+\frac{B}{2bd\tan(c+dx)-2ibd}+\frac{iCdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{Cdx}{2bd\tan(c+dx)-2ibd}+\frac{iC}{2bd\tan(c+dx)-2ibd} & \text{for}\: a=-ib \\ \frac{Bdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{iBdx}{2bd\tan(c+dx)+2ibd}+\frac{iB\log\left(\tan^2(c+dx)+1\right)\tan(c+dx)}{2bd\tan(c+dx)+2ibd}-\frac{B\log\left(\tan^2(c+dx)+1\right)}{2bd\tan(c+dx)+2ibd}-\frac{2iB\log(\tan(c+dx))\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{2B\log(\tan(c+dx))}{2bd\tan(c+dx)+2ibd}+\frac{B}{2bd\tan(c+dx)+2ibd}-\frac{iCdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{Cdx}{2bd\tan(c+dx)+2ibd}-\frac{iC}{2bd\tan(c+dx)+2ibd} & \text{for}\: a=ib \\ \frac{x\left(B\tan(c)+C\tan^2(c)\right)\cot^2(c)}{a+b\tan(c)} & \text{for}\: d=0 \\ -\frac{Ba^2\log\left(\tan^2(c+dx)+1\right)}{2a^3d+2ab^2d}+\frac{2Ba^2\log(\tan(c+dx))}{2a^3d+2ab^2d}-\frac{2Babdx}{2a^3d+2ab^2d}-\frac{2Bb^2\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^3d+2ab^2d}+\frac{2Bb^2\log(\tan(c+dx))}{2a^3d+2ab^2d}+\frac{2Ca^2dx}{2a^3d+2ab^2d}+\frac{2Cab\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^3d+2ab^2d}-\frac{Cab\log\left(\tan^2(c+dx)+1\right)}{2a^3d+2ab^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)**2/tan(c), Eq(a, 0) & Eq(b
, 0) & Eq(d, 0)), ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*log(tan(c + d*x))
/d + C*x)/a, Eq(b, 0)), ((-B*x - B/(d*tan(c + d*x)) - C*log(tan(c + d*x)**2
 + 1)/(2*d) + C*log(tan(c + d*x))/d)/b, Eq(a, 0)), (B*d*x*tan(c + d*x)/(2*b
*d*tan(c + d*x) - 2*I*b*d) - I*B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B*l
og(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - B*log
(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*B*log(tan(c + d*
x))*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*B*log(tan(c + d*x))/(2*
b*d*tan(c + d*x) - 2*I*b*d) + B/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*C*d*x*ta
n(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + C*d*x/(2*b*d*tan(c + d*x) - 2*I
*b*d) + I*C/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (B*d*x*tan(c + d*
x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d)
+ I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d)
- B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*I*B*log(tan
(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*B*log(tan(c + d*
x))/(2*b*d*tan(c + d*x) + 2*I*b*d) + B/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*C
*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*d*x/(2*b*d*tan(c + d*x
) + 2*I*b*d) - I*C/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(B*tan(c
) + C*tan(c)**2)*cot(c)**2/(a + b*tan(c)), Eq(d, 0)), (-B*a**2*log(tan(c +
d*x)**2 + 1)/(2*a**3*d + 2*a*b**2*d) + 2*B*a**2*log(tan(c + d*x))/(2*a**3*d
 + 2*a*b**2*d) - 2*B*a*b*d*x/(2*a**3*d + 2*a*b**2*d) - 2*B*b**2*log(a/b + t
an(c + d*x))/(2*a**3*d + 2*a*b**2*d) + 2*B*b**2*log(tan(c + d*x))/(2*a**3*d
 + 2*a*b**2*d) + 2*C*a**2*d*x/(2*a**3*d + 2*a*b**2*d) + 2*C*a*b*log(a/b + t
an(c + d*x))/(2*a**3*d + 2*a*b**2*d) - C*a*b*log(tan(c + d*x)**2 + 1)/(2*a*
*3*d + 2*a*b**2*d), True))
```

Giac [A]
time = 0.93, size = 113, normalized size = 1.41

$$\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Cab^2-Bb^3)\log(|b\tan(dx+c)+a|)}{a^3b+ab^3}+\frac{2\,B\log(|\tan(dx+c)|)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2 +
 1)/(a^2 + b^2) + 2*(C*a*b^2 - B*b^3)*log(abs(b*tan(d*x + c) + a))/(a^3*b +
 a*b^3) + 2*B*log(abs(tan(d*x + c)))/a)/d
```

Mupad [B]
time = 9.46, size = 115, normalized size = 1.44

$$\frac{B\,\ln\left(\tan(c+d\,x)\right)}{a\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)}-\frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,b-C\,a\right)}{a\,d\,\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
)),x)
```

```
[Out] (B*log(tan(c + d*x)))/(a*d) - (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*1
i - b)) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a - b*1i)) - (b*log(a +
 b*tan(c + d*x))*(B*b - C*a))/(a*d*(a^2 + b^2))
```

3.30 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=103

$$-\frac{(aB+bC)x}{a^2+b^2}-\frac{B\cot(c+dx)}{ad}-\frac{(bB-aC)\log(\sin(c+dx))}{a^2d}+\frac{b^2(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2\left(a^2+b^2\right)d}$$

```
[Out] -(B*a+C*b)*x/(a^2+b^2)-B*cot(d*x+c)/a/d-(B*b-C*a)*ln(sin(d*x+c))/a^2/d+b^2*
(B*b-C*a)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^2/(a^2+b^2)/d
```

Rubi [A]
time = 0.23, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3713, 3690, 3732, 3611, 3556}

$$\frac{b^2(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2d\left(a^2+b^2\right)}-\frac{x(aB+bC)}{a^2+b^2}-\frac{(bB-aC)\log(\sin(c+dx))}{a^2d}-\frac{B\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
]),x]
```

```
[Out] -(((a*B + b*C)*x)/(a^2 + b^2)) - (B*Cot[c + d*x])/(a*d) - ((b*B - a*C)*Log[
Sin[c + d*x]])/(a^2*d) + (b^2*(b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*
x]])/(a^2*(a^2 + b^2)*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
```

```
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$= -\frac{B\cot(c+dx)}{ad} - \frac{\int \frac{\cot(c+dx)(bB-aC+aB\tan(c+dx)+bB\,\mathrm{t}}{a+b\tan(c+dx)}}{a}$$

$$= -\frac{(aB+bC)x}{a^2+b^2} - \frac{B\cot(c+dx)}{ad} - \frac{(bB-aC)\int \cot}{a^2}$$

$$= -\frac{(aB+bC)x}{a^2+b^2} - \frac{B\cot(c+dx)}{ad} - \frac{(bB-aC)\log(\mathrm{si}}{a^2 d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.61, size = 138, normalized size = 1.34

$$\frac{-\frac{2B\cot(c+dx)}{a}+\frac{i(B+iC)\log(i-\tan(c+dx))}{a+ib}+\frac{2(-bB+aC)\log(\tan(c+dx))}{a^2}-\frac{(iB+C)\log(i+\tan(c+dx))}{a-ib}+\frac{2b^2(bB-aC)\log(a+b\tan(c+dx))}{a^2(a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x]),x]

[Out] ((-2*B*Cot[c + d*x])/a + (I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) + (2*(-(b*B) + a*C)*Log[Tan[c + d*x]])/a^2 - ((I*B + C)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)))/(2*d)

Maple [A]
time = 0.48, size = 123, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{B}{a\tan(dx+c)}+\frac{(-Bb+Ca)\ln(\tan(dx+c))}{a^2}+\frac{(Bb-Ca)b^2\ln(a+b\tan(dx+c))}{a^2(a^2+b^2)}+\frac{\frac{(Bb-Ca)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-aB-Cb)\arctan(\tan(dx}{a^2+b^2}}{d}$
default	$\frac{-\frac{B}{a\tan(dx+c)}+\frac{(-Bb+Ca)\ln(\tan(dx+c))}{a^2}+\frac{(Bb-Ca)b^2\ln(a+b\tan(dx+c))}{a^2(a^2+b^2)}+\frac{\frac{(Bb-Ca)\ln\left(1+\tan^2(dx+c)\right)}{2}+(-aB-Cb)\arctan(\tan(dx}{a^2+b^2}}{d}$
norman	$\frac{-\frac{B\tan(dx+c)}{ad}-\frac{(aB+Cb)x\left(\tan^2(dx+c)\right)}{a^2+b^2}}{\tan(dx+c)^2}+\frac{b^2(Bb-Ca)\ln(a+b\tan(dx+c))}{a^2d(a^2+b^2)}-\frac{(Bb-Ca)\ln(\tan(dx+c))}{a^2d}+\frac{(Bb-Ca)\ln(}{2d(}$
risch	$\frac{xB}{ib-a}-\frac{ixC}{ib-a}+\frac{2iBbx}{a^2}+\frac{2iBbc}{a^2d}-\frac{2iCx}{a}-\frac{2iCc}{da}-\frac{2ib^3Bx}{a^2(a^2+b^2)}-\frac{2ib^3Bc}{a^2d(a^2+b^2)}+\frac{2ib^2Cx}{a(a^2+b^2)}+\frac{2ib^2Cc}{ad(a^2+b^2)}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-B/a/tan(d*x+c)+1/a^2*(-B*b+C*a)*ln(tan(d*x+c))+(B*b-C*a)*b^2/a^2/(a^2+b^2)*ln(a+b*tan(d*x+c))+1/(a^2+b^2)*(1/2*(B*b-C*a)*ln(1+tan(d*x+c)^2)+(-B*a-C*b)*arctan(tan(d*x+c))))

Maxima [A]
time = 0.51, size = 131, normalized size = 1.27

$$-\frac{\frac{2(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{2(Cab^2-Bb^3)\log(b\tan(dx+c)+a)}{a^4+a^2b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2(Ca-Bb)\log(\tan(dx+c))}{a^2}+\frac{2B}{a\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a*b^2 - B*b^3)*log(b*tan(d*x + c) + a)/(a^4 + a^2*b^2) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*(C*a - B*b)*log(tan(d*x + c))/a^2 + 2*B/(a*tan(d*x + c)))/d

Fricas [A]
time = 1.19, size = 177, normalized size = 1.72

$$-\frac{2\,Ba^3+2\,Bab^2+2\,(Ba^3+Ca^2b)dx\tan(dx+c)-(Ca^3-Ba^2b+Cab^2-Bb^3)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)+(Cab^2-Bb^3)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)}{2\,(a^4+a^2b^2)d\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -1/2*(2*B*a^3 + 2*B*a*b^2 + 2*(B*a^3 + C*a^2*b)*d*x*tan(d*x + c) - (C*a^3 -
 B*a^2*b + C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*
x + c) + (C*a*b^2 - B*b^3)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a
^2)/(tan(d*x + c)^2 + 1))*tan(d*x + c))/((a^4 + a^2*b^2)*d*tan(d*x + c))
```

Sympy [C] Result contains complex when optimal does not.
time = 5.02, size = 2064, normalized size = 20.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((-B*x - B/(d*t
an(c + d*x)) - C*log(tan(c + d*x)**2 + 1)/(2*d) + C*log(tan(c + d*x))/d)/a,
 Eq(b, 0)), ((B*log(tan(c + d*x)**2 + 1)/(2*d) - B*log(tan(c + d*x))/d - B/
(2*d*tan(c + d*x)**2) - C*x - C/(d*tan(c + d*x)))/b, Eq(a, 0)), (-3*I*B*d*x
*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 3*B*d*x*t
an(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - B*log(tan(c +
d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x))
 + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b
*d*tan(c + d*x)) + 2*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x
)**2 - 2*I*b*d*tan(c + d*x)) - 2*I*B*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*
tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) - 3*I*B*tan(c + d*x)/(2*b*d*tan(c +
 d*x)**2 - 2*I*b*d*tan(c + d*x)) - 2*B/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan
(c + d*x)) + C*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c +
 d*x)) - I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)
) - I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 - 2
*I*b*d*tan(c + d*x)) - C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c
 + d*x)**2 - 2*I*b*d*tan(c + d*x)) + 2*I*C*log(tan(c + d*x))*tan(c + d*x)**
2/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + 2*C*log(tan(c + d*x))*ta
n(c + d*x)/(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)) + C*tan(c + d*x)/
(2*b*d*tan(c + d*x)**2 - 2*I*b*d*tan(c + d*x)), Eq(a, -I*b)), (3*I*B*d*x*ta
n(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - 3*B*d*x*tan(
c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) - B*log(tan(c + d*x
)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) -
```

```
I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*
tan(c + d*x)) + 2*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**
2 + 2*I*b*d*tan(c + d*x)) + 2*I*B*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan
(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 3*I*B*tan(c + d*x)/(2*b*d*tan(c + d*
x)**2 + 2*I*b*d*tan(c + d*x)) - 2*B/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c
+ d*x)) + C*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*
x)) + I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) +
 I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**2 + 2*I*
b*d*tan(c + d*x)) - C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c +
d*x)**2 + 2*I*b*d*tan(c + d*x)) - 2*I*C*log(tan(c + d*x))*tan(c + d*x)**2/(
2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + 2*C*log(tan(c + d*x))*tan(c
 + d*x)/(2*b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)) + C*tan(c + d*x)/(2*
b*d*tan(c + d*x)**2 + 2*I*b*d*tan(c + d*x)), Eq(a, I*b)), (nan, Eq(c, -d*x)
), (x*(B*tan(c) + C*tan(c)**2)*cot(c)**3/(a + b*tan(c)), Eq(d, 0)), (-2*B*a
**3*d*x*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) -
 2*B*a**3/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) + B*a**2*b*l
og(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d
*tan(c + d*x)) - 2*B*a**2*b*log(tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c
+ d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*B*a*b**2/(2*a**4*d*tan(c + d*x) +
2*a**2*b**2*d*tan(c + d*x)) + 2*B*b**3*log(a/b + tan(c + d*x))*tan(c + d*x)
/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*B*b**3*log(tan(c
+ d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) -
 C*a**3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a*
*2*b**2*d*tan(c + d*x)) + 2*C*a**3*log(tan(c + d*x))*tan(c + d*x)/(2*a**4*d
*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*C*a**2*b*d*x*tan(c + d*x)/(
2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d*x)) - 2*C*a*b**2*log(a/b +
tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2*a**2*b**2*d*tan(c + d
*x)) + 2*C*a*b**2*log(tan(c + d*x))*tan(c + d*x)/(2*a**4*d*tan(c + d*x) + 2
*a**2*b**2*d*tan(c + d*x)), True))
```

Giac [A]

time = 1.14, size = 157, normalized size = 1.52

$$-\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Cab^3-Bb^4)\log(|b\tan(dx+c)+a|)}{a^4b+a^2b^3}-\frac{2\,(Ca-Bb)\log(|\tan(dx+c)|)}{a^2}+\frac{2\,(Ca\tan(dx+c)-Bb\tan(dx+c)+Ba)}{a^2\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) + 2*(C*a*b^3 - B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^4*b
+ a^2*b^3) - 2*(C*a - B*b)*log(abs(tan(d*x + c)))/a^2 + 2*(C*a*tan(d*x + c)
 - B*b*tan(d*x + c) + B*a)/(a^2*tan(d*x + c)))/d
```

Mupad [B]

time = 10.34, size = 140, normalized size = 1.36

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,b^3-C\,a\,b^2\right)}{d\,\left(a^4+a^2\,b^2\right)}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,b-C\,a\right)}{a^2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{B\cot(c+d\,x)}{a\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
)),x)
```

```
[Out] (log(a + b*tan(c + d*x))*(B*b^3 - C*a*b^2))/(d*(a^4 + a^2*b^2)) - (log(tan(
c + d*x))*(B*b - C*a))/(a^2*d) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(
a*1i + b)) - (B*cot(c + d*x))/(a*d) + (log(tan(c + d*x) - 1i)*(B*1i - C))/(
2*d*(a + b*1i))
```

3.31 $\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{(bB-aC)x}{a^2+b^2}+\frac{(bB-aC)\cot(c+dx)}{a^2d}-\frac{B\cot^2(c+dx)}{2ad}-\frac{(a^2B-b^2B+abC)\log(\sin(c+dx))}{a^3d}-\frac{b^3(bB-aC)}{}$$

```
[Out] (B*b-C*a)*x/(a^2+b^2)+(B*b-C*a)*cot(d*x+c)/a^2/d-1/2*B*cot(d*x+c)^2/a/d-(B*
a^2-B*b^2+C*a*b)*ln(sin(d*x+c))/a^3/d-b^3*(B*b-C*a)*ln(a*cos(d*x+c)+b*sin(d
*x+c))/a^3/(a^2+b^2)/d
```

Rubi [A]
time = 0.43, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3713, 3690, 3730, 3732, 3611, 3556}

$$\frac{x(bB-aC)}{a^2+b^2}+\frac{(bB-aC)\cot(c+dx)}{a^2d}-\frac{(a^2B+abC-b^2B)\log(\sin(c+dx))}{a^3d}-\frac{b^3(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\,(a^2+b^2)}-\frac{B\cot^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
]),x]
```

```
[Out] ((b*B - a*C)*x)/(a^2 + b^2) + ((b*B - a*C)*Cot[c + d*x])/(a^2*d) - (B*Cot[c
 + d*x]^2)/(2*a*d) - ((a^2*B - b^2*B + a*b*C)*Log[Sin[c + d*x]])/(a^3*d) -
(b^3*(b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
```

```
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \int \frac{\cot^3(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$= -\frac{B\cot^2(c+dx)}{2ad} - \frac{\int \frac{\cot^2(c+dx)(2(bB-aC)+2aB\tan(c+dx)+}{a+b\tan(c+dx)}}{2a}$$

$$= \frac{(bB-aC)\cot(c+dx)}{a^2d} - \frac{B\cot^2(c+dx)}{2ad} + \frac{\int \frac{\cot(c+d}{}}{}$$

$$= \frac{(bB-aC)x}{a^2+b^2} + \frac{(bB-aC)\cot(c+dx)}{a^2d} - \frac{B\cot^2(c+}{2ad}$$

$$= \frac{(bB-aC)x}{a^2+b^2} + \frac{(bB-aC)\cot(c+dx)}{a^2d} - \frac{B\cot^2(c+}{2ad}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.95, size = 163, normalized size = 1.19

$$\frac{\frac{2(bB-aC)\cot(c+dx)}{a^2} - \frac{B\cot^2(c+dx)}{a} + \frac{(B+iC)\log(i-\tan(c+dx))}{a+ib} - \frac{2(a^2B-b^2B+abC)\log(\tan(c+dx))}{a^3} + \frac{(B-iC)\log(i+\tan(c+dx))}{a-ib} + \frac{2b^3(-bB+aC)\log(a+b\tan(c+dx))}{a^3(a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x]),x]
```

```
[Out] ((2*(b*B - a*C)*Cot[c + d*x])/a^2 - (B*Cot[c + d*x]^2)/a + ((B + I*C)*Log[I
 - Tan[c + d*x]])/(a + I*b) - (2*(a^2*B - b^2*B + a*b*C)*Log[Tan[c + d*x]])
/a^3 + ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b^3*(-(b*B) + a*C)*
Log[a + b*Tan[c + d*x]])/(a^3*(a^2 + b^2)))/(2*d)
```

Maple [A]
time = 0.48, size = 152, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{\frac{(aB+Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(Bb-Ca)\arctan(\tan(dx+c))}{a^2+b^2} - \frac{(Bb-Ca)b^3\ln(a+b\tan(dx+c))}{a^3\left(a^2+b^2\right)} - \frac{B}{2a\tan(dx+c)^2} - \frac{-Bb+Ca}{a^2\tan(dx+c)} + \frac{\left(-a^2\right.}{}}{d}$
default	$\frac{\frac{\frac{(aB+Cb)\ln\left(1+\tan^2(dx+c)\right)}{2}+(Bb-Ca)\arctan(\tan(dx+c))}{a^2+b^2} - \frac{(Bb-Ca)b^3\ln(a+b\tan(dx+c))}{a^3\left(a^2+b^2\right)} - \frac{B}{2a\tan(dx+c)^2} - \frac{-Bb+Ca}{a^2\tan(dx+c)} + \frac{\left(-a^2\right.}{}}{d}$
norman	$\frac{\frac{(Bb-Ca)\left(\tan^2(dx+c)\right)}{a^2d}+\frac{(Bb-Ca)x\left(\tan^3(dx+c)\right)}{a^2+b^2}-\frac{B\tan(dx+c)}{2ad}}{\tan(dx+c)^3} + \frac{(aB+Cb)\ln(1+\tan^2(dx+c))}{2d(a^2+b^2)} - \frac{(a^2B-b^2B+Cab)\ln(\mathrm{t}}{a^3d}$
risch	$\frac{2iCbx}{a^2} + \frac{xC}{ib-a} + \frac{2ib^4Bx}{(a^2+b^2)a^3} - \frac{2i\left(iBa\,e^{2i(dx+c)}-Bb\,e^{2i(dx+c)}+Ca\,e^{2i(dx+c)}+Bb-Ca\right)}{a^2d\left(e^{2i(dx+c)}-1\right)^2} - \frac{2ib^3Cc}{(a^2+b^2)a^2d} + \frac{2iCbc}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)*(1/2*(B*a+C*b)*ln(1+tan(d*x+c)^2)+(B*b-C*a)*arctan(tan(d*x
+c)))-(B*b-C*a)*b^3/a^3/(a^2+b^2)*ln(a+b*tan(d*x+c))-1/2*B/a/tan(d*x+c)^2-(
-B*b+C*a)/a^2/tan(d*x+c)+1/a^3*(-B*a^2+B*b^2-C*a*b)*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 158, normalized size = 1.15

$$-\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{2\,(Cab^3-Bb^4)\log(b\tan(dx+c)+a)}{a^5+a^3b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Ba^2+Cab-Bb^2)\log(\tan(dx+c))}{a^3}+\frac{Ba+2\,(Ca-Bb)\tan(dx+c)}{a^2\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a*b^3 - B*b^4)*log(b*tan(d
*x + c) + a)/(a^5 + a^3*b^2) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b
^2) + 2*(B*a^2 + C*a*b - B*b^2)*log(tan(d*x + c))/a^3 + (B*a + 2*(C*a - B*b
)*tan(d*x + c))/(a^2*tan(d*x + c)^2))/d
```

Fricas [A]
time = 4.00, size = 234, normalized size = 1.71

$$-\frac{Ba^4+Ba^2b^2+(Ba^4+Ca^3b+Cab^3-Bb^4)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2-(Cab^3-Bb^4)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+(Ba^4+Ba^2b^2+2\,(Ca^4-Ba^3b)dx)\tan(dx+c)^2+2\,(Ca^4-Ba^3b+Ca^2b^2-Bab^3)\tan(dx+c)}{2\,(a^5+a^3b^2)d\tan(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="fricas")
```

```
[Out] -1/2*(B*a^4 + B*a^2*b^2 + (B*a^4 + C*a^3*b + C*a*b^3 - B*b^4)*log(tan(d*x +
 c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 - (C*a*b^3 - B*b^4)*log((b^2*tan
(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))*tan(d*x + c)^
2 + (B*a^4 + B*a^2*b^2 + 2*(C*a^4 - B*a^3*b)*d*x)*tan(d*x + c)^2 + 2*(C*a^4
 - B*a^3*b + C*a^2*b^2 - B*a*b^3)*tan(d*x + c))/((a^5 + a^3*b^2)*d*tan(d*x
+ c)^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.45, size = 2592, normalized size = 18.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)

[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((B*log(tan(c +
 d*x)**2 + 1)/(2*d) - B*log(tan(c + d*x))/d - B/(2*d*tan(c + d*x)**2) - C*x
 - C/(d*tan(c + d*x)))/a, Eq(b, 0)), ((B*x + B/(d*tan(c + d*x)) - B/(3*d*ta
n(c + d*x)**3) + C*log(tan(c + d*x)**2 + 1)/(2*d) - C*log(tan(c + d*x))/d -
 C/(2*d*tan(c + d*x)**2))/b, Eq(a, 0)), (-3*B*d*x*tan(c + d*x)**3/(2*b*d*ta
n(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) + 3*I*B*d*x*tan(c + d*x)**2/(2*b*d
*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) + 2*I*B*log(tan(c + d*x)**2 + 1
)*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) + 2*B*l
og(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*ta
n(c + d*x)**2) - 4*I*B*log(tan(c + d*x))*tan(c + d*x)**3/(2*b*d*tan(c + d*x
)**3 - 2*I*b*d*tan(c + d*x)**2) - 4*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*
b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) - 3*B*tan(c + d*x)**2/(2*b*d
*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) + I*B*tan(c + d*x)/(2*b*d*tan(c
 + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) - B/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*
tan(c + d*x)**2) - 3*I*C*d*x*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 - 2*I*b
*d*tan(c + d*x)**2) - 3*C*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 - 2*I*
b*d*tan(c + d*x)**2) - C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(2*b*d*ta
n(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) + I*C*log(tan(c + d*x)**2 + 1)*tan
(c + d*x)**2/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**2) + 2*C*log(ta
n(c + d*x))*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(c + d*x)**
2) - 2*I*C*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 - 2*I*b
*d*tan(c + d*x)**2) - 3*I*C*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 - 2*I*b*
d*tan(c + d*x)**2) - 2*C*tan(c + d*x)/(2*b*d*tan(c + d*x)**3 - 2*I*b*d*tan(
c + d*x)**2), Eq(a, -I*b)), (-3*B*d*x*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**
3 + 2*I*b*d*tan(c + d*x)**2) - 3*I*B*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x
)**3 + 2*I*b*d*tan(c + d*x)**2) - 2*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*
x)**3/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2) + 2*B*log(tan(c + d
*x)**2 + 1)*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**
2) + 4*I*B*log(tan(c + d*x))*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 + 2*I*b
*d*tan(c + d*x)**2) - 4*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c +
d*x)**3 + 2*I*b*d*tan(c + d*x)**2) - 3*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x
)**3 + 2*I*b*d*tan(c + d*x)**2) - I*B*tan(c + d*x)/(2*b*d*tan(c + d*x)**3 +
 2*I*b*d*tan(c + d*x)**2) - B/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)
**2) + 3*I*C*d*x*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d
*x)**2) - 3*C*d*x*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c +
d*x)**2) - C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(2*b*d*tan(c + d*x)**
3 + 2*I*b*d*tan(c + d*x)**2) - I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2
/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2) + 2*C*log(tan(c + d*x))*
tan(c + d*x)**3/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2) + 2*I*C*l
og(tan(c + d*x))*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d
*x)**2) + 3*I*C*tan(c + d*x)**2/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*
x)**2) - 2*C*tan(c + d*x)/(2*b*d*tan(c + d*x)**3 + 2*I*b*d*tan(c + d*x)**2)
, Eq(a, I*b)), (nan, Eq(c, -d*x)), (x*(B*tan(c) + C*tan(c)**2)*cot(c)**4/(a
```

```
 + b*tan(c)), Eq(d, 0)), (B*a**4*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(
2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) - 2*B*a**4*log(ta
n(c + d*x))*tan(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c
 + d*x)**2) - B*a**4/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)
**2) + 2*B*a**3*b*d*x*tan(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b*
*2*d*tan(c + d*x)**2) + 2*B*a**3*b*tan(c + d*x)/(2*a**5*d*tan(c + d*x)**2 +
 2*a**3*b**2*d*tan(c + d*x)**2) - B*a**2*b**2/(2*a**5*d*tan(c + d*x)**2 + 2
*a**3*b**2*d*tan(c + d*x)**2) + 2*B*a*b**3*tan(c + d*x)/(2*a**5*d*tan(c + d
*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) - 2*B*b**4*log(a/b + tan(c + d*x))*
tan(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2)
+ 2*B*b**4*log(tan(c + d*x))*tan(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*
a**3*b**2*d*tan(c + d*x)**2) - 2*C*a**4*d*x*tan(c + d*x)**2/(2*a**5*d*tan(c
 + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) - 2*C*a**4*tan(c + d*x)/(2*a**5
*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) + C*a**3*b*log(tan(c +
d*x)**2 + 1)*tan(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(
c + d*x)**2) - 2*C*a**3*b*log(tan(c + d*x))*tan(c + d*x)**2/(2*a**5*d*tan(c
 + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) - 2*C*a**2*b**2*tan(c + d*x)/(2
*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2) + 2*C*a*b**3*log(a
/b + tan(c + d*x))*tan(c + d*x)**2/(2*a**5*d*tan(c + d*x)**2 + 2*a**3*b**2*
d*tan(c + d*x)**2) - 2*C*a*b**3*log(tan(c + d*x))*tan(c + d*x)**2/(2*a**5*d
*tan(c + d*x)**2 + 2*a**3*b**2*d*tan(c + d*x)**2), True))
```

Giac [A]
time = 1.47, size = 214, normalized size = 1.56

$$-\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,(Cab^4-Bb^5)\log(|b\tan(dx+c)+a|)}{a^5b+a^3b^3}+\frac{2\,(Ba^2+Cab-Bb^2)\log(|\tan(dx+c)|)}{a^3}-\frac{3\,Ba^2\tan(dx+c)^2+3\,Cab\tan(dx+c)^2-3\,Bb^2\tan(dx+c)^2-2\,Ca^2\tan(dx+c)+2\,Bab\tan(dx+c)-Ba^2}{a^3\tan(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, al
gorithm="giac")
```

```
[Out] -1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) - 2*(C*a*b^4 - B*b^5)*log(abs(b*tan(d*x + c) + a))/(a^5*b
+ a^3*b^3) + 2*(B*a^2 + C*a*b - B*b^2)*log(abs(tan(d*x + c)))/a^3 - (3*B*a^
2*tan(d*x + c)^2 + 3*C*a*b*tan(d*x + c)^2 - 3*B*b^2*tan(d*x + c)^2 - 2*C*a^
2*tan(d*x + c) + 2*B*a*b*tan(d*x + c) - B*a^2)/(a^3*tan(d*x + c)^2))/d
```

Mupad [B]
time = 10.93, size = 175, normalized size = 1.28

$$-\frac{\cot(c+d\,x)^2\,\left(\frac{B}{2\,a}-\frac{\tan(c+d\,x)\,(B\,b-C\,a)}{a^2}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,(-C+B\,1\mathrm{i})}{2\,d\,(-b+a\,1\mathrm{i})}-\frac{\ln\left(\tan(c+d\,x)\right)\,(B\,a^2+C\,a\,b-B\,b^2)}{a^3\,d}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,(B\,b^4-C\,a\,b^3)}{d\,(a^5+a^3\,b^2)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,(B-C\,1\mathrm{i})}{2\,d\,(a-b\,1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
)),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*1i - b)) - (cot(c + d*x)^2*(B/(
2*a) - (tan(c + d*x)*(B*b - C*a))/a^2))/d - (log(tan(c + d*x))*(B*a^2 - B*b
^2 + C*a*b))/(a^3*d) - (log(a + b*tan(c + d*x))*(B*b^4 - C*a*b^3))/(d*(a^5
+ a^3*b^2)) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a - b*1i))
```

3.32 $\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$-\frac{(2abB-a^2C+b^2C)\,x}{(a^2+b^2)^2}+\frac{(a^2B-b^2B+2abC)\log(\cos(c+dx))}{(a^2+b^2)^2\,d}+\frac{a^2(a^2bB+3b^3B-2a^3C-4ab^2C)\log(a$$

```
[Out] -(2*B*a*b-C*a^2+C*b^2)*x/(a^2+b^2)^2+(B*a^2-B*b^2+2*C*a*b)*ln(cos(d*x+c))/(
a^2+b^2)^2/d+a^2*(B*a^2*b+3*B*b^3-2*C*a^3-4*C*a*b^2)*ln(a+b*tan(d*x+c))/b^3
/(a^2+b^2)^2/d-(B*a*b-2*C*a^2-C*b^2)*tan(d*x+c)/b^2/(a^2+b^2)/d+a*(B*b-C*a)
*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.36, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {3713, 3686, 3728, 3707, 3698, 31, 3556}

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{(-2a^2C+abB-b^2C)\tan(c+dx)}{b^2d\,(a^2+b^2)}+\frac{(a^2B+2abC-b^2B)\log(\cos(c+dx))}{d\,(a^2+b^2)^2}-\frac{x(a^2(-C)+2abB+b^2C)}{(a^2+b^2)^2}+\frac{a^2(-2a^3C+a^2bB-4ab^2C+3b^3B)\log(a+b\tan(c+dx))}{b^3d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^2,x]
```

```
[Out] -(((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2) + ((a^2*B - b^2*B + 2*a*b*C)
*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d) + (a^2*(a^2*b*B + 3*b^3*B - 2*a^3*C -
 4*a*b^2*C)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^2*d) - ((a*b*B - 2*a^
2*C - b^2*C)*Tan[c + d*x])/(b^2*(a^2 + b^2)*d) + (a*(b*B - a*C)*Tan[c + d*x
]^2)/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
```

```
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{a(bB-aC)\tan^2(c+dx)}{b\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\int \frac{\tan(c+dx)(-2a(bB}{}}{}$$

$$= -\frac{\left(abB-2a^2C-b^2C\right)\tan(c+dx)}{b^2\left(a^2+b^2\right)d} + \frac{a(bB-a}{b\left(a^2+b^2\right)d(}$$

$$= -\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} - \frac{\left(abB-2a^2C-b^2C\right)\mathrm{ta}}{b^2\left(a^2+b^2\right)d}$$

$$= -\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} + \frac{\left(a^2B-b^2B+2abC\right)\mathrm{lo}}{\left(a^2+b^2\right)^2}$$

$$= -\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} + \frac{\left(a^2B-b^2B+2abC\right)\mathrm{lo}}{\left(a^2+b^2\right)^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.82, size = 444, normalized size = 2.13

$$\frac{a\left(2(a+ib)^2\left(2ab^2(B+iC)+ia^2b(B+4iC)-2ia^3C+b^3C\right)(c+dx)+2\left(a^2+b^2\right)^2(-bB+2aC)\log(\cos(c+dx))+a^2\left(a^2bB+3b^3B-2a^3C-4ab^2C\right)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)+b\left(2\left(a^3b^2C(3-4ic-4idx)-b^5C(c+dx)+ia^4bB(i+c+dx)-2ia^5C(i+c+dx)+ab^4(C-2B(c+dx))+a^2b^3(C(c+dx)+iB(i+3c+3dx))\right)+2\left(a^2+b^2\right)^2(-bB+2aC)\log(\cos(c+dx))+a^2\left(a^2bB+3b^3B-2a^3C-4ab^2C\right)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)\tan(c+dx)+2b^2\left(a^2+b^2\right)^2C\tan^2(c+dx)+2ia^2\left(-a^2bB-3b^3B+2a^3C+4ab^2C\right)\mathrm{ArcTan}(\tan(c+dx))(a+b\tan(c+dx))}{2b^3\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^2,x]
```

```
[Out] (a*(2*(a + I*b)^2*(2*a*b^2*(B + I*C) + I*a^2*b*(B + (4*I)*C) - (2*I)*a^3*C
+ b^3*C)*(c + d*x) + 2*(a^2 + b^2)^2*(-(b*B) + 2*a*C)*Log[Cos[c + d*x]] + a
^2*(a^2*b*B + 3*b^3*B - 2*a^3*C - 4*a*b^2*C)*Log[(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2]) + b*(2*(a^3*b^2*C*(3 - (4*I)*c - (4*I)*d*x) - b^5*C*(c + d*x) +
 I*a^4*b*B*(I + c + d*x) - (2*I)*a^5*C*(I + c + d*x) + a*b^4*(C - 2*B*(c +
d*x)) + a^2*b^3*(C*(c + d*x) + I*B*(I + 3*c + 3*d*x))) + 2*(a^2 + b^2)^2*(-
(b*B) + 2*a*C)*Log[Cos[c + d*x]] + a^2*(a^2*b*B + 3*b^3*B - 2*a^3*C - 4*a*b
^2*C)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Tan[c + d*x] + 2*b^2*(a^2 +
 b^2)^2*C*Tan[c + d*x]^2 + (2*I)*a^2*(-(a^2*b*B) - 3*b^3*B + 2*a^3*C + 4*a*
b^2*C)*ArcTan[Tan[c + d*x]]*(a + b*Tan[c + d*x]))/(2*b^3*(a^2 + b^2)^2*d*(a
 + b*Tan[c + d*x]))
```

Maple [A]
time = 0.23, size = 172, normalized size = 0.83

method	result

derivativedivides	$\frac{\frac{C\tan(dx+c)}{b^2}+\frac{a^2\left(B\,a^2b+3B\,b^3-2C\,a^3-4Ca\,b^2\right)\ln(a+b\tan(dx+c))}{b^3\left(a^2+b^2\right)^2}+\frac{a^3(Bb-Ca)}{b^3\left(a^2+b^2\right)(a+b\tan(dx+c))}+\frac{\left(-a^2B+b^2B-2Cab\right)\ln\left(1+\tan^2\right.}{2}}{d}$
default	$\frac{\frac{C\tan(dx+c)}{b^2}+\frac{a^2\left(B\,a^2b+3B\,b^3-2C\,a^3-4Ca\,b^2\right)\ln(a+b\tan(dx+c))}{b^3\left(a^2+b^2\right)^2}+\frac{a^3(Bb-Ca)}{b^3\left(a^2+b^2\right)(a+b\tan(dx+c))}+\frac{\left(-a^2B+b^2B-2Cab\right)\ln\left(1+\tan^2\right.}{2}}{d}$
norman	$\frac{\frac{C\left(\tan^2(dx+c)\right)}{bd}+\frac{\left(B\,a^2b-2C\,a^3-Ca\,b^2\right)a}{d\,b^3\left(a^2+b^2\right)}-\frac{a\left(2Bab-C\,a^2+b^2C\right)x}{a^4+2a^2b^2+b^4}-\frac{b\left(2Bab-C\,a^2+b^2C\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}}{a+b\tan(dx+c)}+\frac{a^2\left(B\,a^2b+3B\,b^3-2C\right.}{(a^4+}$
risch	$\frac{2iBx}{b^2}-\frac{xC}{2iba-a^2+b^2}-\frac{2ia^4Bc}{(a^4+2a^2b^2+b^4)d\,b^2}-\frac{6ia^2Bx}{a^4+2a^2b^2+b^4}-\frac{4iCac}{b^3d}-\frac{6ia^2Bc}{(a^4+2a^2b^2+b^4)d}-\frac{4iCax}{b^3}+\frac{8ic}{(a^4+2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(C/b^2*tan(d*x+c)+1/b^3*a^2*(B*a^2*b+3*B*b^3-2*C*a^3-4*C*a*b^2)/(a^2+b^
2)^2*ln(a+b*tan(d*x+c))+1/b^3*a^3*(B*b-C*a)/(a^2+b^2)/(a+b*tan(d*x+c))+1/(a
^2+b^2)^2*(1/2*(-B*a^2+B*b^2-2*C*a*b)*ln(1+tan(d*x+c)^2)+(-2*B*a*b+C*a^2-C*
b^2)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.51, size = 220, normalized size = 1.06

$$\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(2\,Ca^5-Ba^4b+4\,Ca^3b^2-3\,Ba^2b^3)\log(b\tan(dx+c)+a)}{a^4b^3+2\,a^2b^5+b^7}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Ca^4-Ba^3b)}{a^3b^3+ab^5+(a^2b^4+b^6)\tan(dx+c)}+\frac{2\,C\tan(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*C
*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3)*log(b*tan(d*x + c) + a)/(a^4*b^
3 + 2*a^2*b^5 + b^7) - (B*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - 2*(C*a^4 - B*a^3*b)/(a^3*b^3 + a*b^5 + (a^2*b^4 + b
^6)*tan(d*x + c)) + 2*C*tan(d*x + c)/b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(209) = 418.
time = 3.06, size = 434, normalized size = 2.09

$$-\frac{2\,Ca^4b^2-2\,Ba^3b^3-2\,(Ca^3b^3-2\,Ba^2b^4-Cab^5)dx-2\,(Ca^4b^2+2\,Ca^2b^4+Cb^6)\tan(dx+c)^2+(2\,Ca^6-Ba^5b+4\,Ca^4b^2-3\,Ba^3b^3+(2\,Ca^5b-Ba^4b^2+4\,Ca^3b^3-3\,Ba^2b^4)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(2\,Ca^6-Ba^5b+4\,Ca^4b^2-2\,Ba^3b^3+2\,Ca^2b^4-Bab^5+(2\,Ca^5b-Ba^4b^2+4\,Ca^3b^3-2\,Ba^2b^4+2\,Cab^5-Bb^6)\tan(dx+c))\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\,(2\,Ca^5b-Ba^4b^2+2\,Ca^3b^3+Cab^5+(Ca^2b^4-2\,Bab^5-Cb^6)dx)\tan(dx+c)}{2\,((a^4b^4+2\,a^2b^6+b^8)d\tan(dx+c)+(a^5b^3+2\,a^3b^5+ab^7)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/2*(2*C*a^4*b^2 - 2*B*a^3*b^3 - 2*(C*a^3*b^3 - 2*B*a^2*b^4 - C*a*b^5)*d*x
 - 2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*tan(d*x + c)^2 + (2*C*a^6 - B*a^5*b
+ 4*C*a^4*b^2 - 3*B*a^3*b^3 + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^
2*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(t
an(d*x + c)^2 + 1)) - (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 2*B*a^3*b^3 + 2*C*
a^2*b^4 - B*a*b^5 + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 2*B*a^2*b^4 + 2*
C*a*b^5 - B*b^6)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(2*C*a^5*b -
 B*a^4*b^2 + 2*C*a^3*b^3 + C*a*b^5 + (C*a^2*b^4 - 2*B*a*b^5 - C*b^6)*d*x)*t
an(d*x + c))/((a^4*b^4 + 2*a^2*b^6 + b^8)*d*tan(d*x + c) + (a^5*b^3 + 2*a^3
*b^5 + a*b^7)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.20, size = 4541, normalized size = 21.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,
x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)),
 ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d*x)**2/(2*d) + C*x + C*ta
n(c + d*x)**3/(3*d) - C*tan(c + d*x)/d)/a**2, Eq(b, 0)), (3*I*B*d*x*tan(c +
 d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) +
6*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) -
4*b**2*d) - 3*I*B*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) -
 4*b**2*d) + 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c +
 d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*B*log(tan(c + d*x)**2
+ 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b
**2*d) - 2*B*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*
d*tan(c + d*x) - 4*b**2*d) - 5*I*B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 -
 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*B/(4*b**2*d*tan(c + d*x)**2 - 8*I*
b**2*d*tan(c + d*x) - 4*b**2*d) - 9*C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c +
 d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 18*I*C*d*x*tan(c + d*x)/(4
*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 9*C*d*x/(4*
b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*C*log(ta
n(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*t
an(c + d*x) - 4*b**2*d) + 8*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2
*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*C*log(tan(c
+ d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2
*d) + 4*C*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*
x) - 4*b**2*d) + 19*C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*t
an(c + d*x) - 4*b**2*d) - 14*I*C/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan
```

```
(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-3*I*B*d*x*tan(c + d*x)**2/(4*b**2*d*
tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 6*B*d*x*tan(c + d*x
)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 3*I*B*d
*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*lo
g(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2
*d*tan(c + d*x) - 4*b**2*d) + 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(
4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*B*log(ta
n(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*
b**2*d) + 5*I*B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c +
 d*x) - 4*b**2*d) - 4*B/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x)
 - 4*b**2*d) - 9*C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2
*d*tan(c + d*x) - 4*b**2*d) - 18*I*C*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x
)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 9*C*d*x/(4*b**2*d*tan(c + d*x)
**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*C*log(tan(c + d*x)**2 + 1)*
tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**
2*d) + 8*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2
+ 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*C*log(tan(c + d*x)**2 + 1)/(4*b
**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*C*tan(c + d
*x)**3/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 19
*C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**
2*d) + 14*I*C/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*
d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2)*tan(c)**2/(a + b*tan(c))**2, E
q(d, 0)), (2*B*a**5*b*log(a/b + tan(c + d*x))/(2*a**5*b**3*d + 2*a**4*b**4*
d*tan(c + d*x) + 4*a**3*b**5*d + 4*a**2*b**6*d*tan(c + d*x) + 2*a*b**7*d +
2*b**8*d*tan(c + d*x)) + 2*B*a**5*b/(2*a**5*b**3*d + 2*a**4*b**4*d*tan(c +
d*x) + 4*a**3*b**5*d + 4*a**2*b**6*d*tan(c + d*x) + 2*a*b**7*d + 2*b**8*d*t
an(c + d*x)) + 2*B*a**4*b**2*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b
**3*d + 2*a**4*b**4*d*tan(c + d*x) + 4*a**3*b**5*d + 4*a**2*b**6*d*tan(c +
d*x) + 2*a*b**7*d + 2*b**8*d*tan(c + d*x)) + 6*B*a**3*b**3*log(a/b + tan(c
+ d*x))/(2*a**5*b**3*d + 2*a**4*b**4*d*tan(c + d*x) + 4*a**3*b**5*d + 4*a**
2*b**6*d*tan(c + d*x) + 2*a*b**7*d + 2*b**8*d*tan(c + d*x)) - B*a**3*b**3*l
og(tan(c + d*x)**2 + 1)/(2*a**5*b**3*d + 2*a**4*b**4*d*tan(c + d*x) + 4*a**
3*b**5*d + 4*a**2*b**6*d*tan(c + d*x) + 2*a*b**7*d + 2*b**8*d*tan(c + d*x))
 + 2*B*a**3*b**3/(2*a**5*b**3*d + 2*a**4*b**4*d*tan(c + d*x) + 4*a**3*b**5*
d + 4*a**2*b**6*d*tan(c + d*x) + 2*a*b**7*d + 2*b**8*d*tan(c + d*x)) - 4*B*
a**2*b**4*d*x/(2*a**5*b**3*d + 2*a**4*b**4*d*tan(c + d*x) + 4*a**3*b**5*d +
 4*a**2*b**6*d*tan(c + d*x) + 2*a*b**7*d + 2*b**8*d*tan(c + d*x)) + 6*B*a**
2*b**4*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b**3*d + 2*a**4*b**4*d*
tan(c + d*x) + 4*a**3*b**5*d + 4*a**2*b**6*d*tan(c + d*x) + 2*a*b**7*d + 2*
b**8*d*tan(c + d*x)) - B*a**2*b**4*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2
*a**5*b**3*d + 2*a**4*b**4*d*tan(c + d*x) + 4*a**3*b**5*d + 4*a**2*b**6*d*t
an(c + d*x) + 2*a*b**7*d + 2*b**8*d*tan(c + d*x)) - 4*B*a*b**5*d*x*tan(c +
d*x)/(2*a**5*b**3*d + 2*a**4*b**4*d*tan(c + d*x) + 4*a**3*b**5*d + 4*a**2*b
**6*d*tan(c + d*x) + 2*a*b**7*d + 2*b**8*d*tan(...
```

Giac [A]
time = 0.86, size = 290, normalized size = 1.39

$$\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(2\,Ca^5-Ba^4b+4\,Ca^3b^2-3\,Ba^2b^3)\log(|b\tan(dx+c)+a|)}{a^4b^3+2\,a^2b^5+b^7}+\frac{2\,C\tan(dx+c)}{b^2}+\frac{2\,(2\,Ca^5b\tan(dx+c)-Ba^4b^2\tan(dx+c)+4\,Ca^3b^3\tan(dx+c)-3\,Ba^2b^4\tan(dx+c)+Ca^6+3\,Ca^4b^2-2\,Ba^3b^3)}{(a^4b^3+2\,a^2b^5+b^7)(b\tan(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^2
 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*
C*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3)*log(abs(b*tan(d*x + c) + a))/(
a^4*b^3 + 2*a^2*b^5 + b^7) + 2*C*tan(d*x + c)/b^2 + 2*(2*C*a^5*b*tan(d*x +
c) - B*a^4*b^2*tan(d*x + c) + 4*C*a^3*b^3*tan(d*x + c) - 3*B*a^2*b^4*tan(d*
x + c) + C*a^6 + 3*C*a^4*b^2 - 2*B*a^3*b^3)/((a^4*b^3 + 2*a^2*b^5 + b^7)*(b
*tan(d*x + c) + a)))/d
```

Mupad [B]
time = 9.65, size = 210, normalized size = 1.01

$$\frac{C\tan(c+d\,x)}{b^2\,d}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(2\,C\,a^5-B\,a^4\,b+4\,C\,a^3\,b^2-3\,B\,a^2\,b^3\right)}{d\,\left(a^4\,b^3+2\,a^2\,b^5+b^7\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}-\frac{a^2\,\left(C\,a^2-B\,a\,b\right)}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^3+a\,b^2\right)\,\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
))^2,x)
```

```
[Out] (C*tan(c + d*x))/(b^2*d) - (log(a + b*tan(c + d*x))*(2*C*a^5 - 3*B*a^2*b^3
+ 4*C*a^3*b^2 - B*a^4*b))/(d*(b^7 + 2*a^2*b^5 + a^4*b^3)) - (log(tan(c + d*
x) - 1i)*(B + C*1i))/(2*d*(a*b*2i + a^2 - b^2)) - (log(tan(c + d*x) + 1i)*(
B*1i + C))/(2*d*(2*a*b + a^2*1i - b^2*1i)) - (a^2*(C*a^2 - B*a*b))/(b*d*(a*
b^2 + b^3*tan(c + d*x))*(a^2 + b^2))
```

3.33 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$-\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2}-\frac{(2abB-a^2C+b^2C)\log(\cos(c+dx))}{(a^2+b^2)^2\,d}-\frac{a(2b^3B-a^3C-3ab^2C)\log(a+b\tan(c+$$

```
[Out] -(B*a^2-B*b^2+2*C*a*b)*x/(a^2+b^2)^2-(2*B*a*b-C*a^2+C*b^2)*ln(cos(d*x+c))/(
a^2+b^2)^2/d-a*(2*B*b^3-C*a^3-3*C*a*b^2)*ln(a+b*tan(d*x+c))/b^2/(a^2+b^2)^2
/d-a^2*(B*b-C*a)/b^2/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3713, 3685, 3707, 3698, 31, 3556}

$$-\frac{a^2(bB-aC)}{b^2d\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{(a^2(-C)+2abB+b^2C)\log(\cos(c+dx))}{d\,(a^2+b^2)^2}-\frac{x(a^2B+2abC-b^2B)}{(a^2+b^2)^2}-\frac{a(a^3(-C)-3ab^2C+2b^3B)\log(a+b\tan(c+dx))}{b^2d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
^2,x]
```

```
[Out] -(((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2) - ((2*a*b*B - a^2*C + b^2*C)
*Log[Cos[c + d*x]])/((a^2 + b^2)^2*d) - (a*(2*b^3*B - a^3*C - 3*a*b^2*C)*Lo
g[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)^2*d) - (a^2*(b*B - a*C))/(b^2*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
 + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*
```

```
c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$= -\frac{a^2(bB-aC)}{b^2\,(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{-a(bB-aC)+b(b}{}}{}$$

$$= -\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2} - \frac{a^2(bB-aC)}{b^2\,(a^2+b^2)\,d(a+b\tan(}$$

$$= -\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2} - \frac{(2abB-a^2C+b^2C)\log}{(a^2+b^2)^2\,d}$$

$$= -\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2} - \frac{(2abB-a^2C+b^2C)\log}{(a^2+b^2)^2\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.49, size = 324, normalized size = 2.06

$$\frac{a\left(2(a+ib)^2\left(-b^2B+ia^2C+2abC\right)(c+dx)-2\left(a^2+b^2\right)^2C\log(\cos(c+dx))+a\left(-2b^3B+a^3C+3ab^2C\right)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)+b\left(2(a+ib)\left(-ib^3B(c+dx)+ia^3C(i+c+dx)-ab^2(-2iC(c+dx)+B(i+c+dx))+a^2b(B+C(i+c+dx))\right)-2\left(a^2+b^2\right)^2C\log(\cos(c+dx))+a\left(-2b^3B+a^3C+3ab^2C\right)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)\tan(c+dx)-2ia\left(-2b^3B+a^3C+3ab^2C\right)\mathrm{ArcTan}(\tan(c+dx))(a+b\tan(c+dx))}{2b^2\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x])^2,x]
```

```
[Out] (a*(2*(a + I*b)^2*(-(b^2*B) + I*a^2*C + 2*a*b*C)*(c + d*x) - 2*(a^2 + b^2)^
2*C*Log[Cos[c + d*x]] + a*(-2*b^3*B + a^3*C + 3*a*b^2*C)*Log[(a*Cos[c + d*x
] + b*Sin[c + d*x])^2]) + b*(2*(a + I*b)*((-I)*b^3*B*(c + d*x) + I*a^3*C*(I
 + c + d*x) - a*b^2*((-2*I)*C*(c + d*x) + B*(I + c + d*x)) + a^2*b*(B + C*(
I + c + d*x))) - 2*(a^2 + b^2)^2*C*Log[Cos[c + d*x]] + a*(-2*b^3*B + a^3*C
+ 3*a*b^2*C)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Tan[c + d*x] - (2*I)
*a*(-2*b^3*B + a^3*C + 3*a*b^2*C)*ArcTan[Tan[c + d*x]]*(a + b*Tan[c + d*x])
)/(2*b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Maple [A]
time = 0.26, size = 155, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{a^2(Bb-Ca)}{b^2\left(a^2+b^2\right)(a+b\tan(dx+c))}-\frac{a\left(2B\,b^3-C\,a^3-3Ca\,b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2b^2}+\frac{\frac{\left(2Bab-C\,a^2+b^2C\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(-a^2B+b^2B\right.}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{-\frac{a^2(Bb-Ca)}{b^2\left(a^2+b^2\right)(a+b\tan(dx+c))}-\frac{a\left(2B\,b^3-C\,a^3-3Ca\,b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2b^2}+\frac{\frac{\left(2Bab-C\,a^2+b^2C\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(-a^2B+b^2B\right.}{\left(a^2+b^2\right)^2}}{d}$
norman	$\frac{-\frac{a\left(a^2B-b^2B+2Cab\right)x}{a^4+2a^2b^2+b^4}-\frac{b\left(a^2B-b^2B+2Cab\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{\left(Bab-C\,a^2\right)a}{d\,b^2\left(a^2+b^2\right)}}{a+b\tan(dx+c)}+\frac{(2Bab-C\,a^2+b^2C)\ln(1+\tan^2(dx+c))}{2d(a^4+2a^2b^2+b^4)}-\frac{a(}{}$
risch	$\frac{xB}{2iba-a^2+b^2}-\frac{6ia^2Cc}{d(a^4+2a^2b^2+b^4)}+\frac{4iBabx}{a^4+2a^2b^2+b^4}+\frac{4iBabc}{d(a^4+2a^2b^2+b^4)}-\frac{6ia^2Cx}{a^4+2a^2b^2+b^4}+\frac{2iCx}{b^2}+\frac{2iCc}{b^2d}-\frac{}{2iba}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(B*b-C*a)/b^2/(a^2+b^2)/(a+b*tan(d*x+c))-a*(2*B*b^3-C*a^3-3*C*a*b
^2)/(a^2+b^2)^2/b^2*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(2*B*a*b-C*a^2+C*
b^2)*ln(1+tan(d*x+c)^2)+(-B*a^2+B*b^2-2*C*a*b)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.49, size = 197, normalized size = 1.25

$$-\frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Ca^4+3\,Ca^2b^2-2\,Bab^3\right)\log(b\tan(dx+c)+a)}{a^4b^2+2\,a^2b^4+b^6}+\frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Ca^3-Ba^2b\right)}{a^3b^2+ab^4+\left(a^2b^3+b^5\right)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C*
a^4 + 3*C*a^2*b^2 - 2*B*a*b^3)*log(b*tan(d*x + c) + a)/(a^4*b^2 + 2*a^2*b^4
 + b^6) + (C*a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^
2 + b^4) - 2*(C*a^3 - B*a^2*b)/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(d*x +
 c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(153) = 306$.
time = 3.89, size = 311, normalized size = 1.98

$$\frac{2\,Ca^3b^2 - 2\,Ba^2b^3 - 2\,(Ba^3b^2 + 2\,Ca^2b^3 - Bab^4)dx + (Ca^5 + 3\,Ca^3b^2 - 2\,Ba^2b^3 + (Ca^4b + 3\,Ca^2b^3 - 2\,Bab^4)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2 + 2\,ab\tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) - (Ca^5 + 2\,Ca^3b^2 + Cab^4 + (Ca^4b + 2\,Ca^2b^3 + Cb^5)\tan(dx+c))\log\left(\frac{1}{\tan(dx+c)^2 + 1}\right) - 2\,(Ca^4b - Ba^3b^2 + (Ba^2b^3 + 2\,Cab^4 - Bb^5)dx)\tan(dx+c)}{2\,((a^4b^3 + 2\,a^2b^5 + b^7)d\tan(dx+c) + (a^5b^2 + 2\,a^3b^4 + ab^6)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*C*a^3*b^2 - 2*B*a^2*b^3 - 2*(B*a^3*b^2 + 2*C*a^2*b^3 - B*a*b^4)*d*x
+ (C*a^5 + 3*C*a^3*b^2 - 2*B*a^2*b^3 + (C*a^4*b + 3*C*a^2*b^3 - 2*B*a*b^4)*
tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x
+ c)^2 + 1)) - (C*a^5 + 2*C*a^3*b^2 + C*a*b^4 + (C*a^4*b + 2*C*a^2*b^3 + C*
b^5)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(C*a^4*b - B*a^3*b^2 + (
B*a^2*b^3 + 2*C*a*b^4 - B*b^5)*d*x)*tan(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b
^7)*d*tan(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.05, size = 3497, normalized size = 22.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(
d, 0)), ((-B*x + B*tan(c + d*x)/d - C*log(tan(c + d*x)**2 + 1)/(2*d) + C*ta
n(c + d*x)**2/(2*d))/a**2, Eq(b, 0)), (B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(
c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*B*d*x*tan(c + d*x)/
(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - B*d*x/(4*
b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*B*tan(c +
d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*
B/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 3*I*C*d
*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*
```

b**2*d) + 6*C*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*I*C*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*C*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 5*I*C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*C/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - B*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*B/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*I*C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 6*C*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 3*I*C*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*C*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 5*I*C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*C/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2)*tan(c)/(a + b*tan(c))**2, Eq(d, 0)), (-2*B*a**4*b/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - 2*B*a**3*b**2*d*x/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - 2*B*a**2*b**3*d*x*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - 4*B*a**2*b**3*log(a/b + tan(c + d*x))/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*B*a**2*b**3*log(tan(c + d*x)**2 + 1)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - 2*B*a**2*b**3/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*B*a*b**4*d*x/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - 4*B*a*b**4*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*B*a*b**4*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*B*b**5*d*x*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*

```
d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*C*
a**5*log(a/b + tan(c + d*x))/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) +
4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c +
d*x)) + 2*C*a**5/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*
d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*C*
a**4*b*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b**2*d + 2*a**4*b**3*d*
tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x) + 2*a*b**6*d + 2*
b**7*d*tan(c + d*x)) + 6*C*a**3*b**2*log(a/b + tan(c + d*x))/(2*a**5*b**2*d
 + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5*d*tan(c + d*x)
+ 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) - C*a**3*b**2*log(tan(c + d*x)**2 + 1
)/(2*a**5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3*b**4*d + 4*a**2*b**5
*d*tan(c + d*x) + 2*a*b**6*d + 2*b**7*d*tan(c + d*x)) + 2*C*a**3*b**2/(2*a*
*5*b**2*d + 2*a**4*b**3*d*tan(c + d*x) + 4*a**3...
```

Giac [A]
time = 0.71, size = 244, normalized size = 1.55

$$-\frac{\frac{2\,(Ba^2+2\,Cab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{(Ca^2-2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Ca^4+3\,Ca^2b^2-2\,Bab^3)\log(|b\tan(dx+c)+a|)}{a^4b^2+2\,a^2b^4+b^6}+\frac{2\,(Ca^4\tan(dx+c)+3\,Ca^2b^2\tan(dx+c)-2\,Bab^3\tan(dx+c)+Ba^4+2\,Ca^3b-Ba^2b^2)}{(a^4b+2\,a^2b^3+b^5)(b\tan(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="giac")
```

```
[Out] -1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a^
2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C
*a^4 + 3*C*a^2*b^2 - 2*B*a*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b^2 + 2*a
^2*b^4 + b^6) + 2*(C*a^4*tan(d*x + c) + 3*C*a^2*b^2*tan(d*x + c) - 2*B*a*b^
3*tan(d*x + c) + B*a^4 + 2*C*a^3*b - B*a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*
(b*tan(d*x + c) + a)))/d
```

Mupad [B]
time = 9.11, size = 165, normalized size = 1.05

$$\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(C+B\,1{}\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,2{}\mathrm{i}+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1{}\mathrm{i}\right)}{2\,d\,\left(-a^2\,1{}\mathrm{i}+2\,a\,b+b^2\,1{}\mathrm{i}\right)}-\frac{a^2\,\left(B\,b-C\,a\right)}{b^2\,d\,\left(a^2+b^2\right)\,\left(a+b\,\tan\left(c+d\,x\right)\right)}+\frac{a\,\ln\left(a+b\,\tan(c+d\,x)\right)\,\left(C\,a^3+3\,C\,a\,b^2-2\,B\,b^3\right)}{b^2\,d\,{\left(a^2+b^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x))
^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(B*1i + C))/(2*d*(a*b*2i - a^2 + b^2)) + (log(tan(c
 + d*x) - 1i)*(B + C*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) - (a^2*(B*b - C*a
))/(b^2*d*(a^2 + b^2)*(a + b*tan(c + d*x))) + (a*log(a + b*tan(c + d*x))*(C
*a^3 - 2*B*b^3 + 3*C*a*b^2))/(b^2*d*(a^2 + b^2)^2)
```

3.34 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$\frac{(2abB-a^2C+b^2C)\,x}{(a^2+b^2)^2}-\frac{(a^2B-b^2B+2abC)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d}+\frac{a(bB-aC)}{b\,(a^2+b^2)\,d(a+b\tan(c+$$

```
[Out] (2*B*a*b-C*a^2+C*b^2)*x/(a^2+b^2)^2-(B*a^2-B*b^2+2*C*a*b)*ln(a*cos(d*x+c)+b
*sin(d*x+c))/(a^2+b^2)^2/d+a*(B*b-C*a)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}=0.094$, Rules used = {3709, 3612, 3611}

$$\frac{a(bB-aC)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{(a^2B+2abC-b^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}+\frac{x(a^2(-C)+2abB+b^2C)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2 - ((a^2*B - b^2*B + 2*a*b*C)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d) + (a*(b*B - a*C))/(b*
(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
```

```
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{a(bB-aC)}{b\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\int \frac{bB-aC+(aB+bC)\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} + \frac{a(bB-aC)}{b\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{(a^2B$$

$$= \frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} - \frac{\left(a^2B-b^2B+2abC\right)\log(a\cos(c+dx)-}{\left(a^2+b^2\right)^2 d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.51, size = 140, normalized size = 1.22

$$\frac{\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2\left(\left(-a^2B+b^2B-2abC\right)\log(a+b\tan(c+dx))-\frac{a\left(a^2+b^2\right)(-bB+aC)}{b(a+b\tan(c+dx))}\right)}{\left(a^2+b^2\right)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2 + ((B - I*C)*Log[I + Tan[c +
 d*x]])/(a - I*b)^2 + (2*((-(a^2*B) + b^2*B - 2*a*b*C)*Log[a + b*Tan[c + d*
x]] - (a*(a^2 + b^2)*(-(b*B) + a*C))/(b*(a + b*Tan[c + d*x]))))/(a^2 + b^2)
^2)/(2*d)
```

Maple [A]
time = 0.16, size = 145, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{a(Bb-Ca)}{\left(a^2+b^2\right)b(a+b\tan(dx+c))}-\frac{\left(a^2B-b^2B+2Cab\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{\frac{\left(a^2B-b^2B+2Cab\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(2Bab-C\,a^2+b^2C\right)}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{a(Bb-Ca)}{\left(a^2+b^2\right)b(a+b\tan(dx+c))}-\frac{\left(a^2B-b^2B+2Cab\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{\frac{\left(a^2B-b^2B+2Cab\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(2Bab-C\,a^2+b^2C\right)}{\left(a^2+b^2\right)^2}}{d}$
norman	$\frac{\frac{a\left(2Bab-C\,a^2+b^2C\right)x}{a^4+2a^2b^2+b^4}+\frac{b\left(2Bab-C\,a^2+b^2C\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}+\frac{a(Bb-Ca)}{\left(a^2+b^2\right)bd}}{a+b\tan(dx+c)}+\frac{(a^2B-b^2B+2Cab)\ln(1+\tan^2(dx+c))}{2d(a^4+2a^2b^2+b^4)}-(a^2B$

risch	$\frac{ixB}{2iba-a^2+b^2}+\frac{xC}{2iba-a^2+b^2}+\frac{2ia^2Bx}{a^4+2a^2b^2+b^4}-\frac{2iB\,b^2x}{a^4+2a^2b^2+b^4}+\frac{4iCabx}{a^4+2a^2b^2+b^4}+\frac{2ia^2Bc}{(a^4+2a^2b^2+b^4)d}-\frac{2iB}{d(a^4+2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a*(B*b-C*a)/(a^2+b^2)/b/(a+b*tan(d*x+c))-(B*a^2-B*b^2+2*C*a*b)/(a^2+b^
2)^2*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(B*a^2-B*b^2+2*C*a*b)*ln(1+tan(d
*x+c)^2)+(2*B*a*b-C*a^2+C*b^2)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.52, size = 185, normalized size = 1.61

$$-\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ba^2+2\,Cab-Bb^2)\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ca^2-Bab)}{a^3b+ab^3+(a^2b^2+b^4)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*
a^2 + 2*C*a*b - B*b^2)*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - (B
*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2
*(C*a^2 - B*a*b)/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(d*x + c)))/d
```

Fricas [A]
time = 3.61, size = 221, normalized size = 1.92

$$-\frac{2\,Ca^2b-2\,Bab^2+2\,(Ca^3-2\,Ba^2b-Cab^2)dx+(Ba^3+2\,Ca^2b-Bab^2+(Ba^2b+2\,Cab^2-Bb^3)\tan\left(dx+c\right))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-2\,(Ca^3-Ba^2b-(Ca^2b-2\,Bab^2-Cb^3)dx)\tan\left(dx+c\right)}{2\,((a^4b+2\,a^2b^3+b^5)d\tan\left(dx+c\right)+(a^5+2\,a^3b^2+ab^4)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*C*a^2*b - 2*B*a*b^2 + 2*(C*a^3 - 2*B*a^2*b - C*a*b^2)*d*x + (B*a^3
+ 2*C*a^2*b - B*a*b^2 + (B*a^2*b + 2*C*a*b^2 - B*b^3)*tan(d*x + c))*log((b^
2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a
^3 - B*a^2*b - (C*a^2*b - 2*B*a*b^2 - C*b^3)*d*x)*tan(d*x + c))/((a^4*b + 2
*a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.86, size = 2995, normalized size = 26.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)

[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((B*log(tan(c + d*x)**2 + 1)/(2*d) - C*x + C*tan(c + d*x)/d)/a**2, Eq(b, 0)), (I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*B*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*C*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - C*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*C/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*B*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*C*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - C*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*C/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2)/(a + b*tan(c))**2, Eq(d, 0)), (-2*B*a**3*b*log(a/b + tan(c + d*x))/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + B*a**3*b*log(tan(c + d*x)**2 + 1)/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + 2*B*a**3*b/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + 4*B*a**2*b**2*d*x/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) - 2*B*a**2*b**2*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + B*a**2*b**2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + 4*B*a*b**3*d*x*tan(c + d*x)/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + 2*B*a*b**3*log(a/b + tan(c + d*x))/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) - B*a*b**3*log(tan(c + d*x)**2 + 1)/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) +

```
4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c +
d*x)) + 2*B*a*b**3/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d
 + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + 2*B*b
**4*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**5*b*d + 2*a**4*b**2*d*tan(c
+ d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d
*tan(c + d*x)) - B*b**4*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*b*d +
 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) +
2*a*b**5*d + 2*b**6*d*tan(c + d*x)) - 2*C*a**4/(2*a**5*b*d + 2*a**4*b**2*d*
tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*
b**6*d*tan(c + d*x)) - 2*C*a**3*b*d*x/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d
*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*ta
n(c + d*x)) - 2*C*a**2*b**2*d*x*tan(c + d*x)/(2*a**5*b*d + 2*a**4*b**2*d*ta
n(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**5*d + 2*b*
*6*d*tan(c + d*x)) - 4*C*a**2*b**2*log(a/b + tan(c + d*x))/(2*a**5*b*d + 2*
a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a
*b**5*d + 2*b**6*d*tan(c + d*x)) + 2*C*a**2*b**2*log(tan(c + d*x)**2 + 1)/(
2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan
(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) - 2*C*a**2*b**2/(2*a**5*b*d
 + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x)
+ 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + 2*C*a*b**3*d*x/(2*a**5*b*d + 2*a**4
*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b**
5*d + 2*b**6*d*tan(c + d*x)) - 4*C*a*b**3*log(a/b + tan(c + d*x))*tan(c + d
*x)/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a**3*b**3*d + 4*a**2*b**4*
d*tan(c + d*x) + 2*a*b**5*d + 2*b**6*d*tan(c + d*x)) + 2*C*a*b**3*log(tan(c
 + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*b*d + 2*a**4*b**2*d*tan(c + d*x) + 4*a
**3*b**3*d + 4*a**2*b**4*d*tan(c + d*x) + 2*a*b...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(117) = 234$.

time = 0.70, size = 241, normalized size = 2.10

$$-\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ba^2b+2\,Cab^2-Bb^3)\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,(Ba^2b^2\tan(dx+c)+2\,Cab^3\tan(dx+c)-Bb^4\tan(dx+c)-Ca^4+2\,Ba^3b+Ca^2b^2)}{(a^4b+2\,a^2b^3+b^5)(b\tan(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^
2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B
*a^2*b + 2*C*a*b^2 - B*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3
 + b^5) - 2*(B*a^2*b^2*tan(d*x + c) + 2*C*a*b^3*tan(d*x + c) - B*b^4*tan(d*
x + c) - C*a^4 + 2*B*a^3*b + C*a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*tan(d
*x + c) + a)))/d
```

Mupad [B]

time = 9.01, size = 163, normalized size = 1.42

$$\frac{a\,(B\,b-C\,a)}{b\,d\,(a^2+b^2)\,(a+b\tan(c+d\,x))}+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(B+C\,1\mathrm{i})}{2\,d\,(a^2+a\,b\,2\mathrm{i}-b^2)}+\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(C+B\,1\mathrm{i})}{2\,d\,(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i})}-\frac{\ln(a+b\tan(c+d\,x))\left(\frac{B}{a^2+b^2}-\frac{2\,b\,(B\,b-C\,a)}{(a^2+b^2)^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(c + d*x) + C*tan(c + d*x)^2)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(B + C*1i))/(2*d*(a*b*2i + a^2 - b^2)) - (log(a + b
*tan(c + d*x))*(B/(a^2 + b^2) - (2*b*(B*b - C*a))/(a^2 + b^2)^2))/d + (log(
tan(c + d*x) + 1i)*(B*1i + C))/(2*d*(2*a*b + a^2*1i - b^2*1i)) + (a*(B*b -
C*a))/(b*d*(a^2 + b^2)*(a + b*tan(c + d*x)))
```

3.35 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=111

$$\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2}+\frac{(2abB-a^2C+b^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d}-\frac{bB-aC}{(a^2+b^2)\,d(a+b\tan(c+d$$

```
[Out] (B*a^2-B*b^2+2*C*a*b)*x/(a^2+b^2)^2+(2*B*a*b-C*a^2+C*b^2)*ln(a*cos(d*x+c)+b
*sin(d*x+c))/(a^2+b^2)^2/d+(-B*b+C*a)/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3713, 3610, 3612, 3611}

$$-\frac{bB-aC}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{(a^2(-C)+2abB+b^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}+\frac{x(a^2B+2abC-b^2B)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
^2,x]
```

```
[Out] ((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2 + ((2*a*b*B - a^2*C + b^2*C)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d) - (b*B - a*C)/((a^2 +
 b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{B+C\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= -\frac{bB-aC}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\int \frac{aB+bC-(bB-aC)\,t}{a+b\tan(c+d}}{a^2+b^2}$$

$$= \frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2} - \frac{bB-aC}{(a^2+b^2)\,d(a+b\tan(c+}$$

$$= \frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2} + \frac{(2abB-a^2C+b^2C)\log(a}{(a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.52, size = 190, normalized size = 1.71

$$\frac{\frac{C((-ia-b)\log(i-\tan(c+dx))+i(a+ib)\log(i+\tan(c+dx))+2b\log(a+b\tan(c+dx)))}{a^2+b^2}-(bB-aC)\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{i\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2b\left(-2a\log(a+b\tan(c+dx))+\frac{a^2+b^2}{a+b\tan(c+dx)}\right)}{(a^2+b^2)^2}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x])^2,x]
```

```
[Out] ((C*(((-I)*a - b)*Log[I - Tan[c + d*x]] + I*(a + I*b)*Log[I + Tan[c + d*x]]
 + 2*b*Log[a + b*Tan[c + d*x]]))/(a^2 + b^2) - (b*B - a*C)*((I*Log[I - Tan[
c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b*(-2*a
*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan[c + d*x])))/(a^2 + b^2)^2
))/(2*b*d)
```

Maple [A]
time = 0.43, size = 141, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{\frac{(-2Bab+C\,a^2-b^2C)\ln(1+\tan^2(dx+c))}{2}+(a^2B-b^2B+2Cab)\arctan(\tan(dx+c))}{(a^2+b^2)^2}-\frac{Bb-Ca}{(a^2+b^2)(a+b\tan(dx+c))}+\frac{(2Bab-C\,a^2+b^2C)\,\mathrm{l}}{(a^2+b^2}}{d}$
default	$\frac{\frac{\frac{(-2Bab+C\,a^2-b^2C)\ln(1+\tan^2(dx+c))}{2}+(a^2B-b^2B+2Cab)\arctan(\tan(dx+c))}{(a^2+b^2)^2}-\frac{Bb-Ca}{(a^2+b^2)(a+b\tan(dx+c))}+\frac{(2Bab-C\,a^2+b^2C)\,\mathrm{l}}{(a^2+b^2}}{d}$
norman	$\frac{\frac{a(a^2B-b^2B+2Cab)x}{a^4+2a^2b^2+b^4}+\frac{b(a^2B-b^2B+2Cab)x\tan(dx+c)}{a^4+2a^2b^2+b^4}+\frac{(Bb-Ca)b\tan(dx+c)}{ad(a^2+b^2)}}{a+b\tan(dx+c)}+\frac{(2Bab-C\,a^2+b^2C)\ln(a+b\tan(dx+c))}{d(a^4+2a^2b^2+b^4)}-$
risch	$-\frac{xB}{2iba-a^2+b^2}+\frac{ixC}{2iba-a^2+b^2}-\frac{4iBabx}{a^4+2a^2b^2+b^4}+\frac{2ia^2Cx}{a^4+2a^2b^2+b^4}-\frac{2iC\,b^2x}{a^4+2a^2b^2+b^4}-\frac{4iBabc}{d(a^4+2a^2b^2+b^4)}+\frac{2}{d(a^4+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^2*(1/2*(-2*B*a*b+C*a^2-C*b^2)*ln(1+tan(d*x+c)^2)+(B*a^2-B*
b^2+2*C*a*b)*arctan(tan(d*x+c)))-(B*b-C*a)/(a^2+b^2)/(a+b*tan(d*x+c))+(2*B*
a*b-C*a^2+C*b^2)/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

Maxima [A]

time = 0.50, size = 177, normalized size = 1.59

$$\frac{\frac{2\,(Ba^2+2\,Cab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Ca^2-2\,Bab-Cb^2)\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}+\frac{(Ca^2-2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ca-Bb)}{a^3+ab^2+(a^2b+b^3)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C*a
^2 - 2*B*a*b - C*b^2)*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) + (C*
a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*
(C*a - B*b)/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

Fricas [A]

time = 4.13, size = 222, normalized size = 2.00

$$\frac{2\,Cab^2-2\,Bb^3+2\,(Ba^3+2\,Ca^2b-Bab^2)dx-(Ca^3-2\,Ba^2b-Cab^2+(Ca^2b-2\,Bab^2-Cb^3)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-2\,(Ca^2b-Bab^2-(Ba^2b+2\,Cab^2-Bb^3)dx)\tan(dx+c)}{2\,((a^4b+2\,a^2b^3+b^5)d\tan(dx+c)+(a^5+2\,a^3b^2+ab^4)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
gorithm="fricas")
```

```
[Out] 1/2*(2*C*a*b^2 - 2*B*b^3 + 2*(B*a^3 + 2*C*a^2*b - B*a*b^2)*d*x - (C*a^3 - 2
*B*a^2*b - C*a*b^2 + (C*a^2*b - 2*B*a*b^2 - C*b^3)*tan(d*x + c))*log((b^2*t
an(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a^2*
b - B*a*b^2 - (B*a^2*b + 2*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c))/((a^4*b + 2*
a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.23, size = 2895, normalized size = 26.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)/tan(c)**2, Eq(a, 0) & Eq(b
, 0) & Eq(d, 0)), ((B*x + C*log(tan(c + d*x)**2 + 1)/(2*d))/a**2, Eq(b, 0))
, (-B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*
x) - 4*b**2*d) + 2*I*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**
2*d*tan(c + d*x) - 4*b**2*d) + B*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d
*tan(c + d*x) - 4*b**2*d) - B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*
b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*B/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**
2*d*tan(c + d*x) - 4*b**2*d) + I*C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*
x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*C*d*x*tan(c + d*x)/(4*b**2*
d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*C*d*x/(4*b**2*d
*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*C*tan(c + d*x)/(
4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)
), (-B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d
*x) - 4*b**2*d) - 2*I*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b*
*2*d*tan(c + d*x) - 4*b**2*d) + B*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*
d*tan(c + d*x) - 4*b**2*d) - B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I
*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*B/(4*b**2*d*tan(c + d*x)**2 + 8*I*b*
*2*d*tan(c + d*x) - 4*b**2*d) - I*C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d
*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*C*d*x*tan(c + d*x)/(4*b**2
*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*C*d*x/(4*b**2*
d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*C*tan(c + d*x)/
(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, I*b)
), (x*(B*tan(c) + C*tan(c)**2)*cot(c)/(a + b*tan(c))**2, Eq(d, 0)), (2*B*a*
*3*d*x/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*
tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 2*B*a**2*b*d*x*tan(c +
 d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*t
an(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 4*B*a**2*b*log(a/b + ta
n(c + d*x))/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b*
*3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*B*a**2*b*log(ta
n(c + d*x)**2 + 1)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*
a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*B*a**2*b
```

```
/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c
+ d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*B*a*b**2*d*x/(2*a**5*d + 2
*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b
**4*d + 2*b**5*d*tan(c + d*x)) + 4*B*a*b**2*log(a/b + tan(c + d*x))*tan(c +
 d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*t
an(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*B*a*b**2*log(tan(c +
d*x)**2 + 1)*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2
*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*B
*b**3*d*x*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d
+ 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*B*b*
*3/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(
c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - 2*C*a**3*log(a/b + tan(c +
 d*x))/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*
tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + C*a**3*log(tan(c + d*x
)**2 + 1)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3
*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 2*C*a**3/(2*a**5*d
+ 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*
a*b**4*d + 2*b**5*d*tan(c + d*x)) + 4*C*a**2*b*d*x/(2*a**5*d + 2*a**4*b*d*t
an(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b
**5*d*tan(c + d*x)) - 2*C*a**2*b*log(a/b + tan(c + d*x))*tan(c + d*x)/(2*a*
*5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x)
 + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + C*a**2*b*log(tan(c + d*x)**2 + 1)*
tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b
**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 4*C*a*b**2*d*x*t
an(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b*
*3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 2*C*a*b**2*log(a/
b + tan(c + d*x))/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a
**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - C*a*b**2*lo
g(tan(c + d*x)**2 + 1)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d
+ 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 2*C*a*
b**2/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d + 4*a**2*b**3*d*ta
n(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) + 2*C*b**3*log(a/b + tan(c
 + d*x))*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) + 4*a**3*b**2*d +
 4*a**2*b**3*d*tan(c + d*x) + 2*a*b**4*d + 2*b**5*d*tan(c + d*x)) - C*b**3*
log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*d + 2*a**4*b*d*tan(c + d*x) +
 4*a**3*b**2*d + 4*a**2*b**3*d*tan(c + d*x) + 2...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(111) = 222.

time = 0.95, size = 234, normalized size = 2.11

$$\frac{\frac{2\,(Ba^2+2\,Cab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{(Ca^2-2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Ca^2b-2\,Bab^2-Cb^3)\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}+\frac{2\,(Ca^2b\tan(dx+c)-2\,Bab^2\tan(dx+c)-Cb^3\tan(dx+c)+2\,Ca^3-3\,Ba^2b-Bb^3)}{(a^4+2\,a^2b^2+b^4)(b\tan(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, al
```

```
gorithm="giac")
```

```
[Out] 1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a^2
 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C*
a^2*b - 2*B*a*b^2 - C*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3
+ b^5) + 2*(C*a^2*b*tan(d*x + c) - 2*B*a*b^2*tan(d*x + c) - C*b^3*tan(d*x +
 c) + 2*C*a^3 - 3*B*a^2*b - B*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)
 + a)))/d
```

Mupad [B]
time = 9.09, size = 153, normalized size = 1.38

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(-C\,a^2+2\,B\,a\,b+C\,b^2\right)}{d\,{\left(a^2+b^2\right)}^2}-\frac{B\,b-C\,a}{d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x))
^2,x)
```

```
[Out] (log(a + b*tan(c + d*x))*(C*b^2 - C*a^2 + 2*B*a*b))/(d*(a^2 + b^2)^2) - (B*
b - C*a)/(d*(a^2 + b^2)*(a + b*tan(c + d*x))) - (log(tan(c + d*x) + 1i)*(B*
1i + C))/(2*d*(a*b*2i - a^2 + b^2)) - (log(tan(c + d*x) - 1i)*(B + C*1i))/(
2*d*(2*a*b - a^2*1i + b^2*1i))
```

3.36 $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$-\frac{(2abB-a^2C+b^2C)\,x}{(a^2+b^2)^2}+\frac{B\log(\sin(c+dx))}{a^2d}-\frac{b(3a^2bB+b^3B-2a^3C)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2\left(a^2+b^2\right)^2d}+\frac{}{a}$$

```
[Out] -(2*B*a*b-C*a^2+C*b^2)*x/(a^2+b^2)^2+B*ln(sin(d*x+c))/a^2/d-b*(3*B*a^2*b+B*
b^3-2*C*a^3)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^2/(a^2+b^2)^2/d+b*(B*b-C*a)/a/
(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3713, 3690, 3732, 3611, 3556}

$$\frac{b(bB-aC)}{ad\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{x(a^2(-C)+2abB+b^2C)}{(a^2+b^2)^2}+\frac{B\log(\sin(c+dx))}{a^2d}-\frac{b(-2a^3C+3a^2bB+b^3B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^2,x]
```

```
[Out] -(((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)^2) + (B*Log[Sin[c + d*x]])/(a^2
*d) - (b*(3*a^2*b*B + b^3*B - 2*a^3*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/(a^2*(a^2 + b^2)^2*d) + (b*(b*B - a*C))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d
*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
```

```
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cot(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{b(bB-aC)}{a\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\int \frac{\cot(c+dx)\left(\left(a^2+b^2\right)\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} + \frac{b(bB-aC)}{a\left(a^2+b^2\right)d(a+b\tan(\cdots}$$

$$= -\frac{\left(2abB-a^2C+b^2C\right)x}{\left(a^2+b^2\right)^2} + \frac{B\log(\sin(c+dx))}{a^2d} - \frac{b(\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.65, size = 159, normalized size = 1.16

$$-\frac{\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{2B\log(\tan(c+dx))}{a^2}+\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2b(3a^2bB+b^3B-2a^3C)\log(a+b\tan(c+dx))}{a^2(a^2+b^2)^2}+\frac{2b(-bB+aC)}{a(a^2+b^2)(a+b\tan(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^2,x]
```

```
[Out] -1/2*(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (2*B*Log[Tan[c + d*x]
])/a^2 + ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b*(3*a^2*b*B +
b^3*B - 2*a^3*C)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)^2) + (2*b*(-(b*B
) + a*C))/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])))/d
```

Maple [A]
time = 0.50, size = 163, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{B\ln(\tan(dx+c))}{a^2}+\frac{\frac{\left(-a^2B+b^2B-2Cab\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(-2Bab+C\,a^2-b^2C\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{b\left(3B\,a^2b+B\,b^3-2C\,a^3\right)\ln(a}{a^2\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{B\ln(\tan(dx+c))}{a^2}+\frac{\frac{\left(-a^2B+b^2B-2Cab\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(-2Bab+C\,a^2-b^2C\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{b\left(3B\,a^2b+B\,b^3-2C\,a^3\right)\ln(a}{a^2\left(a^2+b^2\right)^2}}{d}$
norman	$\frac{-\frac{a\left(2Bab-C\,a^2+b^2C\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{b\left(2Bab-C\,a^2+b^2C\right)x\left(\tan^2(dx+c)\right)}{a^4+2a^2b^2+b^4}-\frac{\left(b^2B-Cab\right)b\left(\tan^2(dx+c)\right)}{d\,a^2\left(a^2+b^2\right)}}{(a+b\tan(dx+c))\tan(dx+c)}+\frac{B\ln(\tan(dx+c))}{a^2d}-$
risch	$\frac{6iB\,b^2c}{d(a^4+2a^2b^2+b^4)}-\frac{xC}{2iba-a^2+b^2}+\frac{6iB\,b^2x}{a^4+2a^2b^2+b^4}-\frac{4iCabx}{a^4+2a^2b^2+b^4}-\frac{2iBc}{a^2d}+\frac{2ib^4Bx}{(a^4+2a^2b^2+b^4)a^2}-\frac{2ixB}{a^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(B/a^2*ln(tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(-B*a^2+B*b^2-2*C*a*b)*ln(1+ta
n(d*x+c)^2)+(-2*B*a*b+C*a^2-C*b^2)*arctan(tan(d*x+c)))-b*(3*B*a^2*b+B*b^3-2
*C*a^3)/a^2/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+(B*b-C*a)*b/a/(a^2+b^2)/(a+b*tan
(d*x+c)))
```

Maxima [A]
time = 0.51, size = 208, normalized size = 1.52

$$\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(2\,Ca^3b-3\,Ba^2b^2-Bb^4)\log(b\tan(dx+c)+a)}{a^6+2\,a^4b^2+a^2b^4}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Cab-Bb^2)}{a^4+a^2b^2+(a^3b+ab^3)\tan(dx+c)}+\frac{2\,B\log(\tan(dx+c))}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*C
*a^3*b - 3*B*a^2*b^2 - B*b^4)*log(b*tan(d*x + c) + a)/(a^6 + 2*a^4*b^2 + a^
2*b^4) - (B*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2
 + b^4) - 2*(C*a*b - B*b^2)/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x + c))
+ 2*B*log(tan(d*x + c))/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(137) = 274$.
time = 8.04, size = 323, normalized size = 2.36

$$-\frac{2\,Ca^{2}b^{3}-2\,Bab^{4}-2\,(Ca^{5}-2\,Ba^{4}b-Ca^{3}b^{2})dx-(Ba^{5}+2\,Ba^{3}b^{2}+Bab^{4}+(Ba^{4}b+2\,Ba^{2}b^{3}+Bb^{5})\tan\left(dx+c\right))\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)-(2\,Ca^{4}b-3\,Ba^{3}b^{2}-Bab^{4}+(2\,Ca^{3}b^{2}-3\,Ba^{2}b^{3}-Bb^{5})\tan\left(dx+c\right))\log\left(\frac{b^{2}\tan(dx+c)^{2}+2\,ab\tan(dx+c)+a^{2}}{\tan(dx+c)^{2}+1}\right)-2\,(Ca^{3}b^{2}-Ba^{2}b^{3}+(Ca^{4}b-2\,Ba^{3}b^{2}-Ca^{2}b^{3})dx)\tan\left(dx+c\right)}{2\,((a^{6}b+2\,a^{4}b^{3}+a^{2}b^{5})d\tan\left(dx+c\right)+(a^{7}+2\,a^{5}b^{2}+a^{3}b^{4})d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/2*(2*C*a^2*b^3 - 2*B*a*b^4 - 2*(C*a^5 - 2*B*a^4*b - C*a^3*b^2)*d*x - (B*
a^5 + 2*B*a^3*b^2 + B*a*b^4 + (B*a^4*b + 2*B*a^2*b^3 + B*b^5)*tan(d*x + c))
*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - (2*C*a^4*b - 3*B*a^3*b^2 - B*a*
b^4 + (2*C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*tan(d*x + c))*log((b^2*tan(d*x +
c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a^3*b^2 - B*a
^2*b^3 + (C*a^4*b - 2*B*a^3*b^2 - C*a^2*b^3)*d*x)*tan(d*x + c))/((a^6*b + 2
*a^4*b^3 + a^2*b^5)*d*tan(d*x + c) + (a^7 + 2*a^5*b^2 + a^3*b^4)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.02, size = 4461, normalized size = 32.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,
x)
```

```
[Out] Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)**2/tan(c)**2, Eq(a, 0) & E
q(b, 0) & Eq(d, 0)), ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*log(tan(c + d*
x))/d + C*x)/a**2, Eq(b, 0)), ((B*log(tan(c + d*x)**2 + 1)/(2*d) - B*log(ta
n(c + d*x))/d - B/(2*d*tan(c + d*x)**2) - C*x - C/(d*tan(c + d*x)))/b**2, E
q(a, 0)), (3*I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d
*tan(c + d*x) - 4*b**2*d) + 6*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2
- 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*I*B*d*x/(4*b**2*d*tan(c + d*x)**2
 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*log(tan(c + d*x)**2 + 1)*tan(c
 + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d)
- 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8
```

```
*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*B*log(tan(c + d*x)**2 + 1)/(4*b**2*d
*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*B*log(tan(c + d*
x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4
*b**2*d) + 8*I*B*log(tan(c + d*x))*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 -
 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*B*log(tan(c + d*x))/(4*b**2*d*tan(
c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 3*I*B*tan(c + d*x)/(4*b
**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*B/(4*b**2*d
*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - C*d*x*tan(c + d*x)
**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*C
*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b
**2*d) + C*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2
*d) - C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) -
4*b**2*d) + 2*I*C/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b
**2*d), Eq(a, -I*b)), (-3*I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2
 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 6*B*d*x*tan(c + d*x)/(4*b**2*d*tan
(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 3*I*B*d*x/(4*b**2*d*ta
n(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*log(tan(c + d*x)*
*2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x)
 - 4*b**2*d) + 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c
+ d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*B*log(tan(c + d*x)**2 +
 1)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*B*l
og(tan(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan
(c + d*x) - 4*b**2*d) - 8*I*B*log(tan(c + d*x))*tan(c + d*x)/(4*b**2*d*tan(
c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 4*B*log(tan(c + d*x))/(
4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*I*B*tan(
c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) +
4*B/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - C*d*x
*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b*
*2*d) - 2*I*C*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c
 + d*x) - 4*b**2*d) + C*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c +
d*x) - 4*b**2*d) - C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*ta
n(c + d*x) - 4*b**2*d) - 2*I*C/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c
 + d*x) - 4*b**2*d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2)*cot(c)**2/(a
+ b*tan(c))**2, Eq(d, 0)), (-B*a**5*log(tan(c + d*x)**2 + 1)/(2*a**7*d + 2*
a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3
*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 2*B*a**5*log(tan(c + d*x))/(2*a**7*
d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) +
2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - 4*B*a**4*b*d*x/(2*a**7*d + 2*
a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3
*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - B*a**4*b*log(tan(c + d*x)**2 + 1)*t
an(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b*
*3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 2*B*a**4*
b*log(tan(c + d*x))*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a*
*5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(
c + d*x)) - 4*B*a**3*b**2*d*x*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d
```

```
*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b
**5*d*tan(c + d*x)) - 6*B*a**3*b**2*log(a/b + tan(c + d*x))/(2*a**7*d + 2*a
**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a**3*
b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + B*a**3*b**2*log(tan(c + d*x)**2 + 1)
/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*tan(c
+ d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 4*B*a**3*b**2*log(ta
n(c + d*x))/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b*
*3*d*tan(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) + 2*B*a**3*
b**2/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*a**5*b**2*d + 4*a**4*b**3*d*ta
n(c + d*x) + 2*a**3*b**4*d + 2*a**2*b**5*d*tan(c + d*x)) - 6*B*a**2*b**3*lo
g(a/b + tan(c + d*x))*tan(c + d*x)/(2*a**7*d + 2*a**6*b*d*tan(c + d*x) + 4*
a**5*b**2*d + 4*a**4*b**3*d*tan(c + d*x) + 2*a*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(137) = 274.
time = 1.30, size = 279, normalized size = 2.04

$$\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(2\,Ca^3b^2-3\,Ba^2b^3-Bb^5)\log(|b\tan(dx+c)+a|)}{a^6b+2\,a^4b^3+a^2b^5}+\frac{2\,B\log(|\tan(dx+c)|)}{a^2}-\frac{2\,(2\,Ca^3b^2\tan(dx+c)-3\,Ba^2b^3\tan(dx+c)-Bb^5\tan(dx+c)+3\,Ca^4b-4\,Ba^3b^2+Ca^2b^3-2\,Bab^4)}{(a^6+2\,a^4b^2+a^2b^4)(b\tan(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^2
 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*
C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 2*a^
4*b^3 + a^2*b^5) + 2*B*log(abs(tan(d*x + c)))/a^2 - 2*(2*C*a^3*b^2*tan(d*x
+ c) - 3*B*a^2*b^3*tan(d*x + c) - B*b^5*tan(d*x + c) + 3*C*a^4*b - 4*B*a^3*
b^2 + C*a^2*b^3 - 2*B*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4)*(b*tan(d*x + c) +
 a)))/d
```

Mupad [B]
time = 10.69, size = 180, normalized size = 1.31

$$\frac{B\,\ln\left(\tan(c+d\,x)\right)}{a^2\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}+\frac{B\,b^2-C\,a\,b}{a\,d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}-\frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-2\,C\,a^3+3\,B\,a^2\,b+B\,b^3\right)}{a^2\,d\,{\left(a^2+b^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
))^2,x)
```

```
[Out] (B*log(tan(c + d*x)))/(a^2*d) - (log(tan(c + d*x) - 1i)*(B + C*1i))/(2*d*(a
*b*2i + a^2 - b^2)) - (log(tan(c + d*x) + 1i)*(B*1i + C))/(2*d*(2*a*b + a^2
*1i - b^2*1i)) + (B*b^2 - C*a*b)/(a*d*(a^2 + b^2)*(a + b*tan(c + d*x))) - (
b*log(a + b*tan(c + d*x))*(B*b^3 - 2*C*a^3 + 3*B*a^2*b))/(a^2*d*(a^2 + b^2)
^2)
```

3.37 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=192

$$-\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2}-\frac{(2bB-aC)\log(\sin(c+dx))}{a^3d}+\frac{b^2(4a^2bB+2b^3B-3a^3C-ab^2C)\log(a\cos(c+dx}{a^3\,(a^2+b^2)^2\,d}$$

```
[Out] -(B*a^2-B*b^2+2*C*a*b)*x/(a^2+b^2)^2-(2*B*b-C*a)*ln(sin(d*x+c))/a^3/d+b^2*(
4*B*a^2*b+2*B*b^3-3*C*a^3-C*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^3/(a^2+b
^2)^2/d-b*(B*a^2+2*B*b^2-C*a*b)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))-B*cot(d*x+
c)/a/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.43, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3713, 3690, 3730, 3732, 3611, 3556}

$$-\frac{(2bB-aC)\log(\sin(c+dx))}{a^3d}-\frac{b(a^2B-abC+2b^2B)}{a^2d\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{x(a^2B+2abC-b^2B)}{(a^2+b^2)^2}+\frac{b^2(-3a^3C+4a^2bB-ab^2C+2b^3B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\,(a^2+b^2)^2}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^2,x]
```

```
[Out] -(((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2)^2) - ((2*b*B - a*C)*Log[Sin[c +
 d*x]])/(a^3*d) + (b^2*(4*a^2*b*B + 2*b^3*B - 3*a^3*C - a*b^2*C)*Log[a*Cos[
c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)^2*d) - (b*(a^2*B + 2*b^2*B - a
*b*C))/(a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (B*Cot[c + d*x])/(a*d*(a
+ b*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
```

```
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$= -\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))} - \frac{\int \frac{\cot(c+dx)(2bB-aC+aB\tan(c+}{(a+b\tan(c+d}}{a}$$

$$= -\frac{b(a^2B+2b^2B-abC)}{a^2(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{B\cot(c+a}{ad(a+b\tan(c}$$

$$= -\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2} - \frac{b(a^2B+2b^2B-ab}{a^2(a^2+b^2)\,d(a+b\tan(c}$$

$$= -\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2} - \frac{(2bB-aC)\log(\sin(c+a}{a^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.35, size = 193, normalized size = 1.01

$$\frac{-\frac{2B\cot(c+dx)}{a^2}+\frac{i(B+iC)\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{2(-2bB+aC)\log(\tan(c+dx))}{a^3}-\frac{(iB+C)\log(i+\tan(c+dx))}{(a-ib)^2}-\frac{2b^2(-4a^2bB-2b^3B+3a^3C+ab^2C)\log(a+b\tan(c+dx))}{a^3(a^2+b^2)^2}+\frac{2b^2(-bB+aC)}{a^2(a^2+b^2)(a+b\tan(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^2,x]
```

```
[Out] ((-2*B*Cot[c + d*x])/a^2 + (I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2
+ (2*(-2*b*B + a*C)*Log[Tan[c + d*x]])/a^3 - ((I*B + C)*Log[I + Tan[c + d*x
]])/(a - I*b)^2 - (2*b^2*(-4*a^2*b*B - 2*b^3*B + 3*a^3*C + a*b^2*C)*Log[a +
 b*Tan[c + d*x]])/(a^3*(a^2 + b^2)^2) + (2*b^2*(-(b*B) + a*C))/(a^2*(a^2 +
b^2)*(a + b*Tan[c + d*x])))/(2*d)
```

Maple [A]
time = 0.56, size = 196, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{B}{a^2\tan(dx+c)}+\frac{(-2Bb+Ca)\ln(\tan(dx+c))}{a^3}+\frac{b^2\left(4B\,a^2b+2B\,b^3-3C\,a^3-C\,a\,b^2\right)\ln(a+b\tan(dx+c))}{a^3\left(a^2+b^2\right)^2}-\frac{(Bb-Ca)b^2}{a^2\left(a^2+b^2\right)(a+b\tan(dx+c))}+}{d}$
default	$\frac{-\frac{B}{a^2\tan(dx+c)}+\frac{(-2Bb+Ca)\ln(\tan(dx+c))}{a^3}+\frac{b^2\left(4B\,a^2b+2B\,b^3-3C\,a^3-C\,a\,b^2\right)\ln(a+b\tan(dx+c))}{a^3\left(a^2+b^2\right)^2}-\frac{(Bb-Ca)b^2}{a^2\left(a^2+b^2\right)(a+b\tan(dx+c))}+}{d}$
norman	$\frac{\frac{\left(B\,a^2b+2B\,b^3-C\,a\,b^2\right)b\left(\tan^3(dx+c)\right)}{d\,a^3\left(a^2+b^2\right)}-\frac{B\tan(dx+c)}{ad}-\frac{a\left(a^2B-b^2B+2Cab\right)x\left(\tan^2(dx+c)\right)}{a^4+2a^2b^2+b^4}-\frac{b\left(a^2B-b^2B+2Cab\right)x\left(\tan^3(dx+c)\right)}{a^4+2a^2b^2+b^4}}{\tan(dx+c)^2(a+b\tan(dx+c))}-$

risch	$\frac{xB}{2iba-a^2+b^2}-\frac{2iCc}{a^2d}-\frac{ixC}{2iba-a^2+b^2}-\frac{4ib^5Bc}{a^3d(a^4+2a^2b^2+b^4)}+\frac{6iC\,b^2x}{a^4+2a^2b^2+b^4}-\frac{2iCx}{a^2}-\frac{8ib^3Bc}{ad(a^4+2a^2b^2+b^4)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(-B/a^2/tan(d*x+c)+(-2*B*b+C*a)/a^3*ln(tan(d*x+c))+b^2*(4*B*a^2*b+2*B*b
^3-3*C*a^3-C*a*b^2)/a^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))-(B*b-C*a)*b^2/a^2/(a
^2+b^2)/(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(2*B*a*b-C*a^2+C*b^2)*ln(1+tan(
d*x+c)^2)+(-B*a^2+B*b^2-2*C*a*b)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.51, size = 262, normalized size = 1.36

$$-\frac{\frac{2\,(Ba^2+2\,Cab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(3\,Ca^3b^2-4\,Ba^2b^3+Cab^4-2\,Bb^5)\log(b\tan(dx+c)+a)}{a^7+2\,a^5b^2+a^3b^4}+\frac{(Ca^2-2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ba^3+Bab^2+(Ba^2b-Cab^2+2\,Bb^3)\tan(dx+c))}{(a^4b+a^2b^3)\tan(dx+c)^2+(a^5+a^3b^2)\tan(dx+c)}-\frac{2\,(Ca-2\,Bb)\log(\tan(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(3*
C*a^3*b^2 - 4*B*a^2*b^3 + C*a*b^4 - 2*B*b^5)*log(b*tan(d*x + c) + a)/(a^7 +
 2*a^5*b^2 + a^3*b^4) + (C*a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(
a^4 + 2*a^2*b^2 + b^4) + 2*(B*a^3 + B*a*b^2 + (B*a^2*b - C*a*b^2 + 2*B*b^3)
*tan(d*x + c))/((a^4*b + a^2*b^3)*tan(d*x + c)^2 + (a^5 + a^3*b^2)*tan(d*x
+ c)) - 2*(C*a - 2*B*b)*log(tan(d*x + c))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(190) = 380$.
time = 9.19, size = 465, normalized size = 2.42

$$-\frac{2\,Ba^6+4\,Ba^4b^2+2\,Ba^2b^4+2\,(Ca^3b^3-Ba^2b^4+(Ba^5b+2\,Ca^4b^2-Ba^3b^3)dx)\tan(dx+c)^2-((Ca^5b-2\,Ba^4b^2+2\,Ca^3b^3-4\,Ba^2b^4+Cab^5-2\,Bb^6)\tan(dx+c)^2+(Ca^6-2\,Ba^5b+2\,Ca^4b^2-4\,Ba^3b^3+Ca^2b^4-2\,Bab^5)\tan(dx+c))\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+((3\,Ca^3b^3-4\,Ba^2b^4+Cab^5-2\,Bb^6)\tan(dx+c)^2+(3\,Ca^4b^2-4\,Ba^3b^3+Ca^2b^4-2\,Bab^5)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+2\,(Ba^5b+2\,Ba^3b^3-Ca^2b^4+2\,Bab^5+(Ba^6+2\,Ca^5b-Ba^4b^2)dx)\tan(dx+c)}{2\,((a^7b+2\,a^5b^3+a^3b^5)d\tan(dx+c)^2+(a^8+2\,a^6b^2+a^4b^4)d\tan(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
algorithm="fricas")
```

```
[Out] -1/2*(2*B*a^6 + 4*B*a^4*b^2 + 2*B*a^2*b^4 + 2*(C*a^3*b^3 - B*a^2*b^4 + (B*a
^5*b + 2*C*a^4*b^2 - B*a^3*b^3)*d*x)*tan(d*x + c)^2 - ((C*a^5*b - 2*B*a^4*b
^2 + 2*C*a^3*b^3 - 4*B*a^2*b^4 + C*a*b^5 - 2*B*b^6)*tan(d*x + c)^2 + (C*a^6
 - 2*B*a^5*b + 2*C*a^4*b^2 - 4*B*a^3*b^3 + C*a^2*b^4 - 2*B*a*b^5)*tan(d*x +
 c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + ((3*C*a^3*b^3 - 4*B*a^2*b^4
 + C*a*b^5 - 2*B*b^6)*tan(d*x + c)^2 + (3*C*a^4*b^2 - 4*B*a^3*b^3 + C*a^2*b
```

```
^4 - 2*B*a*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
+ a^2)/(tan(d*x + c)^2 + 1)) + 2*(B*a^5*b + 2*B*a^3*b^3 - C*a^2*b^4 + 2*B*a
*b^5 + (B*a^6 + 2*C*a^5*b - B*a^4*b^2)*d*x)*tan(d*x + c))/((a^7*b + 2*a^5*b
^3 + a^3*b^5)*d*tan(d*x + c)^2 + (a^8 + 2*a^6*b^2 + a^4*b^4)*d*tan(d*x + c)
)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.37, size = 8097, normalized size = 42.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,
x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((B*x + B/(d*ta
n(c + d*x)) - B/(3*d*tan(c + d*x)**3) + C*log(tan(c + d*x)**2 + 1)/(2*d) -
C*log(tan(c + d*x))/d - C/(2*d*tan(c + d*x)**2))/b**2, Eq(a, 0)), (9*B*d*x*
tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*
b**2*d*tan(c + d*x)) - 18*I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3
 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 9*B*d*x*tan(c + d*
x)/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c
+ d*x)) - 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c +
d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 8*B*log(tan
(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*ta
n(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 4*I*B*log(tan(c + d*x)**2 + 1)*tan
(c + d*x)/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d
*tan(c + d*x)) + 8*I*B*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c +
d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 16*B*log(ta
n(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c +
d*x)**2 - 4*b**2*d*tan(c + d*x)) - 8*I*B*log(tan(c + d*x))*tan(c + d*x)/(4*
b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)
) + 9*B*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)
**2 - 4*b**2*d*tan(c + d*x)) - 14*I*B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**
3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 4*B/(4*b**2*d*tan
(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 3*I*C*
d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2
- 4*b**2*d*tan(c + d*x)) + 6*C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**
3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 3*I*C*d*x*tan(c +
 d*x)/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan
(c + d*x)) + 2*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c +
 d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 4*I*C*log(
tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d
*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 2*C*log(tan(c + d*x)**2 + 1)*ta
n(c + d*x)/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*
```

```
d*tan(c + d*x)) - 4*C*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c + d
*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 8*I*C*log(ta
n(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c +
d*x)**2 - 4*b**2*d*tan(c + d*x)) + 4*C*log(tan(c + d*x))*tan(c + d*x)/(4*b*
*2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x))
+ 3*I*C*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 - 8*I*b**2*d*tan(c + d*x)
**2 - 4*b**2*d*tan(c + d*x)) + 4*C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**3 -
 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)), Eq(a, -I*b)), (9*B*d*
x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 -
4*b**2*d*tan(c + d*x)) + 18*I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)*
*3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 9*B*d*x*tan(c +
d*x)/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(
c + d*x)) + 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c
+ d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 8*B*log(t
an(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*
tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 4*I*B*log(tan(c + d*x)**2 + 1)*t
an(c + d*x)/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2
*d*tan(c + d*x)) - 8*I*B*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c
+ d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 16*B*log(
tan(c + d*x))*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c
+ d*x)**2 - 4*b**2*d*tan(c + d*x)) + 8*I*B*log(tan(c + d*x))*tan(c + d*x)/(
4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*
x)) + 9*B*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*
x)**2 - 4*b**2*d*tan(c + d*x)) + 14*I*B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)
**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 4*B/(4*b**2*d*t
an(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 3*I*
C*d*x*tan(c + d*x)**3/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**
2 - 4*b**2*d*tan(c + d*x)) + 6*C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)
**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 3*I*C*d*x*tan(c
 + d*x)/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*t
an(c + d*x)) + 2*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*b**2*d*tan(c
 + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) + 4*I*C*lo
g(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2
*d*tan(c + d*x)**2 - 4*b**2*d*tan(c + d*x)) - 2*C*log(tan(c + d*x)**2 + 1)*
tan(c + d*x)/(4*b**2*d*tan(c + d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**
2*d*tan(c + d*x)) - 4*C*log(tan(c + d*x))*tan(c + d*x)**3/(4*b**2*d*tan(c +
 d*x)**3 + 8*I*b**2*d*tan(c + d*x)**2 - 4*b**2*...
```

Giac [A]
time = 1.32, size = 362, normalized size = 1.89

$$-\frac{\frac{2\,(Ba^2+2\,Cab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{(Ca^2-2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(3\,Ca^3b^3-4\,Ba^2b^4+Cab^5-2\,Bb^6)\log(|b\tan(dx+c)+a|)}{a^7b+2\,a^5b^3+a^3b^5}+\frac{Ca^4b\tan(dx+c)^2-2\,Ba^3b^2\tan(dx+c)^2-Ca^2b^3\tan(dx+c)^2+Ca^5\tan(dx+c)-3\,Ca^3b^2\tan(dx+c)+6\,Ba^2b^3\tan(dx+c)-2\,Cab^4\tan(dx+c)+4\,Bb^5\tan(dx+c)+2\,Ba^5+4\,Ba^3b^2+2\,Bab^4}{(a^6+2\,a^4b^2+a^2b^4)\left(b\tan(dx+c)^2+a\tan(dx+c)\right)}-\frac{2\,(Ca-2\,Bb)\log(|\tan(dx+c)|)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
```

```
algorithm="giac")
```

```
[Out] -1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a^
2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(3
*C*a^3*b^3 - 4*B*a^2*b^4 + C*a*b^5 - 2*B*b^6)*log(abs(b*tan(d*x + c) + a))/
(a^7*b + 2*a^5*b^3 + a^3*b^5) + (C*a^4*b*tan(d*x + c)^2 - 2*B*a^3*b^2*tan(d
*x + c)^2 - C*a^2*b^3*tan(d*x + c)^2 + C*a^5*tan(d*x + c) - 3*C*a^3*b^2*tan
(d*x + c) + 6*B*a^2*b^3*tan(d*x + c) - 2*C*a*b^4*tan(d*x + c) + 4*B*b^5*tan
(d*x + c) + 2*B*a^5 + 4*B*a^3*b^2 + 2*B*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4)
*(b*tan(d*x + c)^2 + a*tan(d*x + c))) - 2*(C*a - 2*B*b)*log(abs(tan(d*x + c
)))/a^3)/d
```

Mupad [B]
time = 12.15, size = 230, normalized size = 1.20

$$\frac{b^2\,\ln\left(a+b\,\tan(c+d\,x)\right)\,\left(-3\,C\,a^3+4\,B\,a^2\,b-C\,a\,b^2+2\,B\,b^3\right)}{a^3\,d\,{\left(a^2+b^2\right)}^2}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(2\,B\,b-C\,a\right)}{a^3\,d}+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(C+B\,1{}\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,2{}\mathrm{i}+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1{}\mathrm{i}\right)}{2\,d\,\left(-a^2\,1{}\mathrm{i}+2\,a\,b+b^2\,1{}\mathrm{i}\right)}-\frac{\frac{B}{a}+\frac{\tan(c+d\,x)\,\left(B\,a^2\,b-C\,a\,b^2+2\,B\,b^3\right)}{a^2\,\left(a^2+b^2\right)}}{d\,\left(b\,{\tan\left(c+d\,x\right)}^2+a\,\tan\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
))^2,x)
```

```
[Out] (log(tan(c + d*x) + 1i)*(B*1i + C))/(2*d*(a*b*2i - a^2 + b^2)) - (log(tan(c
 + d*x))*(2*B*b - C*a))/(a^3*d) - (B/a + (tan(c + d*x)*(2*B*b^3 + B*a^2*b -
 C*a*b^2))/(a^2*(a^2 + b^2)))/(d*(a*tan(c + d*x) + b*tan(c + d*x)^2)) + (lo
g(tan(c + d*x) - 1i)*(B + C*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) + (b^2*log
(a + b*tan(c + d*x))*(2*B*b^3 - 3*C*a^3 + 4*B*a^2*b - C*a*b^2))/(a^3*d*(a^2
 + b^2)^2)
```

3.38 $\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=331

$$\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\,x}{(a^2+b^2)^3}+\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\log(\cos(c+dx))}{(a^2+b^2)^3\,d}+\frac{a^2(a^4bB+3a^2b^3B+6$$

```
[Out] (B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*x/(a^2+b^2)^3+(3*B*a^2*b-B*b^3-C*a^3+3*C*
a*b^2)*ln(cos(d*x+c))/(a^2+b^2)^3/d+a^2*(B*a^4*b+3*B*a^2*b^3+6*B*b^5-3*C*a^
5-9*C*a^3*b^2-10*C*a*b^4)*ln(a+b*tan(d*x+c))/b^4/(a^2+b^2)^3/d-(B*a^3*b+3*B
*a*b^3-3*C*a^4-6*C*a^2*b^2-C*b^4)*tan(d*x+c)/b^3/(a^2+b^2)^2/d+1/2*a*(B*b-C
*a)*tan(d*x+c)^3/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2*a*(B*a^2*b+5*B*b^3-3*
C*a^3-7*C*a*b^2)*tan(d*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.61, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3713, 3686, 3726, 3728, 3707, 3698, 31, 3556}

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{a(-3a^3C+a^2bB-7ab^2C+5b^3B)\tan^2(c+dx)}{2b^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{(a^3(-C)+3a^2bB+3ab^2C-b^3B)\log(\cos(c+dx))}{d\left(a^2+b^2\right)^3}+\frac{x(a^3B+3a^2bC-3ab^2B-b^3C)}{\left(a^2+b^2\right)^3}-\frac{(-3a^4C+a^3bB-6a^2b^2C+3ab^3B-b^4C)\tan(c+dx)}{b^3d\left(a^2+b^2\right)^2}+\frac{a^2(-3a^5C+a^4bB-9a^3b^2C+3a^2b^3B-10ab^4C+6b^5B)\log(a+b\tan(c+dx))}{b^4d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^3,x]
```

```
[Out] ((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3 + ((3*a^2*b*B - b
^3*B - a^3*C + 3*a*b^2*C)*Log[Cos[c + d*x]])/((a^2 + b^2)^3*d) + (a^2*(a^4*
b*B + 3*a^2*b^3*B + 6*b^5*B - 3*a^5*C - 9*a^3*b^2*C - 10*a*b^4*C)*Log[a + b
*Tan[c + d*x]])/(b^4*(a^2 + b^2)^3*d) - ((a^3*b*B + 3*a*b^3*B - 3*a^4*C - 6
*a^2*b^2*C - b^4*C)*Tan[c + d*x])/(b^3*(a^2 + b^2)^2*d) + (a*(b*B - a*C)*Ta
n[c + d*x]^3)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a*(a^2*b*B + 5*
b^3*B - 3*a^3*C - 7*a*b^2*C)*Tan[c + d*x]^2)/(2*b^2*(a^2 + b^2)^2*d*(a + b*
Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
```

```
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\tan^4(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\tan^3(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{\tan^2(c+dx)(-3a}{}}{}$$

$$= \frac{a(bB-aC)\tan^3(c+dx)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{a(a^2bB+5b^3B}{2b^2\left(a^2+\right.}$$

$$= -\frac{\left(a^3bB+3ab^3B-3a^4C-6a^2b^2C-b^4C\right)\tan(c+}{b^3\left(a^2+b^2\right)^2d}$$

$$= \frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} - \frac{(a^3bB+3ab^3B}{}$$

$$= \frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} + \frac{(3a^2bB-b^3B}{}$$

$$= \frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} + \frac{(3a^2bB-b^3B}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.61, size = 1146, normalized size = 3.46

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^3,x]
```

```
[Out] (a^4*(-(b*B) + a*C)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*(B + C
*Tan[c + d*x]))/(2*(a - I*b)^2*(a + I*b)^2*b^2*d*(B*Cos[c + d*x] + C*Sin[c
+ d*x])*(a + b*Tan[c + d*x])^3) + ((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*
(c + d*x)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(B + C*Tan[c +
 d*x]))/((a - I*b)^3*(a + I*b)^3*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b
*Tan[c + d*x])^3) + ((I*a^11*b^4*B + a^10*b^5*B + (5*I)*a^9*b^6*B + 5*a^8*b
^7*B + (13*I)*a^7*b^8*B + 13*a^6*b^9*B + (15*I)*a^5*b^10*B + 15*a^4*b^11*B
+ (6*I)*a^3*b^12*B + 6*a^2*b^13*B - (3*I)*a^12*b^3*C - 3*a^11*b^4*C - (15*I
)*a^10*b^5*C - 15*a^9*b^6*C - (31*I)*a^8*b^7*C - 31*a^7*b^8*C - (29*I)*a^6*
b^9*C - 29*a^5*b^10*C - (10*I)*a^4*b^11*C - 10*a^3*b^12*C)*(c + d*x)*Sec[c
+ d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(B + C*Tan[c + d*x]))/((a - I*
b)^6*(a + I*b)^5*b^7*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c + d*x
])^3) - (I*(a^6*b*B + 3*a^4*b^3*B + 6*a^2*b^5*B - 3*a^7*C - 9*a^5*b^2*C - 1
0*a^3*b^4*C)*ArcTan[Tan[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^3*(B + C*Tan[c + d*x]))/(b^4*(a^2 + b^2)^3*d*(B*Cos[c + d*x] + C*Si
n[c + d*x])*(a + b*Tan[c + d*x])^3) + ((-(b*B) + 3*a*C)*Log[Cos[c + d*x]]*S
ec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(B + C*Tan[c + d*x]))/(b^
4*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c + d*x])^3) + ((a^6*b*B +
 3*a^4*b^3*B + 6*a^2*b^5*B - 3*a^7*C - 9*a^5*b^2*C - 10*a^3*b^4*C)*Log[(a*C
os[c + d*x] + b*Sin[c + d*x])^2]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^3*(B + C*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)^3*d*(B*Cos[c + d*x] + C*S
in[c + d*x])*(a + b*Tan[c + d*x])^3) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^2*(-(a^4*b*B*Sin[c + d*x]) - 4*a^2*b^3*B*Sin[c + d*x] + 2*a^5
*C*Sin[c + d*x] + 5*a^3*b^2*C*Sin[c + d*x])*(B + C*Tan[c + d*x]))/((a - I*b
)^2*(a + I*b)^2*b^3*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c + d*x]
)^3) + (C*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*Tan[c + d*x]*(
B + C*Tan[c + d*x]))/(b^3*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c
+ d*x])^3)
```

Maple [A]

time = 0.39, size = 263, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{C\tan(dx+c)}{b^3}+\frac{a^2\left(B a^4 b+3B a^2 b^3+6B b^5-3C a^5-9C a^3 b^2-10C a b^4\right)\ln(a+b\tan(dx+c))}{b^4\left(a^2+b^2\right)^3}-\frac{a^4(Bb-Ca)}{2b^4\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\frac{a^3\left(2B a^2 \cdots\right.}{b^4\left(a^2 \cdots\right.}}{d \cdots}$
default	$\frac{\frac{C\tan(dx+c)}{b^3}+\frac{a^2\left(B a^4 b+3B a^2 b^3+6B b^5-3C a^5-9C a^3 b^2-10C a b^4\right)\ln(a+b\tan(dx+c))}{b^4\left(a^2+b^2\right)^3}-\frac{a^4(Bb-Ca)}{2b^4\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\frac{a^3\left(2B a^2 \cdots\right.}{b^4\left(a^2 \cdots\right.}}{d \cdots}$

norman	$\frac{\frac{C\left(\tan^3(dx+c)\right)}{bd}+\frac{\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)x\left(\tan^2(dx+c)\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{a\left(2B\,a^4b+4B\,a^2b^3-6C\right.}{d\,b^3\left(\right.}}{(a+b\tan}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(C/b^3*tan(d*x+c)+1/b^4*a^2*(B*a^4*b+3*B*a^2*b^3+6*B*b^5-3*C*a^5-9*C*a^
3*b^2-10*C*a*b^4)/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-1/2/b^4*a^4*(B*b-C*a)/(a^2
+b^2)/(a+b*tan(d*x+c))^2+1/b^4*a^3*(2*B*a^2*b+4*B*b^3-3*C*a^3-5*C*a*b^2)/(a
^2+b^2)^2/(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(-3*B*a^2*b+B*b^3+C*a^3-3*C*a
*b^2)*ln(1+tan(d*x+c)^2)+(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*arctan(tan(d*x+c
))))
```

Maxima [A]
time = 0.52, size = 389, normalized size = 1.18

$$\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(3\,Ca^7-Ba^6b+9\,Ca^5b^2-3\,Ba^4b^3+10\,Ca^3b^4-6\,Ba^2b^5)\log(b\tan(dx+c)+a)}{a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10}}+\frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{5\,Ca^7-3\,Ba^6b+9\,Ca^5b^2-7\,Ba^4b^3+2\,(3\,Ca^6b-2\,Ba^5b^2+5\,Ca^4b^3-4\,Ba^3b^4)\tan(dx+c)}{a^6b^4+2\,a^4b^6+a^2b^8+(a^4b^6+2\,a^2b^8+b^{10})\tan(dx+c)^2+2\,(a^5b^5+2\,a^3b^7+ab^9)\tan(dx+c)}+\frac{2\,C\tan(dx+c)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - 2*(3*C*a^7 - B*a^6*b + 9*C*a^5*b^2 - 3*B*a^4*b^3 + 10*C
*a^3*b^4 - 6*B*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^6*b^4 + 3*a^4*b^6 + 3*a^
2*b^8 + b^10) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c)^2
+ 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (5*C*a^7 - 3*B*a^6*b + 9*C*a^5*b
^2 - 7*B*a^4*b^3 + 2*(3*C*a^6*b - 2*B*a^5*b^2 + 5*C*a^4*b^3 - 4*B*a^3*b^4)*
tan(d*x + c))/(a^6*b^4 + 2*a^4*b^6 + a^2*b^8 + (a^4*b^6 + 2*a^2*b^8 + b^10)
*tan(d*x + c)^2 + 2*(a^5*b^5 + 2*a^3*b^7 + a*b^9)*tan(d*x + c)) + 2*C*tan(d
*x + c)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(328) = 656$.
time = 8.00, size = 890, normalized size = 2.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/2*(3*C*a^7*b^2 - B*a^6*b^3 + 9*C*a^5*b^4 - 7*B*a^4*b^5 - 2*(C*a^6*b^3 +
3*C*a^4*b^5 + 3*C*a^2*b^7 + C*b^9)*tan(d*x + c)^3 - 2*(B*a^5*b^4 + 3*C*a^4*
b^5 - 3*B*a^3*b^6 - C*a^2*b^7)*d*x - (9*C*a^7*b^2 - 3*B*a^6*b^3 + 23*C*a^5*
b^4 - 9*B*a^4*b^5 + 12*C*a^3*b^6 + 4*C*a*b^8 + 2*(B*a^3*b^6 + 3*C*a^2*b^7 -
 3*B*a*b^8 - C*b^9)*d*x)*tan(d*x + c)^2 + (3*C*a^9 - B*a^8*b + 9*C*a^7*b^2
- 3*B*a^6*b^3 + 10*C*a^5*b^4 - 6*B*a^4*b^5 + (3*C*a^7*b^2 - B*a^6*b^3 + 9*C
*a^5*b^4 - 3*B*a^4*b^5 + 10*C*a^3*b^6 - 6*B*a^2*b^7)*tan(d*x + c)^2 + 2*(3*
C*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 + 10*C*a^4*b^5 - 6*B*a^3*b^
6)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)) - (3*C*a^9 - B*a^8*b + 9*C*a^7*b^2 - 3*B*a^6*b^3 + 9*C*a^5*
b^4 - 3*B*a^4*b^5 + 3*C*a^3*b^6 - B*a^2*b^7 + (3*C*a^7*b^2 - B*a^6*b^3 + 9*
C*a^5*b^4 - 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 + 3*C*a*b^8 - B*b^9)*ta
n(d*x + c)^2 + 2*(3*C*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 + 9*C*a
^4*b^5 - 3*B*a^3*b^6 + 3*C*a^2*b^7 - B*a*b^8)*tan(d*x + c))*log(1/(tan(d*x
+ c)^2 + 1)) - 2*(3*C*a^8*b - B*a^7*b^2 + 6*C*a^6*b^3 - 3*B*a^5*b^4 - 2*C*a
^4*b^5 + 4*B*a^3*b^6 + C*a^2*b^7 + 2*(B*a^4*b^5 + 3*C*a^3*b^6 - 3*B*a^2*b^7
 - C*a*b^8)*d*x)*tan(d*x + c))/((a^6*b^6 + 3*a^4*b^8 + 3*a^2*b^10 + b^12)*d
*tan(d*x + c)^2 + 2*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*b^9 + a*b^11)*d*tan(d*x +
c) + (a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,
x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 1.10, size = 505, normalized size = 1.53

$$\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(3\,Ca^7-Ba^6b+9\,Ca^5b^2-3\,Ba^4b^3+10\,Ca^3b^4-6\,Ba^2b^5)\log(|b\tan(dx+c)+a|)}{a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10}}+\frac{2\,C\tan(dx+c)}{b^3}+\frac{9\,Ca^7b^2\tan(dx+c)^2-3\,Ba^6b^3\tan(dx+c)^2+27\,Ca^5b^4\tan(dx+c)^2-9\,Ba^4b^5\tan(dx+c)^2+30\,Ca^3b^6\tan(dx+c)^2-18\,Ba^2b^7\tan(dx+c)^2+12\,Ca^8b\tan(dx+c)-2\,Ba^7b^2\tan(dx+c)+38\,Ca^6b^3\tan(dx+c)-6\,Ba^5b^4\tan(dx+c)+50\,Ca^4b^5\tan(dx+c)-28\,Ba^3b^6\tan(dx+c)+4\,Ca^9+13\,Ca^7b^2+Ba^6b^3+21\,Ca^5b^4-11\,Ba^4b^5}{(a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10})(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(3*C*a^7 - B*a^6*b + 9*C*a
^5*b^2 - 3*B*a^4*b^3 + 10*C*a^3*b^4 - 6*B*a^2*b^5)*log(abs(b*tan(d*x + c) +
 a))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10) + 2*C*tan(d*x + c)/b^3 + (9*C
```

```
*a^7*b^2*tan(d*x + c)^2 - 3*B*a^6*b^3*tan(d*x + c)^2 + 27*C*a^5*b^4*tan(d*x
 + c)^2 - 9*B*a^4*b^5*tan(d*x + c)^2 + 30*C*a^3*b^6*tan(d*x + c)^2 - 18*B*a
^2*b^7*tan(d*x + c)^2 + 12*C*a^8*b*tan(d*x + c) - 2*B*a^7*b^2*tan(d*x + c)
+ 38*C*a^6*b^3*tan(d*x + c) - 6*B*a^5*b^4*tan(d*x + c) + 50*C*a^4*b^5*tan(d
*x + c) - 28*B*a^3*b^6*tan(d*x + c) + 4*C*a^9 + 13*C*a^7*b^2 + B*a^6*b^3 +
21*C*a^5*b^4 - 11*B*a^4*b^5)/((a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*(b*t
an(d*x + c) + a)^2))/d
```

Mupad [B]
time = 10.43, size = 335, normalized size = 1.01

$$\frac{C\tan(c+d\,x)}{b^3\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}-\frac{\frac{5\,C\,a^7-3\,B\,a^6\,b+9\,C\,a^5\,b^2-7\,B\,a^4\,b^3}{2\,b\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(3\,C\,a^6-2\,B\,a^5\,b+5\,C\,a^4\,b^2-4\,B\,a^3\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2\,b^3+2\,a\,b^4\tan\left(c+d\,x\right)+b^5\tan\left(c+d\,x\right)^2\right)}+\frac{a^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-3\,C\,a^5+B\,a^4\,b-9\,C\,a^3\,b^2+3\,B\,a^2\,b^3-10\,C\,a\,b^4+6\,B\,b^5\right)}{b^4\,d\,{\left(a^2+b^2\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
))^3,x)
```

```
[Out] (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i
)) - ((5*C*a^7 - 7*B*a^4*b^3 + 9*C*a^5*b^2 - 3*B*a^6*b)/(2*b*(a^4 + b^4 + 2
*a^2*b^2)) + (tan(c + d*x)*(3*C*a^6 - 4*B*a^3*b^3 + 5*C*a^4*b^2 - 2*B*a^5*b
))/(a^4 + b^4 + 2*a^2*b^2))/(d*(a^2*b^3 + b^5*tan(c + d*x)^2 + 2*a*b^4*tan(
c + d*x))) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*b^2*3i - 3*a^2*b -
 a^3*1i + b^3)) + (C*tan(c + d*x))/(b^3*d) + (a^2*log(a + b*tan(c + d*x))*(
6*B*b^5 - 3*C*a^5 + 3*B*a^2*b^3 - 9*C*a^3*b^2 + B*a^4*b - 10*C*a*b^4))/(b^4
*d*(a^2 + b^2)^3)
```

3.39 $\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=250

$$-\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3}+\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\log(\cos(c+dx))}{(a^2+b^2)^3\,d}+\frac{a(a^2b^3B-3b^5B+a^5}{}$$

```
[Out] -(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*x/(a^2+b^2)^3+(B*a^3-3*B*a*b^2+3*C*a^2*b
-C*b^3)*ln(cos(d*x+c))/(a^2+b^2)^3/d+a*(B*a^2*b^3-3*B*b^5+C*a^5+3*C*a^3*b^2
+6*C*a*b^4)*ln(a+b*tan(d*x+c))/b^3/(a^2+b^2)^3/d+1/2*a*(B*b-C*a)*tan(d*x+c)
^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-a^2*(2*B*b^3-C*a^3-3*C*a*b^2)/b^3/(a^2+
b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.37, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {3713, 3686, 3716, 3707, 3698, 31, 3556}

$$\frac{a(bB-aC)\tan^2(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{a^2(a^3(-C)-3ab^2C+2b^3B)}{b^3d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}+\frac{(a^3B+3a^2bC-3ab^2B-b^3C)\log(\cos(c+dx))}{d\,(a^2+b^2)^3}-\frac{x(a^3(-C)+3a^2bB+3ab^2C-b^3B)}{(a^2+b^2)^3}+\frac{a(a^5C+3a^3b^2C+a^2b^3B+6ab^4C-3b^5B)\log(a+b\tan(c+dx))}{b^3d\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^3,x]
```

```
[Out] -(((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3) + ((a^3*B - 3*
a*b^2*B + 3*a^2*b*C - b^3*C)*Log[Cos[c + d*x]])/((a^2 + b^2)^3*d) + (a*(a^2
*b^3*B - 3*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4*C)*Log[a + b*Tan[c + d*x]]
)/(b^3*(a^2 + b^2)^3*d) + (a*(b*B - a*C)*Tan[c + d*x]^2)/(2*b*(a^2 + b^2)*d
*(a + b*Tan[c + d*x])^2) - (a^2*(2*b^3*B - a^3*C - 3*a*b^2*C))/(b^3*(a^2 +
b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3686

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
```

```
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dist[1/(d*(n + 1)*(c^2 + d^2)),
 Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*(
b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
+ 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[e
 + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
+ 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &&
 LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{a(bB-aC)\tan^2(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \int \frac{\tan(c+dx)\,(-2a(b\ldots}{\ldots}$$

$$= \frac{a(bB-aC)\tan^2(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(2b^3B-\ldots}{b^3\,(a^2+b^2)^2\,d(\ldots}$$

$$= -\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3} + \frac{a(bB-a\ldots}{2b\,(a^2+b^2)\,d(\ldots}$$

$$= -\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3} + \frac{(a^3B-3ab^2B\ldots}{\ldots}$$

$$= -\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3} + \frac{(a^3B-3ab^2B\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.23, size = 462, normalized size = 1.85

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*(a^3*b^2*(a^2 + b^2)*(b*B
 - a*C) - 2*a*b*(a^2 + b^2)*(-3*b^3*B + a^3*C + 4*a*b^2*C)*Sin[c + d*x]*(a*
Cos[c + d*x] + b*Sin[c + d*x]) + 2*b^3*(-3*a^2*b*B + b^3*B + a^3*C - 3*a*b^
2*C)*(c + d*x)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (2*I)*a*(a^2*b^3*B - 3
*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4*C)*(c + d*x)*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2 - (2*I)*a*(a^2*b^3*B - 3*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4
*C)*ArcTan[Tan[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 - 2*(a^2 + b^2
)^3*C*Log[Cos[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + a*(a^2*b^3*B
- 3*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4*C)*Log[(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)*(B + C*Tan[c + d*x]))/(2*b^
3*(a^2 + b^2)^3*d*(B*Cos[c + d*x] + C*Sin[c + d*x])*(a + b*Tan[c + d*x])^3)
```

Maple [A]
time = 0.30, size = 242, normalized size = 0.97 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(-B*a^3+3*B*a*b^2-3*C*a^2*b+C*b^3)*ln(1+tan(d*x+c)^
2)+(-3*B*a^2*b+B*b^3+C*a^3-3*C*a*b^2)*arctan(tan(d*x+c)))+a*(B*a^2*b^3-3*B*
b^5+C*a^5+3*C*a^3*b^2+6*C*a*b^4)/(a^2+b^2)^3/b^3*ln(a+b*tan(d*x+c))-a^2*(B*
a^2*b+3*B*b^3-2*C*a^3-4*C*a*b^2)/b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+1/2*a^3*(
B*b-C*a)/b^3/(a^2+b^2)/(a+b*tan(d*x+c))^2)
```

Maxima [**A**]
time $= 0.51$, size $= 366$, normalized size $= 1.46$

$$\frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(Ca^6+3\,Ca^4b^2+Ba^3b^3+6\,Ca^2b^4-3\,Bab^5)\log(b\tan(dx+c)+a)}{a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9}-\frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{3\,Ca^6-Ba^5b+7\,Ca^4b^2-5\,Ba^3b^3+2\,(2\,Ca^5b-Ba^4b^2+4\,Ca^3b^3-3\,Ba^2b^4)\tan(dx+c)}{a^6b^3+2\,a^4b^5+a^2b^7+(a^4b^5+2\,a^2b^7+b^9)\tan(dx+c)^2+2\,(a^5b^4+2\,a^3b^6+ab^8)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + 2*(C*a^6 + 3*C*a^4*b^2 + B*a^3*b^3 + 6*C*a^2*b^4 - 3*B*
a*b^5)*log(b*tan(d*x + c) + a)/(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) - (B
*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6) + (3*C*a^6 - B*a^5*b + 7*C*a^4*b^2 - 5*B*a^3*b^3 + 2
*(2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^2*b^4)*tan(d*x + c))/(a^6*b^3
 + 2*a^4*b^5 + a^2*b^7 + (a^4*b^5 + 2*a^2*b^7 + b^9)*tan(d*x + c)^2 + 2*(a^
5*b^4 + 2*a^3*b^6 + a*b^8)*tan(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(243) = 486$.
time $= 5.07$, size $= 666$, normalized size $= 2.66$

$$\frac{Ca^6b^2+Ba^5b^3+7\,Ca^4b^4-5\,Ba^3b^5+2\,(Ca^5b^3-3\,Ba^4b^4-3\,Ca^3b^5+Ba^2b^6)dx-(3\,Ca^6b^2-Ba^5b^3+9\,Ca^4b^4-7\,Ba^3b^5-2\,(Ca^3b^5-3\,Ba^2b^6-3\,Cab^7+Bb^8)dx)\tan(dx+c)^2+(Ca^8+3\,Ca^6b^2+Ba^5b^3+6\,Ca^4b^4-3\,Ba^3b^5+(Ca^6b^2+3\,Ca^4b^4+Ba^3b^5+6\,Ca^2b^6-3\,Bab^7)\tan(dx+c)^2+2\,(Ca^7b+3\,Ca^5b^3+Ba^4b^4+6\,Ca^3b^5-3\,Ba^2b^6)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(Ca^8+3\,Ca^6b^2+3\,Ca^4b^4+Ca^2b^6+(Ca^6b^2+3\,Ca^4b^4+3\,Ca^2b^6+Cb^8)\tan(dx+c)^2+2\,(Ca^7b+3\,Ca^5b^3+3\,Ca^3b^5+Cab^7)\tan(dx+c))\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-2\,(Ca^7b+3\,Ca^5b^3-3\,Ba^4b^4-4\,Ca^3b^5+3\,Ba^2b^6-2\,(Ca^4b^4-3\,Ba^3b^5-3\,Ca^2b^6+Bab^7)dx)\tan(dx+c)}{2\,((a^6b^5+3\,a^4b^7+3\,a^2b^9+b^{11})d\tan(dx+c)^2+2\,(a^7b^4+3\,a^5b^6+3\,a^3b^8+ab^{10})d\tan(dx+c)+(a^8b^3+3\,a^6b^5+3\,a^4b^7+a^2b^9)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] 1/2*(C*a^6*b^2 + B*a^5*b^3 + 7*C*a^4*b^4 - 5*B*a^3*b^5 + 2*(C*a^5*b^3 - 3*B
*a^4*b^4 - 3*C*a^3*b^5 + B*a^2*b^6)*d*x - (3*C*a^6*b^2 - B*a^5*b^3 + 9*C*a^
4*b^4 - 7*B*a^3*b^5 - 2*(C*a^3*b^5 - 3*B*a^2*b^6 - 3*C*a*b^7 + B*b^8)*d*x)*
tan(d*x + c)^2 + (C*a^8 + 3*C*a^6*b^2 + B*a^5*b^3 + 6*C*a^4*b^4 - 3*B*a^3*b
^5 + (C*a^6*b^2 + 3*C*a^4*b^4 + B*a^3*b^5 + 6*C*a^2*b^6 - 3*B*a*b^7)*tan(d*
x + c)^2 + 2*(C*a^7*b + 3*C*a^5*b^3 + B*a^4*b^4 + 6*C*a^3*b^5 - 3*B*a^2*b^6
)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*
x + c)^2 + 1)) - (C*a^8 + 3*C*a^6*b^2 + 3*C*a^4*b^4 + C*a^2*b^6 + (C*a^6*b^
2 + 3*C*a^4*b^4 + 3*C*a^2*b^6 + C*b^8)*tan(d*x + c)^2 + 2*(C*a^7*b + 3*C*a^
```

```
5*b^3 + 3*C*a^3*b^5 + C*a*b^7)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) -
2*(C*a^7*b + 3*C*a^5*b^3 - 3*B*a^4*b^4 - 4*C*a^3*b^5 + 3*B*a^2*b^6 - 2*(C*a
^4*b^4 - 3*B*a^3*b^5 - 3*C*a^2*b^6 + B*a*b^7)*d*x)*tan(d*x + c))/((a^6*b^5
+ 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d*tan(d*x + c)^2 + 2*(a^7*b^4 + 3*a^5*b^6 +
 3*a^3*b^8 + a*b^10)*d*tan(d*x + c) + (a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^
2*b^9)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,
x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.97, size = 458, normalized size = 1.83

$$\frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(Ca^6+3\,Ca^4b^2+Ba^3b^3+6\,Ca^2b^4-3\,Bab^5)\log(|b\tan(dx+c)+a|)}{a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9}-\frac{3\,Ca^6b\tan(dx+c)^2+9\,Ca^4b^3\tan(dx+c)^2+3\,Ba^3b^4\tan(dx+c)^2+18\,Ca^2b^5\tan(dx+c)^2-9\,Bab^6\tan(dx+c)^2+2\,Ca^7\tan(dx+c)+2\,Ba^6b\tan(dx+c)+6\,Ca^5b^2\tan(dx+c)+14\,Ba^4b^3\tan(dx+c)+28\,Ca^3b^4\tan(dx+c)-12\,Ba^2b^5\tan(dx+c)+Ba^7-Ca^6b+9\,Ba^5b^2+11\,Ca^4b^3-4\,Ba^3b^4}{(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(C*a^6 + 3*C*a^4*b^2 + B*a
^3*b^3 + 6*C*a^2*b^4 - 3*B*a*b^5)*log(abs(b*tan(d*x + c) + a))/(a^6*b^3 + 3
*a^4*b^5 + 3*a^2*b^7 + b^9) - (3*C*a^6*b*tan(d*x + c)^2 + 9*C*a^4*b^3*tan(d
*x + c)^2 + 3*B*a^3*b^4*tan(d*x + c)^2 + 18*C*a^2*b^5*tan(d*x + c)^2 - 9*B*
a*b^6*tan(d*x + c)^2 + 2*C*a^7*tan(d*x + c) + 2*B*a^6*b*tan(d*x + c) + 6*C*
a^5*b^2*tan(d*x + c) + 14*B*a^4*b^3*tan(d*x + c) + 28*C*a^3*b^4*tan(d*x + c
) - 12*B*a^2*b^5*tan(d*x + c) + B*a^7 - C*a^6*b + 9*B*a^5*b^2 + 11*C*a^4*b^
3 - 4*B*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(d*x + c) +
 a)^2))/d
```

Mupad [B]
time = 9.32, size = 307, normalized size = 1.23

$$\frac{\frac{3\,C\,a^6-B\,a^5\,b+7\,C\,a^4\,b^2-5\,B\,a^3\,b^3}{2\,b^3\,(a^4+2\,a^2\,b^2+b^4)}-\frac{a^2\tan(c+d\,x)\,(-2\,C\,a^3+B\,a^2\,b-4\,C\,a\,b^2+3\,B\,b^3)}{b^2\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}+\frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^5+3\,C\,a^3\,b^2+B\,a^2\,b^3+6\,C\,a\,b^4-3\,B\,b^5\right)}{b^3\,d\,{\left(a^2+b^2\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
))^3,x)
```

```
[Out] ((3*C*a^6 - 5*B*a^3*b^3 + 7*C*a^4*b^2 - B*a^5*b)/(2*b^3*(a^4 + b^4 + 2*a^2*
b^2)) - (a^2*tan(c + d*x)*(3*B*b^3 - 2*C*a^3 + B*a^2*b - 4*C*a*b^2))/(b^2*(
a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))
) + (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i -
 b^3)) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(3*a*b^2 + a^2*b*3i - a^3
 - b^3*1i)) + (a*log(a + b*tan(c + d*x))*(C*a^5 - 3*B*b^5 + B*a^2*b^3 + 3*C
*a^3*b^2 + 6*C*a*b^4))/(b^3*d*(a^2 + b^2)^3)
```

3.40 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=189

$$-\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\,x}{(a^2+b^2)^3}-\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}-\frac{}{2b^2(}$$

```
[Out] -(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*x/(a^2+b^2)^3-(3*B*a^2*b-B*b^3-C*a^3+3*C
*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d-1/2*a^2*(B*b-C*a)/b^2/(
a^2+b^2)/d/(a+b*tan(d*x+c))^2+a*(2*B*b^3-C*a^3-3*C*a*b^2)/b^2/(a^2+b^2)^2/d
/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {3713, 3685, 3709, 3612, 3611}

$$-\frac{a^2(bB-aC)}{2b^2d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{a(a^3(-C)-3ab^2C+2b^3B)}{b^2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{(a^3(-C)+3a^2bB+3ab^2C-b^3B)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}-\frac{x(a^3B+3a^2bC-3ab^2B-b^3C)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
^3,x]
```

```
[Out] -(((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3) - ((3*a^2*b*B
- b^3*B - a^3*C + 3*a*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 +
b^2)^3*d) - (a^2*(b*B - a*C))/(2*b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
+ (a*(2*b^3*B - a^3*C - 3*a*b^2*C))/(b^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x
]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3685

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Dist[1/(d*(c^2 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)
*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*c
 + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2)
*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*
c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= -\frac{a^2(bB-aC)}{2b^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{-a(bB-aC)+}{}}{}$$

$$= -\frac{a^2(bB-aC)}{2b^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{a(2b^3B}{b^2\left(a^2+b^2\right)^2}$$

$$= -\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} - \frac{a^2(}{2b^2\left(a^2+b^2\right)a}$$

$$= -\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3} - \frac{\left(3a^2bB-b^3B\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.58, size = 288, normalized size = 1.52

$$\frac{-\frac{bB+aC}{b(a+b\tan(c+dx))^2}-\frac{2C\tan(c+dx)}{(a+b\tan(c+dx))^2}+C\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{i\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2b\left(-2a\log(a+b\tan(c+dx))+\frac{a^2+b^2}{a+b\tan(c+dx)}\right)}{(a^2+b^2)^2}\right)+(bB-aC)\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^3}-\frac{\log(i+\tan(c+dx))}{(ia+b)^3}+\frac{b\left((-6a^2+2b^2)\log(a+b\tan(c+dx))+\frac{(a^2+b^2)(5a^2+b^2+4ab\tan(c+dx))}{(a+b\tan(c+dx))^2}\right)}{(a^2+b^2)^3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x])^3,x]
```

```
[Out] (-((b*B + a*C)/(b*(a + b*Tan[c + d*x])^2)) - (2*C*Tan[c + d*x])/(a + b*Tan[
c + d*x])^2 + C*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c +
 d*x]])/(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a +
 b*Tan[c + d*x])))/(a^2 + b^2)^2) + (b*B - a*C)*((I*Log[I - Tan[c + d*x]])/
(a + I*b)^3 - Log[I + Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*Log[
a + b*Tan[c + d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x]))/(a +
 b*Tan[c + d*x])^2))/(a^2 + b^2)^3))/(2*b*d)
```

Maple [A]
time = 0.24, size = 223, normalized size = 1.18

method	result
derivativedivides	$\frac{-\frac{a^2(Bb-Ca)}{2b^2\left(a^2+b^2\right)(a+b\tan(dx+c))^2}-\frac{\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a\left(2B\,b^3-C\,a^3-3Ca\,b^2\right)}{\left(a^2+b^2\right)^2b^2(a+b\tan(dx+c))}+\frac{\left(3B\,a^2b-B\,b^3\right.}{}}{d}$
default	$\frac{-\frac{a^2(Bb-Ca)}{2b^2\left(a^2+b^2\right)(a+b\tan(dx+c))^2}-\frac{\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a\left(2B\,b^3-C\,a^3-3Ca\,b^2\right)}{\left(a^2+b^2\right)^2b^2(a+b\tan(dx+c))}+\frac{\left(3B\,a^2b-B\,b^3\right.}{}}{d}$
norman	$\frac{-\frac{\left(2Ba\,b^3-C\,a^4-3C\,a^2b^2\right)\left(\tan^2(dx+c)\right)}{2ad\left(a^4+2a^2b^2+b^4\right)}-\frac{a\left(B\,a^3-Ba\,b^2+2C\,a^2b\right)}{2db\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{b^2\left(B\,a^3-3Ba\,b^2+3C\,a^2\right.}{\left(a^4+2a^2b^2+\right.}}{(a+b\tan(dx+c))^2}$
risch	$\frac{xB}{3ia^2b-ib^3-a^3+3a\,b^2}-\frac{ixC}{3ia^2b-ib^3-a^3+3a\,b^2}+\frac{6ia^2bBx}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2iB\,b^3x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2ia^3Cx}{a^6+3a^4b^2+3a^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2*(B*b-C*a)/b^2/(a^2+b^2)/(a+b*tan(d*x+c))^2-(3*B*a^2*b-B*b^3-C
*a^3+3*C*a*b^2)/(a^2+b^2)^3*ln(a+b*tan(d*x+c))+a*(2*B*b^3-C*a^3-3*C*a*b^2)/
(a^2+b^2)^2/b^2/(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(3*B*a^2*b-B*b^3-C*a^3+
3*C*a*b^2)*ln(1+tan(d*x+c)^2)+(-B*a^3+3*B*a*b^2-3*C*a^2*b+C*b^3)*arctan(tan
(d*x+c))))
```

Maxima [A]
time = 0.52, size = 333, normalized size = 1.76

$$-\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{Ca^5+Ba^4b+5\,Ca^3b^2-3\,Ba^2b^3+2\,(Ca^4b+3\,Ca^2b^3-2\,Bab^4)\tan(dx+c)}{a^6b^2+2\,a^4b^4+a^2b^6+(a^4b^4+2\,a^2b^6+b^8)\tan(dx+c)^2+2\,(a^5b^3+2\,a^3b^5+ab^7)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - 2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(b*tan(d*
x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*
a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (C*a^5 + B*a^4*b + 5*C*a^3*b^2 - 3*B*a^2*b^3 + 2*(C*a^4*b + 3*C*a^2*b^3 -
 2*B*a*b^4)*tan(d*x + c))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*b^4 + 2*a^2
*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(d*x + c)))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(184) = 368$.
time = 3.73, size = 478, normalized size = 2.53

$$\frac{Ca^5-3\,Ba^4b-5\,Ca^3b^2+3\,Ba^2b^3-2\,(Ba^5+3\,Ca^4b-3\,Ba^3b^2-Ca^2b^3)dx+(Ca^5+Ba^4b+7\,Ca^3b^2-5\,Ba^2b^3-2\,(Ba^3b^2+3\,Ca^2b^3-3\,Bab^4-Cb^5)dx)\tan(dx+c)^2+\left(Ca^5-3\,Ba^4b-3\,Ca^3b^2+Ba^2b^3+(Ca^3b^2-3\,Ba^2b^3-3\,Cab^4+Bb^5)\tan(dx+c)^2+2\,(Ca^4b-3\,Ba^3b^2-3\,Ca^2b^3+Bab^4)\tan(dx+c)\right)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+2\,(Ba^5+3\,Ca^4b-3\,Ba^3b^2-3\,Ca^2b^3+2\,Bab^4-2\,(Ba^4b+3\,Ca^3b^2-3\,Ba^2b^3-Cab^4)dx)\tan(dx+c)}{2\left((a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)d\tan(dx+c)^2+2\,(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7)d\tan(dx+c)+(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] 1/2*(C*a^5 - 3*B*a^4*b - 5*C*a^3*b^2 + 3*B*a^2*b^3 - 2*(B*a^5 + 3*C*a^4*b -
 3*B*a^3*b^2 - C*a^2*b^3)*d*x + (C*a^5 + B*a^4*b + 7*C*a^3*b^2 - 5*B*a^2*b^
3 - 2*(B*a^3*b^2 + 3*C*a^2*b^3 - 3*B*a*b^4 - C*b^5)*d*x)*tan(d*x + c)^2 + (
C*a^5 - 3*B*a^4*b - 3*C*a^3*b^2 + B*a^2*b^3 + (C*a^3*b^2 - 3*B*a^2*b^3 - 3*
C*a*b^4 + B*b^5)*tan(d*x + c)^2 + 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 +
B*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) + 2*(B*a^5 + 3*C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 +
2*B*a*b^4 - 2*(B*a^4*b + 3*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4)*d*x)*tan(d*x
+ c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 + 2*(a^7*b
+ 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*
b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(184) = 368$.
time = 0.84, size = 410, normalized size = 2.17

$$-\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(Ca^3b-3\,Ba^2b^2-3\,Cab^3+Bb^4)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}+\frac{3\,Ca^3b^4\tan(dx+c)^2-9\,Ba^2b^5\tan(dx+c)^2-9\,Cab^6\tan(dx+c)^2+3\,Bb^7\tan(dx+c)^2+2\,Ca^6b\tan(dx+c)+14\,Ca^4b^3\tan(dx+c)-22\,Ba^3b^4\tan(dx+c)-12\,Ca^2b^5\tan(dx+c)+2\,Bab^6\tan(dx+c)+Ca^7+Ba^6b+9\,Ca^5b^2-11\,Ba^4b^3-4\,Ca^3b^4}{(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] -1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(C*a^3*b - 3*B*a^2*b^2 -
3*C*a*b^3 + B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) + (3*C*a^3*b^4*tan(d*x + c)^2 - 9*B*a^2*b^5*tan(d*x + c)^2 - 9*C
*a*b^6*tan(d*x + c)^2 + 3*B*b^7*tan(d*x + c)^2 + 2*C*a^6*b*tan(d*x + c) + 1
4*C*a^4*b^3*tan(d*x + c) - 22*B*a^3*b^4*tan(d*x + c) - 12*C*a^2*b^5*tan(d*x
 + c) + 2*B*a*b^6*tan(d*x + c) + C*a^7 + B*a^6*b + 9*C*a^5*b^2 - 11*B*a^4*b
^3 - 4*C*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(d*x + c)
+ a)^2))/d
```

Mupad [B]
time = 9.18, size = 280, normalized size = 1.48

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^3-3\,B\,a^2\,b-3\,C\,a\,b^2+B\,b^3\right)}{d\,{\left(a^2+b^2\right)}^3}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}-\frac{\frac{a\,\left(C\,a^4+B\,a^3\,b+5\,C\,a^2\,b^2-3\,B\,a\,b^3\right)}{2\,b^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(C\,a^4+3\,C\,a^2\,b^2-2\,B\,a\,b^3\right)}{b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x))
^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*(B*b^3 + C*a^3 - 3*B*a^2*b - 3*C*a*b^2))/(d*(a^2 +
 b^2)^3) - (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(3*a*b^2 - a^2*b*3i - a
^3 + b^3*1i)) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*b^2*3i - 3*a^2*
b - a^3*1i + b^3)) - ((a*(C*a^4 + 5*C*a^2*b^2 - 3*B*a*b^3 + B*a^3*b))/(2*b^
2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)*(C*a^4 + 3*C*a^2*b^2 - 2*B*a*b^3
))/(b*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c
+ d*x)))
```

3.41 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=179

$$\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3}-\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}+\frac{}{2b\,(a}$$

```
[Out] (3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*x/(a^2+b^2)^3-(B*a^3-3*B*a*b^2+3*C*a^2*b-
C*b^3)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d+1/2*a*(B*b-C*a)/b/(a^2+b
^2)/d/(a+b*tan(d*x+c))^2+(B*a^2-B*b^2+2*C*a*b)/(a^2+b^2)^2/d/(a+b*tan(d*x+c
))
```

Rubi [A]
time = 0.18, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3709, 3610, 3612, 3611}

$$\frac{a(bB-aC)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{a^2B+2abC-b^2B}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{(a^3B+3a^2bC-3ab^2B-b^3C)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{x(a^3(-C)+3a^2bB+3ab^2C-b^3B)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3 - ((a^3*B - 3*a*b
^2*B + 3*a^2*b*C - b^3*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2
)^3*d) + (a*(b*B - a*C))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a^2*
B - b^2*B + 2*a*b*C)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{a(bB-aC)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{\int \frac{bB-aC+(aB+bC)\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{a^2+b^2} \\
&= \frac{a(bB-aC)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{a^2B-b^2B+2abC}{\left(a^2+b^2\right)^2d(a+b\tan(c+dx))} \\
&= \frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3}+\frac{a(bB-aC)}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx)} \\
&= \frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3}-\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)}{(a^2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.73, size = 188, normalized size = 1.05

$$\frac{\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^3}+\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^3}-\frac{2\left(a^3B-3ab^2B+3a^2bC-b^3C\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3}+\frac{a(bB-aC)}{b(a^2+b^2)(a+b\tan(c+dx))^2}+\frac{2\left(a^2B-b^2B+2abC\right)}{(a^2+b^2)^2(a+b\tan(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^3 + ((B - I*C)*Log[I + Tan[c +
 d*x]])/(a - I*b)^3 - (2*(a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*Log[a + b*
Tan[c + d*x]])/(a^2 + b^2)^3 + (a*(b*B - a*C))/(b*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^2) + (2*(a^2*B - b^2*B + 2*a*b*C))/((a^2 + b^2)^2*(a + b*Tan[c + d*
x])))/(2*d)
```

Maple [A]
time = 0.22, size = 213, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{\frac{\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a(Bb-Ca)}{2\left(a^2+b^2\right)b(a+b\tan(dx+c)}}{d}$
default	$\frac{\frac{\frac{\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{2}+\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a(Bb-Ca)}{2\left(a^2+b^2\right)b(a+b\tan(dx+c)}}{d}$
norman	$\frac{\frac{\left(B\,a^2b^2-B\,b^4+2Ca\,b^3\right)\tan(dx+c)}{db\left(a^4+2a^2b^2+b^4\right)}+\frac{\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)x\left(\tan^2(dx+c)\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{a(3B}{}}{(a+b\tan(dx+c))^2}$
risch	$\frac{ixB}{3ia^2b-ib^3-a^3+3a\,b^2}+\frac{xC}{3ia^2b-ib^3-a^3+3a\,b^2}+\frac{2ia^3Bx}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6iBa\,b^2x}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{6ia^2bCa}{a^6+3a^4b^2+3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/(a^2+b^2)^3*(1/2*(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*ln(1+tan(d*x+c)^2
)+(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*arctan(tan(d*x+c)))+1/2*a*(B*b-C*a)/(a^
2+b^2)/b/(a+b*tan(d*x+c))^2+(B*a^2-B*b^2+2*C*a*b)/(a^2+b^2)^2/(a+b*tan(d*x+
c))-(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)/(a^2+b^2)^3*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.50, size = 330, normalized size = 1.84

$$-\frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\left(b\tan\left(dx+c\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{Ca^4-3\,Ba^3b-3\,Ca^2b^2+Bab^3-2\,(Ba^2b^2+2\,Cab^3-Bb^4)\tan\left(dx+c\right)}{a^6b+2\,a^4b^3+a^2b^5+(a^4b^3+2\,a^2b^5+b^7)\tan\left(dx+c\right)^2+2\,(a^5b^2+2\,a^3b^4+ab^6)\tan\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(b*tan(d*
x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*
a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (C*a^4 - 3*B*a^3*b - 3*C*a^2*b^2 + B*a*b^3 - 2*(B*a^2*b^2 + 2*C*a*b^3 - B
*b^4)*tan(d*x + c))/(a^6*b + 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 + 2*a^2*b^5 + b
^7)*tan(d*x + c)^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(176) = 352.
time = 4.13, size = 488, normalized size = 2.73

$$-\frac{3\,Ca^4b-5\,Ba^3b^2-3\,Ca^2b^3+Bab^4+2\,(Ca^5-3\,Ba^4b-3\,Ca^3b^2+Ba^2b^3)dx-(Ca^4b-3\,Ba^3b^2-5\,Ca^2b^3+3\,Bab^4-2\,(Ca^3b^2-3\,Ba^2b^3-3\,Cab^4+Bb^5)dx)\tan\left(dx+c\right)^2+(Ba^5+3\,Ca^4b-3\,Ba^3b^2-Ca^2b^3+(Ba^3b^2+3\,Ca^2b^3-3\,Bab^4-Cb^5)\tan\left(dx+c\right)^2+2\,(Ba^4b+3\,Ca^3b^2-3\,Ba^2b^3-Cab^4)\tan\left(dx+c\right))\log\left(\frac{b^2\tan\left(dx+c\right)^2+2\,ab\tan\left(dx+c\right)+a^2}{\tan\left(dx+c\right)^2+1}\right)-2\,(Ca^5-2\,Ba^4b-3\,Ca^3b^2+3\,Ba^2b^3+2\,Cab^4-Bb^5-2\,(Ca^4b-3\,Ba^3b^2-3\,Ca^2b^3+Bab^4)dx)\tan\left(dx+c\right)}{2\,((a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)d\tan\left(dx+c\right)^2+2\,(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7)d\tan\left(dx+c\right)+(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/2*(3*C*a^4*b - 5*B*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4 + 2*(C*a^5 - 3*B*a^4*
b - 3*C*a^3*b^2 + B*a^2*b^3)*d*x - (C*a^4*b - 3*B*a^3*b^2 - 5*C*a^2*b^3 + 3
*B*a*b^4 - 2*(C*a^3*b^2 - 3*B*a^2*b^3 - 3*C*a*b^4 + B*b^5)*d*x)*tan(d*x + c
)^2 + (B*a^5 + 3*C*a^4*b - 3*B*a^3*b^2 - C*a^2*b^3 + (B*a^3*b^2 + 3*C*a^2*b
^3 - 3*B*a*b^4 - C*b^5)*tan(d*x + c)^2 + 2*(B*a^4*b + 3*C*a^3*b^2 - 3*B*a^2
*b^3 - C*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
+ a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a^5 - 2*B*a^4*b - 3*C*a^3*b^2 + 3*B*a^2
*b^3 + 2*C*a*b^4 - B*b^5 - 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4
)*d*x)*tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c
)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a
^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(176) = 352.
time = 0.84, size = 410, normalized size = 2.29

$$-\frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(Ba^3b+3\,Ca^2b^2-3\,Bab^3-Cb^4)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{3\,Ba^3b^3\tan(dx+c)^2+9\,Ca^2b^4\tan(dx+c)^2-9\,Bab^5\tan(dx+c)^2-3\,Cb^6\tan(dx+c)^2+8\,Ba^4b^2\tan(dx+c)+22\,Ca^3b^3\tan(dx+c)-18\,Ba^2b^4\tan(dx+c)-2\,Cab^5\tan(dx+c)-2\,Bb^6\tan(dx+c)-Ca^6+6\,Ba^5b+11\,Ca^4b^2-7\,Ba^3b^3-Bab^5}{(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(B*a^3*b + 3*C*a^2*b^2 -
3*B*a*b^3 - C*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) - (3*B*a^3*b^3*tan(d*x + c)^2 + 9*C*a^2*b^4*tan(d*x + c)^2 - 9*B
*a*b^5*tan(d*x + c)^2 - 3*C*b^6*tan(d*x + c)^2 + 8*B*a^4*b^2*tan(d*x + c) +
 22*C*a^3*b^3*tan(d*x + c) - 18*B*a^2*b^4*tan(d*x + c) - 2*C*a*b^5*tan(d*x
+ c) - 2*B*b^6*tan(d*x + c) - C*a^6 + 6*B*a^5*b + 11*C*a^4*b^2 - 7*B*a^3*b^
```

```
3 - B*a*b^5)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*tan(d*x + c) + a)^2)
)/d
```

Mupad [B]

time = 9.28, size = 282, normalized size = 1.58

$$\frac{\frac{\tan(c+dx)\,(B\,a^2\,b+2\,C\,a\,b^2-B\,b^3)}{a^4+2\,a^2\,b^2+b^4}-\frac{C\,a^4-3\,B\,a^3\,b-3\,C\,a^2\,b^2+B\,a\,b^3}{2\,b\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{B\,a+3\,C\,b}{{\left(a^2+b^2\right)}^2}-\frac{4\,b^2\,(B\,a+C\,b)}{{\left(a^2+b^2\right)}^3}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*tan(c + d*x) + C*tan(c + d*x)^2)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((tan(c + d*x)*(B*a^2*b - B*b^3 + 2*C*a*b^2))/(a^4 + b^4 + 2*a^2*b^2) - (C*
a^4 - 3*C*a^2*b^2 + B*a*b^3 - 3*B*a^3*b)/(2*b*(a^4 + b^4 + 2*a^2*b^2)))/(d*
(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) - (log(a + b*tan(c + d*x))
*((B*a + 3*C*b)/(a^2 + b^2)^2 - (4*b^2*(B*a + C*b))/(a^2 + b^2)^3))/d - (lo
g(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3))
- (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(3*a*b^2 + a^2*b*3i - a^3 - b^3*
1i))
```

3.42 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=175

$$\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3}+\frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3d}-\frac{}{2\left(a^2-\right.}$$

```
[Out] (B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*x/(a^2+b^2)^3+(3*B*a^2*b-B*b^3-C*a^3+3*C*
a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d+1/2*(-B*b+C*a)/(a^2+b^2)
/d/(a+b*tan(d*x+c))^2+(-2*B*a*b+C*a^2-C*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3713, 3610, 3612, 3611}

$$-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{a^2(-C)+2abB+b^2C}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^3}+\frac{x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x])
^3,x]
```

```
[Out] ((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3 + ((3*a^2*b*B - b
^3*B - a^3*C + 3*a*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2
)^3*d) - (b*B - a*C)/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (2*a*b*B -
a^2*C + b^2*C)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rubi steps

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx=\int \frac{B+C\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$=-\frac{bB-aC}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{\int \frac{aB+bC-(bB-aC\ldots}{(a+b\tan(c\ldots}}{a^2+\ldots}$$

$$=-\frac{bB-aC}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{2abB-\ldots}{\left(a^2+b^2\right)^2d(a\ldots}$$

$$=\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3}-\frac{bB\ldots}{2\left(a^2+b^2\right)d(a\ldots}$$

$$=\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3}+\frac{\left(3a^2bB-b^3B-\ldots\right.}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.79, size = 243, normalized size = 1.39

$$-\frac{C\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{i\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2b\left(-2a\log(a+b\tan(c+dx))+\frac{a^2+b^2}{a+b\tan(c+dx)}\right)}{\left(a^2+b^2\right)^2}\right)+(bB-aC)\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^3}-\frac{\log(i+\tan(c+dx))}{(ia+b)^3}+\frac{b\left(\left(-6a^2+2b^2\right)\log(a+b\tan(c+dx))+\frac{\left(a^2+b^2\right)\left(5a^2+b^2+4ab\tan(c+dx)\right)}{(a+b\tan(c+dx))^2}\right)}{\left(a^2+b^2\right)^3}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c +
 d*x])^3,x]
```

```
[Out] -1/2*(C*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])/
(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan[c
 + d*x])))/(a^2 + b^2)^2) + (b*B - a*C)*((I*Log[I - Tan[c + d*x]])/(a + I*b
```

```
)^3 - Log[I + Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*Log[a + b*Ta
n[c + d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x]))/(a + b*Tan[c
 + d*x])^2))/(a^2 + b^2)^3))/(b*d)
```

Maple [A]
time = 0.51, size = 208, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{\left(3B\,a^{2}b-B\,b^{3}-C\,a^{3}+3Ca\,b^{2}\right)\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}-\frac{Bb-Ca}{2\left(a^{2}+b^{2}\right)(a+b\tan(dx+c))^{2}}-\frac{2Bab-C\,a^{2}+b^{2}C}{\left(a^{2}+b^{2}\right)^{2}(a+b\tan(dx+c))}+\frac{\left(-3B\,a^{2}b+B\,b^{3}+C\,a\right.}{}}{d}$
default	$\frac{\frac{\left(3B\,a^{2}b-B\,b^{3}-C\,a^{3}+3Ca\,b^{2}\right)\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}-\frac{Bb-Ca}{2\left(a^{2}+b^{2}\right)(a+b\tan(dx+c))^{2}}-\frac{2Bab-C\,a^{2}+b^{2}C}{\left(a^{2}+b^{2}\right)^{2}(a+b\tan(dx+c))}+\frac{\left(-3B\,a^{2}b+B\,b^{3}+C\,a\right.}{}}{d}$
norman	$\frac{\frac{\left(B\,a^{3}-3Ba\,b^{2}+3C\,a^{2}b-C\,b^{3}\right)a^{2}x}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}+\frac{b^{2}\left(B\,a^{3}-3Ba\,b^{2}+3C\,a^{2}b-C\,b^{3}\right)x\left(\tan^{2}(dx+c)\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{3B\,a^{2}b^{2}+B\,b^{4}-2C\,a^{3}b}{2bd\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{b\left(2Ba\,b^{2}-C\,a^{2}b+C\right.}{2da\left(a^{4}+2a\right.}}{(a+b\tan(dx+c))^{2}}$
risch	$-\frac{xB}{3ia^{2}b-ib^{3}-a^{3}+3a\,b^{2}}+\frac{ixC}{3ia^{2}b-ib^{3}-a^{3}+3a\,b^{2}}-\frac{6ia^{2}bBx}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}+\frac{2iB\,b^{3}x}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}+\frac{2ia^{3}Cx}{a^{6}+3a^{4}b^{2}+3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/d*((3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-1/2*(
B*b-C*a)/(a^2+b^2)/(a+b*tan(d*x+c))^2-(2*B*a*b-C*a^2+C*b^2)/(a^2+b^2)^2/(a+
b*tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(-3*B*a^2*b+B*b^3+C*a^3-3*C*a*b^2)*ln(1+ta
n(d*x+c)^2)+(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.51, size = 321, normalized size = 1.83

$$\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{3\,Ca^3-5\,Ba^2b-Cab^2-Bb^3+2\,(Ca^2b-2\,Bab^2-Cb^3)\tan(dx+c)}{a^6+2\,a^4b^2+a^2b^4+(a^4b^2+2\,a^2b^4+b^6)\tan(dx+c)^2+2\,(a^5b+2\,a^3b^3+ab^5)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="maxima")
```

```
[Out] 1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - 2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(b*tan(d*x
 + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a
*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (3*C*a^3 - 5*B*a^2*b - C*a*b^2 - B*b^3 + 2*(C*a^2*b - 2*B*a*b^2 - C*b^3)*t
an(d*x + c))/(a^6 + 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*tan(d
*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(171) = 342$.
time = 2.77, size = 482, normalized size = 2.75

$$\frac{5\,Ca^3b^2 - 7\,Ba^2b^3 - Cab^4 - Bb^5 + 2\,(Ba^5 + 3\,Ca^4b - 3\,Ba^3b^2 - Ca^2b^3)dx - (3\,Ca^3b^2 - 5\,Ba^2b^3 - 3\,Cab^4 + Bb^5 - 2\,(Ba^3b^2 + 3\,Ca^2b^3 - 3\,Bab^4 - Cb^5)dx)\tan(dx+c)^2 - (Ca^5 - 3\,Ba^4b - 3\,Ca^3b^2 + Ba^2b^3 + (Ca^3b^2 - 3\,Ba^2b^3 - 3\,Cab^4 + Bb^5)\tan(dx+c)^2 + 2\,(Ca^4b - 3\,Ba^3b^2 - 3\,Ca^2b^3 + Bab^4)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2 + 2\,ab\tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right) - 2\,(2\,Ca^4b - 3\,Ba^3b^2 - 3\,Ca^2b^3 + 3\,Bab^4 + Cb^5 - 2\,(Ba^4b + 3\,Ca^3b^2 - 3\,Ba^2b^3 - Cab^4)dx)\tan(dx+c)}{2\,((a^6b^2 + 3\,a^4b^4 + 3\,a^2b^6 + b^8)d\tan(dx+c)^2 + 2\,(a^7b + 3\,a^5b^3 + 3\,a^3b^5 + ab^7)d\tan(dx+c) + (a^8 + 3\,a^6b^2 + 3\,a^4b^4 + a^2b^6)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="fricas")
```

```
[Out] 1/2*(5*C*a^3*b^2 - 7*B*a^2*b^3 - C*a*b^4 - B*b^5 + 2*(B*a^5 + 3*C*a^4*b - 3
*B*a^3*b^2 - C*a^2*b^3)*d*x - (3*C*a^3*b^2 - 5*B*a^2*b^3 - 3*C*a*b^4 + B*b^
5 - 2*(B*a^3*b^2 + 3*C*a^2*b^3 - 3*B*a*b^4 - C*b^5)*d*x)*tan(d*x + c)^2 - (
C*a^5 - 3*B*a^4*b - 3*C*a^3*b^2 + B*a^2*b^3 + (C*a^3*b^2 - 3*B*a^2*b^3 - 3*
C*a*b^4 + B*b^5)*tan(d*x + c)^2 + 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 +
B*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) - 2*(2*C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 + 3*B*a*b^
4 + C*b^5 - 2*(B*a^4*b + 3*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4)*d*x)*tan(d*x
+ c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 + 2*(a^7*b
+ 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^2 + 3*a^4*
b^4 + a^2*b^6)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(171) = 342$.
time = 1.29, size = 409, normalized size = 2.34

$$\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{2\,(Ca^3b-3\,Ba^2b^2-3\,Cab^3+Bb^4)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} + \frac{3\,Ca^3b^2\tan(dx+c)^2-9\,Ba^2b^3\tan(dx+c)^2-9\,Cab^4\tan(dx+c)^2+3\,Bb^5\tan(dx+c)^2+8\,Ca^4b\tan(dx+c)-22\,Ba^3b^2\tan(dx+c)-18\,Ca^2b^3\tan(dx+c)+2\,Bab^4\tan(dx+c)-2\,Cb^5\tan(dx+c)+6\,Ca^5-14\,Ba^4b-7\,Ca^3b^2-3\,Ba^2b^3-Cab^4-Bb^5}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, al
gorithm="giac")
```

```
[Out] 1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c
```

```
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(C*a^3*b - 3*B*a^2*b^2 - 3
*C*a*b^3 + B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b
^5 + b^7) + (3*C*a^3*b^2*tan(d*x + c)^2 - 9*B*a^2*b^3*tan(d*x + c)^2 - 9*C*
a*b^4*tan(d*x + c)^2 + 3*B*b^5*tan(d*x + c)^2 + 8*C*a^4*b*tan(d*x + c) - 22
*B*a^3*b^2*tan(d*x + c) - 18*C*a^2*b^3*tan(d*x + c) + 2*B*a*b^4*tan(d*x + c
) - 2*C*b^5*tan(d*x + c) + 6*C*a^5 - 14*B*a^4*b - 7*C*a^3*b^2 - 3*B*a^2*b^3
 - C*a*b^4 - B*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*x + c) +
a)^2))/d
```

Mupad [B]
time = 8.94, size = 279, normalized size = 1.59

$$\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,B\,b-C\,a}{{\left(a^2+b^2\right)}^2}-\frac{4\,b^2\,\left(B\,b-C\,a\right)}{{\left(a^2+b^2\right)}^3}\right)}{d}-\frac{\frac{-3\,C\,a^3+5\,B\,a^2\,b+C\,a\,b^2+B\,b^3}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(-C\,a^2\,b+2\,B\,a\,b^2+C\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan{\left(c+d\,x\right)}^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x))
^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((3*B*b - C*a)/(a^2 + b^2)^2 - (4*b^2*(B*b - C*a))
/(a^2 + b^2)^3))/d - ((B*b^3 - 3*C*a^3 + 5*B*a^2*b + C*a*b^2)/(2*(a^4 + b^4
 + 2*a^2*b^2)) + (tan(c + d*x)*(C*b^3 + 2*B*a*b^2 - C*a^2*b))/(a^4 + b^4 +
2*a^2*b^2))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) + (log(tan(
c + d*x) - 1i)*(B*1i - C))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log
(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

3.43 $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=215

$$-\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3}+\frac{B\log(\sin(c+dx))}{a^3d}-\frac{b(6a^4bB+3a^2b^3B+b^5B-3a^5C+a^3b^2C)\log(a}{a^3\left(a^2+b^2\right)^3d}$$

```
[Out] -(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*x/(a^2+b^2)^3+B*ln(sin(d*x+c))/a^3/d-b*(
6*B*a^4*b+3*B*a^2*b^3+B*b^5-3*C*a^5+C*a^3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c)
)/a^3/(a^2+b^2)^3/d+1/2*b*(B*b-C*a)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+b*(3*B
*a^2*b+B*b^3-2*C*a^3)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.47, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3713, 3690, 3730, 3732, 3611, 3556}

$$\frac{B\log(\sin(c+dx))}{a^3d}+\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{b(-2a^3C+3a^2bB+b^3B)}{a^2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{x(a^3(-C)+3a^2bB+3ab^2C-b^3B)}{\left(a^2+b^2\right)^3}-\frac{b(-3a^5C+6a^4bB+a^3b^2C+3a^2b^3B+b^5B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^3,x]
```

```
[Out] -(((3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)^3) + (B*Log[Sin[c
 + d*x]])/(a^3*d) - (b*(6*a^4*b*B + 3*a^2*b^3*B + b^5*B - 3*a^5*C + a^3*b^2
*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^3*(a^2 + b^2)^3*d) + (b*(b*B -
 a*C))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (b*(3*a^2*b*B + b^3*B -
 2*a^3*C))/(a^2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
```

```
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cot(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{b(bB-aC)}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \int \frac{\cot(c+dx)(2(a^2-$$

$$= \frac{b(bB-aC)}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{b(3a^2bB+}{a^2\left(a^2+b^2\right)^2 d}$$

$$= -\frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3} + \frac{b(}{2a\left(a^2+b^2\right)d}$$

$$= -\frac{\left(3a^2bB-b^3B-a^3C+3ab^2C\right)x}{\left(a^2+b^2\right)^3} + \frac{B\log(\sin(c+}{a^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.08, size = 223, normalized size = 1.04

$$\frac{-\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^3}+\frac{2B\log(\tan(c+dx))}{a^3}-\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^3}-\frac{2b\left(6a^4bB+3a^2b^3B+b^5B-3a^5C+a^3b^2C\right)\log(a+b\tan(c+dx))}{a^3\left(a^2+b^2\right)^3}+\frac{b(bB-aC)}{a\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{2b\left(3a^2bB+b^3B-2a^3C\right)}{a^2\left(a^2+b^2\right)^2(a+b\tan(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^3,x]
```

```
[Out] (-(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^3) + (2*B*Log[Tan[c + d*x]])
/a^3 - ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b)^3 - (2*b*(6*a^4*b*B + 3*
a^2*b^3*B + b^5*B - 3*a^5*C + a^3*b^2*C)*Log[a + b*Tan[c + d*x]])/(a^3*(a^2
 + b^2)^3) + (b*(b*B - a*C))/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (2*b*
(3*a^2*b*B + b^3*B - 2*a^3*C))/(a^2*(a^2 + b^2)^2*(a + b*Tan[c + d*x])))/(2
*d)
```

Maple [A]
time = 0.65, size = 243, normalized size = 1.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(B/a^3*ln(tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(-B*a^3+3*B*a*b^2-3*C*a^2*b+C*
b^3)*ln(1+tan(d*x+c)^2)+(-3*B*a^2*b+B*b^3+C*a^3-3*C*a*b^2)*arctan(tan(d*x+c
)))+1/2*b*(B*b-C*a)/a/(a^2+b^2)/(a+b*tan(d*x+c))^2+b*(3*B*a^2*b+B*b^3-2*C*a
^3)/a^2/(a^2+b^2)^2/(a+b*tan(d*x+c))-b*(6*B*a^4*b+3*B*a^2*b^3+B*b^5-3*C*a^5
+C*a^3*b^2)/a^3/(a^2+b^2)^3*ln(a+b*tan(d*x+c)))
```

Maxima [A]
time = 0.52, size = 372, normalized size = 1.73

$$\frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(3\,Ca^5b-6\,Ba^4b^2-Ca^3b^3-3\,Ba^2b^4-Bb^6)\log(b\tan(dx+c)+a)}{a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6}-\frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{5\,Ca^4b-7\,Ba^3b^2+Ca^2b^3-3\,Bab^4+2\,(2\,Ca^3b^2-3\,Ba^2b^3-Bb^5)\tan(dx+c)}{a^8+2\,a^6b^2+a^4b^4+(a^6b^2+2\,a^4b^4+a^2b^6)\tan(dx+c)^2+2\,(a^7b+2\,a^5b^3+a^3b^5)\tan(dx+c)}+\frac{2\,B\log(\tan(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) + 2*(3*C*a^5*b - 6*B*a^4*b^2 - C*a^3*b^3 - 3*B*a^2*b^4 -
B*b^6)*log(b*tan(d*x + c) + a)/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6) - (B
*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6) - (5*C*a^4*b - 7*B*a^3*b^2 + C*a^2*b^3 - 3*B*a*b^4 +
 2*(2*C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*tan(d*x + c))/(a^8 + 2*a^6*b^2 + a^4
*b^4 + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b^
3 + a^3*b^5)*tan(d*x + c)) + 2*B*log(tan(d*x + c))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(213) = 426.
time = 2.35, size = 683, normalized size = 3.18

$$-\frac{7\,Ca^5b^3-9\,Ba^4b^4+Ca^3b^5-3\,Ba^2b^6-2\,(Ca^8-3\,Ba^7b-3\,Ca^6b^2+Ba^5b^3)dx-(5\,Ca^5b^3-7\,Ba^4b^4-Ca^3b^5-Ba^2b^6+2\,(Ca^6b^2-3\,Ba^5b^3-3\,Ca^4b^4+Ba^3b^5)dx)\tan(dx+c)^2-(Ba^8+3\,Ba^6b^2+3\,Ba^4b^4+Ba^2b^6+(Ba^6b^2+3\,Ba^4b^4+3\,Ba^2b^6+Bb^8)\tan(dx+c)^2+2\,(Ba^7b+3\,Ba^5b^3+3\,Ba^3b^5+Bab^7)\tan(dx+c))\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)-(3\,Ca^7b-6\,Ba^6b^2-Ca^5b^3-3\,Ba^4b^4-Ba^2b^6+(3\,Ca^5b^3-6\,Ba^4b^4-Ca^3b^5-3\,Ba^2b^6-Bb^8)\tan(dx+c)^2+2\,(3\,Ca^6b^2-6\,Ba^5b^3-Ca^4b^4-3\,Ba^3b^5-Bab^7)\tan(dx+c))\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-2\,(3\,Ca^6b^2-4\,Ba^5b^3-3\,Ca^4b^4+3\,Ba^3b^5+Bab^7+2\,(Ca^7b-3\,Ba^6b^2-3\,Ca^5b^3+Ba^4b^4)dx)\tan(dx+c)}{2\,((a^9b^2+3\,a^7b^4+3\,a^5b^6+a^3b^8)d\tan(dx+c)^2+2\,(a^{10}b+3\,a^8b^3+3\,a^6b^5+a^4b^7)d\tan(dx+c)+(a^{11}+3\,a^9b^2+3\,a^7b^4+a^5b^6)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/2*(7*C*a^5*b^3 - 9*B*a^4*b^4 + C*a^3*b^5 - 3*B*a^2*b^6 - 2*(C*a^8 - 3*B*
a^7*b - 3*C*a^6*b^2 + B*a^5*b^3)*d*x - (5*C*a^5*b^3 - 7*B*a^4*b^4 - C*a^3*b
^5 - B*a^2*b^6 + 2*(C*a^6*b^2 - 3*B*a^5*b^3 - 3*C*a^4*b^4 + B*a^3*b^5)*d*x)
*tan(d*x + c)^2 - (B*a^8 + 3*B*a^6*b^2 + 3*B*a^4*b^4 + B*a^2*b^6 + (B*a^6*b
^2 + 3*B*a^4*b^4 + 3*B*a^2*b^6 + B*b^8)*tan(d*x + c)^2 + 2*(B*a^7*b + 3*B*a
^5*b^3 + 3*B*a^3*b^5 + B*a*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x +
 c)^2 + 1)) - (3*C*a^7*b - 6*B*a^6*b^2 - C*a^5*b^3 - 3*B*a^4*b^4 - B*a^2*b^
6 + (3*C*a^5*b^3 - 6*B*a^4*b^4 - C*a^3*b^5 - 3*B*a^2*b^6 - B*b^8)*tan(d*x +
 c)^2 + 2*(3*C*a^6*b^2 - 6*B*a^5*b^3 - C*a^4*b^4 - 3*B*a^3*b^5 - B*a*b^7)*t
an(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x +
 c)^2 + 1)) - 2*(3*C*a^6*b^2 - 4*B*a^5*b^3 - 3*C*a^4*b^4 + 3*B*a^3*b^5 + B*
a*b^7 + 2*(C*a^7*b - 3*B*a^6*b^2 - 3*C*a^5*b^3 + B*a^4*b^4)*d*x)*tan(d*x +
c))/((a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*d*tan(d*x + c)^2 + 2*(a^10
*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*tan(d*x + c) + (a^11 + 3*a^9*b^2 +
3*a^7*b^4 + a^5*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,
x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(213) = 426.
time = 1.28, size = 479, normalized size = 2.23

$$\frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(3\,Ca^5b^2-6\,Ba^4b^3-Ca^3b^4-3\,Ba^2b^5-Bb^7)\log(|b\tan(dx+c)+a|)}{a^9b+3\,a^7b^3+3\,a^5b^5+a^3b^7}+\frac{2\,B\log(|\tan(dx+c)|)}{a^3}-\frac{9\,Ca^5b^3\tan(dx+c)^2-18\,Ba^4b^4\tan(dx+c)^2-3\,Ca^3b^5\tan(dx+c)^2-9\,Ba^2b^6\tan(dx+c)^2-3\,Bb^8\tan(dx+c)^2+22\,Ca^6b^2\tan(dx+c)-42\,Ba^5b^3\tan(dx+c)-2\,Ca^4b^4\tan(dx+c)-26\,Ba^3b^5\tan(dx+c)-8\,Bab^7\tan(dx+c)+14\,Ca^7b-25\,Ba^6b^2+3\,Ca^5b^3-19\,Ba^4b^4+Ca^3b^5-6\,Ba^2b^6}{(a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] 1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(3*C*a^5*b^2 - 6*B*a^4*b^3
 - C*a^3*b^4 - 3*B*a^2*b^5 - B*b^7)*log(abs(b*tan(d*x + c) + a))/(a^9*b + 3
*a^7*b^3 + 3*a^5*b^5 + a^3*b^7) + 2*B*log(abs(tan(d*x + c)))/a^3 - (9*C*a^5
*b^3*tan(d*x + c)^2 - 18*B*a^4*b^4*tan(d*x + c)^2 - 3*C*a^3*b^5*tan(d*x + c
)^2 - 9*B*a^2*b^6*tan(d*x + c)^2 - 3*B*b^8*tan(d*x + c)^2 + 22*C*a^6*b^2*ta
n(d*x + c) - 42*B*a^5*b^3*tan(d*x + c) - 2*C*a^4*b^4*tan(d*x + c) - 26*B*a^
3*b^5*tan(d*x + c) - 8*B*a*b^7*tan(d*x + c) + 14*C*a^7*b - 25*B*a^6*b^2 + 3
*C*a^5*b^3 - 19*B*a^4*b^4 + C*a^3*b^5 - 6*B*a^2*b^6)/((a^9 + 3*a^7*b^2 + 3*
a^5*b^4 + a^3*b^6)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 10.98, size = 315, normalized size = 1.47

$$\frac{\frac{-5\,C\,a^3\,b+7\,B\,a^2\,b^2-C\,a\,b^3+3\,B\,b^4}{2\,a\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)\,(-2\,C\,a^3\,b^2+3\,B\,a^2\,b^3+B\,b^5)}{a^2\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}+\frac{B\,\ln\left(\tan(c+d\,x)\right)}{a^3\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}-\frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-3\,C\,a^5+6\,B\,a^4\,b+C\,a^3\,b^2+3\,B\,a^2\,b^3+B\,b^5\right)}{a^3\,d\,\left(a^2+b^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
))^3,x)
```

```
[Out] ((3*B*b^4 + 7*B*a^2*b^2 - C*a*b^3 - 5*C*a^3*b)/(2*a*(a^4 + b^4 + 2*a^2*b^2)
) + (tan(c + d*x)*(B*b^5 + 3*B*a^2*b^3 - 2*C*a^3*b^2))/(a^2*(a^4 + b^4 + 2*
a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) + (B*log(tan
(c + d*x)))/(a^3*d) + (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*b^2*3i +
3*a^2*b - a^3*1i - b^3)) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(3*a*b^
2 + a^2*b*3i - a^3 - b^3*1i)) - (b*log(a + b*tan(c + d*x))*(B*b^5 - 3*C*a^5
 + 3*B*a^2*b^3 + C*a^3*b^2 + 6*B*a^4*b))/(a^3*d*(a^2 + b^2)^3)
```

3.44 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=287

$$-\frac{\left(a^3B-3ab^2B+3a^2bC-b^3C\right)x}{\left(a^2+b^2\right)^3}-\frac{(3bB-aC)\log(\sin(c+dx))}{a^4d}+\frac{b^2\left(10a^4bB+9a^2b^3B+3b^5B-6a^5C-3\right.}{a^4\,(}$$

```
[Out] -(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*x/(a^2+b^2)^3-(3*B*b-C*a)*ln(sin(d*x+c))
/a^4/d+b^2*(10*B*a^4*b+9*B*a^2*b^3+3*B*b^5-6*C*a^5-3*C*a^3*b^2-C*a*b^4)*ln(
a*cos(d*x+c)+b*sin(d*x+c))/a^4/(a^2+b^2)^3/d-1/2*b*(2*B*a^2+3*B*b^2-C*a*b)/
a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-B*cot(d*x+c)/a/d/(a+b*tan(d*x+c))^2-b*(B
*a^4+6*B*a^2*b^2+3*B*b^4-3*C*a^3*b-C*a*b^3)/a^3/(a^2+b^2)^2/d/(a+b*tan(d*x+
c))
```

Rubi [A]
time = 0.65, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3713, 3690, 3730, 3732, 3611, 3556}

$$-\frac{(3bB-aC)\log(\sin(c+dx))}{a^4d}-\frac{b\left(2a^2B-abC+3b^2B\right)}{2a^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{\left(a^2+b^2\right)^3}-\frac{b\left(a^4B-3a^3bC+6a^2b^2B-ab^3C+3b^4B\right)}{a^3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{b^2\left(-6a^5C+10a^4bB-3a^3b^2C+9a^2b^3B-ab^4C+3b^5B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4d\left(a^2+b^2\right)^3}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x
])^3,x]
```

```
[Out] -(((a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2)^3) - ((3*b*B - a*
C)*Log[Sin[c + d*x]])/(a^4*d) + (b^2*(10*a^4*b*B + 9*a^2*b^3*B + 3*b^5*B -
6*a^5*C - 3*a^3*b^2*C - a*b^4*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a^4
*(a^2 + b^2)^3*d) - (b*(2*a^2*B + 3*b^2*B - a*b*C))/(2*a^2*(a^2 + b^2)*d*(a
 + b*Tan[c + d*x])^2) - (B*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^2) - (b*
(a^4*B + 6*a^2*b^2*B + 3*b^4*B - 3*a^3*b*C - a*b^3*C))/(a^3*(a^2 + b^2)^2*d
*(a + b*Tan[c + d*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3690

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*B
*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)
) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n +
 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &
& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
|| (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3713

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b^2, Int[(a + b*Tan[e + f*x])^(m +
 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 - a
*b*B + a^2*C, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
```

```
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= -\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2} - \frac{\int \frac{\cot(c+dx)(3bB-aC+aB\tan(c \ldots}{(a+b\tan(c+ \ldots}}{a}$$

$$= -\frac{b(2a^2B+3b^2B-abC)}{2a^2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{B\cot(c \ldots}{ad(a+b\tan(\ldots}$$

$$= -\frac{b(2a^2B+3b^2B-abC)}{2a^2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{B\cot(c \ldots}{ad(a+b\tan(\ldots}$$

$$= -\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\,x}{(a^2+b^2)^3} - \frac{b(2a^2B+ \ldots}{2a^2(a^2+b^2)\,d \ldots}$$

$$= -\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\,x}{(a^2+b^2)^3} - \frac{(3bB-aC)\log \ldots}{a^4 \ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.28, size = 288, normalized size = 1.00

$$-\frac{B\cot(c+dx)}{a^3d}+\frac{(B+iC)\log(i-\tan(c+dx))}{2(ia-b)^3d}-\frac{(3bB-aC)\log(\tan(c+dx))}{a^4d}-\frac{(iB+C)\log(i+\tan(c+dx))}{2(a-ib)^3d}+\frac{b^2(10a^4bB+9a^2b^3B+3b^5B-6a^5C-3a^3b^2C-ab^4C)\log(a+b\tan(c+dx))}{a^4(a^2+b^2)^3d}-\frac{b^2(bB-aC)}{2a^2(a^2+b^2)d(a+b\tan(c+dx))^2}-\frac{b^2(4a^2bB+2b^3B-3a^3C-ab^2C)}{a^3(a^2+b^2)^2d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
 + d*x])^3,x]
```

```
[Out] -((B*Cot[c + d*x])/(a^3*d)) + ((B + I*C)*Log[I - Tan[c + d*x]])/(2*(I*a - b
)^3*d) - ((3*b*B - a*C)*Log[Tan[c + d*x]])/(a^4*d) - ((I*B + C)*Log[I + Tan
[c + d*x]])/(2*(a - I*b)^3*d) + (b^2*(10*a^4*b*B + 9*a^2*b^3*B + 3*b^5*B -
6*a^5*C - 3*a^3*b^2*C - a*b^4*C)*Log[a + b*Tan[c + d*x]])/(a^4*(a^2 + b^2)^
3*d) - (b^2*(b*B - a*C))/(2*a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (b^
2*(4*a^2*b*B + 2*b^3*B - 3*a^3*C - a*b^2*C))/(a^3*(a^2 + b^2)^2*d*(a + b*Ta
n[c + d*x]))
```

Maple [A]
time = 0.68, size = 289, normalized size = 1.01

method	result

derivativedivides	$\frac{-\frac{b^2\left(4B\,a^2b+2B\,b^3-3C\,a^3-C\,a\,b^2\right)}{a^3\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{b^2\left(10B\,a^4b+9B\,a^2b^3+3B\,b^5-6C\,a^5-3C\,a^3b^2-C\,a\,b^4\right)\ln(a+b\tan(dx+c))}{a^4\left(a^2+b^2\right)^3}-\frac{(Bb-C}{2a^2\left(a^2+b^2\right)(a+}}{}$
default	$\frac{-\frac{b^2\left(4B\,a^2b+2B\,b^3-3C\,a^3-C\,a\,b^2\right)}{a^3\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{b^2\left(10B\,a^4b+9B\,a^2b^3+3B\,b^5-6C\,a^5-3C\,a^3b^2-C\,a\,b^4\right)\ln(a+b\tan(dx+c))}{a^4\left(a^2+b^2\right)^3}-\frac{(Bb-C}{2a^2\left(a^2+b^2\right)(a+}}{}$
norman	$\frac{\frac{b\left(3B\,a^4b+11B\,a^2b^3+6B\,b^5-4C\,a^3b^2-2Ca\,b^4\right)\left(\tan^3(dx+c)\right)}{d\,a^3\left(a^4+2a^2b^2+b^4\right)}-\frac{B\tan(dx+c)}{ad}-\frac{b^2\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)x\left(\tan^4(dx+c)\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^2*(4*B*a^2*b+2*B*b^3-3*C*a^3-C*a*b^2)/a^3/(a^2+b^2)^2/(a+b*tan(d*x+
c))+b^2*(10*B*a^4*b+9*B*a^2*b^3+3*B*b^5-6*C*a^5-3*C*a^3*b^2-C*a*b^4)/a^4/(a
^2+b^2)^3*ln(a+b*tan(d*x+c))-1/2*(B*b-C*a)*b^2/a^2/(a^2+b^2)/(a+b*tan(d*x+c
))^2+1/(a^2+b^2)^3*(1/2*(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*ln(1+tan(d*x+c)^2
)+(-B*a^3+3*B*a*b^2-3*C*a^2*b+C*b^3)*arctan(tan(d*x+c)))-B/a^3/tan(d*x+c)+(
-3*B*b+C*a)/a^4*ln(tan(d*x+c)))
```

Maxima [A]
time = 0.51, size = 454, normalized size = 1.58

$$-\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(6\,Ca^5b^2-10\,Ba^4b^3+3\,Ca^3b^4-9\,Ba^2b^5+Cab^6-3\,Bb^7\right)\log\left(b\tan(dx+c)+a\right)}{a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,Ba^6+4\,Ba^4b^2+2\,Ba^2b^4+2\left(Ba^4b^2-3\,Ca^3b^3+6\,Ba^2b^4-Cab^5+3\,Bb^6\right)\tan(dx+c)^2+\left(4\,Ba^5b-7\,Ca^4b^2+17\,Ba^3b^3-3\,Ca^2b^4+9\,Bab^5\right)\tan(dx+c)}{\left(a^7b^2+2\,a^5b^4+a^3b^6\right)\tan(dx+c)^3+2\left(a^8b+2\,a^6b^3+a^4b^5\right)\tan(dx+c)^2+\left(a^9+2\,a^7b^2+a^5b^4\right)\tan(dx+c)}-\frac{2\left(Ca-3\,Bb\right)\log\left(\tan(dx+c)\right)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="maxima")
```

```
[Out] -1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(6*C*a^5*b^2 - 10*B*a^4*b^3 + 3*C*a^3*b^4 - 9*B*a^2*
b^5 + C*a*b^6 - 3*B*b^7)*log(b*tan(d*x + c) + a)/(a^10 + 3*a^8*b^2 + 3*a^6*
b^4 + a^4*b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c)^2
 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*B*a^6 + 4*B*a^4*b^2 + 2*B*a^
2*b^4 + 2*(B*a^4*b^2 - 3*C*a^3*b^3 + 6*B*a^2*b^4 - C*a*b^5 + 3*B*b^6)*tan(d
*x + c)^2 + (4*B*a^5*b - 7*C*a^4*b^2 + 17*B*a^3*b^3 - 3*C*a^2*b^4 + 9*B*a*b
^5)*tan(d*x + c))/((a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*tan(d*x + c)^3 + 2*(a^8*
b + 2*a^6*b^3 + a^4*b^5)*tan(d*x + c)^2 + (a^9 + 2*a^7*b^2 + a^5*b^4)*tan(d
*x + c)) - 2*(C*a - 3*B*b)*log(tan(d*x + c))/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(283) = 566$.

time = 3.22, size = 917, normalized size = 3.20

$$-\frac{2 Ba^{9} + 6 Ba^{7}b^{2} + 6 Ba^{5}b^{4} + 2 Ba^{3}b^{6} + \left(7 Ca^{5}b^{4} - 9 Ba^{4}b^{5} + Ca^{3}b^{6} - 3 Ba^{2}b^{7} + 2 \left(Ba^{7}b^{2} + 3 Ca^{6}b^{3} - 3 Ba^{5}b^{4} - Ca^{4}b^{5}\right)dx\right)\tan\left(dx + c\right)^{3} + 2 \left(Ba^{7}b^{2} + 4 Ca^{6}b^{3} - 2 Ba^{5}b^{4} - 3 Ca^{4}b^{5} + 6 Ba^{3}b^{6} - Ca^{2}b^{7} + 3 Bab^{8} + 2 \left(Ba^{8}b + 3 Ca^{7}b^{2} - 3 Ba^{6}b^{3} - Ca^{5}b^{4}\right)dx\right)\tan\left(dx + c\right)^{2} - \left(\left(Ca^{7}b^{2} - 3 Ba^{6}b^{3} + 3 Ca^{5}b^{4} - 9 Ba^{4}b^{5} + 3 Ca^{3}b^{6} - 9 Ba^{2}b^{7} + Cab^{8} - 3 Bb^{9}\right)\tan\left(dx + c\right)^{3} + 2 \left(Ca^{8}b - 3 Ba^{7}b^{2} + 3 Ca^{6}b^{3} - 9 Ba^{5}b^{4} + 3 Ca^{4}b^{5} - 9 Ba^{3}b^{6} + Ca^{2}b^{7} - 3 Bab^{8}\right)\tan\left(dx + c\right)^{2} + \left(Ca^{9} - 3 Ba^{8}b + 3 Ca^{7}b^{2} - 9 Ba^{6}b^{3} + 3 Ca^{5}b^{4} - 9 Ba^{4}b^{5} + Ca^{3}b^{6} - 3 Ba^{2}b^{7}\right)\tan\left(dx + c\right)\right)\log\left(\frac{\tan\left(dx + c\right)^{2}}{\tan\left(dx + c\right)^{2} + 1}\right) + \left(\left(6 Ca^{5}b^{4} - 10 Ba^{4}b^{5} + 3 Ca^{3}b^{6} - 9 Ba^{2}b^{7} + Cab^{8} - 3 Bb^{9}\right)\tan\left(dx + c\right)^{3} + 2 \left(6 Ca^{6}b^{3} - 10 Ba^{5}b^{4} + 3 Ca^{4}b^{5} - 9 Ba^{3}b^{6} + Ca^{2}b^{7} - 3 Bab^{8}\right)\tan\left(dx + c\right)^{2} + \left(6 Ca^{7}b^{2} - 10 Ba^{6}b^{3} + 3 Ca^{5}b^{4} - 9 Ba^{4}b^{5} + Ca^{3}b^{6} - 3 Ba^{2}b^{7}\right)\tan\left(dx + c\right)\right)\log\left(\frac{b^{2}\tan\left(dx + c\right)^{2} + 2 ab\tan\left(dx + c\right) + a^{2}}{\tan\left(dx + c\right)^{2} + 1}\right) + \left(4 Ba^{8}b + 12 Ba^{6}b^{3} - 9 Ca^{5}b^{4} + 23 Ba^{4}b^{5} - 3 Ca^{3}b^{6} + 9 Ba^{2}b^{7} + 2 \left(Ba^{9} + 3 Ca^{8}b - 3 Ba^{7}b^{2} - Ca^{6}b^{3}\right)dx\right)\tan\left(dx + c\right)}{2 \left(\left(a^{10}b^{2} + 3 a^{8}b^{4} + 3 a^{6}b^{6} + a^{4}b^{8}\right)d\tan\left(dx + c\right)^{3} + 2 \left(a^{11}b + 3 a^{9}b^{3} + 3 a^{7}b^{5} + a^{5}b^{7}\right)d\tan\left(dx + c\right)^{2} + \left(a^{12} + 3 a^{10}b^{2} + 3 a^{8}b^{4} + a^{6}b^{6}\right)d\tan\left(dx + c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="fricas")
```

```
[Out] -1/2*(2*B*a^9 + 6*B*a^7*b^2 + 6*B*a^5*b^4 + 2*B*a^3*b^6 + (7*C*a^5*b^4 - 9*
B*a^4*b^5 + C*a^3*b^6 - 3*B*a^2*b^7 + 2*(B*a^7*b^2 + 3*C*a^6*b^3 - 3*B*a^5*
b^4 - C*a^4*b^5)*d*x)*tan(d*x + c)^3 + 2*(B*a^7*b^2 + 4*C*a^6*b^3 - 2*B*a^5
*b^4 - 3*C*a^4*b^5 + 6*B*a^3*b^6 - C*a^2*b^7 + 3*B*a*b^8 + 2*(B*a^8*b + 3*C
*a^7*b^2 - 3*B*a^6*b^3 - C*a^5*b^4)*d*x)*tan(d*x + c)^2 - ((C*a^7*b^2 - 3*B
*a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7 + C*a*b^8
- 3*B*b^9)*tan(d*x + c)^3 + 2*(C*a^8*b - 3*B*a^7*b^2 + 3*C*a^6*b^3 - 9*B*a^
5*b^4 + 3*C*a^4*b^5 - 9*B*a^3*b^6 + C*a^2*b^7 - 3*B*a*b^8)*tan(d*x + c)^2 +
 (C*a^9 - 3*B*a^8*b + 3*C*a^7*b^2 - 9*B*a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5
 + C*a^3*b^6 - 3*B*a^2*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^
2 + 1)) + ((6*C*a^5*b^4 - 10*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7 + C*a*b^
8 - 3*B*b^9)*tan(d*x + c)^3 + 2*(6*C*a^6*b^3 - 10*B*a^5*b^4 + 3*C*a^4*b^5 -
 9*B*a^3*b^6 + C*a^2*b^7 - 3*B*a*b^8)*tan(d*x + c)^2 + (6*C*a^7*b^2 - 10*B*
a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5 + C*a^3*b^6 - 3*B*a^2*b^7)*tan(d*x + c)
)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))
 + (4*B*a^8*b + 12*B*a^6*b^3 - 9*C*a^5*b^4 + 23*B*a^4*b^5 - 3*C*a^3*b^6 + 9
*B*a^2*b^7 + 2*(B*a^9 + 3*C*a^8*b - 3*B*a^7*b^2 - C*a^6*b^3)*d*x)*tan(d*x +
 c))/((a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8)*d*tan(d*x + c)^3 + 2*(a^
11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d*tan(d*x + c)^2 + (a^12 + 3*a^10*b
^2 + 3*a^8*b^4 + a^6*b^6)*d*tan(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,
x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 1.62, size = 560, normalized size = 1.95

$$-\frac{\frac{2 \left(Ba^{3}+3 Ca^{2}b-3 Bab^{2}-Cb^{3}\right)\left(dx+c\right)}{a^{6}+3 a^{4}b^{2}+3 a^{2}b^{4}+b^{6}}+\frac{\left(Ca^{3}-3 Ba^{2}b-3 Cab^{2}+Bb^{3}\right)\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{6}+3 a^{4}b^{2}+3 a^{2}b^{4}+b^{6}}+\frac{2 \left(6 Ca^{5}b^{3}-10 Ba^{4}b^{4}+3 Ca^{3}b^{5}-9 Ba^{2}b^{6}+Cab^{7}-3 Bb^{8}\right)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^{10}b+3 a^{8}b^{3}+3 a^{6}b^{5}+a^{4}b^{7}}-\frac{18 Ca^{5}b^{4}\tan\left(dx+c\right)^{2}-30 Ba^{4}b^{5}\tan\left(dx+c\right)^{2}+9 Ca^{3}b^{6}\tan\left(dx+c\right)^{2}-27 Ba^{2}b^{7}\tan\left(dx+c\right)^{2}+3 Cab^{8}\tan\left(dx+c\right)^{2}-9 Bb^{9}\tan\left(dx+c\right)^{2}+42 Ca^{6}b^{3}\tan\left(dx+c\right)-68 Ba^{5}b^{4}\tan\left(dx+c\right)+26 Ca^{4}b^{5}\tan\left(dx+c\right)-66 Ba^{3}b^{6}\tan\left(dx+c\right)+8 Ca^{2}b^{7}\tan\left(dx+c\right)-22 Bab^{8}\tan\left(dx+c\right)+25 Ca^{7}b^{2}-39 Ba^{6}b^{3}+19 Ca^{5}b^{4}-41 Ba^{4}b^{5}+6 Ca^{3}b^{6}-14 Ba^{2}b^{7}}{\left(a^{10}+3 a^{8}b^{2}+3 a^{6}b^{4}+a^{4}b^{6}\right)\left(b\tan\left(dx+c\right)+a\right)^{2}}-\frac{2 \left(Ca-3 Bb\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^{4}}+\frac{2 \left(Ca\tan\left(dx+c\right)-3 Bb\tan\left(dx+c\right)+Ba\right)}{a^{4}\tan\left(dx+c\right)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
algorithm="giac")
```

```
[Out] -1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(6*C*a^5*b^3 - 10*B*a^4*b
^4 + 3*C*a^3*b^5 - 9*B*a^2*b^6 + C*a*b^7 - 3*B*b^8)*log(abs(b*tan(d*x + c)
+ a))/(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7) - (18*C*a^5*b^4*tan(d*x +
c)^2 - 30*B*a^4*b^5*tan(d*x + c)^2 + 9*C*a^3*b^6*tan(d*x + c)^2 - 27*B*a^2*
b^7*tan(d*x + c)^2 + 3*C*a*b^8*tan(d*x + c)^2 - 9*B*b^9*tan(d*x + c)^2 + 42
*C*a^6*b^3*tan(d*x + c) - 68*B*a^5*b^4*tan(d*x + c) + 26*C*a^4*b^5*tan(d*x
+ c) - 66*B*a^3*b^6*tan(d*x + c) + 8*C*a^2*b^7*tan(d*x + c) - 22*B*a*b^8*ta
n(d*x + c) + 25*C*a^7*b^2 - 39*B*a^6*b^3 + 19*C*a^5*b^4 - 41*B*a^4*b^5 + 6*
C*a^3*b^6 - 14*B*a^2*b^7)/((a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6)*(b*tan(
d*x + c) + a)^2) - 2*(C*a - 3*B*b)*log(abs(tan(d*x + c)))/a^4 + 2*(C*a*tan(
d*x + c) - 3*B*b*tan(d*x + c) + B*a)/(a^4*tan(d*x + c)))/d
```

Mupad [B]
time = 13.99, size = 380, normalized size = 1.32

$$\frac{b^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-6\,C\,a^5+10\,B\,a^4\,b-3\,C\,a^3\,b^2+9\,B\,a^2\,b^3-C\,a\,b^4+3\,B\,b^5\right)}{a^4\,d\,{\left(a^2+b^2\right)}^3}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(3\,B\,b-C\,a\right)}{a^4\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}-\frac{\frac{B}{a}+\frac{{\tan(c+d\,x)}^2\,\left(B\,a^4\,b^2-3\,C\,a^3\,b^3+6\,B\,a^2\,b^4-C\,a\,b^5+3\,B\,b^6\right)}{a^3\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(4\,B\,a^4\,b-7\,C\,a^3\,b^2+17\,B\,a^2\,b^3-3\,C\,a\,b^4+9\,B\,b^5\right)}{2\,a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\,\tan\left(c+d\,x\right)+2\,a\,b\,{\tan\left(c+d\,x\right)}^2+b^2\,{\tan\left(c+d\,x\right)}^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x
))^3,x)
```

```
[Out] (b^2*log(a + b*tan(c + d*x))*(3*B*b^5 - 6*C*a^5 + 9*B*a^2*b^3 - 3*C*a^3*b^2
 + 10*B*a^4*b - C*a*b^4))/(a^4*d*(a^2 + b^2)^3) - (log(tan(c + d*x) - 1i)*(
B*1i - C))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(c + d*x))*(
3*B*b - C*a))/(a^4*d) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*b^2*3i
- 3*a^2*b - a^3*1i + b^3)) - (B/a + (tan(c + d*x)^2*(3*B*b^6 + 6*B*a^2*b^4
+ B*a^4*b^2 - 3*C*a^3*b^3 - C*a*b^5))/(a^3*(a^4 + b^4 + 2*a^2*b^2)) + (tan(
c + d*x)*(9*B*b^5 + 17*B*a^2*b^3 - 7*C*a^3*b^2 + 4*B*a^4*b - 3*C*a*b^4))/(2
*a^2*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2*tan(c + d*x) + b^2*tan(c + d*x)^3 +
2*a*b*tan(c + d*x)^2))
```

3.45 $\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\text{ t}\right.$

Optimal. Leaf size=132

$$\frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)}+\frac{(A-C)\,{}_2F_1\left(1,\frac{3+n}{2};\frac{5+n}{2};-\tan^2(c+dx)\right)(b\tan(c+dx))^{3+n}}{b^3d(3+n)}+\frac{B\,{}_2F_1\left(1,\frac{4+n}{2};\frac{6+n}{2};-\tan\right.}{b^4d}$$

```
[Out] C*(b*tan(d*x+c))^(3+n)/b^3/d/(3+n)+(A-C)*hypergeom([1, 3/2+1/2*n],[5/2+1/2*
n],-tan(d*x+c)^2)*(b*tan(d*x+c))^(3+n)/b^3/d/(3+n)+B*hypergeom([1, 2+1/2*n]
,[3+1/2*n],-tan(d*x+c)^2)*(b*tan(d*x+c))^(4+n)/b^4/d/(4+n)
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {16, 3711, 3619, 3557, 371}

$$\frac{(A-C)(b\tan(c+dx))^{n+3}\,{}_2F_1\left(1,\frac{n+3}{2};\frac{n+5}{2};-\tan^2(c+dx)\right)}{b^3d(n+3)}+\frac{B(b\tan(c+dx))^{n+4}\,{}_2F_1\left(1,\frac{n+4}{2};\frac{n+6}{2};-\tan^2(c+dx)\right)}{b^4d(n+4)}+\frac{C(b\tan(c+dx))^{n+3}}{b^3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (C*(b*Tan[c + d*x])^(3 + n))/(b^3*d*(3 + n)) + ((A - C)*Hypergeometric2F1[1
, (3 + n)/2, (5 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(3 + n))/(b^3*d*(
3 + n)) + (B*Hypergeometric2F1[1, (4 + n)/2, (6 + n)/2, -Tan[c + d*x]^2]*(b
*Tan[c + d*x])^(4 + n))/(b^4*d*(4 + n))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] && !IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

Rubi steps

$$\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{\int (b\tan(c+dx))^{2+n}\left(A+B\,\mathrm{ta}\right.}{}$$

$$= \frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)} + \frac{\int (b\,\mathrm{ta}}{}$$

$$= \frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)} + \frac{B\int (b}{}$$

$$= \frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)} + \frac{B\mathrm{Subs}}{}$$

$$= \frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)} + \frac{(A-C}{}$$

Mathematica [A]
time = 0.31, size = 110, normalized size = 0.83

$$\frac{\tan^3(c+dx)(b\tan(c+dx))^n\left(C(4+n)+(A-C)(4+n)\,{}_2F_1\left(1,\frac{3+n}{2};\frac{5+n}{2};-\tan^2(c+dx)\right)+B(3+n)\,{}_2F_1\left(1,\frac{4+n}{2};\frac{6+n}{2};-\tan^2(c+dx)\right)\tan(c+dx)\right)}{d(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (Tan[c + d*x]^3*(b*Tan[c + d*x])^n*(C*(4 + n) + (A - C)*(4 + n)*Hypergeomet
ric2F1[1, (3 + n)/2, (5 + n)/2, -Tan[c + d*x]^2] + B*(3 + n)*Hypergeometric
2F1[1, (4 + n)/2, (6 + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]))/(d*(3 + n)*(4
+ n))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(\tan^2\left(dx+c\right)\right)\left(b\tan\left(dx+c\right)\right)^n\left(A+B\tan\left(dx+c\right)+C\left(\tan^2\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] int(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^4 + B*tan(d*x + c)^3 + A*tan(d*x + c)^2)*(b*tan(d*
x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(c+dx\right)\right)^n\left(A+B\tan\left(c+dx\right)+C\tan^2\left(c+dx\right)\right)\tan^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2*(b*tan(d*x+c))**n*(A+B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Integral((b*tan(c + d*x))**n*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c
 + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^2\,(b\tan(c+d\,x))^n\,\left(C\tan(c+d\,x)^2+B\tan(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(b*tan(c + d*x))^n*(A + B*tan(c + d*x) + C*tan(c + d*x)^
2),x)
```

```
[Out] int(tan(c + d*x)^2*(b*tan(c + d*x))^n*(A + B*tan(c + d*x) + C*tan(c + d*x)^
2), x)
```

3.46 $\int \tan^m(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\right.$

Optimal. Leaf size=154

$$\frac{C\tan^{1+m}(c+dx)(b\tan(c+dx))^n}{d(1+m+n)}+\frac{(A-C)\,{}_2F_1\left(1,\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);-\tan^2(c+dx)\right)\tan^{1+m}(c+}{d(1+m+n)}$$

```
[Out] C*tan(d*x+c)^(1+m)*(b*tan(d*x+c))^n/d/(1+m+n)+(A-C)*hypergeom([1, 1/2+1/2*m
+1/2*n],[3/2+1/2*m+1/2*n],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)*(b*tan(d*x+c))^n/
d/(1+m+n)+B*hypergeom([1, 1+1/2*m+1/2*n],[2+1/2*m+1/2*n],-tan(d*x+c)^2)*tan
(d*x+c)^(2+m)*(b*tan(d*x+c))^n/d/(2+m+n)
```

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {20, 3711, 3619, 3557, 371}

$$\frac{(A-C)\tan^{m+1}(c+dx)(b\tan(c+dx))^n\,{}_2F_1\left(1,\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);-\tan^2(c+dx)\right)}{d(m+n+1)}+\frac{B\tan^{m+2}(c+dx)(b\tan(c+dx))^n\,{}_2F_1\left(1,\frac{1}{2}(m+n+2);\frac{1}{2}(m+n+4);-\tan^2(c+dx)\right)}{d(m+n+2)}+\frac{C\tan^{m+1}(c+dx)(b\tan(c+dx))^n}{d(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^
2),x]
```

```
[Out] (C*Tan[c + d*x]^(1 + m)*(b*Tan[c + d*x])^n)/(d*(1 + m + n)) + ((A - C)*Hype
rgeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m)*(b*Tan[c + d*x])^n)/(d*(1 + m + n)) + (B*Hypergeometric2F1[1, (2
+ m + n)/2, (4 + m + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m)*(b*Tan[c +
 d*x])^n)/(d*(2 + m + n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \left(\tan^{-n}(c+dx)(b\tan(c+dx)\right.$$

$$= \frac{C\tan^{1+m}(c+dx)(b\tan(c+}{d(1+m+n)}$$

$$= \frac{C\tan^{1+m}(c+dx)(b\tan(c+}{d(1+m+n)}$$

$$= \frac{C\tan^{1+m}(c+dx)(b\tan(c+}{d(1+m+n)}$$

$$= \frac{C\tan^{1+m}(c+dx)(b\tan(c+}{d(1+m+n)}$$

Mathematica [A]
time = 0.27, size = 115, normalized size = 0.75

$$\frac{\tan^{1+m}(c+dx)(b\tan(c+dx))^n\left(\frac{C}{1+m+n}+\frac{(A-C)\,{}_2F_1\left(1,\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);-\tan^2(c+dx)\right)}{1+m+n}+\frac{B\,{}_2F_1\left(1,\frac{1}{2}(2+m+n);\frac{1}{2}(4+m+n);-\tan^2(c+dx)\right)\tan(c+dx)}{2+m+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^m*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c +
 d*x]^2),x]
```

```
[Out] (Tan[c + d*x]^(1 + m)*(b*Tan[c + d*x])^n*(C/(1 + m + n) + ((A - C)*Hypergeo
metric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[c + d*x]^2])/(1 + m + n) +
```

```
(B*Hypergeometric2F1[1, (2 + m + n)/2, (4 + m + n)/2, -Tan[c + d*x]^2]*Tan[
c + d*x])/(2 + m + n)))/d
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int \left(\tan^{m}\left(dx+c\right)\right)\left(b\tan\left(dx+c\right)\right)^{n}\left(A+B\tan\left(dx+c\right)+C\left(\tan^{2}\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
[Out] int(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="maxima")
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="fricas")
[Out] integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*x
 + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(c+dx\right)\right)^{n}\left(A+B\tan\left(c+dx\right)+C\tan^{2}\left(c+dx\right)\right)\tan^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(b*tan(d*x+c))**n*(A+B*tan(d*x+c)+C*tan(d*x+c)**2),
x)
```

```
[Out] Integral((b*tan(c + d*x))**n*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c
 + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^m\,{\left(b\tan(c+d\,x)\right)}^n\,\left(C\,{\tan(c+d\,x)}^2+B\tan(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(b*tan(c + d*x))^n*(A + B*tan(c + d*x) + C*tan(c + d*x)^
2),x)
```

```
[Out] int(tan(c + d*x)^m*(b*tan(c + d*x))^n*(A + B*tan(c + d*x) + C*tan(c + d*x)^
2), x)
```

3.47 $\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\,(A+B\tan(c+dx)+C$

Optimal. Leaf size=170

$$\frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}}{d(3+2m)}+\frac{2(A-C)\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(c+dx)\right)\tan^{1+m}(c+dx}{d(3+2m)}$$

```
[Out] 2*C*(b*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/d/(3+2*m)+2*(A-C)*hypergeom([1, 3
/4+1/2*m],[7/4+1/2*m],-tan(d*x+c)^2)*(b*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/
d/(3+2*m)+2*B*hypergeom([1, 5/4+1/2*m],[9/4+1/2*m],-tan(d*x+c)^2)*(b*tan(d*
x+c))^(1/2)*tan(d*x+c)^(2+m)/d/(5+2*m)
```

Rubi [A]
time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$,
Rules used = {20, 3711, 3619, 3557, 371}

$$\frac{2(A-C)\sqrt{b\tan(c+dx)}\,\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);-\tan^2(c+dx)\right)}{d(2m+3)}+\frac{2B\sqrt{b\tan(c+dx)}\,\tan^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2m+5);\frac{1}{4}(2m+9);-\tan^2(c+dx)\right)}{d(2m+5)}+\frac{2C\sqrt{b\tan(c+dx)}\,\tan^{m+1}(c+dx)}{d(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^m*Sqrt[b*Tan[c + d*x]]*(A + B*Tan[c + d*x] + C*Tan[c + d*x
]^2),x]
```

```
[Out] (2*C*Tan[c + d*x]^(1 + m)*Sqrt[b*Tan[c + d*x]])/(d*(3 + 2*m)) + (2*(A - C)*
Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m)*Sqrt[b*Tan[c + d*x]])/(d*(3 + 2*m)) + (2*B*Hypergeometric2F1[1, (
5 + 2*m)/4, (9 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m)*Sqrt[b*Tan[c
 + d*x]])/(d*(5 + 2*m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
```

```
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{\sqrt{b\tan(c+dx)}\int \tan^{\frac{1}{2}+m}(c$$

$$= \frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c}{d(3+2m)}$$

$$= \frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c}{d(3+2m)}$$

$$= \frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c}{d(3+2m)}$$

$$= \frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c}{d(3+2m)}$$

Mathematica [A]
time = 0.38, size = 133, normalized size = 0.78

$$\frac{2\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}\left(C(5+2m)+(A-C)(5+2m)\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(c+dx)\right)+B(3+2m)\,{}_2F_1\left(1,\frac{1}{4}(5+2m);\frac{1}{4}(9+2m);-\tan^2(c+dx)\right)\tan(c+dx)\right)}{d(3+2m)(5+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^m*Sqrt[b*Tan[c + d*x]]*(A + B*Tan[c + d*x] + C*Tan[c
 + d*x]^2),x]
```

```
[Out] (2*Tan[c + d*x]^(1 + m)*Sqrt[b*Tan[c + d*x]]*(C*(5 + 2*m) + (A - C)*(5 + 2*
m)*Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2] + B*(3 +
 2*m)*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c
 + d*x]))/(d*(3 + 2*m)*(5 + 2*m))
```

Maple [F]
time = 0.65, size = 0, normalized size = 0.00

$$\int \left(\tan^{m}\left(dx+c\right)\right) \sqrt{b\tan\left(dx+c\right)}\, \left(A+B\tan\left(dx+c\right)+C\left(\tan^{2}\left(dx+c\right)\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

```
[Out] int(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c))*tan(d
*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan\left(c+dx\right)}\, \left(A+B\tan\left(c+dx\right)+C\tan^{2}\left(c+dx\right)\right)\tan^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)*
*2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x))*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(
c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c))*tan(
d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c+d\,x)^m\,\sqrt{b\tan\left(c+d\,x\right)}\,\left(C\tan(c+d\,x)^2+B\tan(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(b*tan(c + d*x))^(1/2)*(A + B*tan(c + d*x) + C*tan(c + d
*x)^2),x)
```

```
[Out] int(tan(c + d*x)^m*(b*tan(c + d*x))^(1/2)*(A + B*tan(c + d*x) + C*tan(c + d
*x)^2), x)
```

3.48 $\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx$

Optimal. Leaf size=170

$$\frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}}+\frac{2(A-C)\,{}_2F_1\left(1,\frac{1}{4}(1+2m);\frac{1}{4}(5+2m);-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}}+\frac{2B\,{}_2F$$

```
[Out] 2*C*tan(d*x+c)^(1+m)/d/(1+2*m)/(b*tan(d*x+c))^(1/2)+2*(A-C)*hypergeom([1, 1
/4+1/2*m],[5/4+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/(1+2*m)/(b*tan(d*x+
c))^(1/2)+2*B*hypergeom([1, 3/4+1/2*m],[7/4+1/2*m],-tan(d*x+c)^2)*tan(d*x+c
)^(2+m)/d/(3+2*m)/(b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {20, 3711, 3619, 3557, 371}

$$\frac{2(A-C)\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2m+1);\frac{1}{4}(2m+5);-\tan^2(c+dx)\right)}{d(2m+1)\sqrt{b\tan(c+dx)}}+\frac{2B\tan^{m+2}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);-\tan^2(c+dx)\right)}{d(2m+3)\sqrt{b\tan(c+dx)}}+\frac{2C\tan^{m+1}(c+dx)}{d(2m+1)\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[b*Tan[c +
 d*x]],x]
```

```
[Out] (2*C*Tan[c + d*x]^(1 + m))/(d*(1 + 2*m)*Sqrt[b*Tan[c + d*x]]) + (2*(A - C)*
Hypergeometric2F1[1, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(1 + m))/(d*(1 + 2*m)*Sqrt[b*Tan[c + d*x]]) + (2*B*Hypergeometric2F1[1, (
3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(3 + 2*m
)*Sqrt[b*Tan[c + d*x]])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3619

```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Tan[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^2
+ d^2, 0] &&  !IntegerQ[2*m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx &= \frac{\sqrt{\tan(c+dx)}\int \tan^{-\frac{1}{2}+m}(c+dx)\,(A+B\tan\cdots}{\sqrt{b\tan(c+dx)\cdots}}\\ &= \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}}+\frac{\sqrt{\tan(c+dx)}\cdots}{\cdots}\\ &= \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}}+\frac{\left(B\sqrt{\tan(c+d\cdots}\right.}{\sqrt{\cdots}}\\ &= \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}}+\frac{\left(B\sqrt{\tan(c+d\cdots}\right.}{\cdots}\\ &= \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}}+\frac{2(A-C)\,{}_2F_1\big(1\cdots}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 133, normalized size = 0.78

$$\frac{2\tan^{1+m}(c+dx)\left(C(3+2m)+(A-C)(3+2m)\,{}_2F_1\left(1,\tfrac{1}{4}(1+2m);\tfrac{1}{4}(5+2m);-\tan^2(c+dx)\right)+B(1+2m)\,{}_2F_1\left(1,\tfrac{1}{4}(3+2m);\tfrac{1}{4}(7+2m);-\tan^2(c+dx)\right)\tan(c+dx)\right)}{d(1+2m)(3+2m)\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[b*T
an[c + d*x]],x]
```

```
[Out] (2*Tan[c + d*x]^(1 + m)*(C*(3 + 2*m) + (A - C)*(3 + 2*m)*Hypergeometric2F1[
1, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[c + d*x]^2] + B*(1 + 2*m)*Hypergeometric2
F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]))/(d*(1 + 2*m
)*(3 + 2*m)*Sqrt[b*Tan[c + d*x]])
```

Maple [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(\tan^{m}\left(dx+c\right)\right)\left(A+B\tan\left(dx+c\right)+C\left(\tan^{2}\left(dx+c\right)\right)\right)}{\sqrt{b\tan\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c))*tan(d
*x + c)^m/(b*tan(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\tan\left(c+dx\right)+C\tan^{2}\left(c+dx\right)\right)\tan^{m}\left(c+dx\right)}{\sqrt{b\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(A+B*tan(d*x+c)+C*tan(d*x+c)**2)/(b*tan(d*x+c))**(1
/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c + d*x)**m/sqrt(b*ta
n(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(c+d\,x)^m\,\left(C\tan(c+d\,x)^2+B\tan(c+d\,x)+A\right)}{\sqrt{b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^m*(A + B*tan(c + d*x) + C*tan(c + d*x)^2))/(b*tan(c + d*x
))^(1/2),x)
```

```
[Out] int((tan(c + d*x)^m*(A + B*tan(c + d*x) + C*tan(c + d*x)^2))/(b*tan(c + d*x
))^(1/2), x)
```

3.49 $\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal. Leaf size=328

$$-\frac{\left(bB+\sqrt{-b^2}\,(A-C)\right)F_1\left(\frac{1}{2};1,-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},1+\frac{b\tan(c+dx)}{a}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}\sqrt{a+b}}{b\left(a-\sqrt{-b^2}\right)d}$$

```
[Out] 2*C*hypergeom([1/2, -m],[3/2],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1/2)*tan(
d*x+c)^m/b/d/((-b*tan(d*x+c)/a)^m)-AppellF1(1/2,1,-m,3/2,(a+b*tan(d*x+c))/(
a+(-b^2)^(1/2)),1+b*tan(d*x+c)/a)*(b*B-(A-C)*(-b^2)^(1/2))*(a+b*tan(d*x+c))
^(1/2)*tan(d*x+c)^m/b/d/(a+(-b^2)^(1/2))/((-b*tan(d*x+c)/a)^m)-AppellF1(1/2
,1,-m,3/2,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),1+b*tan(d*x+c)/a)*(b*B+(A-C)*(-
b^2)^(1/2))*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^m/b/d/(a-(-b^2)^(1/2))/((-b*t
an(d*x+c)/a)^m)
```

Rubi [A]
time = 1.13, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3736, 6852, 1706, 252, 251, 441, 440}

$$-\frac{\left(\sqrt{-b^2}\,(A-C)+bB\right)\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}F_1\left(\frac{1}{2};1,-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{b\tan(c+dx)}{a}+1\right)}{bd\left(a-\sqrt{-b^2}\right)}-\frac{\left(bB-\sqrt{-b^2}\,(A-C)\right)\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}F_1\left(\frac{1}{2};1,-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}},\frac{b\tan(c+dx)}{a}+1\right)}{bd\left(a+\sqrt{-b^2}\right)}+\frac{2C\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b\tan(c+dx)}{a}+1\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a + b*Tan
[c + d*x]],x]
```

```
[Out] -(((b*B + Sqrt[-b^2]*(A - C))*AppellF1[1/2, 1, -m, 3/2, (a + b*Tan[c + d*x]
)/(a - Sqrt[-b^2]), 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c
 + d*x]])/(b*(a - Sqrt[-b^2])*d*(-((b*Tan[c + d*x])/a))^m)) - ((b*B - Sqrt[
-b^2]*(A - C))*AppellF1[1/2, 1, -m, 3/2, (a + b*Tan[c + d*x])/(a + Sqrt[-b^
2]), 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]])/(b*(a
 + Sqrt[-b^2])*d*(-((b*Tan[c + d*x])/a))^m) + (2*C*Hypergeometric2F1[1/2, -
m, 3/2, 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]])/(b
*d*(-((b*Tan[c + d*x])/a))^m)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^m(A+Bx+Cx^2)}{\sqrt{a+bx}\,(1+x^2)}\,dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \frac{\left(\frac{-a+x^2}{b}\right)^m(Ab^2+(a-x^2)(-bB+C(a-x^2)))}{a^2+b^2-2ax^2+x^4}\,dx \right.}{bd}$$

$$= \frac{(2\tan^m(c+dx)(b\tan(c+dx))^{-m})\,\text{Subst}\left(\int \frac{(-}{}\right.}{}$$

$$= \frac{(2\tan^m(c+dx)(b\tan(c+dx))^{-m})\,\text{Subst}\left(\int \left(C\right.\right.}{}$$

$$= \frac{(2\tan^m(c+dx)(b\tan(c+dx))^{-m})\,\text{Subst}\left(\int \frac{(-}{}\right.}{}$$

$$= \frac{(2\tan^m(c+dx)(b\tan(c+dx))^{-m})\,\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= \frac{2C\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a}\right)\tan^m(c+dx)\left(\right.}{bd}$$

$$= \frac{2C\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{a+b\tan(c+dx)}{a}\right)\tan^m(c+dx)\left(\right.}{bd}$$

$$= -\frac{\left(bB+\sqrt{-b^2}\,(A-C)\right)F_1\left(\frac{1}{2};1,-m;\frac{3}{2};\frac{a+b\,\mathrm{ta}}{a-\sqrt{}}\right.}{}$$

Mathematica [F]
time = 45.84, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a +
 b*Tan[c + d*x]],x]
```

```
[Out] Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a +
 b*Tan[c + d*x]], x]
```

Maple [F]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{(\tan^m (dx+c)) (A + B\tan (dx+c) + C(\tan^2 (dx+c)))}{\sqrt{a+b\tan (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/2),x)
```

```
[Out] int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan
(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan(
d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\tan (c+dx) + C\tan^2 (c+dx)) \tan^m (c+dx)}{\sqrt{a+b\tan (c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m*(A+B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**
(1/2),x)
```

```
[Out] Integral((A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c + d*x)**m/sqrt(a +
b*tan(c + d*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(c+d\,x)^m\,\left(C\tan(c+d\,x)^2+B\tan(c+d\,x)+A\right)}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(c + d*x)^m*(A + B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c +
 d*x))^(1/2),x)
```

```
[Out] int((tan(c + d*x)^m*(A + B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c +
 d*x))^(1/2), x)
```

3.50 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+\right.$

Optimal. Leaf size=353

$$\left(a^3(Ac-cC-Bd)-3ab^2(Ac-cC-Bd)-3a^2b(Bc+(A-C)d)+b^3(Bc+(A-C)d)\right)x-\frac{(3a^2b(Ac-}{}$$

```
[Out] (a^3*(A*c-B*d-C*c)-3*a*b^2*(A*c-B*d-C*c)-3*a^2*b*(B*c+(A-C)*d)+b^3*(B*c+(A-
C)*d))*x-(3*a^2*b*(A*c-B*d-C*c)-b^3*(A*c-B*d-C*c)+a^3*(B*c+(A-C)*d)-3*a*b^2
*(B*c+(A-C)*d))*ln(cos(f*x+e))/f+b*(2*a*b*(A*c-B*d-C*c)+a^2*(B*c+(A-C)*d)-b
^2*(B*c+(A-C)*d))*tan(f*x+e)/f+1/2*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*(a
+b*tan(f*x+e))^2/f+1/3*(B*c+(A-C)*d)*(a+b*tan(f*x+e))^3/f-1/20*(a*C*d-5*b*(
B*d+C*c))*(a+b*tan(f*x+e))^4/b^2/f+1/5*C*d*tan(f*x+e)*(a+b*tan(f*x+e))^4/b/
f
```

Rubi [A]
time = 0.55, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3718, 3711, 3609, 3606, 3556}

$$\frac{b\tan(e+fx)\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}-\frac{\log(\cos(e+fx))\left(a^3(d(A-C)+Bc)+3a^2b(Ac-Bd-cC)-3ab^2(d(A-C)+Bc)-b^3(Ac-Bd-cC)\right)}{f}+x\left(a^3(Ac-Bd-cC)-3a^2b(d(A-C)+Bc)-3ab^2(Ac-Bd-cC)+b^3(d(A-C)+Bc)\right)+\frac{(d(A-C)+Bc)(a+b\tan(e+fx))^3}{3f}+\frac{(a+b\tan(e+fx))^2(aAd+aBc-aCd+Abc-bBd-bcC)}{2f}-\frac{(aCd-5b(Bd+cC))(a+b\tan(e+fx))^4}{20b^2f}+\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```

```
[Out] (a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) - 3*a^2*b*(B*c + (A - C)
*d) + b^3*(B*c + (A - C)*d))*x - ((3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c
*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))*Log[Cos[e +
f*x]])/f + (b*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c +
 (A - C)*d))*Tan[e + f*x])/f + ((A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*
C*d)*(a + b*Tan[e + f*x])^2)/(2*f) + ((B*c + (A - C)*d)*(a + b*Tan[e + f*x]
)^3)/(3*f) - ((a*C*d - 5*b*(c*C + B*d))*(a + b*Tan[e + f*x])^4)/(20*b^2*f)
+ (C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^4)/(5*b*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{Cd\tan(e+fx)(a+}{5bf}$$

$$= -\frac{(aCd-5b(cC+B}{}$$

$$= \frac{(Bc+(A-C)d)(a}{3f}$$

$$= \frac{(Abc+aBc-bcC+}{}$$

$$= \left(a^3(Ac-cC-Bd)\right.$$

$$= \left(a^3(Ac-cC-Bd)\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.85, size = 280, normalized size = 0.79

$$\frac{\frac{3(-aCd+5b(cC+Bd))(a+b\tan(e+fx))^4}{b}+12Cd\tan(e+fx)(a+b\tan(e+fx))^4+30(Abc-aBc-bcC-aAd-bBd+aCd)\left((ia-b)^3\log(i-\tan(e+fx))-(ia+b)^3\log(i+\tan(e+fx))+6ab^2\tan(e+fx)+b^3\tan^2(e+fx)\right)-10(Bc+(A-C)d)\left(3i(a+ib)^4\log(i-\tan(e+fx))-3i(a-ib)^4\log(i+\tan(e+fx))+6b^2(-6a^2+b^2)\tan(e+fx)-12ab^3\tan^2(e+fx)-2b^4\tan^3(e+fx)\right)}{60bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] ((3*(-(a*C*d) + 5*b*(c*C + B*d))*(a + b*Tan[e + f*x])^4)/b + 12*C*d*Tan[e +
 f*x]*(a + b*Tan[e + f*x])^4 + 30*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d +
a*C*d)*((I*a - b)^3*Log[I - Tan[e + f*x]] - (I*a + b)^3*Log[I + Tan[e + f*x
]] + 6*a*b^2*Tan[e + f*x] + b^3*Tan[e + f*x]^2) - 10*(B*c + (A - C)*d)*((3*
I)*(a + I*b)^4*Log[I - Tan[e + f*x]] - (3*I)*(a - I*b)^4*Log[I + Tan[e + f*
x]] + 6*b^2*(-6*a^2 + b^2)*Tan[e + f*x] - 12*a*b^3*Tan[e + f*x]^2 - 2*b^4*T
an[e + f*x]^3))/(60*b*f)
```

Maple [A]
time = 0.21, size = 639, normalized size = 1.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*(A*a^3*d+3*A*a^2*b*c-3*A*a*b^2*d-A*b^3*c+B*a^3*c-3*B*a^2*b*d-3*B*a
*b^2*c+B*b^3*d-C*a^3*d-3*C*a^2*b*c+3*C*a*b^2*d+C*b^3*c)*ln(1+tan(f*x+e)^2)+
(A*a^3*c-3*A*a^2*b*d-3*A*a*b^2*c+A*b^3*d-B*a^3*d-3*B*a^2*b*c+3*B*a*b^2*d+B*
b^3*c-C*a^3*c+3*C*a^2*b*d+3*C*a*b^2*c-C*b^3*d)*arctan(tan(f*x+e))+C*b^3*d*t
an(f*x+e)-1/2*C*b^3*c*tan(f*x+e)^2+1/5*C*b^3*d*tan(f*x+e)^5+1/4*B*b^3*d*tan
(f*x+e)^4+1/4*C*b^3*c*tan(f*x+e)^4+1/3*A*b^3*d*tan(f*x+e)^3+1/3*B*b^3*c*tan
(f*x+e)^3-1/3*C*b^3*d*tan(f*x+e)^3+1/2*A*b^3*c*tan(f*x+e)^2-1/2*B*b^3*d*tan
(f*x+e)^2+B*a^3*d*tan(f*x+e)-B*b^3*c*tan(f*x+e)+C*a^3*c*tan(f*x+e)-A*b^3*d*
tan(f*x+e)+1/2*C*a^3*d*tan(f*x+e)^2-3/2*C*a*b^2*d*tan(f*x+e)^2+B*a*b^2*d*ta
n(f*x+e)^3+C*a^2*b*d*tan(f*x+e)^3+C*a*b^2*c*tan(f*x+e)^3+3*A*a^2*b*d*tan(f*
x+e)+3*A*a*b^2*c*tan(f*x+e)+3*B*a^2*b*c*tan(f*x+e)-3*B*a*b^2*d*tan(f*x+e)-3
*C*a^2*b*d*tan(f*x+e)-3*C*a*b^2*c*tan(f*x+e)+3/4*C*a*b^2*d*tan(f*x+e)^4+3/2
*A*a*b^2*d*tan(f*x+e)^2+3/2*B*a^2*b*d*tan(f*x+e)^2+3/2*B*a*b^2*c*tan(f*x+e)
^2+3/2*C*a^2*b*c*tan(f*x+e)^2)
```

Maxima [A]
time = 0.51, size = 423, normalized size = 1.20

$$\frac{12\,Cb^3d\tan(fx+e)^5+15\,(Cb^3c+(3\,Cab^2+Bb^3)d)\tan(fx+e)^4+20\,((3\,Cab^2+Bb^3)c+(3\,Ca^2b+3\,Bab^2+(A-C)b^3)d)\tan(fx+e)^3+30\,((3\,Ca^2b+3\,Bab^2+(A-C)b^3)c+(Ca^3+3\,Ba^2b+3\,(A-C)ab^2-Bb^3)d)\tan(fx+e)^2+60\,(((A-C)a^3-3\,Ba^2b-3\,(A-C)ab^2+Bb^3)c-(Ba^3+3\,(A-C)a^2b-3\,Bab^2-(A-C)b^3)d)(fx+e)+30\,((Ba^3+3\,(A-C)a^2b-3\,Bab^2-(A-C)b^3)c+((A-C)a^3-3\,Ba^2b-3\,(A-C)ab^2+Bb^3)d)\log(\tan(fx+e)^2+1)+60\,((Ca^3+3\,Ba^2b+3\,(A-C)ab^2-Bb^3)c+(Ba^3+3\,(A-C)a^2b-3\,Bab^2-(A-C)b^3)d)\tan(fx+e)}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] 1/60*(12*C*b^3*d*tan(f*x + e)^5 + 15*(C*b^3*c + (3*C*a*b^2 + B*b^3)*d)*tan(
f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b
^3)*d)*tan(f*x + e)^3 + 30*((3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c + (C*a^
3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*d)*tan(f*x + e)^2 + 60*(((A - C)*a
^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c - (B*a^3 + 3*(A - C)*a^2*b - 3*
B*a*b^2 - (A - C)*b^3)*d)*(f*x + e) + 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*
b^2 - (A - C)*b^3)*c + ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*
d)*log(tan(f*x + e)^2 + 1) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b
^3)*c + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*tan(f*x + e)
)/f
```

Fricas [A]
time = 1.48, size = 421, normalized size = 1.19

$$\frac{12 \, C b^{3} d \tan\left(f x + e\right)^{5} + 15 \, {\left(C b^{3} c + {\left(3 \, C a b^{2} + B b^{3}\right)} d\right)} \tan\left(f x + e\right)^{4} + 20 \, {\left({\left(3 \, C a b^{2} + B b^{3}\right)} c + {\left(3 \, C a^{2} b + 3 \, B a b^{2} + {\left(A - C\right)} b^{3}\right)} d\right)} \tan\left(f x + e\right)^{3} + 60 \, {\left({\left({\left(A - C\right)} a^{3} - 3 \, B a^{2} b - 3 \, {\left(A - C\right)} a b^{2} + B b^{3}\right)} c - {\left(B a^{3} + 3 \, {\left(A - C\right)} a^{2} b - 3 \, B a b^{2} - {\left(A - C\right)} b^{3}\right)} d\right)} f x + 30 \, {\left({\left(3 \, C a^{2} b + 3 \, B a b^{2} + {\left(A - C\right)} b^{3}\right)} c + {\left(C a^{3} + 3 \, B a^{2} b + 3 \, {\left(A - C\right)} a b^{2} - B b^{3}\right)} d\right)} \tan\left(f x + e\right)^{2} - 30 \, {\left({\left(B a^{3} + 3 \, {\left(A - C\right)} a^{2} b - 3 \, B a b^{2} - {\left(A - C\right)} b^{3}\right)} c + {\left({\left(A - C\right)} a^{3} - 3 \, B a^{2} b - 3 \, {\left(A - C\right)} a b^{2} + B b^{3}\right)} d\right)} \log\left(\frac{1}{\tan\left(f x + e\right)^{2} + 1}\right) + 60 \, {\left({\left(C a^{3} + 3 \, B a^{2} b + 3 \, {\left(A - C\right)} a b^{2} - B b^{3}\right)} c + {\left(B a^{3} + 3 \, {\left(A - C\right)} a^{2} b - 3 \, B a b^{2} - {\left(A - C\right)} b^{3}\right)} d\right)} \tan\left(f x + e\right)}{60 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*b^3*d*tan(f*x + e)^5 + 15*(C*b^3*c + (3*C*a*b^2 + B*b^3)*d)*tan(
f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b
^3)*d)*tan(f*x + e)^3 + 60*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*
b^3)*c - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*f*x + 30*((
3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c + (C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b
^2 - B*b^3)*d)*tan(f*x + e)^2 - 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 -
(A - C)*b^3)*c + ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d)*log
(1/(tan(f*x + e)^2 + 1)) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3
)*c + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*tan(f*x + e))/
f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(316) = 632$.
time = 0.35, size = 1001, normalized size = 2.84

$$\begin{cases} \text{[illegible]} \\ x \left(a + b \tan{\left(e \right)}\right)^{3} \left(c + d \tan{\left(e \right)}\right) \left(A + B \tan{\left(e \right)} + C \tan^{2}{\left(e \right)}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Piecewise((A*a**3*c*x + A*a**3*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*a**2*
b*c*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*a**2*b*d*x + 3*A*a**2*b*d*tan(e +
```

```
f*x)/f - 3*A*a*b**2*c*x + 3*A*a*b**2*c*tan(e + f*x)/f - 3*A*a*b**2*d*log(ta
n(e + f*x)**2 + 1)/(2*f) + 3*A*a*b**2*d*tan(e + f*x)**2/(2*f) - A*b**3*c*lo
g(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*c*tan(e + f*x)**2/(2*f) + A*b**3*d*x
+ A*b**3*d*tan(e + f*x)**3/(3*f) - A*b**3*d*tan(e + f*x)/f + B*a**3*c*log(t
an(e + f*x)**2 + 1)/(2*f) - B*a**3*d*x + B*a**3*d*tan(e + f*x)/f - 3*B*a**2
*b*c*x + 3*B*a**2*b*c*tan(e + f*x)/f - 3*B*a**2*b*d*log(tan(e + f*x)**2 + 1
)/(2*f) + 3*B*a**2*b*d*tan(e + f*x)**2/(2*f) - 3*B*a*b**2*c*log(tan(e + f*x
)**2 + 1)/(2*f) + 3*B*a*b**2*c*tan(e + f*x)**2/(2*f) + 3*B*a*b**2*d*x + B*a
*b**2*d*tan(e + f*x)**3/f - 3*B*a*b**2*d*tan(e + f*x)/f + B*b**3*c*x + B*b*
*3*c*tan(e + f*x)**3/(3*f) - B*b**3*c*tan(e + f*x)/f + B*b**3*d*log(tan(e +
 f*x)**2 + 1)/(2*f) + B*b**3*d*tan(e + f*x)**4/(4*f) - B*b**3*d*tan(e + f*x
)**2/(2*f) - C*a**3*c*x + C*a**3*c*tan(e + f*x)/f - C*a**3*d*log(tan(e + f*
x)**2 + 1)/(2*f) + C*a**3*d*tan(e + f*x)**2/(2*f) - 3*C*a**2*b*c*log(tan(e
+ f*x)**2 + 1)/(2*f) + 3*C*a**2*b*c*tan(e + f*x)**2/(2*f) + 3*C*a**2*b*d*x
+ C*a**2*b*d*tan(e + f*x)**3/f - 3*C*a**2*b*d*tan(e + f*x)/f + 3*C*a*b**2*c
*x + C*a*b**2*c*tan(e + f*x)**3/f - 3*C*a*b**2*c*tan(e + f*x)/f + 3*C*a*b**
2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a*b**2*d*tan(e + f*x)**4/(4*f) - 3
*C*a*b**2*d*tan(e + f*x)**2/(2*f) + C*b**3*c*log(tan(e + f*x)**2 + 1)/(2*f)
 + C*b**3*c*tan(e + f*x)**4/(4*f) - C*b**3*c*tan(e + f*x)**2/(2*f) - C*b**3
*d*x + C*b**3*d*tan(e + f*x)**5/(5*f) - C*b**3*d*tan(e + f*x)**3/(3*f) + C*
b**3*d*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))**3*(c + d*tan(e))*(A +
B*tan(e) + C*tan(e)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 11805 vs. $2(352) = 704$.
time = 8.91, size = 11805, normalized size = 33.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] 1/60*(60*A*a^3*c*f*x*tan(f*x)^5*tan(e)^5 - 60*C*a^3*c*f*x*tan(f*x)^5*tan(e)
^5 - 180*B*a^2*b*c*f*x*tan(f*x)^5*tan(e)^5 - 180*A*a*b^2*c*f*x*tan(f*x)^5*t
an(e)^5 + 180*C*a*b^2*c*f*x*tan(f*x)^5*tan(e)^5 + 60*B*b^3*c*f*x*tan(f*x)^5
*tan(e)^5 - 60*B*a^3*d*f*x*tan(f*x)^5*tan(e)^5 - 180*A*a^2*b*d*f*x*tan(f*x)
^5*tan(e)^5 + 180*C*a^2*b*d*f*x*tan(f*x)^5*tan(e)^5 + 180*B*a*b^2*d*f*x*tan
(f*x)^5*tan(e)^5 + 60*A*b^3*d*f*x*tan(f*x)^5*tan(e)^5 - 60*C*b^3*d*f*x*tan(
f*x)^5*tan(e)^5 - 30*B*a^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
1))*tan(f*x)^5*tan(e)^5 - 90*A*a^2*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(t
an(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*C*a^2*b*c*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
```

e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*B*a*b^2*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 30*A*b^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 30*C*b^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 30*A*a^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 30*C*a^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*B*a^2*b*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*A*a*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 90*C*a*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 30*B*b^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 300*A*a^3*c*f*x*tan(f*x)^4*tan(e)^4 + 300*C*a^3*c*f*x*tan(f*x)^4*tan(e)^4 + 900*B*a^2*b*c*f*x*tan(f*x)^4*tan(e)^4 + 900*A*a*b^2*c*f*x*tan(f*x)^4*tan(e)^4 - 900*C*a*b^2*c*f*x*tan(f*x)^4*tan(e)^4 - 300*B*b^3*c*f*x*tan(f*x)^4*tan(e)^4 + 300*B*a^3*d*f*x*tan(f*x)^4*tan(e)^4 + 900*A*a^2*b*d*f*x*tan(f*x)^4*tan(e)^4 - 900*C*a^2*b*d*f*x*tan(f*x)^4*tan(e)^4 - 900*B*a*b^2*d*f*x*tan(f*x)^4*tan(e)^4 - 300*A*b^3*d*f*x*tan(f*x)^4*tan(e)^4 + 300*C*b^3*d*f*x*tan(f*x)^4*tan(e)^4 + 90*C*a^2*b*c*tan(f*x)^5*tan(e)^5 + 90*B*a*b^2*c*tan(f*x)^5*tan(e)^5 + 30*A*b^3*c*tan(f*x)^5*tan(e)^5 - 45*C*b^3*c*tan(f*x)^5*tan(e)^5 + 30*C*a^3*d*tan(f*x)^5*tan(e)^5 + 90*B*a^2*b*d*tan(f*x)^5*tan(e)^5 + 90*A*a*b^2*d*tan(f*x)^5*tan(e)^5 - 135*C*a*b^2*d*tan(f*x)^5*tan(e)^5 - 45*B*b^3*d*tan(f*x)^5*tan(e)^5 + 150*B*a^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 450*A*a^2*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 450*C*a^2*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 450*B*a*b^2*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 150*A*b^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 150*C*b^3*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 150*A*a^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 150*

```
C*a^3*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)
^4 - 450*B*a^2*b*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)^4*tan(e)^4 - 450*A*a*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan
(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
 1))*tan(f*x)^4*tan(e)^4 + 450*C*a*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/
(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 150*B*b^3*d*log(4*(tan(f*x)^4*tan(e)^
2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +...
```

Mupad [B]
time = 9.00, size = 477, normalized size = 1.35

$$x\,\left(A\,a^3\,c+A\,b^3\,d-B\,a^3\,d+B\,b^3\,c-C\,a^3\,c-C\,b^3\,d-3\,A\,a\,b^2\,c-3\,A\,a^2\,b\,d-3\,B\,a^2\,b\,c+3\,B\,a\,b^2\,d+3\,C\,a\,b^2\,c+3\,C\,a^2\,b\,d\right)+\frac{{\mathrm{tan}\left(e+f\,x\right)}^4\,\left(\frac{B\,b^3\,d}{4}+\frac{C\,b^3\,c}{4}+\frac{3\,C\,a\,b^2\,d}{4}\right)}{f}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^3\,\left(\frac{A\,b^3\,d}{3}+\frac{B\,b^3\,c}{3}-\frac{C\,b^3\,d}{3}+B\,a\,b^2\,d+C\,a\,b^2\,c+C\,a^2\,b\,d\right)}{f}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(\frac{A\,b^3\,c}{2}-\frac{B\,b^3\,d}{2}+\frac{C\,a^3\,d}{2}-\frac{C\,b^3\,c}{2}+\frac{3\,A\,a\,b^2\,d}{2}+\frac{3\,B\,a\,b^2\,c}{2}+\frac{3\,B\,a^2\,b\,d}{2}+\frac{3\,C\,a^2\,b\,c}{2}-\frac{3\,C\,a\,b^2\,d}{2}\right)}{f}+\frac{\ln\left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)\,\left(\frac{A\,a^3\,d}{2}-\frac{A\,b^3\,c}{2}+\frac{B\,a^3\,c}{2}+\frac{B\,b^3\,d}{2}-\frac{C\,a^3\,d}{2}+\frac{C\,b^3\,c}{2}+\frac{3\,A\,a^2\,b\,c}{2}-\frac{3\,A\,a\,b^2\,d}{2}-\frac{3\,B\,a\,b^2\,c}{2}-\frac{3\,B\,a^2\,b\,d}{2}-\frac{3\,C\,a^2\,b\,c}{2}+\frac{3\,C\,a\,b^2\,d}{2}\right)}{f}+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(B\,a^3\,d-A\,b^3\,d-B\,b^3\,c+C\,a^3\,c+C\,b^3\,d+3\,A\,a\,b^2\,c+3\,A\,a^2\,b\,d+3\,B\,a^2\,b\,c-3\,B\,a\,b^2\,d-3\,C\,a\,b^2\,c-3\,C\,a^2\,b\,d\right)}{f}+\frac{C\,b^3\,d\,{\mathrm{tan}\left(e+f\,x\right)}^5}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan
(e + f*x)^2),x)
```

```
[Out] x*(A*a^3*c + A*b^3*d - B*a^3*d + B*b^3*c - C*a^3*c - C*b^3*d - 3*A*a*b^2*c
- 3*A*a^2*b*d - 3*B*a^2*b*c + 3*B*a*b^2*d + 3*C*a*b^2*c + 3*C*a^2*b*d) + (t
an(e + f*x)^4*((B*b^3*d)/4 + (C*b^3*c)/4 + (3*C*a*b^2*d)/4))/f + (tan(e + f
*x)^3*((A*b^3*d)/3 + (B*b^3*c)/3 - (C*b^3*d)/3 + B*a*b^2*d + C*a*b^2*c + C*
a^2*b*d))/f + (tan(e + f*x)^2*((A*b^3*c)/2 - (B*b^3*d)/2 + (C*a^3*d)/2 - (C
*b^3*c)/2 + (3*A*a*b^2*d)/2 + (3*B*a*b^2*c)/2 + (3*B*a^2*b*d)/2 + (3*C*a^2*
b*c)/2 - (3*C*a*b^2*d)/2))/f + (log(tan(e + f*x)^2 + 1)*((A*a^3*d)/2 - (A*b
^3*c)/2 + (B*a^3*c)/2 + (B*b^3*d)/2 - (C*a^3*d)/2 + (C*b^3*c)/2 + (3*A*a^2*
b*c)/2 - (3*A*a*b^2*d)/2 - (3*B*a*b^2*c)/2 - (3*B*a^2*b*d)/2 - (3*C*a^2*b*c
)/2 + (3*C*a*b^2*d)/2))/f + (tan(e + f*x)*(B*a^3*d - A*b^3*d - B*b^3*c + C*
a^3*c + C*b^3*d + 3*A*a*b^2*c + 3*A*a^2*b*d + 3*B*a^2*b*c - 3*B*a*b^2*d - 3
*C*a*b^2*c - 3*C*a^2*b*d))/f + (C*b^3*d*tan(e + f*x)^5)/(5*f)
```

3.51 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx)) \left(A + B\tan(e + f\right.$

Optimal. Leaf size=248

$$\left(a^2(Ac - cC - Bd) - b^2(Ac - cC - Bd) - 2ab(Bc + (A - C)d)\right) x - \frac{(2ab(Ac - cC - Bd) + a^2(Bc + (A - C}{}$$

```
[Out] (a^2*(A*c-B*d-C*c)-b^2*(A*c-B*d-C*c)-2*a*b*(B*c+(A-C)*d))*x-(2*a*b*(A*c-B*d
-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*d))*ln(cos(f*x+e))/f+b*(A*a*d+A*b*c+
B*a*c-B*b*d-C*a*d-C*b*c)*tan(f*x+e)/f+1/2*(B*c+(A-C)*d)*(a+b*tan(f*x+e))^2/
f-1/12*(a*C*d-4*b*(B*d+C*c))*(a+b*tan(f*x+e))^3/b^2/f+1/4*C*d*tan(f*x+e)*(a
+b*tan(f*x+e))^3/b/f
```

Rubi [A]
time = 0.31, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3718, 3711, 3609, 3606, 3556}

$$-\frac{\log(\cos(e+fx))\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}+x\left(a^2(Ac-Bd-cC)-2ab(d(A-C)+Bc)-b^2(Ac-Bd-cC)\right)+\frac{(d(A-C)+Bc)(a+b\tan(e+fx))^2}{2f}+\frac{b\tan(e+fx)(aAd+aBc-aCd+Abc-bBd-bcC)}{f}-\frac{(aCd-4b(Bd+cC))(a+b\tan(e+fx))^3}{12b^2f}+\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```

```
[Out] (a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*c + (A - C)*d))*x
 - ((2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d
))*Log[Cos[e + f*x]])/f + (b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d
)*Tan[e + f*x])/f + ((B*c + (A - C)*d)*(a + b*Tan[e + f*x])^2)/(2*f) - ((a*
C*d - 4*b*(c*C + B*d))*(a + b*Tan[e + f*x])^3)/(12*b^2*f) + (C*d*Tan[e + f*
x]*(a + b*Tan[e + f*x])^3)/(4*b*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{Cd\tan(e+fx)(a}{4b}$$

$$= -\frac{(aCd-4b(cC+}{}$$

$$= \frac{(Bc+(A-C)d)(}{2}$$

$$= (a^2(Ac-cC-Bd$$

$$= (a^2(Ac-cC-Bd$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.27, size = 243, normalized size = 0.98

$$\frac{\frac{(-aCd+4b(cC+Bd))(a+b\tan(e+fx))^3}{b}+3Cd\tan(e+fx)(a+b\tan(e+fx))^3-6(Abc-aBc-bcC-aAd-bBd+aCd)\left(i\left((a+ib)^2\log(i-\tan(e+fx))-(a-ib)^2\log(i+\tan(e+fx))\right)-2b^2\tan(e+fx)\right)+6(Bc+(A-C)d)\left((ia-b)^3\log(i-\tan(e+fx))-(ia+b)^3\log(i+\tan(e+fx))+6ab^2\tan(e+fx)+b^3\tan^2(e+fx)\right)}{12bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] (((-(a*C*d) + 4*b*(c*C + B*d))*(a + b*Tan[e + f*x])^3)/b + 3*C*d*Tan[e + f*
x]*(a + b*Tan[e + f*x])^3 - 6*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*
d)*(I*((a + I*b)^2*Log[I - Tan[e + f*x]] - (a - I*b)^2*Log[I + Tan[e + f*x]
]) - 2*b^2*Tan[e + f*x]) + 6*(B*c + (A - C)*d)*((I*a - b)^3*Log[I - Tan[e +
 f*x]] - (I*a + b)^3*Log[I + Tan[e + f*x]] + 6*a*b^2*Tan[e + f*x] + b^3*Tan
[e + f*x]^2))/(12*b*f)
```

Maple [A]
time = 0.12, size = 386, normalized size = 1.56

method	result
norman	$\left(A\,a^{2} c-2 A a b d-A\,b^{2} c-B\,a^{2} d-2 B a b c+B\,b^{2} d-C\,a^{2} c+2 C a b d+C\,b^{2} c\right) x+\frac{(2Aabd-$
derivativedivides	$\frac{C\,b^{2} d\left(\tan^{4}\left(f x+e\right)\right)}{4}+\frac{B\,b^{2} d\left(\tan^{3}\left(f x+e\right)\right)}{3}+\frac{2 C a b d\left(\tan^{3}\left(f x+e\right)\right)}{3}+\frac{C\,b^{2} c\left(\tan^{3}\left(f x+e\right)\right)}{3}+\frac{A\,b^{2} d\left(\tan^{2}\left(f x+e\right)\right)}{2}+B a b d(\tan^{2}(f x+e)$
default	$\frac{C\,b^{2} d\left(\tan^{4}\left(f x+e\right)\right)}{4}+\frac{B\,b^{2} d\left(\tan^{3}\left(f x+e\right)\right)}{3}+\frac{2 C a b d\left(\tan^{3}\left(f x+e\right)\right)}{3}+\frac{C\,b^{2} c\left(\tan^{3}\left(f x+e\right)\right)}{3}+\frac{A\,b^{2} d\left(\tan^{2}\left(f x+e\right)\right)}{2}+B a b d(\tan^{2}(f x+e)$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*C*b^2*d*tan(f*x+e)^4+1/3*B*b^2*d*tan(f*x+e)^3+2/3*C*a*b*d*tan(f*x+
e)^3+1/3*C*b^2*c*tan(f*x+e)^3+1/2*A*b^2*d*tan(f*x+e)^2+B*a*b*d*tan(f*x+e)^2
+1/2*B*b^2*c*tan(f*x+e)^2+1/2*C*a^2*d*tan(f*x+e)^2+C*a*b*c*tan(f*x+e)^2-1/2
*C*b^2*d*tan(f*x+e)^2+2*A*a*b*d*tan(f*x+e)+A*b^2*c*tan(f*x+e)+B*a^2*d*tan(f
*x+e)+2*B*a*b*c*tan(f*x+e)-B*b^2*d*tan(f*x+e)+C*a^2*c*tan(f*x+e)-2*C*a*b*d*
tan(f*x+e)-C*b^2*c*tan(f*x+e)+1/2*(A*a^2*d+2*A*a*b*c-A*b^2*d+B*a^2*c-2*B*a*
b*d-B*b^2*c-C*a^2*d-2*C*a*b*c+C*b^2*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c-2*A*a*b*
d-A*b^2*c-B*a^2*d-2*B*a*b*c+B*b^2*d-C*a^2*c+2*C*a*b*d+C*b^2*c)*arctan(tan(f
*x+e)))
```

Maxima [A]
time = 0.50, size = 280, normalized size = 1.13

$$\frac{3\,Cb^{2}d\tan\left(fx+e\right)^{4}+4\,\left(Cb^{2}c+\left(2\,Cab+Bb^{2}\right)d\right)\tan\left(fx+e\right)^{3}+6\,\left(\left(2\,Cab+Bb^{2}\right)c+\left(Ca^{2}+2\,Bab+\left(A-C\right)b^{2}\right)d\right)\tan\left(fx+e\right)^{2}+12\,\left(\left(\left(A-C\right)a^{2}-2\,Bab-\left(A-C\right)b^{2}\right)c-\left(Ba^{2}+2\,\left(A-C\right)ab-Bb^{2}\right)d\right)\left(fx+e\right)+6\,\left(\left(Ba^{2}+2\,\left(A-C\right)ab-Bb^{2}\right)c+\left(\left(A-C\right)a^{2}-2\,Bab-\left(A-C\right)b^{2}\right)d\right)\log\left(\tan\left(fx+e\right)^{2}+1\right)+12\,\left(\left(Ca^{2}+2\,Bab+\left(A-C\right)b^{2}\right)c+\left(Ba^{2}+2\,\left(A-C\right)ab-Bb^{2}\right)d\right)\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*C*b^2*d*tan(f*x + e)^4 + 4*(C*b^2*c + (2*C*a*b + B*b^2)*d)*tan(f*x
+ e)^3 + 6*((2*C*a*b + B*b^2)*c + (C*a^2 + 2*B*a*b + (A - C)*b^2)*d)*tan(f*
x + e)^2 + 12*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c - (B*a^2 + 2*(A - C)
*a*b - B*b^2)*d)*(f*x + e) + 6*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c + ((A - C
)*a^2 - 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 + 1) + 12*((C*a^2 + 2*
B*a*b + (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d)*tan(f*x + e))/f
```

Fricas [A]
time = 3.35, size = 278, normalized size = 1.12

$$\frac{3 C b^{2} d \tan\left(f x + e\right)^{4} + 4 \left(C b^{2} c + \left(2 C a b + B b^{2}\right) d\right) \tan\left(f x + e\right)^{3} + 12 \left(\left(\left(A - C\right) a^{2} - 2 B a b - \left(A - C\right) b^{2}\right) c - \left(B a^{2} + 2 \left(A - C\right) a b - B b^{2}\right) d\right) f x + 6 \left(\left(2 C a b + B b^{2}\right) c + \left(C a^{2} + 2 B a b + \left(A - C\right) b^{2}\right) d\right) \tan\left(f x + e\right)^{2} - 6 \left(\left(B a^{2} + 2 \left(A - C\right) a b - B b^{2}\right) c + \left(\left(A - C\right) a^{2} - 2 B a b - \left(A - C\right) b^{2}\right) d\right) \log\left(\frac{1}{\tan\left(f x + e\right)^{2} + 1}\right) + 12 \left(\left(C a^{2} + 2 B a b + \left(A - C\right) b^{2}\right) c + \left(B a^{2} + 2 \left(A - C\right) a b - B b^{2}\right) d\right) \tan\left(f x + e\right)}{12 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*b^2*d*tan(f*x + e)^4 + 4*(C*b^2*c + (2*C*a*b + B*b^2)*d)*tan(f*x
+ e)^3 + 12*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c - (B*a^2 + 2*(A - C)*a
*b - B*b^2)*d)*f*x + 6*((2*C*a*b + B*b^2)*c + (C*a^2 + 2*B*a*b + (A - C)*b^
2)*d)*tan(f*x + e)^2 - 6*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c + ((A - C)*a^2
- 2*B*a*b - (A - C)*b^2)*d)*log(1/(tan(f*x + e)^2 + 1)) + 12*((C*a^2 + 2*B*
a*b + (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(218) = 436$.
time = 0.23, size = 617, normalized size = 2.49

$$\begin{cases} A a^{2} c x + \frac{A a^{2} d \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{2 f} + \frac{A a b c \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{f} - 2 A a b d x + \frac{2 A a b d \tan{\left(e + f x \right)}}{f} - A b^{2} c x + \frac{A b^{2} c \tan{\left(e + f x \right)}}{f} - \frac{A b^{2} d \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{2 f} + \frac{A b^{2} d \tan^{2}{\left(e + f x \right)}}{2 f} + \frac{B a^{2} c \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{2 f} - B a^{2} d x + \frac{B a^{2} d \tan{\left(e + f x \right)}}{f} - 2 B a b c x + \frac{2 B a b c \tan{\left(e + f x \right)}}{f} - \frac{B a b d \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{f} + \frac{B a b d \tan^{2}{\left(e + f x \right)}}{f} - \frac{B b^{2} c \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{2 f} + \frac{B b^{2} c \tan^{2}{\left(e + f x \right)}}{2 f} + B b^{2} d x + \frac{B b^{2} d \tan^{3}{\left(e + f x \right)}}{3 f} - \frac{B b^{2} d \tan{\left(e + f x \right)}}{f} - C a^{2} c x + \frac{C a^{2} c \tan{\left(e + f x \right)}}{f} - \frac{C a^{2} d \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{2 f} + \frac{C a^{2} d \tan^{2}{\left(e + f x \right)}}{2 f} - \frac{C a b c \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{f} + \frac{C a b c \tan^{2}{\left(e + f x \right)}}{f} + 2 C a b d x + \frac{2 C a b d \tan^{3}{\left(e + f x \right)}}{3 f} - \frac{2 C a b d \tan{\left(e + f x \right)}}{f} + C b^{2} c x + \frac{C b^{2} c \tan^{3}{\left(e + f x \right)}}{3 f} - \frac{C b^{2} c \tan{\left(e + f x \right)}}{f} + \frac{C b^{2} d \log{\left(\tan^{2}{\left(e + f x \right)} + 1 \right)}}{2 f} + \frac{C b^{2} d \tan^{4}{\left(e + f x \right)}}{4 f} - \frac{C b^{2} d \tan^{2}{\left(e + f x \right)}}{2 f} & \text{for}\: f \neq 0 \\ x \left(a + b \tan{\left(e \right)}\right)^{2} \left(c + d \tan{\left(e \right)}\right) \left(A + B \tan{\left(e \right)} + C \tan^{2}{\left(e \right)}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Piecewise((A*a**2*c*x + A*a**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + A*a*b*c*l
og(tan(e + f*x)**2 + 1)/f - 2*A*a*b*d*x + 2*A*a*b*d*tan(e + f*x)/f - A*b**2
*c*x + A*b**2*c*tan(e + f*x)/f - A*b**2*d*log(tan(e + f*x)**2 + 1)/(2*f) +
A*b**2*d*tan(e + f*x)**2/(2*f) + B*a**2*c*log(tan(e + f*x)**2 + 1)/(2*f) -
B*a**2*d*x + B*a**2*d*tan(e + f*x)/f - 2*B*a*b*c*x + 2*B*a*b*c*tan(e + f*x)
/f - B*a*b*d*log(tan(e + f*x)**2 + 1)/f + B*a*b*d*tan(e + f*x)**2/f - B*b**
2*c*log(tan(e + f*x)**2 + 1)/(2*f) + B*b**2*c*tan(e + f*x)**2/(2*f) + B*b**
2*d*x + B*b**2*d*tan(e + f*x)**3/(3*f) - B*b**2*d*tan(e + f*x)/f - C*a**2*c
*x + C*a**2*c*tan(e + f*x)/f - C*a**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*
a**2*d*tan(e + f*x)**2/(2*f) - C*a*b*c*log(tan(e + f*x)**2 + 1)/f + C*a*b*c
*tan(e + f*x)**2/f + 2*C*a*b*d*x + 2*C*a*b*d*tan(e + f*x)**3/(3*f) - 2*C*a*
b*d*tan(e + f*x)/f + C*b**2*c*x + C*b**2*c*tan(e + f*x)**3/(3*f) - C*b**2*c
*tan(e + f*x)/f + C*b**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**2*d*tan(e
```

```
+ f*x)**4/(4*f) - C*b**2*d*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(
e))**2*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6502 vs. $2(248) = 496$.
time = 3.76, size = 6502, normalized size = 26.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] 1/12*(12*A*a^2*c*f*x*tan(f*x)^4*tan(e)^4 - 12*C*a^2*c*f*x*tan(f*x)^4*tan(e)
^4 - 24*B*a*b*c*f*x*tan(f*x)^4*tan(e)^4 - 12*A*b^2*c*f*x*tan(f*x)^4*tan(e)^
4 + 12*C*b^2*c*f*x*tan(f*x)^4*tan(e)^4 - 12*B*a^2*d*f*x*tan(f*x)^4*tan(e)^4
 - 24*A*a*b*d*f*x*tan(f*x)^4*tan(e)^4 + 24*C*a*b*d*f*x*tan(f*x)^4*tan(e)^4
+ 12*B*b^2*d*f*x*tan(f*x)^4*tan(e)^4 - 6*B*a^2*c*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 12*A*a*b*c*log(4*(tan(f*x)^4*
tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f
*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 12*C*a*b*c*log(4*(tan
(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 6*B*b^2*c*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 6*A*a^
2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 +
 6*C*a^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan
(e)^4 + 12*B*a*b*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)^4*tan(e)^4 + 6*A*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^4*tan(e)^4 - 6*C*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*
tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^
2 + 1))*tan(f*x)^4*tan(e)^4 - 48*A*a^2*c*f*x*tan(f*x)^3*tan(e)^3 + 48*C*a^2
*c*f*x*tan(f*x)^3*tan(e)^3 + 96*B*a*b*c*f*x*tan(f*x)^3*tan(e)^3 + 48*A*b^2*
c*f*x*tan(f*x)^3*tan(e)^3 - 48*C*b^2*c*f*x*tan(f*x)^3*tan(e)^3 + 48*B*a^2*d
*f*x*tan(f*x)^3*tan(e)^3 + 96*A*a*b*d*f*x*tan(f*x)^3*tan(e)^3 - 96*C*a*b*d*
f*x*tan(f*x)^3*tan(e)^3 - 48*B*b^2*d*f*x*tan(f*x)^3*tan(e)^3 + 12*C*a*b*c*t
an(f*x)^4*tan(e)^4 + 6*B*b^2*c*tan(f*x)^4*tan(e)^4 + 6*C*a^2*d*tan(f*x)^4*t
an(e)^4 + 12*B*a*b*d*tan(f*x)^4*tan(e)^4 + 6*A*b^2*d*tan(f*x)^4*tan(e)^4 -
9*C*b^2*d*tan(f*x)^4*tan(e)^4 + 24*B*a^2*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
```

```
)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 48*A*a*b*c*log(4*(tan(f*x)^4*tan(e)
^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*ta
n(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 48*C*a*b*c*log(4*(tan(f*x)^
4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan
(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 24*B*b^2*c*log(4*(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 24*A*a^2*d*
log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + ta
n(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 24*
C*a^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)
^3 - 48*B*a*b*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
3*tan(e)^3 - 24*A*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*t
an(f*x)^3*tan(e)^3 + 24*C*b^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)^3*tan(e)^3 - 12*C*a^2*c*tan(f*x)^4*tan(e)^3 - 24*B*a*b*c*ta
n(f*x)^4*tan(e)^3 - 12*A*b^2*c*tan(f*x)^4*tan(e)^3 + 12*C*b^2*c*tan(f*x)^4*
tan(e)^3 - 12*B*a^2*d*tan(f*x)^4*tan(e)^3 - 24*A*a*b*d*tan(f*x)^4*tan(e)^3
+ 24*C*a*b*d*tan(f*x)^4*tan(e)^3 + 12*B*b^2*d*tan(f*x)^4*tan(e)^3 - 12*C*a^
2*c*tan(f*x)^3*tan(e)^4 - 24*B*a*b*c*tan(f*x)^3*tan(e)^4 - 12*A*b^2*c*tan(f
*x)^3*tan(e)^4 + 12*C*b^2*c*tan(f*x)^3*tan(e)^4 - 12*B*a^2*d*tan(f*x)^3*tan
(e)^4 - 24*A*a*b*d*tan(f*x)^3*tan(e)^4 + 24*C*a*b*d*tan(f*x)^3*tan(e)^4 + 1
2*B*b^2*d*tan(f*x)^3*tan(e)^4 + 72*A*a^2*c*f*x*tan(f*x)^2*tan(e)^2 - 72*C*a
^2*c*f*x*tan(f*x)^2*tan(e)^2 - 144*B*a*b*c*f*x*tan(f*x)^2*tan(e)^2 - 72*A*b
^2*c*f*x*tan(f*x)^2*tan(e)^2 + 72*C*b^2*c*f*x*tan(f*x)^2*tan(e)^2 - 72*B*a^
2*d*f*x*tan(f*x)^2*tan(e)^2 - 144*A*a*b*d*f*x*tan(f*x)^2*tan(e)^2 + 144*C*a
*b*d*f*x*tan(f*x)^2*tan(e)^2 + 72*B*b^2*d*f*x*tan(f*x)^2*tan(e)^2 + 12*C*a*
b*c*tan(f*x)^4*tan(e)^2 + 6*B*b^2*c*tan(f*x)^4*tan(e)^2 + 6*C*a^2*d*tan(f*x
)^4*tan(e)^2 + 12*B*a*b*d*tan(f*x)^4*tan(e)^2 + 6*A*b^2*d*tan(f*x)^4*tan(e)
^2 - 6*C*b^2*d*tan(f*x)^4*tan(e)^2 - 24*C*a*b*c*tan(f*x)^3*tan(e)^3 - 12*B*
b^2*c*tan(f*x)^3*tan(e)^3 - 12*C*a^2*d*tan(f*x)^3*tan(e)^3 - 24*B*a*b*d*tan
(f*x)^3*tan(e)^3 - 12*A*b^2*d*tan(f*x)^3*tan(e)^3 + 24*C*b^2*d*tan(f*x)^3*t
an(e)^3 + 12*C*a*b*c*tan(f*x)^2*tan(e)^4 + 6*B*...
```

Mupad [B]
time = 8.98, size = 300, normalized size = 1.21

$$\frac{\tan(e+fx)^2\left(\frac{Ab^2d}{2}+\frac{Bb^2c}{2}+\frac{Ca^2d}{2}-\frac{Cb^2d}{2}+Babd+Cabc\right)}{f}-x\left(Ab^2c-Aa^2c+Ba^2d+Ca^2c-Bb^2d-Cb^2c+2Aabd+2Babc-2Cabd\right)-\frac{\ln\left(\tan(e+fx)^2+1\right)\left(\frac{Ab^2d}{2}-\frac{Ba^2c}{2}-\frac{Aa^2d}{2}+\frac{Bb^2c}{2}+\frac{Ca^2d}{2}-\frac{Cb^2d}{2}-Aabc+Babd+Cabc\right)}{f}+\frac{\tan(e+fx)\left(Ab^2c+Ba^2d+Ca^2c-Bb^2d-Cb^2c+2Aabd+2Babc-2Cabd\right)}{f}+\frac{\tan(e+fx)^3\left(\frac{Bb^2d}{3}+\frac{Cb^2c}{3}+\frac{2Cabd}{3}\right)}{f}+\frac{Cb^2d\tan(e+fx)^4}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan
(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^2*((A*b^2*d)/2 + (B*b^2*c)/2 + (C*a^2*d)/2 - (C*b^2*d)/2 + B*
a*b*d + C*a*b*c))/f - x*(A*b^2*c - A*a^2*c + B*a^2*d + C*a^2*c - B*b^2*d -
C*b^2*c + 2*A*a*b*d + 2*B*a*b*c - 2*C*a*b*d) - (log(tan(e + f*x)^2 + 1)*((A
*b^2*d)/2 - (B*a^2*c)/2 - (A*a^2*d)/2 + (B*b^2*c)/2 + (C*a^2*d)/2 - (C*b^2*
d)/2 - A*a*b*c + B*a*b*d + C*a*b*c))/f + (tan(e + f*x)*(A*b^2*c + B*a^2*d +
 C*a^2*c - B*b^2*d - C*b^2*c + 2*A*a*b*d + 2*B*a*b*c - 2*C*a*b*d))/f + (tan
(e + f*x)^3*((B*b^2*d)/3 + (C*b^2*c)/3 + (2*C*a*b*d)/3))/f + (C*b^2*d*tan(e
 + f*x)^4)/(4*f)
```

3.52 $\int (a+b\tan(e+fx))(c+d\tan(e+fx)) \left(A + B\tan(e + f\right.$

Optimal. Leaf size=161

$$(a(Ac-cC-Bd)-b(Bc+(A-C)d))x-\frac{(Abc+aBc-bcC+aAd-bBd-aCd)\log(\cos(e+fx))}{f}+\frac{(Ab+a\ldots}{}$$

```
[Out] (a*(A*c-B*d-C*c)-b*(B*c+(A-C)*d))*x-(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*l
n(cos(f*x+e))/f+(A*b+B*a-C*b)*d*tan(f*x+e)/f-1/6*(-3*B*b*d-3*C*a*d+C*b*c)*(
c+d*tan(f*x+e))^2/d^2/f+1/3*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^2/d/f
```

Rubi [A]
time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3718, 3711, 3606, 3556}

$$-\frac{\log(\cos(e+fx))(aAd+aBc-aCd+Abc-bBd-bcC)}{f}-x(-a(Ac-Bd-cC)+bd(A-C)+bBc)+\frac{d\tan(e+fx)(aB+Ab-bC)}{f}-\frac{(-3aCd-3bBd+bcC)(c+d\tan(e+fx))^2}{6d^2f}+\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e
 + f*x]^2),x]
```

```
[Out] -((b*B*c + b*(A - C)*d - a*(A*c - c*C - B*d))*x) - ((A*b*c + a*B*c - b*c*C
+ a*A*d - b*B*d - a*C*d)*Log[Cos[e + f*x]])/f + ((A*b + a*B - b*C)*d*Tan[e
+ f*x])/f - ((b*c*C - 3*b*B*d - 3*a*C*d)*(c + d*Tan[e + f*x])^2)/(6*d^2*f)
+ (b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^2)/(3*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{bC\tan(e+fx)(c+d}{3df}$$

$$= -\frac{(bcC-3bBd-3aC}{}$$

$$= -(bBc+b(A-C)d$$

$$= -(bBc+b(A-C)d$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.07, size = 161, normalized size = 1.00

$$\frac{3(a+ib)(A+iB-C)(-ic+d)\log(i-\tan(e+fx))+3(a-ib)(A-iB-C)(ic+d)\log(i+\tan(e+fx))+6(Ab+aB-bC)d\tan(e+fx)+\frac{(-bcC+3bBd+3aCd)(c+d\tan(e+fx))^2}{d^2}+\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^2}{d}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```

```
[Out] (3*(a + I*b)*(A + I*B - C)*((-I)*c + d)*Log[I - Tan[e + f*x]] + 3*(a - I*b)
*(A - I*B - C)*(I*c + d)*Log[I + Tan[e + f*x]] + 6*(A*b + a*B - b*C)*d*Tan[
e + f*x] + ((-(b*c*C) + 3*b*B*d + 3*a*C*d)*(c + d*Tan[e + f*x])^2)/d^2 + (2
*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^2)/d)/(6*f)
```

Maple [A]
time = 0.12, size = 187, normalized size = 1.16

method	result
norman	$(Aac-Abd-Bad-Bbc-Cac+Cbd)\,x+\frac{(Abd+Bad+Bbc+Cac-Cbd)\tan(fx+e)}{f}+\frac{(Bbd+aCd+}{}$

derivativedivides	$\frac{Cbd\left(\tan^3(fx+e)\right)}{3}+\frac{B\left(\tan^2(fx+e)\right)bd}{2}+\frac{C\left(\tan^2(fx+e)\right)ad}{2}+\frac{C\left(\tan^2(fx+e)\right)bc}{2}+A\tan(fx+e)bd+B\tan(fx+e)ad+B\tan(f$
default	$\frac{Cbd\left(\tan^3(fx+e)\right)}{3}+\frac{B\left(\tan^2(fx+e)\right)bd}{2}+\frac{C\left(\tan^2(fx+e)\right)ad}{2}+\frac{C\left(\tan^2(fx+e)\right)bc}{2}+A\tan(fx+e)bd+B\tan(fx+e)ad+B\tan(f$
risch	$\frac{2iBace}{f}-\frac{2iBbde}{f}-\frac{2iCade}{f}-\frac{2iCbce}{f}+\frac{2iAade}{f}+\frac{2iAbce}{f}-iCbcx-\frac{\ln(e^{2i(fx+e)}+1)Abc}{f}-\frac{\ln(e^{2i(fx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*C*tan(f*x+e)^3*b*d+1/2*B*tan(f*x+e)^2*b*d+1/2*C*tan(f*x+e)^2*a*d+1
/2*C*tan(f*x+e)^2*b*c+A*tan(f*x+e)*b*d+B*tan(f*x+e)*a*d+B*tan(f*x+e)*b*c+C*
tan(f*x+e)*a*c-C*b*d*tan(f*x+e)+1/2*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*l
n(1+tan(f*x+e)^2)+(A*a*c-A*b*d-B*a*d-B*b*c-C*a*c+C*b*d)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.51, size = 156, normalized size = 0.97

$$\frac{2\,Cbd\tan\left(fx+e\right)^{3}+3\,(Cbc+(Ca+Bb)d)\tan\left(fx+e\right)^{2}+6\,(((A-C)a-Bb)c-(Ba+(A-C)b)d)(fx+e)+3\,((Ba+(A-C)b)c+((A-C)a-Bb)d)\log\left(\tan\left(fx+e\right)^{2}+1\right)+6\,((Ca+Bb)c+(Ba+(A-C)b)d)\tan\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x, algorithm="maxima")
```

```
[Out] 1/6*(2*C*b*d*tan(f*x + e)^3 + 3*(C*b*c + (C*a + B*b)*d)*tan(f*x + e)^2 + 6*
(((A - C)*a - B*b)*c - (B*a + (A - C)*b)*d)*(f*x + e) + 3*((B*a + (A - C)*b
)*c + ((A - C)*a - B*b)*d)*log(tan(f*x + e)^2 + 1) + 6*((C*a + B*b)*c + (B*
a + (A - C)*b)*d)*tan(f*x + e))/f
```

Fricas [A]
time = 2.70, size = 154, normalized size = 0.96

$$\frac{2\,Cbd\tan\left(fx+e\right)^{3}+6\,(((A-C)a-Bb)c-(Ba+(A-C)b)d)fx+3\,(Cbc+(Ca+Bb)d)\tan\left(fx+e\right)^{2}-3\,((Ba+(A-C)b)c+((A-C)a-Bb)d)\log\left(\frac{1}{\tan(fx+e)^{2}+1}\right)+6\,((Ca+Bb)c+(Ba+(A-C)b)d)\tan\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*b*d*tan(f*x + e)^3 + 6*(((A - C)*a - B*b)*c - (B*a + (A - C)*b)*d)
*f*x + 3*(C*b*c + (C*a + B*b)*d)*tan(f*x + e)^2 - 3*((B*a + (A - C)*b)*c +
((A - C)*a - B*b)*d)*log(1/(tan(f*x + e)^2 + 1)) + 6*((C*a + B*b)*c + (B*a
 + (A - C)*b)*d)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(148) = 296$.
time = 0.14, size = 326, normalized size = 2.02

$$\begin{cases} Aacx + \frac{Aad\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{Abc\log\left(\tan^2(e+fx)+1\right)}{2f} - Abdx + \frac{Abd\tan(e+fx)}{f} + \frac{Bac\log\left(\tan^2(e+fx)+1\right)}{2f} - Badx + \frac{Bad\tan(e+fx)}{f} - Bbcx + \frac{Bbc\tan(e+fx)}{f} - \frac{Bbd\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{Bbd\tan^2(e+fx)}{2f} - Cacx + \frac{Cac\tan(e+fx)}{f} - \frac{Cad\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{Cad\tan^2(e+fx)}{2f} - \frac{Cbc\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{Cbc\tan^2(e+fx)}{2f} + Cbdx + \frac{Cbd\tan^3(e+fx)}{3f} - \frac{Cbd\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x(a+b\tan(e))(c+d\tan(e))(A+B\tan(e)+C\tan^2(e)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2
),x)
```

```
[Out] Piecewise((A*a*c*x + A*a*d*log(tan(e + f*x)**2 + 1)/(2*f) + A*b*c*log(tan(e
 + f*x)**2 + 1)/(2*f) - A*b*d*x + A*b*d*tan(e + f*x)/f + B*a*c*log(tan(e +
f*x)**2 + 1)/(2*f) - B*a*d*x + B*a*d*tan(e + f*x)/f - B*b*c*x + B*b*c*tan(e
 + f*x)/f - B*b*d*log(tan(e + f*x)**2 + 1)/(2*f) + B*b*d*tan(e + f*x)**2/(2
*f) - C*a*c*x + C*a*c*tan(e + f*x)/f - C*a*d*log(tan(e + f*x)**2 + 1)/(2*f)
 + C*a*d*tan(e + f*x)**2/(2*f) - C*b*c*log(tan(e + f*x)**2 + 1)/(2*f) + C*b
*c*tan(e + f*x)**2/(2*f) + C*b*d*x + C*b*d*tan(e + f*x)**3/(3*f) - C*b*d*ta
n(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))*(c + d*tan(e))*(A + B*tan(e) + C
*tan(e)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2918 vs. $2(162) = 324$.
time = 1.70, size = 2918, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x, algorithm="giac")
```

```
[Out] 1/6*(6*A*a*c*f*x*tan(f*x)^3*tan(e)^3 - 6*C*a*c*f*x*tan(f*x)^3*tan(e)^3 - 6*
B*b*c*f*x*tan(f*x)^3*tan(e)^3 - 6*B*a*d*f*x*tan(f*x)^3*tan(e)^3 - 6*A*b*d*f
*x*tan(f*x)^3*tan(e)^3 + 6*C*b*d*f*x*tan(f*x)^3*tan(e)^3 - 3*B*a*c*log(4*(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 3*A*b*c*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 3*C*b*
c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 3
*A*a*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^
3 + 3*C*a*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*ta
n(e)^3 + 3*B*b*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)
```

```
^3*tan(e)^3 - 18*A*a*c*f*x*tan(f*x)^2*tan(e)^2 + 18*C*a*c*f*x*tan(f*x)^2*ta
n(e)^2 + 18*B*b*c*f*x*tan(f*x)^2*tan(e)^2 + 18*B*a*d*f*x*tan(f*x)^2*tan(e)^
2 + 18*A*b*d*f*x*tan(f*x)^2*tan(e)^2 - 18*C*b*d*f*x*tan(f*x)^2*tan(e)^2 + 3
*C*b*c*tan(f*x)^3*tan(e)^3 + 3*C*a*d*tan(f*x)^3*tan(e)^3 + 3*B*b*d*tan(f*x)
^3*tan(e)^3 + 9*B*a*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ta
n(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan
(f*x)^2*tan(e)^2 + 9*A*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
 + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)
)*tan(f*x)^2*tan(e)^2 - 9*C*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)^2*tan(e)^2 + 9*A*a*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x
)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan
(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 9*C*a*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 9*B*b*d*log(4*(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
 + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 6*C*a*c*tan(f*x)^3*tan(e)^2 - 6
*B*b*c*tan(f*x)^3*tan(e)^2 - 6*B*a*d*tan(f*x)^3*tan(e)^2 - 6*A*b*d*tan(f*x)
^3*tan(e)^2 + 6*C*b*d*tan(f*x)^3*tan(e)^2 - 6*C*a*c*tan(f*x)^2*tan(e)^3 - 6
*B*b*c*tan(f*x)^2*tan(e)^3 - 6*B*a*d*tan(f*x)^2*tan(e)^3 - 6*A*b*d*tan(f*x)
^2*tan(e)^3 + 6*C*b*d*tan(f*x)^2*tan(e)^3 + 18*A*a*c*f*x*tan(f*x)*tan(e) -
18*C*a*c*f*x*tan(f*x)*tan(e) - 18*B*b*c*f*x*tan(f*x)*tan(e) - 18*B*a*d*f*x*
tan(f*x)*tan(e) - 18*A*b*d*f*x*tan(f*x)*tan(e) + 18*C*b*d*f*x*tan(f*x)*tan(
e) + 3*C*b*c*tan(f*x)^3*tan(e) + 3*C*a*d*tan(f*x)^3*tan(e) + 3*B*b*d*tan(f*
x)^3*tan(e) - 3*C*b*c*tan(f*x)^2*tan(e)^2 - 3*C*a*d*tan(f*x)^2*tan(e)^2 - 3
*B*b*d*tan(f*x)^2*tan(e)^2 + 3*C*b*c*tan(f*x)*tan(e)^3 + 3*C*a*d*tan(f*x)*t
an(e)^3 + 3*B*b*d*tan(f*x)*tan(e)^3 - 2*C*b*d*tan(f*x)^3 - 9*B*a*c*log(4*(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 9*A*b*c*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 9*C*b*c*log(4*
(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 9*A*a*d*log(4
*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x
)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 9*C*a*d*log(
4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 9*B*b*d*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 12*C*a*c*t
an(f*x)^2*tan(e) + 12*B*b*c*tan(f*x)^2*tan(e) + 12*B*a*d*tan(f*x)^2*tan(e)
+ 12*A*b*d*tan(f*x)^2*tan(e) - 18*C*b*d*tan(f*x)^2*tan(e) + 12*C*a*c*tan(f*
x)*tan(e)^2 + 12*B*b*c*tan(f*x)*tan(e)^2 + 12*B*a*d*tan(f*x)*tan(e)^2 + 12*
A*b*d*tan(f*x)*tan(e)^2 - 18*C*b*d*tan(f*x)*tan(e)^2 - 2*C*b*d*tan(e)^3 - 6
*A*a*c*f*x + 6*C*a*c*f*x + 6*B*b*c*f*x + 6*B*a*d*f*x + 6*A*b*d*f*x - 6*C*b*
d*f*x - 3*C*b*c*tan(f*x)^2 - 3*C*a*d*tan(f*x)^2 - 3*B*b*d*tan(f*x)^2 + 3*C*
```

```
b*c*tan(f*x)*tan(e) + 3*C*a*d*tan(f*x)*tan(e) + 3*B*b*d*tan(f*x)*tan(e) - 3
*C*b*c*tan(e)^2 - 3*C*a*d*tan(e)^2 - 3*B*b*d*tan(e)^2 + 3*B*a*c*log(4*(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) + 3*A*b*c*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)/(tan(e)^2 + 1)) - 3*C*b*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*...
```

Mupad [B]
time = 8.84, size = 153, normalized size = 0.95

$$\frac{\ln\left(\tan(e+fx)^2+1\right)\left(\frac{Aad}{2}+\frac{Abc}{2}+\frac{Bac}{2}-\frac{Bbd}{2}-\frac{Cad}{2}-\frac{Cbc}{2}\right)}{f}-x\,(Abd-Aac+Bad+Bbc+Cac-Cbd)+\frac{\tan(e+fx)^2\left(\frac{Bbd}{2}+\frac{Cad}{2}+\frac{Cbc}{2}\right)}{f}+\frac{\tan(e+fx)\,(Abd+Bad+Bbc+Cac-Cbd)}{f}+\frac{Cbd\tan(e+fx)^3}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e
 + f*x)^2),x)
```

```
[Out] (log(tan(e + f*x)^2 + 1)*((A*a*d)/2 + (A*b*c)/2 + (B*a*c)/2 - (B*b*d)/2 - (
C*a*d)/2 - (C*b*c)/2))/f - x*(A*b*d - A*a*c + B*a*d + B*b*c + C*a*c - C*b*d
) + (tan(e + f*x)^2*((B*b*d)/2 + (C*a*d)/2 + (C*b*c)/2))/f + (tan(e + f*x)*
(A*b*d + B*a*d + B*b*c + C*a*c - C*b*d))/f + (C*b*d*tan(e + f*x)^3)/(3*f)
```

3.53 $\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+\right.$

Optimal. Leaf size=73

$$(Ac-cC-Bd)x-\frac{(Bc+(A-C)d)\log(\cos(e+fx))}{f}+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e+fx))^2}{2df}$$

```
[Out] (A*c-B*d-C*c)*x-(B*c+(A-C)*d)*ln(cos(f*x+e))/f+B*d*tan(f*x+e)/f+1/2*C*(c+d*
tan(f*x+e))^2/d/f
```

Rubi [A]

time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3711, 3606, 3556}

$$-\frac{(d(A-C)+Bc)\log(\cos(e+fx))}{f}+x(Ac-Bd-cC)+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e+fx))^2}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (A*c - c*C - B*d)*x - ((B*c + (A - C)*d)*Log[Cos[e + f*x]])/f + (B*d*Tan[e
+ f*x])/f + (C*(c + d*Tan[e + f*x])^2)/(2*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C(c+d\tan(e+fx))^2}{2df} + \int (A-C+B$$

$$= (Ac-cC-Bd)x + \frac{Bd\tan(e+fx)}{f} + \frac{C}{}$$

$$= (Ac-cC-Bd)x - \frac{(Bc+(A-C)d)\log}{f}$$

Mathematica [A]
time = 0.33, size = 76, normalized size = 1.04

$$\frac{2Acfx-2(cC+Bd)\text{ArcTan}(\tan(e+fx))-2(Bc+(A-C)d)\log(\cos(e+fx))+2(cC+Bd)\tan(e+fx)+Cd\tan^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*A*c*f*x - 2*(c*C + B*d)*ArcTan[Tan[e + f*x]] - 2*(B*c + (A - C)*d)*Log[C
os[e + f*x]] + 2*(c*C + B*d)*Tan[e + f*x] + C*d*Tan[e + f*x]^2)/(2*f)
```

Maple [A]
time = 0.07, size = 80, normalized size = 1.10

method	result
norman	$(Ac-Bd-cC)\,x+\frac{(Bd+cC)\tan(fx+e)}{f}+\frac{Cd(\tan^2(fx+e))}{2f}+\frac{(Ad+Bc-Cd)\ln(1+\tan^2(fx+e))}{2f}$
derivativedivides	$\frac{\frac{Cd(\tan^2(fx+e))}{2}+B\tan(fx+e)d+C\tan(fx+e)c+\frac{(Ad+Bc-Cd)\ln(1+\tan^2(fx+e))}{2}+(Ac-Bd-cC)\arctan(\tan(fx+e))}{f}$
default	$\frac{\frac{Cd(\tan^2(fx+e))}{2}+B\tan(fx+e)d+C\tan(fx+e)c+\frac{(Ad+Bc-Cd)\ln(1+\tan^2(fx+e))}{2}+(Ac-Bd-cC)\arctan(\tan(fx+e))}{f}$
risch	$-\frac{2iCde}{f}-iCdx+\frac{2iAde}{f}+Acx-Bdx-Ccx+iBcx+\frac{2iBce}{f}+iAdx+\frac{2i(-iCd\,e^{2i(fx+e)}+Bc}{f(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(1/2*C*tan(f*x+e)^2*d+B*tan(f*x+e)*d+C*tan(f*x+e)*c+1/2*(A*d+B*c-C*d)*l
n(1+tan(f*x+e)^2)+(A*c-B*d-C*c)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.49, size = 78, normalized size = 1.07

$$\frac{Cd\tan(fx+e)^2+2((A-C)c-Bd)(fx+e)+(Bc+(A-C)d)\log\left(\tan(fx+e)^2+1\right)+2(Cc+Bd)\tan(fx+e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="ma
xima")
```

```
[Out] 1/2*(C*d*tan(f*x + e)^2 + 2*((A - C)*c - B*d)*(f*x + e) + (B*c + (A - C)*d)
*log(tan(f*x + e)^2 + 1) + 2*(C*c + B*d)*tan(f*x + e))/f
```

Fricas [A]
time = 3.15, size = 77, normalized size = 1.05

$$\frac{Cd\tan\left(fx+e\right)^{2}+2\left((A-C)c-Bd\right)fx-\left(Bc+(A-C)d\right)\log\left(\frac{1}{\tan(fx+e)^{2}+1}\right)+2\left(Cc+Bd\right)\tan\left(fx+e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="fr
icas")
```

```
[Out] 1/2*(C*d*tan(f*x + e)^2 + 2*((A - C)*c - B*d)*f*x - (B*c + (A - C)*d)*log(1
/(tan(f*x + e)^2 + 1)) + 2*(C*c + B*d)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(60) = 120$.
time = 0.10, size = 131, normalized size = 1.79

$$\begin{cases} Acx+\frac{Ad\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{Bc\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}-Bdx+\frac{Bd\tan\left(e+fx\right)}{f}-Ccx+\frac{Cc\tan\left(e+fx\right)}{f}-\frac{Cd\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{Cd\tan^{2}\left(e+fx\right)}{2f} & \text{for}\ f\neq 0 \\ x\left(c+d\tan\left(e\right)\right)\left(A+B\tan\left(e\right)+C\tan^{2}\left(e\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((A*c*x + A*d*log(tan(e + f*x)**2 + 1)/(2*f) + B*c*log(tan(e + f*x
)**2 + 1)/(2*f) - B*d*x + B*d*tan(e + f*x)/f - C*c*x + C*c*tan(e + f*x)/f -
 C*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*d*tan(e + f*x)**2/(2*f), Ne(f, 0)),
 (x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(74) = 148$.
time = 0.86, size = 918, normalized size = 12.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*A*c*f*x*tan(f*x)^2*tan(e)^2 - 2*C*c*f*x*tan(f*x)^2*tan(e)^2 - 2*B*d*
f*x*tan(f*x)^2*tan(e)^2 - B*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan
(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
 1))*tan(f*x)^2*tan(e)^2 - A*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1))*tan(f*x)^2*tan(e)^2 + C*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)^2*tan(e)^2 - 4*A*c*f*x*tan(f*x)*tan(e) + 4*C*c*f*x*tan(f*x)
*tan(e) + 4*B*d*f*x*tan(f*x)*tan(e) + C*d*tan(f*x)^2*tan(e)^2 + 2*B*c*log(4
*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x
)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 2*A*d*log(4*
(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 2*C*d*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 2*C*c*tan(f*x)
^2*tan(e) - 2*B*d*tan(f*x)^2*tan(e) - 2*C*c*tan(f*x)*tan(e)^2 - 2*B*d*tan(f
*x)*tan(e)^2 + 2*A*c*f*x - 2*C*c*f*x - 2*B*d*f*x + C*d*tan(f*x)^2 + C*d*tan
(e)^2 - B*c*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - A*d*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) + C*d*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)/(tan(e)^2 + 1)) + 2*C*c*tan(f*x) + 2*B*d*tan(f*x) + 2*C*c*tan(e) +
2*B*d*tan(e) + C*d)/(f*tan(f*x)^2*tan(e)^2 - 2*f*tan(f*x)*tan(e) + f)
```

Mupad [B]
time = 8.68, size = 75, normalized size = 1.03

$$\frac{\tan(e+f\,x)\,\left(B\,d+C\,c\right)}{f}-x\,\left(B\,d-A\,c+C\,c\right)+\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{A\,d}{2}+\frac{B\,c}{2}-\frac{C\,d}{2}\right)}{f}+\frac{C\,d\tan(e+f\,x)^2}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)*(B*d + C*c))/f - x*(B*d - A*c + C*c) + (log(tan(e + f*x)^2 +
1)*((A*d)/2 + (B*c)/2 - (C*d)/2))/f + (C*d*tan(e + f*x)^2)/(2*f)
```

3.54 $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=156

$$\frac{(a(Ac-cC-Bd)+b(Bc+(A-C)d))x}{a^2+b^2}+\frac{(Abc-aBc-bcC-aAd-bBd+aCd)\log(\cos(e+fx))}{(a^2+b^2)f}+\frac{(A$$

```
[Out] (a*(A*c-B*d-C*c)+b*(B*c+(A-C)*d))*x/(a^2+b^2)+(-A*a*d+A*b*c-B*a*c-B*b*d+C*a
*d-C*b*c)*ln(cos(f*x+e))/(a^2+b^2)/f+(A*b^2-a*(B*b-C*a))*(-a*d+b*c)*ln(a+b*
tan(f*x+e))/b^2/(a^2+b^2)/f+C*d*tan(f*x+e)/b/f
```

Rubi [A]
time = 0.24, antiderivative size = 155, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3718, 3707, 3698, 31, 3556}

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\log(a+b\tan(e+fx))}{b^2f\left(a^2+b^2\right)}+\frac{\log(\cos(e+fx))(-aAd-aBc+aCd+Abc-bBd-bcC)}{f\left(a^2+b^2\right)}+\frac{x(a(Ac-Bd-cC)+bd(A-C)+bBc)}{a^2+b^2}+\frac{Cd\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*T
an[e + f*x]),x]
```

```
[Out] ((b*B*c + b*(A - C)*d + a*(A*c - c*C - B*d))*x)/(a^2 + b^2) + ((A*b*c - a*B
*c - b*c*C - a*A*d - b*B*d + a*C*d)*Log[Cos[e + f*x]])/((a^2 + b^2)*f) + ((
A*b^2 - a*(b*B - a*C))*(b*c - a*d)*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2
)*f) + (C*d*Tan[e + f*x])/(b*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
```

```
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx &= \frac{Cd\tan(e+fx)}{bf}-\frac{\int \frac{-Abc+aCd-b(Bc+(A-C}{}}{}\\ &= \frac{(bBc+b(A-C)d+a(Ac-cC-Bd))}{a^2+b^2}\\ &= \frac{(bBc+b(A-C)d+a(Ac-cC-Bd))}{a^2+b^2}\\ &= \frac{(bBc+b(A-C)d+a(Ac-cC-Bd))}{a^2+b^2}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.79, size = 148, normalized size = 0.95

$$\frac{\frac{(A+iB-C)(-ic+d)\log(i-\tan(e+fx))}{a+ib}+\frac{(A-iB-C)(ic+d)\log(i+\tan(e+fx))}{a-ib}+\frac{2\left(Ab^2+a(-bB+aC)\right)(bc-ad)\log(a+b\tan(e+fx))}{b^2(a^2+b^2)}+\frac{2Cd\tan(e+fx)}{b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x]),x]
```

```
[Out] (((A + I*B - C)*((-I)*c + d)*Log[I - Tan[e + f*x]])/(a + I*b) + ((A - I*B -
 C)*(I*c + d)*Log[I + Tan[e + f*x]])/(a - I*b) + (2*(A*b^2 + a*(-(b*B) + a*
```

```
C))*(b*c - a*d)*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)) + (2*C*d*Tan[e +
 f*x])/b)/(2*f)
```

Maple [A]

time = 0.21, size = 173, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{Cd\tan(fx+e)}{b}+\frac{\frac{(Aad-Abc+Bac+Bbd-aCd+Cbc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(Aac+Abd-Bad+Bbc-Cac-Cbd)\arctan(\tan(fx+e))}{a^2+b^2}+\left(-\right.}{f}$
default	$\frac{\frac{Cd\tan(fx+e)}{b}+\frac{\frac{(Aad-Abc+Bac+Bbd-aCd+Cbc)\ln\left(1+\tan^2(fx+e)\right)}{2}+(Aac+Abd-Bad+Bbc-Cac-Cbd)\arctan(\tan(fx+e))}{a^2+b^2}+\left(-\right.}{f}$
norman	$\frac{(Aac+Abd-Bad+Bbc-Cac-Cbd)x}{a^2+b^2}+\frac{Cd\tan(fx+e)}{bf}+\frac{(Aad-Abc+Bac+Bbd-aCd+Cbc)\ln(1+\tan^2(fx+e))}{2(a^2+b^2)f}-$
risch	$-\frac{2iB\,a^2de}{bf(a^2+b^2)}+\frac{2ia^3Cde}{b^2f(a^2+b^2)}-\frac{2iC\,a^2ce}{bf(a^2+b^2)}+\frac{\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{ib+a}{ib-a}\right)B\,a^2d}{bf(a^2+b^2)}-\frac{\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{ib+a}{ib-a}\right)a^3Cd}{b^2f(a^2+b^2)}+\frac{\ln\left(\mathrm{e}^{2i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/f*(C*d/b*tan(f*x+e)+1/(a^2+b^2)*(1/2*(A*a*d-A*b*c+B*a*c+B*b*d-C*a*d+C*b*c
)*ln(1+tan(f*x+e)^2)+(A*a*c+A*b*d-B*a*d+B*b*c-C*a*c-C*b*d)*arctan(tan(f*x+e
)))+(-A*a*b^2*d+A*b^3*c+B*a^2*b*d-B*a*b^2*c-C*a^3*d+C*a^2*b*c)/b^2/(a^2+b^2
)*ln(a+b*tan(f*x+e)))
```

Maxima [A]

time = 0.51, size = 187, normalized size = 1.20

$$\frac{\frac{2\,Cd\tan(fx+e)}{b}+\frac{2\,(((A-C)a+Bb)c-(Ba-(A-C)b)d)(fx+e)}{a^2+b^2}+\frac{2\,((Ca^2b-Bab^2+Ab^3)c-(Ca^3-Ba^2b+Aab^2)d)\log(b\tan(fx+e)+a)}{a^2b^2+b^4}+\frac{((Ba-(A-C)b)c+((A-C)a+Bb)d)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*d*tan(f*x + e)/b + 2*(((A - C)*a + B*b)*c - (B*a - (A - C)*b)*d)*(
f*x + e)/(a^2 + b^2) + 2*((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 - B*a^2*b
+ A*a*b^2)*d)*log(b*tan(f*x + e) + a)/(a^2*b^2 + b^4) + ((B*a - (A - C)*b)*
c + ((A - C)*a + B*b)*d)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2))/f
```

Fricas [A]

time = 5.58, size = 231, normalized size = 1.48

$$\frac{2\,(((A-C)ab^2+Bb^3)c-(Bab^2-(A-C)b^3)d)fx+2\,(Ca^2b+Cb^3)d\tan(fx+e)+((Ca^2b-Bab^2+Ab^3)c-(Ca^3-Ba^2b+Aab^2)d)\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)-((Ca^2b+Cb^3)c-(Ca^3-Ba^2b+Cab^2-Bb^3)d)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,(a^2b^2+b^4)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a*b^2 + B*b^3)*c - (B*a*b^2 - (A - C)*b^3)*d)*f*x + 2*(C*a
^2*b + C*b^3)*d*tan(f*x + e) + ((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 - B*
a^2*b + A*a*b^2)*d)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(ta
n(f*x + e)^2 + 1)) - ((C*a^2*b + C*b^3)*c - (C*a^3 - B*a^2*b + C*a*b^2 - B*
b^3)*d)*log(1/(tan(f*x + e)^2 + 1)))/((a^2*b^2 + b^4)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.02, size = 2387, normalized size = 15.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e)
),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(a,
0) & Eq(b, 0) & Eq(f, 0)), ((A*c*x + A*d*log(tan(e + f*x)**2 + 1)/(2*f) + B
*c*log(tan(e + f*x)**2 + 1)/(2*f) - B*d*x + B*d*tan(e + f*x)/f - C*c*x + C*
c*tan(e + f*x)/f - C*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*d*tan(e + f*x)**2
/(2*f))/a, Eq(b, 0)), (I*A*c*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f
) + A*c*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*A*c/(2*b*f*tan(e + f*x) - 2*
I*b*f) + A*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*A*d*f*x/(2
*b*f*tan(e + f*x) - 2*I*b*f) - A*d/(2*b*f*tan(e + f*x) - 2*I*b*f) + B*c*f*x
*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*B*c*f*x/(2*b*f*tan(e + f*x
) - 2*I*b*f) - B*c/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*B*d*f*x*tan(e + f*x)/
(2*b*f*tan(e + f*x) - 2*I*b*f) + B*d*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + B
*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I
*B*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*B*d/(2*b*f
*tan(e + f*x) - 2*I*b*f) + I*C*c*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I
*b*f) + C*c*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + C*c*log(tan(e + f*x)**2 +
1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*C*c*log(tan(e + f*x)**2
+ 1)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*C*c/(2*b*f*tan(e + f*x) - 2*I*b*f)
- 3*C*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + 3*I*C*d*f*x/(2*b*
f*tan(e + f*x) - 2*I*b*f) + I*C*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*
b*f*tan(e + f*x) - 2*I*b*f) + C*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f
*x) - 2*I*b*f) + 2*C*d*tan(e + f*x)**2/(2*b*f*tan(e + f*x) - 2*I*b*f) + 3*C
*d/(2*b*f*tan(e + f*x) - 2*I*b*f), Eq(a, -I*b)), (-I*A*c*f*x*tan(e + f*x)/(
2*b*f*tan(e + f*x) + 2*I*b*f) + A*c*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*
A*c/(2*b*f*tan(e + f*x) + 2*I*b*f) + A*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*
x) + 2*I*b*f) + I*A*d*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - A*d/(2*b*f*tan(e
 + f*x) + 2*I*b*f) + B*c*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) +
```

```
I*B*c*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - B*c/(2*b*f*tan(e + f*x) + 2*I*b*
f) - I*B*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + B*d*f*x/(2*b*f
*tan(e + f*x) + 2*I*b*f) + B*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f
*tan(e + f*x) + 2*I*b*f) + I*B*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*
x) + 2*I*b*f) + I*B*d/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*C*c*f*x*tan(e + f*
x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + C*c*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f)
+ C*c*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f)
+ I*C*c*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*C*c/(2*
b*f*tan(e + f*x) + 2*I*b*f) - 3*C*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) +
2*I*b*f) - 3*I*C*d*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*C*d*log(tan(e + f
*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + C*d*log(tan(e + f
*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*C*d*tan(e + f*x)**2/(2*b*f*t
an(e + f*x) + 2*I*b*f) + 3*C*d/(2*b*f*tan(e + f*x) + 2*I*b*f), Eq(a, I*b)),
 (x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2)/(a + b*tan(e)), Eq(f, 0)),
(2*A*a*b**2*c*f*x/(2*a**2*b**2*f + 2*b**4*f) - 2*A*a*b**2*d*log(a/b + tan(e
 + f*x))/(2*a**2*b**2*f + 2*b**4*f) + A*a*b**2*d*log(tan(e + f*x)**2 + 1)/(
2*a**2*b**2*f + 2*b**4*f) + 2*A*b**3*c*log(a/b + tan(e + f*x))/(2*a**2*b**2
*f + 2*b**4*f) - A*b**3*c*log(tan(e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b**4*
f) + 2*A*b**3*d*f*x/(2*a**2*b**2*f + 2*b**4*f) + 2*B*a**2*b*d*log(a/b + tan
(e + f*x))/(2*a**2*b**2*f + 2*b**4*f) - 2*B*a*b**2*c*log(a/b + tan(e + f*x)
)/(2*a**2*b**2*f + 2*b**4*f) + B*a*b**2*c*log(tan(e + f*x)**2 + 1)/(2*a**2*
b**2*f + 2*b**4*f) - 2*B*a*b**2*d*f*x/(2*a**2*b**2*f + 2*b**4*f) + 2*B*b**3
*c*f*x/(2*a**2*b**2*f + 2*b**4*f) + B*b**3*d*log(tan(e + f*x)**2 + 1)/(2*a*
*2*b**2*f + 2*b**4*f) - 2*C*a**3*d*log(a/b + tan(e + f*x))/(2*a**2*b**2*f +
 2*b**4*f) + 2*C*a**2*b*c*log(a/b + tan(e + f*x))/(2*a**2*b**2*f + 2*b**4*f
) + 2*C*a**2*b*d*tan(e + f*x)/(2*a**2*b**2*f + 2*b**4*f) - 2*C*a*b**2*c*f*x
/(2*a**2*b**2*f + 2*b**4*f) - C*a*b**2*d*log(tan(e + f*x)**2 + 1)/(2*a**2*b
**2*f + 2*b**4*f) + C*b**3*c*log(tan(e + f*x)**2 + 1)/(2*a**2*b**2*f + 2*b*
*4*f) - 2*C*b**3*d*f*x/(2*a**2*b**2*f + 2*b**4*f) + 2*C*b**3*d*tan(e + f*x)
/(2*a**2*b**2*f + 2*b**4*f), True))
```

Giac [A]
time = 0.67, size = 186, normalized size = 1.19

$$\frac{\frac{2\,Cd\tan(fx+e)}{b}+\frac{2\,(Aac-Cac+Bbc-Bad+Abd-Cbd)(fx+e)}{a^2+b^2}+\frac{(Bac-Abc+Cbc+Aad-Cad+Bbd)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}+\frac{2\,(Ca^2bc-Bab^2c+Ab^3c-Ca^3d+Ba^2bd-Aab^2d)\log(|b\tan(fx+e)+a|)}{a^2b^2+b^4}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*d*tan(f*x + e)/b + 2*(A*a*c - C*a*c + B*b*c - B*a*d + A*b*d - C*b*
d)*(f*x + e)/(a^2 + b^2) + (B*a*c - A*b*c + C*b*c + A*a*d - C*a*d + B*b*d)*
log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 2*(C*a^2*b*c - B*a*b^2*c + A*b^3*c -
C*a^3*d + B*a^2*b*d - A*a*b^2*d)*log(abs(b*tan(f*x + e) + a))/(a^2*b^2 + b^
4))/f
```

Mupad [B]
time = 10.13, size = 186, normalized size = 1.19

$$\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A\,d+B\,c-C\,d-A\,c\,1\mathrm{i}+B\,d\,1\mathrm{i}+C\,c\,1\mathrm{i}\right)}{2\,f\,\left(a+b\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(B\,d+A\,d\,1\mathrm{i}+B\,c\,1\mathrm{i}-A\,c+C\,c-C\,d\,1\mathrm{i}\right)}{2\,f\,\left(b+a\,1\mathrm{i}\right)}-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(b^2\,\left(A\,a\,d+B\,a\,c\right)-b\,\left(B\,a^2\,d+C\,a^2\,c\right)-A\,b^3\,c+C\,a^3\,d\right)}{f\,\left(a^2\,b^2+b^4\right)}+\frac{C\,d\tan(e+f\,x)}{b\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b*t
an(e + f*x)),x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(A*d - A*c*1i + B*c + B*d*1i + C*c*1i - C*d))/(2*f*
(a + b*1i)) + (log(tan(e + f*x) + 1i)*(A*d*1i - A*c + B*c*1i + B*d + C*c -
C*d*1i))/(2*f*(a*1i + b)) - (log(a + b*tan(e + f*x))*(b^2*(A*a*d + B*a*c) -
 b*(B*a^2*d + C*a^2*c) - A*b^3*c + C*a^3*d))/(f*(b^4 + a^2*b^2)) + (C*d*tan
(e + f*x))/(b*f)
```

3.55 $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=265

$$\frac{\left(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd)+2ab(Bc+(A-C)d)\right)x}{\left(a^2+b^2\right)^2}+\frac{(2ab(Ac-cC-Bd)-a^2(Bc+(A-}{(a}$$

```
[Out] (a^2*(A*c-B*d-C*c)-b^2*(A*c-B*d-C*c)+2*a*b*(B*c+(A-C)*d))*x/(a^2+b^2)^2+(2*
a*b*(A*c-B*d-C*c)-a^2*(B*c+(A-C)*d)+b^2*(B*c+(A-C)*d))*ln(cos(f*x+e))/(a^2+
b^2)^2/f+(a^4*C*d+b^4*(A*d+B*c)+2*a*b^3*(A*c-B*d-C*c)-a^2*b^2*(B*c+(A-3*C)*
d))*ln(a+b*tan(f*x+e))/b^2/(a^2+b^2)^2/f-(A*b^2-a*(B*b-C*a))*(-a*d+b*c)/b^2
/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]

time = 0.32, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3716, 3707, 3698, 31, 3556}

$$-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{\log(\cos(e+fx))\left(-\left(a^2(d(A-C)+Bc)\right)+2ab(Ac-Bd-cC)+b^2(d(A-C)+Bc)\right)}{f\left(a^2+b^2\right)^2}+\frac{x\left(a^2(Ac-Bd-cC)+2ab(d(A-C)+Bc)-b^2(Ac-Bd-cC)\right)}{\left(a^2+b^2\right)^2}+\frac{\left(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)\right)\log(a+b\tan(e+fx))}{b^2f\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*T
an[e + f*x])^2,x]
```

```
[Out] ((a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) + 2*a*b*(B*c + (A - C)*d))*
x)/(a^2 + b^2)^2 + ((2*a*b*(A*c - c*C - B*d) - a^2*(B*c + (A - C)*d) + b^2*
(B*c + (A - C)*d))*Log[Cos[e + f*x]])/((a^2 + b^2)^2*f) + ((a^4*C*d + b^4*(
B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Log[a
 + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)^2*f) - ((A*b^2 - a*(b*B - a*C))*(b*c -
 a*d))/(b^2*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
```

```
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = -\frac{(Ab^2-a(bB-aC))\,(bc-ad)}{b^2\,(a^2+b^2)\,f(a+b\tan(e+fx))}+\frac{\int \cdots}{\cdots}$$

$$= \frac{(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd\cdots}{(a^2+b^2)^2}$$

$$= \frac{(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd\cdots}{(a^2+b^2)^2}$$

$$= \frac{(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd\cdots}{(a^2+b^2)^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.49, size = 589, normalized size = 2.22

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^2,x]
```

```
[Out] (a^2*(2*(a + I*b)^2*(A*b^2*(c - I*d) + I*a^2*C*d + 2*a*b*C*d + b^2*((-I)*B*
c - c*C - B*d))*(e + f*x) - 2*(a^2 + b^2)^2*C*d*Log[Cos[e + f*x]] + (a^4*C*
d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*
d))*Log[(a*Cos[e + f*x] + b*Sin[e + f*x])^2]) + b*(2*(a + I*b)*((-I)*A*b^4*
c + I*a^4*C*d*(I + e + f*x) + a*b^3*(A*c*(1 + I*e + I*f*x) - I*c*C*(e + f*x
) - I*B*d*(e + f*x) + B*c*(I + e + f*x) + A*d*(I + e + f*x)) - I*a^2*b^2*(I
*A*c*(e + f*x) - 2*C*d*(e + f*x) + B*c*(-I + e + f*x) + A*d*(-I + e + f*x)
- I*c*C*(I + e + f*x) - I*B*d*(I + e + f*x)) + a^3*b*(c*C + d*(B + C*(I + e
 + f*x)))) - 2*a*(a^2 + b^2)^2*C*d*Log[Cos[e + f*x]] + a*(a^4*C*d + b^4*(B*
c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Log[(a*
Cos[e + f*x] + b*Sin[e + f*x])^2])*Tan[e + f*x] - (2*I)*a*(a^4*C*d + b^4*(B
*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*ArcTan
[Tan[e + f*x]]*(a + b*Tan[e + f*x]))/(2*a*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e
+ f*x]))
```

Maple [A]

time = 0.32, size = 321, normalized size = 1.21

method	result
derivativedivides	$-\frac{-A a\,b^{2} d+A\,b^{3} c+B\,a^{2} b d-B a\,b^{2} c-a^{3} C d+C\,a^{2} b c}{b^{2} \left(a^{2}+b^{2}\right) \left(a+b \tan \left(f x+e\right)\right)}+\frac{\left(-A\,a^{2} b^{2} d+2 A a\,b^{3} c+A\,b^{4} d-B\,a^{2} b^{2} c-2 B a\,b^{3} d+B\,b^{4} c+a^{4} C d+3 C\,a^{2} b^{2} d-2 C \cdots\right.}{\left(a^{2}+b^{2}\right)^{2} b^{2}}$
default	$-\frac{-A a\,b^{2} d+A\,b^{3} c+B\,a^{2} b d-B a\,b^{2} c-a^{3} C d+C\,a^{2} b c}{b^{2} \left(a^{2}+b^{2}\right) \left(a+b \tan \left(f x+e\right)\right)}+\frac{\left(-A\,a^{2} b^{2} d+2 A a\,b^{3} c+A\,b^{4} d-B\,a^{2} b^{2} c-2 B a\,b^{3} d+B\,b^{4} c+a^{4} C d+3 C\,a^{2} b^{2} d-2 C \cdots\right.}{\left(a^{2}+b^{2}\right)^{2} b^{2}}$
norman	$\frac{\frac{a \left(A\,a^{2} c+2 A a b d-A\,b^{2} c-B\,a^{2} d+2 B a b c+B\,b^{2} d-C\,a^{2} c-2 C a b d+C\,b^{2} c\right) x}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{A a\,b^{2} d-A\,b^{3} c-B\,a^{2} b d+B a\,b^{2} c+a^{3} C d-C\,a^{2} b c}{b^{2} f \left(a^{2}+b^{2}\right)}+\frac{b \left(A\,a^{2} \cdots\right.}{\cdots}}{a+b \tan \left(f x+e\right)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(-A*a*b^2*d+A*b^3*c+B*a^2*b*d-B*a*b^2*c-C*a^3*d+C*a^2*b*c)/b^2/(a^2+b
^2)/(a+b*tan(f*x+e))+1/(a^2+b^2)^2*(-A*a^2*b^2*d+2*A*a*b^3*c+A*b^4*d-B*a^2*
b^2*c-2*B*a*b^3*d+B*b^4*c+C*a^4*d+3*C*a^2*b^2*d-2*C*a*b^3*c)/b^2*ln(a+b*tan
(f*x+e))+1/(a^2+b^2)^2*(1/2*(A*a^2*d-2*A*a*b*c-A*b^2*d+B*a^2*c+2*B*a*b*d-B*
b^2*c-C*a^2*d+2*C*a*b*c+C*b^2*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c+2*A*a*b*d-A*b^
2*c-B*a^2*d+2*B*a*b*c+B*b^2*d-C*a^2*c-2*C*a*b*d+C*b^2*c)*arctan(tan(f*x+e))
))
```

Maxima [A]

time = 0.52, size = 342, normalized size = 1.29

$$\frac{\frac{2\,(((A-C)a^2+2\,Bab-(A-C)b^2)c-(Ba^2-2\,(A-C)ab-Bb^2)d)(fx+e)}{a^4+2\,a^2b^2+b^4}-\frac{2\,((Ba^2b^2-2\,(A-C)ab^3-Bb^4)c-(Ca^4-(A-3\,C)a^2b^2-2\,Bab^3+Ab^4)d)\log(b\tan(fx+e)+a)}{a^4b^2+2\,a^2b^4+b^6}+\frac{((Ba^2-2\,(A-C)ab-Bb^2)c+((A-C)a^2+2\,Bab-(A-C)b^2)d)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,((Ca^2b-Bab^2+Ab^3)c-(Ca^3-Ba^2b+Aab^2)d)}{a^3b^2+ab^4+(a^2b^3+b^5)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c - (B*a^2 - 2*(A - C)*a*b -
B*b^2)*d)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a^2*b^2 - 2*(A - C)*a*b
^3 - B*b^4)*c - (C*a^4 - (A - 3*C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*d)*log(b*ta
n(f*x + e) + a)/(a^4*b^2 + 2*a^2*b^4 + b^6) + ((B*a^2 - 2*(A - C)*a*b - B*b
^2)*c + ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - 2*((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 - B*a^2*b
 + A*a*b^2)*d)/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(268) = 536$.
time = 6.44, size = 564, normalized size = 2.13

$$\frac{2\,(((A-C)a^3b^2+2\,Ba^2b^3-(A-C)ab^4)c-(Ba^3b^2-2\,(A-C)a^2b^3-Bab^4)d)fx-2\,(Ca^2b^3-Bab^4+Ab^5)c+2\,(Ca^3b^2-Ba^2b^3+Aab^4)d-((Ba^3b^2-2\,(A-C)a^2b^3-Bab^4)c-(Ca^5-(A-3\,C)a^3b^2-2\,Ba^2b^3+Aab^4)d+((Ba^2b^3-2\,(A-C)ab^4-Bb^5)c-(Ca^4b-(A-3\,C)a^2b^3-2\,Bab^4+Ab^5)d)\tan(fx+e))\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)-((Ca^4b+2\,Ca^2b^3+Cb^5)d\tan(fx+e)+(Ca^5+2\,Ca^3b^2+Cab^4)d)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+2\,((((A-C)a^2b^3+2\,Bab^4-(A-C)b^5)c-(Ba^2b^3-2\,(A-C)ab^4-Bb^5)d)fx+(Ca^3b^2-Ba^2b^3+Aab^4)c-(Ca^4b-Ba^3b^2+Aa^2b^3)d)\tan(fx+e)}{2\,((a^4b^3+2\,a^2b^5+b^7)f\tan(fx+e)+(a^5b^2+2\,a^3b^4+ab^6)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a^3*b^2 + 2*B*a^2*b^3 - (A - C)*a*b^4)*c - (B*a^3*b^2 - 2*
(A - C)*a^2*b^3 - B*a*b^4)*d)*f*x - 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c + 2*(
C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*d - ((B*a^3*b^2 - 2*(A - C)*a^2*b^3 - B*a*
b^4)*c - (C*a^5 - (A - 3*C)*a^3*b^2 - 2*B*a^2*b^3 + A*a*b^4)*d + ((B*a^2*b^
3 - 2*(A - C)*a*b^4 - B*b^5)*c - (C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*b^4 +
 A*b^5)*d)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2
)/(tan(f*x + e)^2 + 1)) - ((C*a^4*b + 2*C*a^2*b^3 + C*b^5)*d*tan(f*x + e) +
 (C*a^5 + 2*C*a^3*b^2 + C*a*b^4)*d)*log(1/(tan(f*x + e)^2 + 1)) + 2*((((A -
 C)*a^2*b^3 + 2*B*a*b^4 - (A - C)*b^5)*c - (B*a^2*b^3 - 2*(A - C)*a*b^4 - B
*b^5)*d)*f*x + (C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c - (C*a^4*b - B*a^3*b^2 +
 A*a^2*b^3)*d)*tan(f*x + e))/((a^4*b^3 + 2*a^2*b^5 + b^7)*f*tan(f*x + e) +
(a^5*b^2 + 2*a^3*b^4 + a*b^6)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.65, size = 9721, normalized size = 36.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e)
)**2,x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2, Eq(
a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c*x + A*d*log(tan(e + f*x)**2 + 1)/(2*f)
+ B*c*log(tan(e + f*x)**2 + 1)/(2*f) - B*d*x + B*d*tan(e + f*x)/f - C*c*x +
 C*c*tan(e + f*x)/f - C*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*d*tan(e + f*x)
**2/(2*f))/a**2, Eq(b, 0)), (-A*c*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x
)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c*f*x*tan(e + f*x)/(4*b*
*2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + A*c*f*x/(4*b**
2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - A*c*tan(e + f*x
)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c
/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*A*d*f*
x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b
**2*f) + 2*A*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(
e + f*x) - 4*b**2*f) - I*A*d*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan
(e + f*x) - 4*b**2*f) + I*A*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*
b**2*f*tan(e + f*x) - 4*b**2*f) + I*B*c*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e
 + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*B*c*f*x*tan(e + f*x)/(
4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - I*B*c*f*x/
(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*B*c*tan
(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) +
 B*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x
) - 4*b**2*f) - 2*I*B*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b*
*2*f*tan(e + f*x) - 4*b**2*f) - B*d*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**
2*f*tan(e + f*x) - 4*b**2*f) - 3*B*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2
 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*B*d/(4*b**2*f*tan(e + f*x)**2
- 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + C*c*f*x*tan(e + f*x)**2/(4*b**2*f*t
an(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*C*c*f*x*tan(e +
f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - C*c*
f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*C*c
*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*
f) + 2*I*C*c/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f
) + 3*I*C*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(
e + f*x) - 4*b**2*f) + 6*C*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8
*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*I*C*d*f*x/(4*b**2*f*tan(e + f*x)**2
- 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*C*d*log(tan(e + f*x)**2 + 1)*tan(
e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f)
 - 4*I*C*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2
- 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*C*d*log(tan(e + f*x)**2 + 1)/(4*b
**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 5*I*C*d*tan(e
 + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 4
*C*d/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f), Eq(a,
 -I*b)), (-A*c*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*t
an(e + f*x) - 4*b**2*f) - 2*I*A*c*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**
```

```
2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + A*c*f*x/(4*b**2*f*tan(e + f*x)**2
 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - A*c*tan(e + f*x)/(4*b**2*f*tan(e +
 f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*A*c/(4*b**2*f*tan(e +
f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - I*A*d*f*x*tan(e + f*x)**2/(
4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*A*d*f*x*
tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f
) + I*A*d*f*x/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*
f) - I*A*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x)
 - 4*b**2*f) - I*B*c*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b*
*2*f*tan(e + f*x) - 4*b**2*f) + 2*B*c*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*
x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*B*c*f*x/(4*b**2*f*tan(e + f
*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - I*B*c*tan(e + f*x)/(4*b**2*f
*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + B*d*f*x*tan(e + f*
x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I
*B*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) -
 4*b**2*f) - B*d*f*x/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) -
4*b**2*f) - 3*B*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e
 + f*x) - 4*b**2*f) - 2*I*B*d/(4*b**2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e
+ f*x) - 4*b**2*f) + C*c*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*
I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*C*c*f*x*tan(e + f*x)/(4*b**2*f*tan(
e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - C*c*f*x/(4*b**2*f*tan(e
 + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*C*c*tan(e + f*x)/(4*b*
*2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*C*c/(4*b**
2*f*tan(e + f*x)**2 + 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*I*C*d*f*x*tan
(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 + 8*I*b*...
```

Giac [A]
time = 0.76, size = 531, normalized size = 2.00

$$\frac{\frac{2\,(Aa^2c-Ca^2c+2\,Babc-Ab^2c+Cb^2c-Ba^2d+2\,Aabd-2\,Cabd+Bb^2d)\,(fx+e)}{a^4+2\,a^2b^2+b^4}+\frac{(Ba^2c-2\,Aabc+2\,Cabc-Bb^2c+Aa^2d-Ca^2d+2\,Babd-Ab^2d+Cb^2d)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Ba^2b^2c-2\,Aab^3c+2\,Cab^3c-Bb^4c-Ca^4d+Aa^2b^2d-3\,Ca^2b^2d+2\,Bab^3d-Ab^4d)\log(|b\tan(fx+e)+a|)}{a^4b^2+2\,a^2b^4+b^6}+\frac{2\,(Ba^2b^2c\tan(fx+e)-2\,Aab^3c\tan(fx+e)+2\,Cab^3c\tan(fx+e)-Bb^4c\tan(fx+e)-Ca^4d\tan(fx+e)+Aa^2b^2d\tan(fx+e)-3\,Ca^2b^2d\tan(fx+e)+2\,Bab^3d\tan(fx+e)-Ab^4d\tan(fx+e)-Ca^4c+2\,Ba^3bc-3\,Aa^2b^2c+Ca^2b^2c-Ab^4c-Ba^4d+2\,Aa^3bd-2\,Ca^3bd+Ba^2b^2d)}{(a^4b+2\,a^2b^3+b^5)(b\tan(fx+e)+a)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c - C*a^2*c + 2*B*a*b*c - A*b^2*c + C*b^2*c - B*a^2*d + 2*A*a
*b*d - 2*C*a*b*d + B*b^2*d)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) + (B*a^2*c -
2*A*a*b*c + 2*C*a*b*c - B*b^2*c + A*a^2*d - C*a^2*d + 2*B*a*b*d - A*b^2*d +
 C*b^2*d)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(B*a^2*b^2*c
- 2*A*a*b^3*c + 2*C*a*b^3*c - B*b^4*c - C*a^4*d + A*a^2*b^2*d - 3*C*a^2*b^2
*d + 2*B*a*b^3*d - A*b^4*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^2 + 2*a^2*b
^4 + b^6) + 2*(B*a^2*b^2*c*tan(f*x + e) - 2*A*a*b^3*c*tan(f*x + e) + 2*C*a*
b^3*c*tan(f*x + e) - B*b^4*c*tan(f*x + e) - C*a^4*d*tan(f*x + e) + A*a^2*b^
2*d*tan(f*x + e) - 3*C*a^2*b^2*d*tan(f*x + e) + 2*B*a*b^3*d*tan(f*x + e) -
A*b^4*d*tan(f*x + e) - C*a^4*c + 2*B*a^3*b*c - 3*A*a^2*b^2*c + C*a^2*b^2*c
```

```
- A*b^4*c - B*a^4*d + 2*A*a^3*b*d - 2*C*a^3*b*d + B*a^2*b^2*d)/((a^4*b + 2*
a^2*b^3 + b^5)*(b*tan(f*x + e) + a)))/f
```

Mupad [B]
time = 21.14, size = 1875, normalized size = 7.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b*t
an(e + f*x))^2,x)
```

```
[Out] (log(a + b*tan(e + f*x))*(b^4*(A*d + B*c) - b^3*(2*B*a*d - 2*A*a*c + 2*C*a*
c) - b^2*(A*a^2*d + B*a^2*c - 3*C*a^2*d) + C*a^4*d))/(f*(b^6 + 2*a^2*b^4 +
a^4*b^2)) - (log((A*B*b^4*d^2 - A*B*b^4*c^2 + B*C*a^4*d^2 + B*C*b^4*c^2 - A
^2*b^4*c*d + B^2*b^4*c*d + C^2*a^4*c*d - A^2*a*b^3*c^2 + A^2*a*b^3*d^2 + B^
2*a*b^3*c^2 - B^2*a*b^3*d^2 - C^2*a*b^3*c^2 + C^2*a*b^3*d^2 + A*B*a^2*b^2*c
^2 - A*B*a^2*b^2*d^2 - B*C*a^2*b^2*c^2 + 3*B*C*a^2*b^2*d^2 + A^2*a^2*b^2*c*
d - B^2*a^2*b^2*c*d + 3*C^2*a^2*b^2*c*d - A*C*a^4*c*d + A*C*b^4*c*d + 2*A*C
*a*b^3*c^2 - 2*A*C*a*b^3*d^2 - 4*A*C*a^2*b^2*c*d + 4*A*B*a*b^3*c*d - 4*B*C*
a*b^3*c*d)/(b*(a^2 + b^2)^2) + (tan(e + f*x)*(A^2*b^4*c^2 + B^2*b^4*d^2 + C
^2*a^4*d^2 + C^2*b^4*c^2 + C^2*b^4*d^2 + A^2*a^2*b^2*d^2 + B^2*a^2*b^2*c^2
+ 3*C^2*a^2*b^2*d^2 - A*C*a^4*d^2 - 2*A*C*b^4*c^2 - A*C*b^4*d^2 - 4*A*C*a^2
*b^2*d^2 - 2*A*B*b^4*c*d - B*C*a^4*c*d + B*C*b^4*c*d - 2*A*B*a*b^3*c^2 + 2*
A*B*a*b^3*d^2 + 2*B*C*a*b^3*c^2 - 2*B*C*a*b^3*d^2 - 2*A^2*a*b^3*c*d + 2*B^2
*a*b^3*c*d - 2*C^2*a*b^3*c*d + 2*A*B*a^2*b^2*c*d - 4*B*C*a^2*b^2*c*d + 4*A*
C*a*b^3*c*d))/(b*(a^2 + b^2)^2) + ((c + d*1i)*(A + B*1i - C)*(A*b*c - B*b*d
 - 4*C*a*d - C*b*c + (tan(e + f*x)*(3*A*b^4*d + 3*B*b^4*c + 2*C*a^4*d - 5*C
*b^4*d + 4*A*a*b^3*c - 4*B*a*b^3*d - 4*C*a*b^3*c - A*a^2*b^2*d - B*a^2*b^2*
c + C*a^2*b^2*d))/(b*(a^2 + b^2)) + (b*(c + d*1i)*(4*a*b - a^2*tan(e + f*x)
 + 3*b^2*tan(e + f*x))*(A + B*1i - C)*1i)/(a*1i - b)^2)*1i)/(2*(a*1i - b)^2
))*(A*c + A*d*1i + B*c*1i - B*d - C*c - C*d*1i))/(2*f*(2*a*b - a^2*1i + b^2
*1i)) - (log((A*B*b^4*d^2 - A*B*b^4*c^2 + B*C*a^4*d^2 + B*C*b^4*c^2 - A^2*b
^4*c*d + B^2*b^4*c*d + C^2*a^4*c*d - A^2*a*b^3*c^2 + A^2*a*b^3*d^2 + B^2*a*
b^3*c^2 - B^2*a*b^3*d^2 - C^2*a*b^3*c^2 + C^2*a*b^3*d^2 + A*B*a^2*b^2*c^2 -
 A*B*a^2*b^2*d^2 - B*C*a^2*b^2*c^2 + 3*B*C*a^2*b^2*d^2 + A^2*a^2*b^2*c*d -
B^2*a^2*b^2*c*d + 3*C^2*a^2*b^2*c*d - A*C*a^4*c*d + A*C*b^4*c*d + 2*A*C*a*b
^3*c^2 - 2*A*C*a*b^3*d^2 - 4*A*C*a^2*b^2*c*d + 4*A*B*a*b^3*c*d - 4*B*C*a*b^
3*c*d)/(b*(a^2 + b^2)^2) + (tan(e + f*x)*(A^2*b^4*c^2 + B^2*b^4*d^2 + C^2*a
^4*d^2 + C^2*b^4*c^2 + C^2*b^4*d^2 + A^2*a^2*b^2*d^2 + B^2*a^2*b^2*c^2 + 3*
C^2*a^2*b^2*d^2 - A*C*a^4*d^2 - 2*A*C*b^4*c^2 - A*C*b^4*d^2 - 4*A*C*a^2*b^2
*d^2 - 2*A*B*b^4*c*d - B*C*a^4*c*d + B*C*b^4*c*d - 2*A*B*a*b^3*c^2 + 2*A*B*
a*b^3*d^2 + 2*B*C*a*b^3*c^2 - 2*B*C*a*b^3*d^2 - 2*A^2*a*b^3*c*d + 2*B^2*a*b
^3*c*d - 2*C^2*a*b^3*c*d + 2*A*B*a^2*b^2*c*d - 4*B*C*a^2*b^2*c*d + 4*A*C*a*
b^3*c*d))/(b*(a^2 + b^2)^2) + ((c*1i + d)*(B*1i - A + C)*(A*b*c - B*b*d - 4
```

```
*C*a*d - C*b*c + (tan(e + f*x)*(3*A*b^4*d + 3*B*b^4*c + 2*C*a^4*d - 5*C*b^4
*d + 4*A*a*b^3*c - 4*B*a*b^3*d - 4*C*a*b^3*c - A*a^2*b^2*d - B*a^2*b^2*c +
C*a^2*b^2*d))/(b*(a^2 + b^2)) + (b*(c*1i + d)*(4*a*b - a^2*tan(e + f*x) + 3
*b^2*tan(e + f*x))*(B*1i - A + C))/(a*1i + b)^2))/(2*(a*1i + b)^2))*(A*c*1i
 + A*d + B*c - B*d*1i - C*c*1i - C*d))/(2*f*(a*b*2i - a^2 + b^2)) - (A*b^3*
c - C*a^3*d - A*a*b^2*d - B*a*b^2*c + B*a^2*b*d + C*a^2*b*c)/(b^2*f*(a^2 +
b^2)*(a + b*tan(e + f*x)))
```

3.56 $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=320

$$\frac{(a^3(Ac-cC-Bd)-3ab^2(Ac-cC-Bd)+3a^2b(Bc+(A-C)d)-b^3(Bc+(A-C)d))\,x}{(a^2+b^2)^3}+\frac{(3a^2b(Ac-$$

```
[Out] (a^3*(A*c-B*d-C*c)-3*a*b^2*(A*c-B*d-C*c)+3*a^2*b*(B*c+(A-C)*d)-b^3*(B*c+(A-
C)*d))*x/(a^2+b^2)^3+(3*a^2*b*(A*c-B*d-C*c)-b^3*(A*c-B*d-C*c)-a^3*(B*c+(A-C
)*d)+3*a*b^2*(B*c+(A-C)*d))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^3/f-1/2
*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)/b^2/(a^2+b^2)/f/(a+b*tan(f*x+e))^2+(-a^4*C*
d-b^4*(A*d+B*c)-2*a*b^3*(A*c-B*d-C*c)+a^2*b^2*(B*c+(A-3*C)*d))/b^2/(a^2+b^2
)^2/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.50, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {3716, 3709, 3612, 3611}

$$-\frac{(bc-ad)\,(Ab^2-a(bB-aC))}{2b^2f\,(a^2+b^2)\,(a+b\tan(e+fx))^2}-\frac{a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)}{b^2f\,(a^2+b^2)^2\,(a+b\tan(e+fx))}+\frac{(-(a^3(d(A-C)+Bc))+3a^2b(Ac-Bd-cC)+3ab^2(d(A-C)+Bc)-b^3(Ac-Bd-cC))\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)^3}+\frac{x(a^3(Ac-Bd-cC)+3a^2b(d(A-C)+Bc)-3ab^2(Ac-Bd-cC)-b^3(d(A-C)+Bc))}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*T
an[e + f*x])^3,x]
```

```
[Out] ((a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) + 3*a^2*b*(B*c + (A - C
)*d) - b^3*(B*c + (A - C)*d))*x)/(a^2 + b^2)^3 + ((3*a^2*b*(A*c - c*C - B*d
) - b^3*(A*c - c*C - B*d) - a^3*(B*c + (A - C)*d) + 3*a*b^2*(B*c + (A - C)*
d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*f) - ((A*b^2 - a*(
b*B - a*C))*(b*c - a*d))/(2*b^2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (a^
4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3
*C)*d))/(b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = -\frac{(Ab^2-a(bB-aC))\,(bc-ad)}{2b^2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}+\cdots$$

$$= -\frac{(Ab^2-a(bB-aC))\,(bc-ad)}{2b^2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}-\cdots$$

$$= \frac{(a^3(Ac-cC-Bd)-3ab^2(Ac-cC-\cdots}{\cdots}$$

$$= \frac{(a^3(Ac-cC-Bd)-3ab^2(Ac-cC-\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.62, size = 331, normalized size = 1.03

$$\frac{\frac{bcC-bBd-aCd}{(a+b\tan(e+fx))^2}-\frac{2bC(c+d\tan(e+fx))}{(a+b\tan(e+fx))^2}+2b(Bc+(A-C)d)\left(-\frac{i\log(i-\tan(e+fx))}{2(a+ib)^2}+\frac{i\log(i+\tan(e+fx))}{2(a-ib)^2}+\frac{b\left(2a\log(a+b\tan(e+fx))-\frac{a^2+b^2}{a+b\tan(e+fx)}\right)}{(a^2+b^2)^2}\right)-b(-Abc+aBc+bcC+aAd+bBd-aCd)\left(\frac{\log(i-\tan(e+fx))}{(-ia+b)^3}+\frac{\log(i+\tan(e+fx))}{(ia+b)^3}+\frac{b\left((6a^2-2b^2)\log(a+b\tan(e+fx))-\frac{(a^2+b^2)(5a^2+b^2+4ab\tan(e+fx))}{(a+b\tan(e+fx))^2}\right)}{(a^2+b^2)^3}\right)}{2b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^3,x]
```

```
[Out] ((b*c*C - b*B*d - a*C*d)/(a + b*Tan[e + f*x])^2 - (2*b*C*(c + d*Tan[e + f*x
]))/(a + b*Tan[e + f*x])^2 + 2*b*(B*c + (A - C)*d)*(((-1/2*I)*Log[I - Tan[e
 + f*x]])/(a + I*b)^2 + ((I/2)*Log[I + Tan[e + f*x]])/(a - I*b)^2 + (b*(2*a
*Log[a + b*Tan[e + f*x]] - (a^2 + b^2)/(a + b*Tan[e + f*x])))/(a^2 + b^2)^2
) - b*(-(A*b*c) + a*B*c + b*c*C + a*A*d + b*B*d - a*C*d)*(Log[I - Tan[e + f
*x]]/((-I)*a + b)^3 + Log[I + Tan[e + f*x]]/(I*a + b)^3 + (b*((6*a^2 - 2*b^
2)*Log[a + b*Tan[e + f*x]] - ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[e + f*x]
))/(a + b*Tan[e + f*x])^2))/(a^2 + b^2)^3))/(2*b^2*f)
```

Maple [A]
time = 0.30, size = 494, normalized size = 1.54

method	result
derivativedivides	$-\frac{-A a b^2 d+A b^3 c+B a^2 b d-B a b^2 c-a^3 C d+C a^2 b c}{2b^2\left(a^2+b^2\right)(a+b\tan(fx+e))^2}-\frac{-A a^2 b^2 d+2A a b^3 c+A b^4 d-B a^2 b^2 c-2B a b^3 d+B b^4 c+a^4 C d+3C a^2 b^2 d-2C a}{\left(a^2+b^2\right)^2 b^2(a+b\tan(fx+e))}$
default	$-\frac{-A a b^2 d+A b^3 c+B a^2 b d-B a b^2 c-a^3 C d+C a^2 b c}{2b^2\left(a^2+b^2\right)(a+b\tan(fx+e))^2}-\frac{-A a^2 b^2 d+2A a b^3 c+A b^4 d-B a^2 b^2 c-2B a b^3 d+B b^4 c+a^4 C d+3C a^2 b^2 d-2C a}{\left(a^2+b^2\right)^2 b^2(a+b\tan(fx+e))}$
norman	$\frac{\left(A a^2 b^2 d-2A a b^3 c-A b^4 d+B a^2 b^2 c+2B a b^3 d-B b^4 c-a^4 C d-3C a^2 b^2 d+2C a b^3 c\right)\tan(fx+e)}{f b\left(a^4+2a^2 b^2+b^4\right)}+\frac{\left(A a^3 c+3A a^2 b d-3A a b^2 c-A b^3 d-\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*(-A*a*b^2*d+A*b^3*c+B*a^2*b*d-B*a*b^2*c-C*a^3*d+C*a^2*b*c)/b^2/(a
^2+b^2)/(a+b*tan(f*x+e))^2-(-A*a^2*b^2*d+2*A*a*b^3*c+A*b^4*d-B*a^2*b^2*c-2*
B*a*b^3*d+B*b^4*c+C*a^4*d+3*C*a^2*b^2*d-2*C*a*b^3*c)/(a^2+b^2)^2/b^2/(a+b*t
an(f*x+e))-(A*a^3*d-3*A*a^2*b*c-3*A*a*b^2*d+A*b^3*c+B*a^3*c+3*B*a^2*b*d-3*B
*a*b^2*c-B*b^3*d-C*a^3*d+3*C*a^2*b*c+3*C*a*b^2*d-C*b^3*c)/(a^2+b^2)^3*ln(a+
b*tan(f*x+e))+1/(a^2+b^2)^3*(1/2*(A*a^3*d-3*A*a^2*b*c-3*A*a*b^2*d+A*b^3*c+B
*a^3*c+3*B*a^2*b*d-3*B*a*b^2*c-B*b^3*d-C*a^3*d+3*C*a^2*b*c+3*C*a*b^2*d-C*b^
3*c)*ln(1+tan(f*x+e)^2)+(A*a^3*c+3*A*a^2*b*d-3*A*a*b^2*c-A*b^3*d-B*a^3*d+3*
B*a^2*b*c+3*B*a*b^2*d-B*b^3*c-C*a^3*c-3*C*a^2*b*d+3*C*a*b^2*c+C*b^3*d)*arct
an(tan(f*x+e))))
```

Maxima [A]
time = 0.53, size = 580, normalized size = 1.81

$$\frac{\frac{2\left(\left((A-C)a^3+3Ba^2b-3(A-C)ab^2-Bb^3\right)c-\left(Ba^3-3(A-C)a^2b-3Bab^2+(A-C)b^3\right)d\right)(fx+e)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2\left(\left(Ba^3-3(A-C)a^2b-3Bab^2+(A-C)b^3\right)c+\left((A-C)a^3+3Ba^2b-3(A-C)ab^2-Bb^3\right)d\right)\log\left(b\tan(fx+e)+a\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(\left(Ba^3-3(A-C)a^2b-3Bab^2+(A-C)b^3\right)c+\left((A-C)a^3+3Ba^2b-3(A-C)ab^2-Bb^3\right)d\right)\log\left(\tan(fx+e)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(Ca^4b-3Ba^3b^2+(5A-3C)a^2b^3+Bab^4+Ab^5\right)c+\left(Ca^5+Ba^4b-(3A-5C)a^3b^2-3Ba^2b^3+Aab^4\right)d-2\left(\left(Ba^2b^3-2(A-C)ab^4-Bb^5\right)c-\left(Ca^4b-(A-3C)a^2b^3-2Bab^4+Ab^5\right)d\right)\tan(fx+e)}{a^6b^2+2a^4b^4+a^2b^6+\left(a^4b^4+2a^2b^6+b^8\right)\tan(fx+e)^2+2\left(a^5b^3+2a^3b^5+ab^7\right)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c - (B*a^3 - 3*
(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6) - 2*((B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c +
 ((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*d)*log(b*tan(f*x + e)
+ a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((B*a^3 - 3*(A - C)*a^2*b - 3*B*
a*b^2 + (A - C)*b^3)*c + ((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3
)*d)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((C*a^4*
b - 3*B*a^3*b^2 + (5*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c + (C*a^5 + B*a^4
*b - (3*A - 5*C)*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*d - 2*((B*a^2*b^3 - 2*(A
- C)*a*b^4 - B*b^5)*c - (C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*b^4 + A*b^5)*d
)*tan(f*x + e))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*b^4 + 2*a^2*b^6 + b^8
)*tan(f*x + e)^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(320) = 640$.
time = 7.21, size = 996, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a^5 + 3*B*a^4*b - 3*(A - C)*a^3*b^2 - B*a^2*b^3)*c - (B*a^
5 - 3*(A - C)*a^4*b - 3*B*a^3*b^2 + (A - C)*a^2*b^3)*d)*f*x + (2*(((A - C)*
a^3*b^2 + 3*B*a^2*b^3 - 3*(A - C)*a*b^4 - B*b^5)*c - (B*a^3*b^2 - 3*(A - C)
*a^2*b^3 - 3*B*a*b^4 + (A - C)*b^5)*d)*f*x + (C*a^4*b - 3*B*a^3*b^2 + 5*(A
- C)*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c + (C*a^5 + B*a^4*b - (3*A - 7*C)*a^3*b^
2 - 5*B*a^2*b^3 + 3*A*a*b^4)*d)*tan(f*x + e)^2 - (3*C*a^4*b - 5*B*a^3*b^2 +
 (7*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c + (C*a^5 - 3*B*a^4*b + 5*(A - C)*
a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*d - (((B*a^3*b^2 - 3*(A - C)*a^2*b^3 - 3*B
*a*b^4 + (A - C)*b^5)*c + ((A - C)*a^3*b^2 + 3*B*a^2*b^3 - 3*(A - C)*a*b^4
- B*b^5)*d)*tan(f*x + e)^2 + (B*a^5 - 3*(A - C)*a^4*b - 3*B*a^3*b^2 + (A -
C)*a^2*b^3)*c + ((A - C)*a^5 + 3*B*a^4*b - 3*(A - C)*a^3*b^2 - B*a^2*b^3)*d
 + 2*((B*a^4*b - 3*(A - C)*a^3*b^2 - 3*B*a^2*b^3 + (A - C)*a*b^4)*c + ((A -
 C)*a^4*b + 3*B*a^3*b^2 - 3*(A - C)*a^2*b^3 - B*a*b^4)*d)*tan(f*x + e))*log
((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) + 2*
(2*(((A - C)*a^4*b + 3*B*a^3*b^2 - 3*(A - C)*a^2*b^3 - B*a*b^4)*c - (B*a^4*
b - 3*(A - C)*a^3*b^2 - 3*B*a^2*b^3 + (A - C)*a*b^4)*d)*f*x + (C*a^5 - 2*B*
a^4*b + 3*(A - C)*a^3*b^2 + 3*B*a^2*b^3 - (3*A - 2*C)*a*b^4 - B*b^5)*c + (B
*a^5 - (2*A - 3*C)*a^4*b - 3*B*a^3*b^2 + 3*(A - C)*a^2*b^3 + 2*B*a*b^4 - A*
```

```
b^5)*d)*tan(f*x + e))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*f*tan(f*x +
e)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*f*tan(f*x + e) + (a^8 + 3*
a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e)
)**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(320) = 640$.
time = 0.95, size = 1037, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c - C*a^3*c + 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c - B*b^3
*c - B*a^3*d + 3*A*a^2*b*d - 3*C*a^2*b*d + 3*B*a*b^2*d - A*b^3*d + C*b^3*d)
*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (B*a^3*c - 3*A*a^2*b*c + 3
*C*a^2*b*c - 3*B*a*b^2*c + A*b^3*c - C*b^3*c + A*a^3*d - C*a^3*d + 3*B*a^2*
b*d - 3*A*a*b^2*d + 3*C*a*b^2*d - B*b^3*d)*log(tan(f*x + e)^2 + 1)/(a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b*c - 3*A*a^2*b^2*c + 3*C*a^2*b^2*c
- 3*B*a*b^3*c + A*b^4*c - C*b^4*c + A*a^3*b*d - C*a^3*b*d + 3*B*a^2*b^2*d -
 3*A*a*b^3*d + 3*C*a*b^3*d - B*b^4*d)*log(abs(b*tan(f*x + e) + a))/(a^6*b +
 3*a^4*b^3 + 3*a^2*b^5 + b^7) + (3*B*a^3*b^4*c*tan(f*x + e)^2 - 9*A*a^2*b^5
*c*tan(f*x + e)^2 + 9*C*a^2*b^5*c*tan(f*x + e)^2 - 9*B*a*b^6*c*tan(f*x + e)
^2 + 3*A*b^7*c*tan(f*x + e)^2 - 3*C*b^7*c*tan(f*x + e)^2 + 3*A*a^3*b^4*d*ta
n(f*x + e)^2 - 3*C*a^3*b^4*d*tan(f*x + e)^2 + 9*B*a^2*b^5*d*tan(f*x + e)^2
- 9*A*a*b^6*d*tan(f*x + e)^2 + 9*C*a*b^6*d*tan(f*x + e)^2 - 3*B*b^7*d*tan(f
*x + e)^2 + 8*B*a^4*b^3*c*tan(f*x + e) - 22*A*a^3*b^4*c*tan(f*x + e) + 22*C
*a^3*b^4*c*tan(f*x + e) - 18*B*a^2*b^5*c*tan(f*x + e) + 2*A*a*b^6*c*tan(f*x
 + e) - 2*C*a*b^6*c*tan(f*x + e) - 2*B*b^7*c*tan(f*x + e) - 2*C*a^6*b*d*tan
(f*x + e) + 8*A*a^4*b^3*d*tan(f*x + e) - 14*C*a^4*b^3*d*tan(f*x + e) + 22*B
*a^3*b^4*d*tan(f*x + e) - 18*A*a^2*b^5*d*tan(f*x + e) + 12*C*a^2*b^5*d*tan(
f*x + e) - 2*B*a*b^6*d*tan(f*x + e) - 2*A*b^7*d*tan(f*x + e) - C*a^6*b*c +
6*B*a^5*b^2*c - 14*A*a^4*b^3*c + 11*C*a^4*b^3*c - 7*B*a^3*b^4*c - 3*A*a^2*b
```

```
^5*c - B*a*b^6*c - A*b^7*c - C*a^7*d - B*a^6*b*d + 6*A*a^5*b^2*d - 9*C*a^5*
b^2*d + 11*B*a^4*b^3*d - 7*A*a^3*b^4*d + 4*C*a^3*b^4*d - A*a*b^6*d)/((a^6*b
^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(f*x + e) + a)^2))/f
```

Mupad [B]
time = 15.89, size = 502, normalized size = 1.57

$$-\frac{\frac{A\,b^5\,c+C\,a^5\,d+A\,a\,b^4\,d+B\,a\,b^4\,c+B\,a^4\,b\,d+C\,a^4\,b\,c+5\,A\,a^2\,b^3\,c-3\,A\,a^3\,b^2\,d-3\,B\,a^3\,b^2\,c-3\,B\,a^2\,b^3\,d-3\,C\,a^2\,b^3\,c+5\,C\,a^3\,b^2\,d}{2\,b^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan\left(e+f\,x\right)\,\left(A\,b^4\,d+B\,b^4\,c+C\,a^4\,d+2\,A\,a\,b^3\,c-2\,B\,a\,b^3\,d-2\,C\,a\,b^3\,c-A\,a^2\,b^2\,d-B\,a^2\,b^2\,c+3\,C\,a^2\,b^2\,d\right)}{b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{f\,\left(a^2+2\,a\,b\,\tan\left(e+f\,x\right)+b^2\,{\tan\left(e+f\,x\right)}^2\right)}-\frac{\ln\left(\tan\left(e+f\,x\right)+1{}\mathrm{i}\right)\,\left(B\,d+A\,d\,1{}\mathrm{i}+B\,c\,1{}\mathrm{i}-A\,c+C\,c-C\,d\,1{}\mathrm{i}\right)}{2\,f\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}-\frac{\ln\left(\tan\left(e+f\,x\right)-\mathrm{i}\right)\,\left(A\,d+B\,c-C\,d-A\,c\,1{}\mathrm{i}+B\,d\,1{}\mathrm{i}+C\,c\,1{}\mathrm{i}\right)}{2\,f\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}-\frac{\ln\left(a+b\,\tan\left(e+f\,x\right)\right)\,\left(\left(A\,d+B\,c-C\,d\right)\,a^3+\left(3\,B\,d-3\,A\,c+3\,C\,c\right)\,a^2\,b+\left(3\,C\,d-3\,B\,c-3\,A\,d\right)\,a\,b^2+\left(A\,c-B\,d-C\,c\right)\,b^3\right)}{f\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b*t
an(e + f*x))^3,x)
```

```
[Out] - ((A*b^5*c + C*a^5*d + A*a*b^4*d + B*a*b^4*c + B*a^4*b*d + C*a^4*b*c + 5*A
*a^2*b^3*c - 3*A*a^3*b^2*d - 3*B*a^3*b^2*c - 3*B*a^2*b^3*d - 3*C*a^2*b^3*c
+ 5*C*a^3*b^2*d)/(2*b^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(e + f*x)*(A*b^4*d +
 B*b^4*c + C*a^4*d + 2*A*a*b^3*c - 2*B*a*b^3*d - 2*C*a*b^3*c - A*a^2*b^2*d
- B*a^2*b^2*c + 3*C*a^2*b^2*d))/(b*(a^4 + b^4 + 2*a^2*b^2)))/(f*(a^2 + b^2*
tan(e + f*x)^2 + 2*a*b*tan(e + f*x))) - (log(tan(e + f*x) + 1i)*(A*d*1i - A
*c + B*c*1i + B*d + C*c - C*d*1i))/(2*f*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)
) - (log(tan(e + f*x) - 1i)*(A*d - A*c*1i + B*c + B*d*1i + C*c*1i - C*d))/(
2*f*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(a + b*tan(e + f*x))*(a^3*(A
*d + B*c - C*d) - b^3*(B*d - A*c + C*c) + a^2*b*(3*B*d - 3*A*c + 3*C*c) - a
*b^2*(3*A*d + 3*B*c - 3*C*d)))/(f*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
```

3.57 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\,(A+B\tan(e+$

Optimal. Leaf size=661

$$-((a^3(c^2C+2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C+2Bcd-Cd^2-A(c^2-d^2))+3a^2b(2c(A-C)d+$$

```
[Out] -(a^3*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-3*a*b^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2
-d^2))+3*a^2*b*(2*c*(A-C)*d+B*(c^2-d^2))-b^3*(2*c*(A-C)*d+B*(c^2-d^2)))*x+(
3*a^2*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b^3*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d
^2))-a^3*(2*c*(A-C)*d+B*(c^2-d^2))+3*a*b^2*(2*c*(A-C)*d+B*(c^2-d^2)))*ln(co
s(f*x+e))/f+d*(3*a^2*b*(A*c-B*d-C*c)-b^3*(A*c-B*d-C*c)+a^3*(B*c+(A-C)*d)-3*
a*b^2*(B*c+(A-C)*d))*tan(f*x+e)/f+1/2*(a^3*B-3*a*b^2*B+3*a^2*b*(A-C)-b^3*(A
-C))*(c+d*tan(f*x+e))^2/f+1/60*(4*a^3*C*d^3-3*a^2*b*d^2*(-16*B*d+3*C*c)+3*a
*b^2*d*(2*c^2*C-5*B*c*d+20*(A-C)*d^2)-b^3*(c^3*C-2*B*c^2*d+5*c*(A-C)*d^2+20
*B*d^3))*(c+d*tan(f*x+e))^3/d^4/f+1/20*b*(5*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*
(-2*B*b*d-C*a*d+C*b*c))*tan(f*x+e)*(c+d*tan(f*x+e))^3/d^3/f-1/10*(-2*B*b*d-
C*a*d+C*b*c)*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3/d^2/f+1/6*C*(a+b*tan(f*x
+e))^3*(c+d*tan(f*x+e))^3/d/f
```

Rubi [A]
time = 1.43, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3711, 3609, 3606, 3556}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] -((a^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C + 2*B*c*d
 - C*d^2 - A*(c^2 - d^2)) + 3*a^2*b*(2*c*(A - C)*d + B*(c^2 - d^2)) - b^3*(
2*c*(A - C)*d + B*(c^2 - d^2)))*x) + ((3*a^2*b*(c^2*C + 2*B*c*d - C*d^2 - A
*(c^2 - d^2)) - b^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^3*(2*c*(A
 - C)*d + B*(c^2 - d^2)) + 3*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos
[e + f*x]])/f + (d*(3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^3
*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))*Tan[e + f*x])/f + ((a^3*B -
 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C))*(c + d*Tan[e + f*x])^2)/(2*f) +
 ((4*a^3*C*d^3 - 3*a^2*b*d^2*(3*c*C - 16*B*d) + 3*a*b^2*d*(2*c^2*C - 5*B*c*
d + 20*(A - C)*d^2) - b^3*(c^3*C - 2*B*c^2*d + 5*c*(A - C)*d^2 + 20*B*d^3))
*(c + d*Tan[e + f*x])^3)/(60*d^4*f) + (b*(5*b*(A*b + a*B - b*C)*d^2 + (b*c
- a*d)*(b*c*C - 2*b*B*d - a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(20*
d^3*f) - ((b*c*C - 2*b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f
*x])^3)/(10*d^2*f) + (C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3)/(6*d
*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
```

```
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C(a+b\tan(e+f \ldots}{\ldots}$$

$$= -\frac{(bcC-2bBd- \ldots}{\ldots}$$

$$= \frac{b(5b(Ab+aB-b \ldots}{\ldots}$$

$$= \frac{(4a^3Cd^3-3a^2bd^2 \ldots}{\ldots}$$

$$= \frac{(a^3B-3ab^2B+3 \ldots}{\ldots}$$

$$= -\left(a^3\left(c^2C+2Bcd \ldots\right.\right.$$

$$= -\left(a^3\left(c^2C+2Bcd \ldots\right.\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.45, size = 573, normalized size = 0.87

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df}+\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3)/(6*d*f) + ((-3*(b*c*C - 2
*b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f) + ((
3*b*(5*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*d))*Tan
[e + f*x]*(c + d*Tan[e + f*x])^3)/(2*d*f) - (((-24*a*d*(5*b*(A*b + a*B - b*
C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*d)) + b*(-120*(a^2*B - b^2*B +
2*a*b*(A - C))*d^3 + 6*c*(5*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C -
2*b*B*d - a*C*d))))*(c + d*Tan[e + f*x])^3)/(3*d*f) - (60*(d^2*(3*a^2*b*(A*
c - c*C + B*d) - b^3*(A*c - c*C + B*d) + a^3*(B*c - (A - C)*d) - 3*a*b^2*(B
```

```
*c - (A - C)*d))*(I*(c + I*d)^2*Log[I - Tan[e + f*x]] - I*(c - I*d)^2*Log[I
 + Tan[e + f*x]] - 2*d^2*Tan[e + f*x]) + (a^3*B - 3*a*b^2*B + 3*a^2*b*(A -
C) - b^3*(A - C))*d^2*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[
I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2)))/f)/(4*d))/
(5*d))/(6*d)
```

Maple [A]
time = 0.24, size = 1239, normalized size = 1.87

method	result
norman	$(A\,a^3c^2 - A\,a^3d^2 - 6A\,a^2bcd - 3Aa\,b^2c^2 + 3Aa\,b^2d^2 + 2A\,b^3cd - 2B\,a^3cd - 3B\,a^2b\,c^2 +$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(3*A*a*b^2*c*d*tan(f*x+e)^2+3*B*a^2*b*c*d*tan(f*x+e)^2-3*C*a*b^2*c*d*ta
n(f*x+e)^2+3/2*C*a*b^2*c*d*tan(f*x+e)^4+2*B*a*b^2*c*d*tan(f*x+e)^3+2*C*a^2*
b*c*d*tan(f*x+e)^3+6*A*a^2*b*c*d*tan(f*x+e)-6*B*a*b^2*c*d*tan(f*x+e)-6*C*a^
2*b*c*d*tan(f*x+e)+3*B*a^2*b*c^2*tan(f*x+e)-3*B*a^2*b*d^2*tan(f*x+e)-3*C*a*
b^2*c^2*tan(f*x+e)+3*C*a*b^2*d^2*tan(f*x+e)+2*C*b^3*c*d*tan(f*x+e)+1/2*(2*A
*a^3*c*d+3*A*a^2*b*c^2-3*A*a^2*b*d^2-6*A*a*b^2*c*d-A*b^3*c^2+A*b^3*d^2+B*a^
3*c^2-B*a^3*d^2-6*B*a^2*b*c*d-3*B*a*b^2*c^2+3*B*a*b^2*d^2+2*B*b^3*c*d-2*C*a
^3*c*d-3*C*a^2*b*c^2+3*C*a^2*b*d^2+6*C*a*b^2*c*d+C*b^3*c^2-C*b^3*d^2)*ln(1+
tan(f*x+e)^2)+(A*a^3*c^2-A*a^3*d^2-6*A*a^2*b*c*d-3*A*a*b^2*c^2+3*A*a*b^2*d^
2+2*A*b^3*c*d-2*B*a^3*c*d-3*B*a^2*b*c^2+3*B*a^2*b*d^2+6*B*a*b^2*c*d+B*b^3*c
^2-B*b^3*d^2-C*a^3*c^2+C*a^3*d^2+6*C*a^2*b*c*d+3*C*a*b^2*c^2-3*C*a*b^2*d^2-
2*C*b^3*c*d)*arctan(tan(f*x+e))+1/4*A*b^3*d^2*tan(f*x+e)^4+1/4*C*b^3*c^2*ta
n(f*x+e)^4-1/4*C*b^3*d^2*tan(f*x+e)^4+1/3*B*b^3*c^2*tan(f*x+e)^3-1/3*B*b^3*
d^2*tan(f*x+e)^3+1/3*C*a^3*d^2*tan(f*x+e)^3+1/2*A*b^3*c^2*tan(f*x+e)^2-1/2*
A*b^3*d^2*tan(f*x+e)^2+1/2*B*a^3*d^2*tan(f*x+e)^2-1/2*C*b^3*c^2*tan(f*x+e)^
2+1/2*C*b^3*d^2*tan(f*x+e)^2+1/6*C*b^3*d^2*tan(f*x+e)^6+A*a^3*d^2*tan(f*x+e
)-B*b^3*c^2*tan(f*x+e)+B*b^3*d^2*tan(f*x+e)+C*a^3*c^2*tan(f*x+e)-C*a^3*d^2*
tan(f*x+e)+1/5*B*b^3*d^2*tan(f*x+e)^5+C*a^3*c*d*tan(f*x+e)^2+A*a*b^2*d^2*ta
n(f*x+e)^3+B*a^2*b*d^2*tan(f*x+e)^3+C*a*b^2*c^2*tan(f*x+e)^3+3/5*C*a*b^2*d^
2*tan(f*x+e)^5+2/5*C*b^3*c*d*tan(f*x+e)^5+3/4*B*a*b^2*d^2*tan(f*x+e)^4+1/2*
B*b^3*c*d*tan(f*x+e)^4+3/4*C*a^2*b*d^2*tan(f*x+e)^4+2/3*A*b^3*c*d*tan(f*x+e
)^3-C*a*b^2*d^2*tan(f*x+e)^3-2/3*C*b^3*c*d*tan(f*x+e)^3+3/2*A*a^2*b*d^2*tan
(f*x+e)^2+3/2*B*a*b^2*c^2*tan(f*x+e)^2-3/2*B*a*b^2*d^2*tan(f*x+e)^2-B*b^3*c
*d*tan(f*x+e)^2+3/2*C*a^2*b*c^2*tan(f*x+e)^2-3/2*C*a^2*b*d^2*tan(f*x+e)^2+3
```

```
*A*a*b^2*c^2*tan(f*x+e)-3*A*a*b^2*d^2*tan(f*x+e)-2*A*b^3*c*d*tan(f*x+e)+2*B
*a^3*c*d*tan(f*x+e))
```

Maxima [A]
time = 0.51, size = 699, normalized size = 1.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(10*C*b^3*d^2*tan(f*x + e)^6 + 12*(2*C*b^3*c*d + (3*C*a*b^2 + B*b^3)*d
^2)*tan(f*x + e)^5 + 15*(C*b^3*c^2 + 2*(3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2*b
 + 3*B*a*b^2 + (A - C)*b^3)*d^2)*tan(f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c
^2 + 2*(3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c*d + (C*a^3 + 3*B*a^2*b + 3*(
A - C)*a*b^2 - B*b^3)*d^2)*tan(f*x + e)^3 + 30*((3*C*a^2*b + 3*B*a*b^2 + (A
 - C)*b^3)*c^2 + 2*(C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c*d + (B*a
^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*tan(f*x + e)^2 + 60*((
(A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^2 - 2*(B*a^3 + 3*(A -
C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2*b - 3*(A -
 C)*a*b^2 + B*b^3)*d^2)*(f*x + e) + 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^
2 - (A - C)*b^3)*c^2 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3
)*c*d - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*log(tan(f*
x + e)^2 + 1) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c^2 + 2*(
B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d + ((A - C)*a^3 - 3*B
*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*tan(f*x + e))/f
```

Fricas [A]
time = 7.55, size = 697, normalized size = 1.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*b^3*d^2*tan(f*x + e)^6 + 12*(2*C*b^3*c*d + (3*C*a*b^2 + B*b^3)*d
^2)*tan(f*x + e)^5 + 15*(C*b^3*c^2 + 2*(3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2*b
 + 3*B*a*b^2 + (A - C)*b^3)*d^2)*tan(f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c
^2 + 2*(3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c*d + (C*a^3 + 3*B*a^2*b + 3*(
A - C)*a*b^2 - B*b^3)*d^2)*tan(f*x + e)^3 + 60*(((A - C)*a^3 - 3*B*a^2*b -
3*(A - C)*a*b^2 + B*b^3)*c^2 - 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A
- C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*f*
x + 30*((3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c^2 + 2*(C*a^3 + 3*B*a^2*b +
3*(A - C)*a*b^2 - B*b^3)*c*d + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A -
```

```
C)*b^3)*d^2)*tan(f*x + e)^2 - 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A
 - C)*b^3)*c^2 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d
- (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*log(1/(tan(f*x +
 e)^2 + 1)) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c^2 + 2*(B*
a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d + ((A - C)*a^3 - 3*B*a
^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(604) = 1208$.
time = 0.50, size = 1819, normalized size = 2.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Piecewise((A*a**3*c**2*x + A*a**3*c*d*log(tan(e + f*x)**2 + 1)/f - A*a**3*d
**2*x + A*a**3*d**2*tan(e + f*x)/f + 3*A*a**2*b*c**2*log(tan(e + f*x)**2 +
1)/(2*f) - 6*A*a**2*b*c*d*x + 6*A*a**2*b*c*d*tan(e + f*x)/f - 3*A*a**2*b*d*
*2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*a**2*b*d**2*tan(e + f*x)**2/(2*f) -
 3*A*a*b**2*c**2*x + 3*A*a*b**2*c**2*tan(e + f*x)/f - 3*A*a*b**2*c*d*log(ta
n(e + f*x)**2 + 1)/f + 3*A*a*b**2*c*d*tan(e + f*x)**2/f + 3*A*a*b**2*d**2*x
 + A*a*b**2*d**2*tan(e + f*x)**3/f - 3*A*a*b**2*d**2*tan(e + f*x)/f - A*b**
3*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*c**2*tan(e + f*x)**2/(2*f) +
 2*A*b**3*c*d*x + 2*A*b**3*c*d*tan(e + f*x)**3/(3*f) - 2*A*b**3*c*d*tan(e +
 f*x)/f + A*b**3*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*d**2*tan(e +
f*x)**4/(4*f) - A*b**3*d**2*tan(e + f*x)**2/(2*f) + B*a**3*c**2*log(tan(e +
 f*x)**2 + 1)/(2*f) - 2*B*a**3*c*d*x + 2*B*a**3*c*d*tan(e + f*x)/f - B*a**3
*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*a**3*d**2*tan(e + f*x)**2/(2*f) -
3*B*a**2*b*c**2*x + 3*B*a**2*b*c**2*tan(e + f*x)/f - 3*B*a**2*b*c*d*log(tan
(e + f*x)**2 + 1)/f + 3*B*a**2*b*c*d*tan(e + f*x)**2/f + 3*B*a**2*b*d**2*x
+ B*a**2*b*d**2*tan(e + f*x)**3/f - 3*B*a**2*b*d**2*tan(e + f*x)/f - 3*B*a*
b**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*b**2*c**2*tan(e + f*x)**2/
(2*f) + 6*B*a*b**2*c*d*x + 2*B*a*b**2*c*d*tan(e + f*x)**3/f - 6*B*a*b**2*c*
d*tan(e + f*x)/f + 3*B*a*b**2*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*b
**2*d**2*tan(e + f*x)**4/(4*f) - 3*B*a*b**2*d**2*tan(e + f*x)**2/(2*f) + B*
b**3*c**2*x + B*b**3*c**2*tan(e + f*x)**3/(3*f) - B*b**3*c**2*tan(e + f*x)/
f + B*b**3*c*d*log(tan(e + f*x)**2 + 1)/f + B*b**3*c*d*tan(e + f*x)**4/(2*f
) - B*b**3*c*d*tan(e + f*x)**2/f - B*b**3*d**2*x + B*b**3*d**2*tan(e + f*x)
**5/(5*f) - B*b**3*d**2*tan(e + f*x)**3/(3*f) + B*b**3*d**2*tan(e + f*x)/f
- C*a**3*c**2*x + C*a**3*c**2*tan(e + f*x)/f - C*a**3*c*d*log(tan(e + f*x)*
*2 + 1)/f + C*a**3*c*d*tan(e + f*x)**2/f + C*a**3*d**2*x + C*a**3*d**2*tan(
e + f*x)**3/(3*f) - C*a**3*d**2*tan(e + f*x)/f - 3*C*a**2*b*c**2*log(tan(e
+ f*x)**2 + 1)/(2*f) + 3*C*a**2*b*c**2*tan(e + f*x)**2/(2*f) + 6*C*a**2*b*c
```

```
*d*x + 2*C*a**2*b*c*d*tan(e + f*x)**3/f - 6*C*a**2*b*c*d*tan(e + f*x)/f + 3
*C*a**2*b*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a**2*b*d**2*tan(e + f*x
)**4/(4*f) - 3*C*a**2*b*d**2*tan(e + f*x)**2/(2*f) + 3*C*a*b**2*c**2*x + C*
a*b**2*c**2*tan(e + f*x)**3/f - 3*C*a*b**2*c**2*tan(e + f*x)/f + 3*C*a*b**2
*c*d*log(tan(e + f*x)**2 + 1)/f + 3*C*a*b**2*c*d*tan(e + f*x)**4/(2*f) - 3*
C*a*b**2*c*d*tan(e + f*x)**2/f - 3*C*a*b**2*d**2*x + 3*C*a*b**2*d**2*tan(e
+ f*x)**5/(5*f) - C*a*b**2*d**2*tan(e + f*x)**3/f + 3*C*a*b**2*d**2*tan(e +
 f*x)/f + C*b**3*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**3*c**2*tan(e +
f*x)**4/(4*f) - C*b**3*c**2*tan(e + f*x)**2/(2*f) - 2*C*b**3*c*d*x + 2*C*b*
*3*c*d*tan(e + f*x)**5/(5*f) - 2*C*b**3*c*d*tan(e + f*x)**3/(3*f) + 2*C*b**
3*c*d*tan(e + f*x)/f - C*b**3*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**3*
d**2*tan(e + f*x)**6/(6*f) - C*b**3*d**2*tan(e + f*x)**4/(4*f) + C*b**3*d**
2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e))**3*(c + d*tan(e))**2*
(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 24014 vs. $2(660) = 1320$.
time = 18.53, size = 24014, normalized size = 36.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] 1/60*(60*A*a^3*c^2*f*x*tan(f*x)^6*tan(e)^6 - 60*C*a^3*c^2*f*x*tan(f*x)^6*ta
n(e)^6 - 180*B*a^2*b*c^2*f*x*tan(f*x)^6*tan(e)^6 - 180*A*a*b^2*c^2*f*x*tan(
f*x)^6*tan(e)^6 + 180*C*a*b^2*c^2*f*x*tan(f*x)^6*tan(e)^6 + 60*B*b^3*c^2*f*
x*tan(f*x)^6*tan(e)^6 - 120*B*a^3*c*d*f*x*tan(f*x)^6*tan(e)^6 - 360*A*a^2*b
*c*d*f*x*tan(f*x)^6*tan(e)^6 + 360*C*a^2*b*c*d*f*x*tan(f*x)^6*tan(e)^6 + 36
0*B*a*b^2*c*d*f*x*tan(f*x)^6*tan(e)^6 + 120*A*b^3*c*d*f*x*tan(f*x)^6*tan(e)
^6 - 120*C*b^3*c*d*f*x*tan(f*x)^6*tan(e)^6 - 60*A*a^3*d^2*f*x*tan(f*x)^6*ta
n(e)^6 + 60*C*a^3*d^2*f*x*tan(f*x)^6*tan(e)^6 + 180*B*a^2*b*d^2*f*x*tan(f*x
)^6*tan(e)^6 + 180*A*a*b^2*d^2*f*x*tan(f*x)^6*tan(e)^6 - 180*C*a*b^2*d^2*f*
x*tan(f*x)^6*tan(e)^6 - 60*B*b^3*d^2*f*x*tan(f*x)^6*tan(e)^6 - 30*B*a^3*c^2
*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + t
an(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 90
*A*a^2*b*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*
tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*t
an(e)^6 + 90*C*a^2*b*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
 tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*
tan(f*x)^6*tan(e)^6 + 90*B*a*b^2*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x
)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan
(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 30*A*b^3*c^2*log(4*(tan(f*x)^4*tan(e)^2 -
 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
```

+ 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 30*C*b^3*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 60*A*a^3*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 60*C*a^3*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 180*B*a^2*b*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 180*A*a*b^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 180*C*a*b^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 60*B*b^3*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 30*B*a^3*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 90*A*a^2*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 90*C*a^2*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 90*B*a*b^2*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 30*A*b^3*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 30*C*b^3*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 360*A*a^3*c^2*f*x*tan(f*x)^5*tan(e)^5 + 360*C*a^3*c^2*f*x*tan(f*x)^5*tan(e)^5 + 1080*B*a^2*b*c^2*f*x*tan(f*x)^5*tan(e)^5 + 1080*A*a*b^2*c^2*f*x*tan(f*x)^5*tan(e)^5 - 1080*C*a*b^2*c^2*f*x*tan(f*x)^5*tan(e)^5 - 360*B*b^3*c^2*f*x*tan(f*x)^5*tan(e)^5 + 720*B*a^3*c*d*f*x*tan(f*x)^5*tan(e)^5 + 2160*A*a^2*b*c*d*f*x*tan(f*x)^5*tan(e)^5 - 2160*C*a^2*b*c*d*f*x*tan(f*x)^5*tan(e)^5 - 2160*B*a*b^2*c*d*f*x*tan(f*x)^5*tan(e)^5 - 720*A*b^3*c*d*f*x*tan(f*x)^5*tan(e)^5 + 720*C*b^3*c*d*f*x*tan(f*x)^5*tan(e)^5 + 360*A*a^3*d^2*f*x*tan(f*x)^5*tan(e)^5 - 360*C*a^3*d^2*f*x*tan(f*x)^5*tan(e)^5 - 1080*B*a^2*b*d^2*f*x*tan(f*x)^5*tan(e)^5 - 1080*A*a*b^2*d^2*f*x*tan(f*x)^5*tan(e)^5 + 1080*C*a*b^2*d^2*f*x*tan(f*x)^5*tan(e)^5 + 360*B*b^3*d^2*f*x*tan(f*x)^5*tan(e)^5 + 90*C*a^2*b*c^2*tan(f*x)^6*tan(e)^6 + 90*B*a*b^2*c^2*tan(f*x)^6*tan(e)^6 + 30*A*b^3*c^2*tan(f*x)^6*tan(e)^6 - 45*C*b^3*c^2*tan(f*x)^6*tan(e)^6 + 60*C*a^3*c*d*tan(f*x)^6*tan(e)^6 + 180*B*a^2*b*c*d*tan(f*x)^6*tan(e)^6 + 180*A*a*b^2*c*d*tan(f*x)^6*tan(e)^6 - 270*C*a*b^2*c*d*tan(f*x)^6*tan(e)^6 - 90*B*b^3*c*d*tan(f*x)^6*tan(e)^6 + 30*B*a^3*d^2*tan(f*x)^6*tan(e)^6 + 90*A*a^2*b*d^2*tan(f*x)^6*tan(e)^6 - 135*C*a^2*b*d^2*tan(f*x)^6*tan(e)^6 - 135*B*a*b^2*d^2*tan(f*x)^6*tan(e)^6 - 45*A*b^3*d^2*

```
tan(f*x)^6*tan(e)^6 + 55*C*b^3*d^2*tan(f*x)^6*tan(e)^6 + 180*B*a^3*c^2*log(
4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ...
```

Mupad [B]
time = 9.29, size = 891, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2),x)
```

```
[Out] x*(A*a^3*c^2 - A*a^3*d^2 + B*b^3*c^2 - C*a^3*c^2 - B*b^3*d^2 + C*a^3*d^2 +
2*A*b^3*c*d - 2*B*a^3*c*d - 2*C*b^3*c*d - 3*A*a*b^2*c^2 + 3*A*a*b^2*d^2 - 3
*B*a^2*b*c^2 + 3*B*a^2*b*d^2 + 3*C*a*b^2*c^2 - 3*C*a*b^2*d^2 - 6*A*a^2*b*c*
d + 6*B*a*b^2*c*d + 6*C*a^2*b*c*d) - (tan(e + f*x)*(B*b^3*c^2 - A*a^3*d^2 -
 b^2*d*(B*b*d + 3*C*a*d + 2*C*b*c) - C*a^3*c^2 + C*a^3*d^2 + 2*A*b^3*c*d -
2*B*a^3*c*d - 3*A*a*b^2*c^2 + 3*A*a*b^2*d^2 - 3*B*a^2*b*c^2 + 3*B*a^2*b*d^2
 + 3*C*a*b^2*c^2 - 6*A*a^2*b*c*d + 6*B*a*b^2*c*d + 6*C*a^2*b*c*d))/f - (log
(tan(e + f*x)^2 + 1)*((A*b^3*c^2)/2 - (B*a^3*c^2)/2 - (A*b^3*d^2)/2 + (B*a^
3*d^2)/2 - (C*b^3*c^2)/2 + (C*b^3*d^2)/2 - A*a^3*c*d - B*b^3*c*d + C*a^3*c*
d - (3*A*a^2*b*c^2)/2 + (3*A*a^2*b*d^2)/2 + (3*B*a*b^2*c^2)/2 - (3*B*a*b^2*
d^2)/2 + (3*C*a^2*b*c^2)/2 - (3*C*a^2*b*d^2)/2 + 3*A*a*b^2*c*d + 3*B*a^2*b*
c*d - 3*C*a*b^2*c*d))/f + (tan(e + f*x)^4*((A*b^3*d^2)/4 + (C*b^3*c^2)/4 -
(C*b^3*d^2)/4 + (B*b^3*c*d)/2 + (3*B*a*b^2*d^2)/4 + (3*C*a^2*b*d^2)/4 + (3*
C*a*b^2*c*d)/2))/f + (tan(e + f*x)^3*((B*b^3*c^2)/3 - (b^2*d*(B*b*d + 3*C*a
*d + 2*C*b*c))/3 + (C*a^3*d^2)/3 + (2*A*b^3*c*d)/3 + A*a*b^2*d^2 + B*a^2*b*
d^2 + C*a*b^2*c^2 + 2*B*a*b^2*c*d + 2*C*a^2*b*c*d))/f + (tan(e + f*x)^2*((A
*b^3*c^2)/2 - (A*b^3*d^2)/2 + (B*a^3*d^2)/2 - (C*b^3*c^2)/2 + (C*b^3*d^2)/2
 - B*b^3*c*d + C*a^3*c*d + (3*A*a^2*b*d^2)/2 + (3*B*a*b^2*c^2)/2 - (3*B*a*b
^2*d^2)/2 + (3*C*a^2*b*c^2)/2 - (3*C*a^2*b*d^2)/2 + 3*A*a*b^2*c*d + 3*B*a^2
*b*c*d - 3*C*a*b^2*c*d))/f + (b^2*d*tan(e + f*x)^5*(B*b*d + 3*C*a*d + 2*C*b
*c))/(5*f) + (C*b^3*d^2*tan(e + f*x)^6)/(6*f)
```

3.58 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2 \left(A+B\tan(e+\right.$

Optimal. Leaf size=443

$$-\left(\left(a^2\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b^2\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)+2ab\left(2c(A-C)d+B\left(c^2\right.\right.\right.\right.$$

```
[Out] -(a^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2
))+2*a*b*(2*c*(A-C)*d+B*(c^2-d^2)))*x+(2*a*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^
2))-a^2*(2*c*(A-C)*d+B*(c^2-d^2))+b^2*(2*c*(A-C)*d+B*(c^2-d^2)))*ln(cos(f*x
+e))/f+d*(2*a*b*(A*c-B*d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*d))*tan(f*x+
e)/f+1/2*(a^2*B-b^2*B+2*a*b*(A-C))*(c+d*tan(f*x+e))^2/f+1/60*(8*a^2*C*d^2-1
0*a*b*d*(-4*B*d+C*c)+b^2*(2*c^2*C-5*B*c*d+20*(A-C)*d^2))*(c+d*tan(f*x+e))^3
/d^3/f-1/20*b*(-5*B*b*d-2*C*a*d+2*C*b*c)*tan(f*x+e)*(c+d*tan(f*x+e))^3/d^2/
f+1/5*C*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3/d/f
```

Rubi [A]

time = 0.83, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3711, 3609, 3606, 3556}

$$\frac{\log(\cos(e+fx))\left(-\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+b^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)}{f}-x\left(a^2\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+2ab\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-b^2\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)\right)+\frac{(c+d\tan(e+fx))^3\left(8a^2Cd^2-10abd(cC-4Bd)+b^2\left(20d^2(A-C)-5Bcd+2c^2C\right)\right)}{60d^3f}+\frac{\left(a^2B+2ab(A-C)-b^2B\right)(c+d\tan(e+fx))^2}{2f}+\frac{d\tan(e+fx)\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}-\frac{b\tan(e+fx)(-2aCd-5bBd+2bcC)(c+d\tan(e+fx))^3}{20d^2f}+\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] -((a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d - C
*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x) + ((2*a*b
*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*(c^2 -
d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*x]])/f + (d*(2*a
*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*Tan[e
 + f*x])/f + ((a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x])^2)/(2*f)
 + ((8*a^2*C*d^2 - 10*a*b*d*(c*C - 4*B*d) + b^2*(2*c^2*C - 5*B*c*d + 20*(A
- C)*d^2))*(c + d*Tan[e + f*x])^3)/(60*d^3*f) - (b*(2*b*c*C - 5*b*B*d - 2*a
*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(20*d^2*f) + (C*(a + b*Tan[e + f
*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
```

```
f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C(a+b\tan(e+fx}{\ldots}$$

$$= -\frac{b(2bcC-5bBd-}{\ldots}$$

$$= \frac{(8a^2Cd^2-10abd(c}{\ldots}$$

$$= \frac{(a^2B-b^2B+2ab(}{\ldots}$$

$$= -\left(a^2\left(c^2C+2Bcd\right.\right.$$

$$= -\left(a^2\left(c^2C+2Bcd\right.\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.18, size = 352, normalized size = 0.79

$$\frac{(8a^2Cd^2+10abd(-cC+4Bd)+b^2(2c^2C-5Bcd+20(A-C)d^2))(c+d\tan(e+fx))^3+3bd(-2bcC+5bBd+2aCd)\tan(e+fx)(c+d\tan(e+fx))^3+12Cd^2(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3+30d(d(2ab(Ac-cC+Bd)+a^2(Bc+(-A+C)d)-b^2(Bc+(-A+C)d))(i((c+id)^2\log(i-\tan(e+fx))-(c-id)^2\log(i+\tan(e+fx)))-2d^2\tan(e+fx))+(a^2B-b^2B+2ab(A-C))d((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6cd^2\tan(e+fx)+d^3\tan^2(e+fx)))}{60d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] ((8*a^2*C*d^2 + 10*a*b*d*(-(c*C) + 4*B*d) + b^2*(2*c^2*C - 5*B*c*d + 20*(A
- C)*d^2))*(c + d*Tan[e + f*x])^3 + 3*b*d*(-2*b*c*C + 5*b*B*d + 2*a*C*d)*Ta
n[e + f*x]*(c + d*Tan[e + f*x])^3 + 12*C*d^2*(a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^3 + 30*d*(d*(2*a*b*(A*c - c*C + B*d) + a^2*(B*c + (-A + C)*d)
 - b^2*(B*c + (-A + C)*d))*(I*((c + I*d)^2*Log[I - Tan[e + f*x]] - (c - I*d
)^2*Log[I + Tan[e + f*x]]) - 2*d^2*Tan[e + f*x]) + (a^2*B - b^2*B + 2*a*b*(
A - C))*d*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[I + Tan[e +
f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2)))/(60*d^3*f)
```

Maple [A]
time = 0.18, size = 770, normalized size = 1.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*B*a*b*c*d*tan(f*x+e)^2+4/3*C*a*b*c*d*tan(f*x+e)^3-4*C*a*b*c*d*tan(f*
x+e)+4*A*a*b*c*d*tan(f*x+e)+(A*a^2*c^2-A*a^2*d^2-4*A*a*b*c*d-A*b^2*c^2+A*b^
2*d^2-2*B*a^2*c*d-2*B*a*b*c^2+2*B*a*b*d^2+2*B*b^2*c*d-C*a^2*c^2+C*a^2*d^2+4
```

```
*C*a*b*c*d+C*b^2*c^2-C*b^2*d^2)*arctan(tan(f*x+e))+1/2*(2*A*a^2*c*d+2*A*a*b
*c^2-2*A*a*b*d^2-2*A*b^2*c*d+B*a^2*c^2-B*a^2*d^2-4*B*a*b*c*d-B*b^2*c^2+B*b^
2*d^2-2*C*a^2*c*d-2*C*a*b*c^2+2*C*a*b*d^2+2*C*b^2*c*d)*ln(1+tan(f*x+e)^2)-A
*b^2*d^2*tan(f*x+e)-C*b^2*c^2*tan(f*x+e)+C*b^2*d^2*tan(f*x+e)-a^2*C*d^2*tan
(f*x+e)+1/3*A*b^2*d^2*tan(f*x+e)^3+1/3*C*a^2*d^2*tan(f*x+e)^3+1/3*C*b^2*c^2
*tan(f*x+e)^3+1/2*B*a^2*d^2*tan(f*x+e)^2+1/2*B*b^2*c^2*tan(f*x+e)^2-1/2*B*b
^2*d^2*tan(f*x+e)^2+1/5*C*b^2*d^2*tan(f*x+e)^5+A*a^2*d^2*tan(f*x+e)+A*b^2*c
^2*tan(f*x+e)+C*a^2*c^2*tan(f*x+e)-1/3*C*b^2*d^2*tan(f*x+e)^3+1/4*B*b^2*d^2
*tan(f*x+e)^4+2/3*B*b^2*c*d*tan(f*x+e)^3-C*a*b*d^2*tan(f*x+e)^2-C*b^2*c*d*t
an(f*x+e)^2+A*a*b*d^2*tan(f*x+e)^2+A*b^2*c*d*tan(f*x+e)^2+C*a^2*c*d*tan(f*x
+e)^2+C*a*b*c^2*tan(f*x+e)^2-2*B*a*b*d^2*tan(f*x+e)-2*B*b^2*c*d*tan(f*x+e)+
2*B*a^2*c*d*tan(f*x+e)+2*B*a*b*c^2*tan(f*x+e)+1/2*C*a*b*d^2*tan(f*x+e)^4+1/
2*C*b^2*c*d*tan(f*x+e)^4+2/3*B*a*b*d^2*tan(f*x+e)^3)
```

Maxima [**A**]
time = 0.51, size = 470, normalized size = 1.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(12*C*b^2*d^2*tan(f*x + e)^5 + 15*(2*C*b^2*c*d + (2*C*a*b + B*b^2)*d^2
)*tan(f*x + e)^4 + 20*(C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C*a^2 + 2*B*a
*b + (A - C)*b^2)*d^2)*tan(f*x + e)^3 + 30*((2*C*a*b + B*b^2)*c^2 + 2*(C*a^
2 + 2*B*a*b + (A - C)*b^2)*c*d + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*tan(f
*x + e)^2 + 60*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^2 - 2*(B*a^2 + 2*(A
 - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*(f*x +
e) + 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2 + 2*((A - C)*a^2 - 2*B*a*b - (
A - C)*b^2)*c*d - (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 +
 1) + 60*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2 + 2*(B*a^2 + 2*(A - C)*a*b -
B*b^2)*c*d + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*tan(f*x + e))/f
```

Fricas [**A**]
time = 5.08, size = 468, normalized size = 1.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*b^2*d^2*tan(f*x + e)^5 + 15*(2*C*b^2*c*d + (2*C*a*b + B*b^2)*d^2
)*tan(f*x + e)^4 + 20*(C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C*a^2 + 2*B*a
*b + (A - C)*b^2)*d^2)*tan(f*x + e)^3 + 60*(((A - C)*a^2 - 2*B*a*b - (A - C
```

```
)*b^2)*c^2 - 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B*a*b
 - (A - C)*b^2)*d^2)*f*x + 30*((2*C*a*b + B*b^2)*c^2 + 2*(C*a^2 + 2*B*a*b +
 (A - C)*b^2)*c*d + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*tan(f*x + e)^2 - 3
0*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2 + 2*((A - C)*a^2 - 2*B*a*b - (A - C)
*b^2)*c*d - (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*log(1/(tan(f*x + e)^2 + 1)
) + 60*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2 + 2*(B*a^2 + 2*(A - C)*a*b - B*
b^2)*c*d + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(396) = 792$.
time = 0.35, size = 1134, normalized size = 2.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Piecewise((A*a**2*c**2*x + A*a**2*c*d*log(tan(e + f*x)**2 + 1)/f - A*a**2*d
**2*x + A*a**2*d**2*tan(e + f*x)/f + A*a*b*c**2*log(tan(e + f*x)**2 + 1)/f
- 4*A*a*b*c*d*x + 4*A*a*b*c*d*tan(e + f*x)/f - A*a*b*d**2*log(tan(e + f*x)*
*2 + 1)/f + A*a*b*d**2*tan(e + f*x)**2/f - A*b**2*c**2*x + A*b**2*c**2*tan(
e + f*x)/f - A*b**2*c*d*log(tan(e + f*x)**2 + 1)/f + A*b**2*c*d*tan(e + f*x
)**2/f + A*b**2*d**2*x + A*b**2*d**2*tan(e + f*x)**3/(3*f) - A*b**2*d**2*ta
n(e + f*x)/f + B*a**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*B*a**2*c*d*x
+ 2*B*a**2*c*d*tan(e + f*x)/f - B*a**2*d**2*log(tan(e + f*x)**2 + 1)/(2*f)
+ B*a**2*d**2*tan(e + f*x)**2/(2*f) - 2*B*a*b*c**2*x + 2*B*a*b*c**2*tan(e +
 f*x)/f - 2*B*a*b*c*d*log(tan(e + f*x)**2 + 1)/f + 2*B*a*b*c*d*tan(e + f*x)
**2/f + 2*B*a*b*d**2*x + 2*B*a*b*d**2*tan(e + f*x)**3/(3*f) - 2*B*a*b*d**2*
tan(e + f*x)/f - B*b**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*b**2*c**2*t
an(e + f*x)**2/(2*f) + 2*B*b**2*c*d*x + 2*B*b**2*c*d*tan(e + f*x)**3/(3*f)
- 2*B*b**2*c*d*tan(e + f*x)/f + B*b**2*d**2*log(tan(e + f*x)**2 + 1)/(2*f)
+ B*b**2*d**2*tan(e + f*x)**4/(4*f) - B*b**2*d**2*tan(e + f*x)**2/(2*f) - C
*a**2*c**2*x + C*a**2*c**2*tan(e + f*x)/f - C*a**2*c*d*log(tan(e + f*x)**2
+ 1)/f + C*a**2*c*d*tan(e + f*x)**2/f + C*a**2*d**2*x + C*a**2*d**2*tan(e +
 f*x)**3/(3*f) - C*a**2*d**2*tan(e + f*x)/f - C*a*b*c**2*log(tan(e + f*x)**
2 + 1)/f + C*a*b*c**2*tan(e + f*x)**2/f + 4*C*a*b*c*d*x + 4*C*a*b*c*d*tan(e
 + f*x)**3/(3*f) - 4*C*a*b*c*d*tan(e + f*x)/f + C*a*b*d**2*log(tan(e + f*x)
**2 + 1)/f + C*a*b*d**2*tan(e + f*x)**4/(2*f) - C*a*b*d**2*tan(e + f*x)**2/
f + C*b**2*c**2*x + C*b**2*c**2*tan(e + f*x)**3/(3*f) - C*b**2*c**2*tan(e +
 f*x)/f + C*b**2*c*d*log(tan(e + f*x)**2 + 1)/f + C*b**2*c*d*tan(e + f*x)**
4/(2*f) - C*b**2*c*d*tan(e + f*x)**2/f - C*b**2*d**2*x + C*b**2*d**2*tan(e
+ f*x)**5/(5*f) - C*b**2*d**2*tan(e + f*x)**3/(3*f) + C*b**2*d**2*tan(e + f
*x)/f, Ne(f, 0)), (x*(a + b*tan(e))**2*(c + d*tan(e))**2*(A + B*tan(e) + C*
tan(e)**2), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 13549 vs. 2(444) = 888.
time = 9.57, size = 13549, normalized size = 30.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] 1/60*(60*A*a^2*c^2*f*x*tan(f*x)^5*tan(e)^5 - 60*C*a^2*c^2*f*x*tan(f*x)^5*ta
n(e)^5 - 120*B*a*b*c^2*f*x*tan(f*x)^5*tan(e)^5 - 60*A*b^2*c^2*f*x*tan(f*x)^
5*tan(e)^5 + 60*C*b^2*c^2*f*x*tan(f*x)^5*tan(e)^5 - 120*B*a^2*c*d*f*x*tan(f
*x)^5*tan(e)^5 - 240*A*a*b*c*d*f*x*tan(f*x)^5*tan(e)^5 + 240*C*a*b*c*d*f*x*
tan(f*x)^5*tan(e)^5 + 120*B*b^2*c*d*f*x*tan(f*x)^5*tan(e)^5 - 60*A*a^2*d^2*
f*x*tan(f*x)^5*tan(e)^5 + 60*C*a^2*d^2*f*x*tan(f*x)^5*tan(e)^5 + 120*B*a*b*
d^2*f*x*tan(f*x)^5*tan(e)^5 + 60*A*b^2*d^2*f*x*tan(f*x)^5*tan(e)^5 - 60*C*b
^2*d^2*f*x*tan(f*x)^5*tan(e)^5 - 30*B*a^2*c^2*log(4*(tan(f*x)^4*tan(e)^2 -
2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e)
+ 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 60*A*a*b*c^2*log(4*(tan(f*x)^4*t
an(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*
x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 60*C*a*b*c^2*log(4*(ta
n(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
- 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 30*B*b^2*c^2
*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + t
an(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 60
*A*a^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan
(e)^5 + 60*C*a^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)^5*tan(e)^5 + 120*B*a*b*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)^5*tan(e)^5 + 60*A*b^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 60*C*b^2*c*d*log(4*(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 30*B*a^2*d^2*log(4*(tan(f*
x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*
tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 60*A*a*b*d^2*log
(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 60*C*a
*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)
^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^
5 - 30*B*b^2*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)
```

```
^5*tan(e)^5 - 300*A*a^2*c^2*f*x*tan(f*x)^4*tan(e)^4 + 300*C*a^2*c^2*f*x*tan
(f*x)^4*tan(e)^4 + 600*B*a*b*c^2*f*x*tan(f*x)^4*tan(e)^4 + 300*A*b^2*c^2*f*
x*tan(f*x)^4*tan(e)^4 - 300*C*b^2*c^2*f*x*tan(f*x)^4*tan(e)^4 + 600*B*a^2*c
*d*f*x*tan(f*x)^4*tan(e)^4 + 1200*A*a*b*c*d*f*x*tan(f*x)^4*tan(e)^4 - 1200*
C*a*b*c*d*f*x*tan(f*x)^4*tan(e)^4 - 600*B*b^2*c*d*f*x*tan(f*x)^4*tan(e)^4 +
 300*A*a^2*d^2*f*x*tan(f*x)^4*tan(e)^4 - 300*C*a^2*d^2*f*x*tan(f*x)^4*tan(e
)^4 - 600*B*a*b*d^2*f*x*tan(f*x)^4*tan(e)^4 - 300*A*b^2*d^2*f*x*tan(f*x)^4*
tan(e)^4 + 300*C*b^2*d^2*f*x*tan(f*x)^4*tan(e)^4 + 60*C*a*b*c^2*tan(f*x)^5*
tan(e)^5 + 30*B*b^2*c^2*tan(f*x)^5*tan(e)^5 + 60*C*a^2*c*d*tan(f*x)^5*tan(e
)^5 + 120*B*a*b*c*d*tan(f*x)^5*tan(e)^5 + 60*A*b^2*c*d*tan(f*x)^5*tan(e)^5
- 90*C*b^2*c*d*tan(f*x)^5*tan(e)^5 + 30*B*a^2*d^2*tan(f*x)^5*tan(e)^5 + 60*
A*a*b*d^2*tan(f*x)^5*tan(e)^5 - 90*C*a*b*d^2*tan(f*x)^5*tan(e)^5 - 45*B*b^2
*d^2*tan(f*x)^5*tan(e)^5 + 150*B*a^2*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/
(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 300*A*a*b*c^2*log(4*(tan(f*x)^4*tan(e
)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*t
an(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 300*C*a*b*c^2*log(4*(tan(f
*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2
*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 150*B*b^2*c^2*l
og(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 300*
A*a^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan
(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(
e)^4 - 300*C*a^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)^4*tan(e)^4 - 600*B*a*b*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)^4*tan(e)^4 - 300*A*b^2*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 300*C*b^2*c*d*log(4*(tan(f*x)^4*tan
(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)
*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^...
```

Mupad [B]
time = 9.12, size = 561, normalized size = 1.27

$$x\,\left(A\,a^2\,c^2-A\,a^2\,d^2-A\,b^2\,c^2+A\,b^2\,d^2-C\,a^2\,c^2+C\,a^2\,d^2+C\,b^2\,c^2-C\,b^2\,d^2-2\,B\,a\,b\,c^2+2\,B\,a\,b\,d^2-2\,B\,a^2\,c\,d+2\,B\,b^2\,c\,d-4\,A\,a\,b\,c\,d+4\,C\,a\,b\,c\,d\right)-\frac{\ln\left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)\,\left(\frac{B\,a^2\,d^2}{2}-\frac{B\,a^2\,c^2}{2}+\frac{B\,b^2\,c^2}{2}-\frac{B\,b^2\,d^2}{2}-A\,a\,b\,c^2+A\,a\,b\,d^2-A\,a^2\,c\,d+C\,a\,b\,c^2+A\,b^2\,c\,d-C\,a\,b\,d^2+C\,a^2\,c\,d-C\,b^2\,c\,d+2\,B\,a\,b\,c\,d\right)}{f}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(\frac{B\,a^2\,d^2}{2}+\frac{B\,b^2\,c^2}{2}-\frac{b\,d\,\left(B\,b\,d+2\,C\,a\,d+2\,C\,b\,c\right)}{2}+A\,a\,b\,d^2+C\,a\,b\,c^2+A\,b^2\,c\,d+C\,a^2\,c\,d+2\,B\,a\,b\,c\,d\right)}{f}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^3\,\left(\frac{A\,b^2\,d^2}{3}+\frac{C\,a^2\,d^2}{3}+\frac{C\,b^2\,c^2}{3}-\frac{C\,b^2\,d^2}{3}+\frac{2\,B\,a\,b\,d^2}{3}+\frac{2\,B\,b^2\,c\,d}{3}+\frac{4\,C\,a\,b\,c\,d}{3}\right)}{f}+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(A\,a^2\,d^2+A\,b^2\,c^2-A\,b^2\,d^2+C\,a^2\,c^2-C\,a^2\,d^2-C\,b^2\,c^2+C\,b^2\,d^2+2\,B\,a\,b\,c^2-2\,B\,a\,b\,d^2+2\,B\,a^2\,c\,d-2\,B\,b^2\,c\,d+4\,A\,a\,b\,c\,d-4\,C\,a\,b\,c\,d\right)}{f}+\frac{b\,d\,{\mathrm{tan}\left(e+f\,x\right)}^4\,\left(B\,b\,d+2\,C\,a\,d+2\,C\,b\,c\right)}{4\,f}+\frac{C\,b^2\,d^2\,{\mathrm{tan}\left(e+f\,x\right)}^5}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2),x)
```

```
[Out] x*(A*a^2*c^2 - A*a^2*d^2 - A*b^2*c^2 + A*b^2*d^2 - C*a^2*c^2 + C*a^2*d^2 +
C*b^2*c^2 - C*b^2*d^2 - 2*B*a*b*c^2 + 2*B*a*b*d^2 - 2*B*a^2*c*d + 2*B*b^2*c
*d - 4*A*a*b*c*d + 4*C*a*b*c*d) - (log(tan(e + f*x)^2 + 1)*((B*a^2*d^2)/2 -
```

```
 (B*a^2*c^2)/2 + (B*b^2*c^2)/2 - (B*b^2*d^2)/2 - A*a*b*c^2 + A*a*b*d^2 - A*
a^2*c*d + C*a*b*c^2 + A*b^2*c*d - C*a*b*d^2 + C*a^2*c*d - C*b^2*c*d + 2*B*a
*b*c*d))/f + (tan(e + f*x)^2*((B*a^2*d^2)/2 + (B*b^2*c^2)/2 - (b*d*(B*b*d +
 2*C*a*d + 2*C*b*c))/2 + A*a*b*d^2 + C*a*b*c^2 + A*b^2*c*d + C*a^2*c*d + 2*
B*a*b*c*d))/f + (tan(e + f*x)^3*((A*b^2*d^2)/3 + (C*a^2*d^2)/3 + (C*b^2*c^2
)/3 - (C*b^2*d^2)/3 + (2*B*a*b*d^2)/3 + (2*B*b^2*c*d)/3 + (4*C*a*b*c*d)/3))
/f + (tan(e + f*x)*(A*a^2*d^2 + A*b^2*c^2 - A*b^2*d^2 + C*a^2*c^2 - C*a^2*d
^2 - C*b^2*c^2 + C*b^2*d^2 + 2*B*a*b*c^2 - 2*B*a*b*d^2 + 2*B*a^2*c*d - 2*B*
b^2*c*d + 4*A*a*b*c*d - 4*C*a*b*c*d))/f + (b*d*tan(e + f*x)^4*(B*b*d + 2*C*
a*d + 2*C*b*c))/(4*f) + (C*b^2*d^2*tan(e + f*x)^5)/(5*f)
```

3.59 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+f$

Optimal. Leaf size=266

$$-\left(\left(a\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)x\right)-\frac{\left(a\left(Bc^2-2cCd-Bd^2\right)-b\left(c^2\right.\right.}{}$$

```
[Out] -(a*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))+b*(2*c*(A-C)*d+B*(c^2-d^2)))*x-(a*(B*
c^2-B*d^2-2*C*c*d)-b*(2*B*c*d+C*c^2-C*d^2)+A*(2*a*c*d+b*(c^2-d^2)))*ln(cos(
f*x+e))/f+d*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*tan(f*x+e)/f+1/2*(A*b+B*a
-C*b)*(c+d*tan(f*x+e))^2/f-1/12*(-4*B*b*d-4*C*a*d+C*b*c)*(c+d*tan(f*x+e))^3
/d^2/f+1/4*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^3/d/f
```

Rubi [A]

time = 0.32, antiderivative size = 264, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3718, 3711, 3609, 3606, 3556}

$$-\frac{\log(\cos(e+fx))\left(2aAcd+aB(c^2-d^2)-2acCd+Ab(c^2-d^2)-b(2Bcd+c^2C-Cd^2)\right)}{f}-x\left(a\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+b\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+\frac{(aB+Ab-bC)(c+d\tan(e+fx))^2}{2f}+\frac{d\tan(e+fx)(aAd+aBc-aCd+Abc-bBd-bcC)}{f}-\frac{(-4aCd-4bBd+bcC)(c+d\tan(e+fx))^3}{12d^2f}+\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```

```
[Out] -((a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d + B*(c^2
- d^2)))*x) - ((2*a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) -
 b*(c^2*C + 2*B*c*d - C*d^2))*Log[Cos[e + f*x]])/f + (d*(A*b*c + a*B*c - b*
c*C + a*A*d - b*B*d - a*C*d)*Tan[e + f*x])/f + ((A*b + a*B - b*C)*(c + d*Ta
n[e + f*x])^2)/(2*f) - ((b*c*C - 4*b*B*d - 4*a*C*d)*(c + d*Tan[e + f*x])^3)
/(12*d^2*f) + (b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(4*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{bC\tan(e+fx)(c+}{4d}$$

$$= -\frac{(bcC-4bBd-4}{}$$

$$= \frac{(Ab+aB-bC)(c}{2}$$

$$= -\left(a\left(c^2C+2Bcd-\right.\right.$$

$$= -\left(a\left(c^2C+2Bcd-\right.\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.77, size = 241, normalized size = 0.91

$$\frac{\frac{(-bcC+4bBd+4aCd)(c+d\tan(e+fx))^3}{d}+3bC\tan(e+fx)(c+d\tan(e+fx))^3+6(Abc+aBc-bcC-aAd+bBd+aCd)\left(i\left((c+id)^2\log(i-\tan(e+fx))-(c-id)^2\log(i+\tan(e+fx))\right)-2d^2\tan(e+fx)\right)+6(Ab+aB-bC)\left((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6cd^2\tan(e+fx)+d^3\tan^2(e+fx)\right)}{12df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] (((-(b*c*C) + 4*b*B*d + 4*a*C*d)*(c + d*Tan[e + f*x])^3)/d + 3*b*C*Tan[e +
f*x]*(c + d*Tan[e + f*x])^3 + 6*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*
C*d)*(I*((c + I*d)^2*Log[I - Tan[e + f*x]] - (c - I*d)^2*Log[I + Tan[e + f*
x]]) - 2*d^2*Tan[e + f*x]) + 6*(A*b + a*B - b*C)*((I*c - d)^3*Log[I - Tan[e
 + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*T
an[e + f*x]^2))/(12*d*f)
```

Maple [A]
time = 0.12, size = 386, normalized size = 1.45

method	result
norman	$(Aa\,c^2 - Aa\,d^2 - 2Abcd - 2Bacd - Bb\,c^2 + Bb\,d^2 - Ca\,c^2 + Ca\,d^2 + 2Cbcd)\,x + \frac{(Aa\,d^2+}{}$
derivativedivides	$\frac{Cb\,d^2\left(\tan^4(fx+e)\right)}{4} + \frac{Bb\,d^2\left(\tan^3(fx+e)\right)}{3} + \frac{Ca\,d^2\left(\tan^3(fx+e)\right)}{3} + \frac{2Cbcd\left(\tan^3(fx+e)\right)}{3} + \frac{Ab\,d^2\left(\tan^2(fx+e)\right)}{2} + \frac{Ba\,d^2\left(\tan^2(fx+e)\right)}{2}$
default	$\frac{Cb\,d^2\left(\tan^4(fx+e)\right)}{4} + \frac{Bb\,d^2\left(\tan^3(fx+e)\right)}{3} + \frac{Ca\,d^2\left(\tan^3(fx+e)\right)}{3} + \frac{2Cbcd\left(\tan^3(fx+e)\right)}{3} + \frac{Ab\,d^2\left(\tan^2(fx+e)\right)}{2} + \frac{Ba\,d^2\left(\tan^2(fx+e)\right)}{2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*C*b*d^2*tan(f*x+e)^4+1/3*B*b*d^2*tan(f*x+e)^3+1/3*C*a*d^2*tan(f*x+
e)^3+2/3*C*b*c*d*tan(f*x+e)^3+1/2*A*b*d^2*tan(f*x+e)^2+1/2*B*a*d^2*tan(f*x+
e)^2+B*b*c*d*tan(f*x+e)^2+C*a*c*d*tan(f*x+e)^2+1/2*C*b*c^2*tan(f*x+e)^2-1/2
*C*b*d^2*tan(f*x+e)^2+A*a*d^2*tan(f*x+e)+2*A*b*c*d*tan(f*x+e)+2*B*a*c*d*tan
(f*x+e)+B*b*c^2*tan(f*x+e)-B*b*d^2*tan(f*x+e)+C*a*c^2*tan(f*x+e)-C*a*d^2*ta
n(f*x+e)-2*C*b*c*d*tan(f*x+e)+1/2*(2*A*a*c*d+A*b*c^2-A*b*d^2+B*a*c^2-B*a*d^
2-2*B*b*c*d-2*C*a*c*d-C*b*c^2+C*b*d^2)*ln(1+tan(f*x+e)^2)+(A*a*c^2-A*a*d^2-
2*A*b*c*d-2*B*a*c*d-B*b*c^2+B*b*d^2-C*a*c^2+C*a*d^2+2*C*b*c*d)*arctan(tan(f
*x+e)))
```

Maxima [A]
time = 0.52, size = 266, normalized size = 1.00

$$\frac{3\,Cbd^2\tan(fx+e)^4 + 4\,(2\,Cbcd + (Ca+Bb)d^2)\tan(fx+e)^3 + 6\,(Cbc^2 + 2\,(Ca+Bb)cd + (Ba+(A-C)b)d^2)\tan(fx+e)^2 + 12\,(((A-C)a-Bb)c^2 - 2\,(Ba+(A-C)b)cd - ((A-C)a-Bb)d^2)(fx+e) + 6\,((Ba+(A-C)b)c^2 + 2\,((A-C)a-Bb)cd - (Ba+(A-C)b)d^2)\log\left(\tan(fx+e)^2+1\right) + 12\,((Ca+Bb)c^2 + 2\,(Ba+(A-C)b)cd + ((A-C)a-Bb)d^2)\tan(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*C*b*d^2*tan(f*x + e)^4 + 4*(2*C*b*c*d + (C*a + B*b)*d^2)*tan(f*x +
e)^3 + 6*(C*b*c^2 + 2*(C*a + B*b)*c*d + (B*a + (A - C)*b)*d^2)*tan(f*x + e)
^2 + 12*(((A - C)*a - B*b)*c^2 - 2*(B*a + (A - C)*b)*c*d - ((A - C)*a - B*b
)*d^2)*(f*x + e) + 6*((B*a + (A - C)*b)*c^2 + 2*((A - C)*a - B*b)*c*d - (B*
a + (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1) + 12*((C*a + B*b)*c^2 + 2*(B*a
+ (A - C)*b)*c*d + ((A - C)*a - B*b)*d^2)*tan(f*x + e))/f
```

Fricas [A]
time = 5.50, size = 264, normalized size = 0.99

$$\frac{3\,Cbd^2\tan\left(fx+e\right)^4+4\left(2\,Cbcd+(Ca+Bb)d^2\right)\tan\left(fx+e\right)^3+12\left(\left((A-C)a-Bb\right)c^2-2\left(Ba+(A-C)b\right)cd-\left((A-C)a-Bb\right)d^2\right)fx+6\left(Cbc^2+2\left(Ca+Bb\right)cd+\left(Ba+(A-C)b\right)d^2\right)\tan\left(fx+e\right)^2-6\left(\left(Ba+(A-C)b\right)c^2+2\left((A-C)a-Bb\right)cd-\left(Ba+(A-C)b\right)d^2\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+12\left(\left(Ca+Bb\right)c^2+2\left(Ba+(A-C)b\right)cd+\left((A-C)a-Bb\right)d^2\right)\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*b*d^2*tan(f*x + e)^4 + 4*(2*C*b*c*d + (C*a + B*b)*d^2)*tan(f*x +
e)^3 + 12*(((A - C)*a - B*b)*c^2 - 2*(B*a + (A - C)*b)*c*d - ((A - C)*a - B
*b)*d^2)*f*x + 6*(C*b*c^2 + 2*(C*a + B*b)*c*d + (B*a + (A - C)*b)*d^2)*tan(
f*x + e)^2 - 6*((B*a + (A - C)*b)*c^2 + 2*((A - C)*a - B*b)*c*d - (B*a + (A
 - C)*b)*d^2)*log(1/(tan(f*x + e)^2 + 1)) + 12*((C*a + B*b)*c^2 + 2*(B*a +
(A - C)*b)*c*d + ((A - C)*a - B*b)*d^2)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(246) = 492$.
time = 0.23, size = 617, normalized size = 2.32

$$\begin{cases} Aac^2x+\frac{Aacd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-Aad^2x+\frac{Aad^2\tan\left(e+fx\right)}{f}+\frac{Abc^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-2Abcdx+\frac{2Abcd\tan\left(e+fx\right)}{f}-\frac{Abd^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Abd^2\tan^2\left(e+fx\right)}{2f}+\frac{Bac^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-2Bacdx+\frac{2Bacd\tan\left(e+fx\right)}{f}-\frac{Bad^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Bad^2\tan^2\left(e+fx\right)}{2f}-Bbc^2x+\frac{Bbc^2\tan\left(e+fx\right)}{f}-\frac{Bbcd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{Bbcd\tan^2\left(e+fx\right)}{f}+Bbd^2x+\frac{Bbd^2\tan^3\left(e+fx\right)}{3f}-\frac{Bbd^2\tan\left(e+fx\right)}{f}-Cac^2x+\frac{Cac^2\tan\left(e+fx\right)}{f}-\frac{Cacd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{Cacd\tan^2\left(e+fx\right)}{f}+Cad^2x+\frac{Cad^2\tan^3\left(e+fx\right)}{3f}-\frac{Cad^2\tan\left(e+fx\right)}{f}-\frac{Cbc^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Cbc^2\tan^2\left(e+fx\right)}{2f}+2Cbcdx+\frac{2Cbcd\tan^3\left(e+fx\right)}{3f}-\frac{2Cbcd\tan\left(e+fx\right)}{f}+\frac{Cbd^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Cbd^2\tan^4\left(e+fx\right)}{4f}-\frac{Cbd^2\tan^2\left(e+fx\right)}{2f} & \text{for}\: f \neq 0 \\ x\left(a+b\tan\left(e\right)\right)\left(c+d\tan\left(e\right)\right)^2\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Piecewise((A*a*c**2*x + A*a*c*d*log(tan(e + f*x)**2 + 1)/f - A*a*d**2*x + A
*a*d**2*tan(e + f*x)/f + A*b*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*A*b*c*
d*x + 2*A*b*c*d*tan(e + f*x)/f - A*b*d**2*log(tan(e + f*x)**2 + 1)/(2*f) +
A*b*d**2*tan(e + f*x)**2/(2*f) + B*a*c**2*log(tan(e + f*x)**2 + 1)/(2*f) -
2*B*a*c*d*x + 2*B*a*c*d*tan(e + f*x)/f - B*a*d**2*log(tan(e + f*x)**2 + 1)/
(2*f) + B*a*d**2*tan(e + f*x)**2/(2*f) - B*b*c**2*x + B*b*c**2*tan(e + f*x)
/f - B*b*c*d*log(tan(e + f*x)**2 + 1)/f + B*b*c*d*tan(e + f*x)**2/f + B*b*d
**2*x + B*b*d**2*tan(e + f*x)**3/(3*f) - B*b*d**2*tan(e + f*x)/f - C*a*c**2
*x + C*a*c**2*tan(e + f*x)/f - C*a*c*d*log(tan(e + f*x)**2 + 1)/f + C*a*c*d
*tan(e + f*x)**2/f + C*a*d**2*x + C*a*d**2*tan(e + f*x)**3/(3*f) - C*a*d**2
*tan(e + f*x)/f - C*b*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*c**2*tan(e
+ f*x)**2/(2*f) + 2*C*b*c*d*x + 2*C*b*c*d*tan(e + f*x)**3/(3*f) - 2*C*b*c*d
*tan(e + f*x)/f + C*b*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*d**2*tan(e
```

```
+ f*x)**4/(4*f) - C*b*d**2*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(
e))*(c + d*tan(e))**2*(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6502 vs. $2(266) = 532$.
time = 3.88, size = 6502, normalized size = 24.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] 1/12*(12*A*a*c^2*f*x*tan(f*x)^4*tan(e)^4 - 12*C*a*c^2*f*x*tan(f*x)^4*tan(e)
^4 - 12*B*b*c^2*f*x*tan(f*x)^4*tan(e)^4 - 24*B*a*c*d*f*x*tan(f*x)^4*tan(e)^
4 - 24*A*b*c*d*f*x*tan(f*x)^4*tan(e)^4 + 24*C*b*c*d*f*x*tan(f*x)^4*tan(e)^4
 - 12*A*a*d^2*f*x*tan(f*x)^4*tan(e)^4 + 12*C*a*d^2*f*x*tan(f*x)^4*tan(e)^4
+ 12*B*b*d^2*f*x*tan(f*x)^4*tan(e)^4 - 6*B*a*c^2*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 6*A*b*c^2*log(4*(tan(f*x)^4*t
an(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*
x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 6*C*b*c^2*log(4*(tan(f
*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2
*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 12*A*a*c*d*log(
4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 12*C*a*
c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 +
 12*B*b*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*ta
n(e)^4 + 6*B*a*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*
x)^4*tan(e)^4 + 6*A*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^4*tan(e)^4 - 6*C*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*
tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^
2 + 1))*tan(f*x)^4*tan(e)^4 - 48*A*a*c^2*f*x*tan(f*x)^3*tan(e)^3 + 48*C*a*c
^2*f*x*tan(f*x)^3*tan(e)^3 + 48*B*b*c^2*f*x*tan(f*x)^3*tan(e)^3 + 96*B*a*c*
d*f*x*tan(f*x)^3*tan(e)^3 + 96*A*b*c*d*f*x*tan(f*x)^3*tan(e)^3 - 96*C*b*c*d
*f*x*tan(f*x)^3*tan(e)^3 + 48*A*a*d^2*f*x*tan(f*x)^3*tan(e)^3 - 48*C*a*d^2*
f*x*tan(f*x)^3*tan(e)^3 - 48*B*b*d^2*f*x*tan(f*x)^3*tan(e)^3 + 6*C*b*c^2*ta
n(f*x)^4*tan(e)^4 + 12*C*a*c*d*tan(f*x)^4*tan(e)^4 + 12*B*b*c*d*tan(f*x)^4*
tan(e)^4 + 6*B*a*d^2*tan(f*x)^4*tan(e)^4 + 6*A*b*d^2*tan(f*x)^4*tan(e)^4 -
9*C*b*d^2*tan(f*x)^4*tan(e)^4 + 24*B*a*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
```

```
)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 24*A*b*c^2*log(4*(tan(f*x)^4*tan(e)
^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*ta
n(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 24*C*b*c^2*log(4*(tan(f*x)^
4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan
(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 48*A*a*c*d*log(4*(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 48*C*a*c*d*
log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + ta
n(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 48*
B*b*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)
^3 - 24*B*a*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^
3*tan(e)^3 - 24*A*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) +
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*t
an(f*x)^3*tan(e)^3 + 24*C*b*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)^3*tan(e)^3 - 12*C*a*c^2*tan(f*x)^4*tan(e)^3 - 12*B*b*c^2*ta
n(f*x)^4*tan(e)^3 - 24*B*a*c*d*tan(f*x)^4*tan(e)^3 - 24*A*b*c*d*tan(f*x)^4*
tan(e)^3 + 24*C*b*c*d*tan(f*x)^4*tan(e)^3 - 12*A*a*d^2*tan(f*x)^4*tan(e)^3
+ 12*C*a*d^2*tan(f*x)^4*tan(e)^3 + 12*B*b*d^2*tan(f*x)^4*tan(e)^3 - 12*C*a*
c^2*tan(f*x)^3*tan(e)^4 - 12*B*b*c^2*tan(f*x)^3*tan(e)^4 - 24*B*a*c*d*tan(f
*x)^3*tan(e)^4 - 24*A*b*c*d*tan(f*x)^3*tan(e)^4 + 24*C*b*c*d*tan(f*x)^3*tan
(e)^4 - 12*A*a*d^2*tan(f*x)^3*tan(e)^4 + 12*C*a*d^2*tan(f*x)^3*tan(e)^4 + 1
2*B*b*d^2*tan(f*x)^3*tan(e)^4 + 72*A*a*c^2*f*x*tan(f*x)^2*tan(e)^2 - 72*C*a
*c^2*f*x*tan(f*x)^2*tan(e)^2 - 72*B*b*c^2*f*x*tan(f*x)^2*tan(e)^2 - 144*B*a
*c*d*f*x*tan(f*x)^2*tan(e)^2 - 144*A*b*c*d*f*x*tan(f*x)^2*tan(e)^2 + 144*C*
b*c*d*f*x*tan(f*x)^2*tan(e)^2 - 72*A*a*d^2*f*x*tan(f*x)^2*tan(e)^2 + 72*C*a
*d^2*f*x*tan(f*x)^2*tan(e)^2 + 72*B*b*d^2*f*x*tan(f*x)^2*tan(e)^2 + 6*C*b*c
^2*tan(f*x)^4*tan(e)^2 + 12*C*a*c*d*tan(f*x)^4*tan(e)^2 + 12*B*b*c*d*tan(f*
x)^4*tan(e)^2 + 6*B*a*d^2*tan(f*x)^4*tan(e)^2 + 6*A*b*d^2*tan(f*x)^4*tan(e)
^2 - 6*C*b*d^2*tan(f*x)^4*tan(e)^2 - 12*C*b*c^2*tan(f*x)^3*tan(e)^3 - 24*C*
a*c*d*tan(f*x)^3*tan(e)^3 - 24*B*b*c*d*tan(f*x)^3*tan(e)^3 - 12*B*a*d^2*tan
(f*x)^3*tan(e)^3 - 12*A*b*d^2*tan(f*x)^3*tan(e)^3 + 24*C*b*d^2*tan(f*x)^3*t
an(e)^3 + 6*C*b*c^2*tan(f*x)^2*tan(e)^4 + 12*C*...
```

Mupad [B]
time = 9.01, size = 300, normalized size = 1.13

$$\frac{\tan(e+f\,x)^2\,\left(\frac{A\,b\,d^2}{2}+\frac{B\,a\,d^2}{2}+\frac{C\,b\,c^2}{2}-\frac{C\,b\,d^2}{2}+B\,b\,c\,d+C\,a\,c\,d\right)}{f}-x\,\left(A\,a\,d^2-A\,a\,c^2+B\,b\,c^2+C\,a\,c^2-B\,b\,d^2-C\,a\,d^2+2\,A\,b\,c\,d+2\,B\,a\,c\,d-2\,C\,b\,c\,d\right)-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{A\,b\,d^2}{2}-\frac{B\,a\,c^2}{2}-\frac{A\,b\,c^2}{2}+\frac{B\,a\,d^2}{2}+\frac{C\,b\,c^2}{2}-\frac{C\,b\,d^2}{2}-A\,a\,c\,d+B\,b\,c\,d+C\,a\,c\,d\right)}{f}+\frac{\tan(e+f\,x)\,\left(A\,a\,d^2+B\,b\,c^2+C\,a\,c^2-B\,b\,d^2-C\,a\,d^2+2\,A\,b\,c\,d+2\,B\,a\,c\,d-2\,C\,b\,c\,d\right)}{f}+\frac{\tan(e+f\,x)^3\,\left(\frac{B\,b\,d^2}{3}+\frac{C\,a\,d^2}{3}+\frac{2\,C\,b\,c\,d}{3}\right)}{f}+\frac{C\,b\,d^2\,\tan(e+f\,x)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan
(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^2*((A*b*d^2)/2 + (B*a*d^2)/2 + (C*b*c^2)/2 - (C*b*d^2)/2 + B*
b*c*d + C*a*c*d))/f - x*(A*a*d^2 - A*a*c^2 + B*b*c^2 + C*a*c^2 - B*b*d^2 -
C*a*d^2 + 2*A*b*c*d + 2*B*a*c*d - 2*C*b*c*d) - (log(tan(e + f*x)^2 + 1)*((A
*b*d^2)/2 - (B*a*c^2)/2 - (A*b*c^2)/2 + (B*a*d^2)/2 + (C*b*c^2)/2 - (C*b*d^
2)/2 - A*a*c*d + B*b*c*d + C*a*c*d))/f + (tan(e + f*x)*(A*a*d^2 + B*b*c^2 +
 C*a*c^2 - B*b*d^2 - C*a*d^2 + 2*A*b*c*d + 2*B*a*c*d - 2*C*b*c*d))/f + (tan
(e + f*x)^3*((B*b*d^2)/3 + (C*a*d^2)/3 + (2*C*b*c*d)/3))/f + (C*b*d^2*tan(e
 + f*x)^4)/(4*f)
```

3.60 $\int (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+\right.$

Optimal. Leaf size=131

$$-\left(\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)x\right)-\frac{\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\log(\cos(e+fx))}{f}+\frac{d(Bc+(A-C)d}{f}$$

```
[Out] -(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))*x-(2*c*(A-C)*d+B*(c^2-d^2))*ln(cos(f*x+e
))/f+d*(B*c+(A-C)*d)*tan(f*x+e)/f+1/2*B*(c+d*tan(f*x+e))^2/f+1/3*C*(c+d*tan
(f*x+e))^3/d/f
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3711, 3609, 3606, 3556}

$$-\frac{(2cd(A-C)+B(c^2-d^2))\log(\cos(e+fx))}{f}-x\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+\frac{d\tan(e+fx)(d(A-C)+Bc)}{f}+\frac{B(c+d\tan(e+fx))^2}{2f}+\frac{C(c+d\tan(e+fx))^3}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -((c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2))*x) - ((2*c*(A - C)*d + B*(c^2 -
 d^2))*Log[Cos[e + f*x]])/f + (d*(B*c + (A - C)*d)*Tan[e + f*x])/f + (B*(c
+ d*Tan[e + f*x])^2)/(2*f) + (C*(c + d*Tan[e + f*x])^3)/(3*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \frac{C(c+d\tan(e+fx))^3}{3df} + \int (A-C+B$$

$$= \frac{B(c+d\tan(e+fx))^2}{2f} + \frac{C(c+d\tan(e$$

$$= -\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)x$$

$$= -\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)x$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 176, normalized size = 1.34

$$\frac{2C(c+d\tan(e+fx))^3+3(Bc+(-A+C)d)\left(i\left((c+id)^2\log(i-\tan(e+fx))-(c-id)^2\log(i+\tan(e+fx))\right)-2d^2\tan(e+fx)\right)+3B\left((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6cd^2\tan(e+fx)+d^3\tan^2(e+fx)\right)}{6df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*C*(c + d*Tan[e + f*x])^3 + 3*(B*c + (-A + C)*d)*(I*((c + I*d)^2*Log[I -
Tan[e + f*x]] - (c - I*d)^2*Log[I + Tan[e + f*x]]) - 2*d^2*Tan[e + f*x]) +
3*B*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]]
+ 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2))/(6*d*f)
```

Maple [A]
time = 0.08, size = 162, normalized size = 1.24

method	result
norman	$\left(A c^2-A d^2-2Bcd-c^2C+C d^2\right)x+\frac{\left(A d^2+2Bcd+c^2C-C d^2\right)\tan(fx+e)}{f}+\frac{C d^2\left(\tan^3(fx+e)\right)}{3f}+$
derivativedivides	$\frac{\frac{C d^2\left(\tan^3(fx+e)\right)}{3}+\frac{B d^2\left(\tan^2(fx+e)\right)}{2}+Ccd\left(\tan^2(fx+e)\right)+A d^2\tan(fx+e)+2Bcd\tan(fx+e)+c^2C\tan(fx+e)-C d^2\tan}{f}$
default	$\frac{\frac{C d^2\left(\tan^3(fx+e)\right)}{3}+\frac{B d^2\left(\tan^2(fx+e)\right)}{2}+Ccd\left(\tan^2(fx+e)\right)+A d^2\tan(fx+e)+2Bcd\tan(fx+e)+c^2C\tan(fx+e)-C d^2\tan}{f}$

risch	$-2iCcdx+iB\,c^2x+\frac{2iB\,c^2e}{f}+2iAcdx+A\,c^2x-A\,d^2x-2Bcdx-C\,c^2x+C\,d^2x-\frac{4i\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/f*(1/3*C*d^2*tan(f*x+e)^3+1/2*B*d^2*tan(f*x+e)^2+C*c*d*tan(f*x+e)^2+A*d^2
*tan(f*x+e)+2*B*c*d*tan(f*x+e)+c^2*C*tan(f*x+e)-C*d^2*tan(f*x+e)+1/2*(2*A*c
*d+B*c^2-B*d^2-2*C*c*d)*ln(1+tan(f*x+e)^2)+(A*c^2-A*d^2-2*B*c*d-C*c^2+C*d^2
)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.52, size = 140, normalized size = 1.07

$$\frac{2\,Cd^2\tan\left(fx+e\right)^3+3\left(2\,Ccd+Bd^2\right)\tan\left(fx+e\right)^2+6\left((A-C)c^2-2\,Bcd-(A-C)d^2\right)(fx+e)+3\left(Bc^2+2\,(A-C)cd-Bd^2\right)\log\left(\tan\left(fx+e\right)^2+1\right)+6\left(Cc^2+2\,Bcd+(A-C)d^2\right)\tan\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
maxima")
```

```
[Out] 1/6*(2*C*d^2*tan(f*x + e)^3 + 3*(2*C*c*d + B*d^2)*tan(f*x + e)^2 + 6*((A -
C)*c^2 - 2*B*c*d - (A - C)*d^2)*(f*x + e) + 3*(B*c^2 + 2*(A - C)*c*d - B*d^
2)*log(tan(f*x + e)^2 + 1) + 6*(C*c^2 + 2*B*c*d + (A - C)*d^2)*tan(f*x + e)
)/f
```

Fricas [A]
time = 5.63, size = 138, normalized size = 1.05

$$\frac{2\,Cd^2\tan\left(fx+e\right)^3+6\left((A-C)c^2-2\,Bcd-(A-C)d^2\right)fx+3\left(2\,Ccd+Bd^2\right)\tan\left(fx+e\right)^2-3\left(Bc^2+2\,(A-C)cd-Bd^2\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+6\left(Cc^2+2\,Bcd+(A-C)d^2\right)\tan\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*C*d^2*tan(f*x + e)^3 + 6*((A - C)*c^2 - 2*B*c*d - (A - C)*d^2)*f*x +
 3*(2*C*c*d + B*d^2)*tan(f*x + e)^2 - 3*(B*c^2 + 2*(A - C)*c*d - B*d^2)*log
(1/(tan(f*x + e)^2 + 1)) + 6*(C*c^2 + 2*B*c*d + (A - C)*d^2)*tan(f*x + e))/
f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(107) = 214$.
time = 0.14, size = 241, normalized size = 1.84

$$\begin{cases} Ac^2x+\frac{Acd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-Ad^2x+\frac{Ad^2\tan\left(e+fx\right)}{f}+\frac{Bc^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-2Bcdx+\frac{2Bcd\tan\left(e+fx\right)}{f}-\frac{Bd^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Bd^2\tan^2\left(e+fx\right)}{2f}-Cc^2x+\frac{Cc^2\tan\left(e+fx\right)}{f}-\frac{Ccd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}+\frac{Ccd\tan^2\left(e+fx\right)}{f}+Cd^2x+\frac{Cd^2\tan^3\left(e+fx\right)}{3f}-\frac{Cd^2\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\left(c+d\tan\left(e\right)\right)^2\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((A*c**2*x + A*c*d*log(tan(e + f*x)**2 + 1)/f - A*d**2*x + A*d**2*
tan(e + f*x)/f + B*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*B*c*d*x + 2*B*c*
d*tan(e + f*x)/f - B*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*d**2*tan(e + f
*x)**2/(2*f) - C*c**2*x + C*c**2*tan(e + f*x)/f - C*c*d*log(tan(e + f*x)**2
 + 1)/f + C*c*d*tan(e + f*x)**2/f + C*d**2*x + C*d**2*tan(e + f*x)**3/(3*f)
 - C*d**2*tan(e + f*x)/f, Ne(f, 0)), (x*(c + d*tan(e))**2*(A + B*tan(e) + C
*tan(e)**2), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2128 vs. $2(131) = 262$.
time = 1.41, size = 2128, normalized size = 16.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] 1/6*(6*A*c^2*f*x*tan(f*x)^3*tan(e)^3 - 6*C*c^2*f*x*tan(f*x)^3*tan(e)^3 - 12
*B*c*d*f*x*tan(f*x)^3*tan(e)^3 - 6*A*d^2*f*x*tan(f*x)^3*tan(e)^3 + 6*C*d^2*
f*x*tan(f*x)^3*tan(e)^3 - 3*B*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)^3*tan(e)^3 - 6*A*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(t
an(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 6*C*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*
tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 3*B*d^2*log(4*(tan(f*x)^4*tan(e)^2
 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(
e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 18*A*c^2*f*x*tan(f*x)^2*tan(e
)^2 + 18*C*c^2*f*x*tan(f*x)^2*tan(e)^2 + 36*B*c*d*f*x*tan(f*x)^2*tan(e)^2 +
 18*A*d^2*f*x*tan(f*x)^2*tan(e)^2 - 18*C*d^2*f*x*tan(f*x)^2*tan(e)^2 + 6*C*
c*d*tan(f*x)^3*tan(e)^3 + 3*B*d^2*tan(f*x)^3*tan(e)^3 + 9*B*c^2*log(4*(tan(
f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 18*A*c*d*log(4
*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x
)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 18*C*c*d
*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + t
an(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 9*
B*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2
 - 6*C*c^2*tan(f*x)^3*tan(e)^2 - 12*B*c*d*tan(f*x)^3*tan(e)^2 - 6*A*d^2*tan
(f*x)^3*tan(e)^2 + 6*C*d^2*tan(f*x)^3*tan(e)^2 - 6*C*c^2*tan(f*x)^2*tan(e)^
```

```
3 - 12*B*c*d*tan(f*x)^2*tan(e)^3 - 6*A*d^2*tan(f*x)^2*tan(e)^3 + 6*C*d^2*ta
n(f*x)^2*tan(e)^3 + 18*A*c^2*f*x*tan(f*x)*tan(e) - 18*C*c^2*f*x*tan(f*x)*ta
n(e) - 36*B*c*d*f*x*tan(f*x)*tan(e) - 18*A*d^2*f*x*tan(f*x)*tan(e) + 18*C*d
^2*f*x*tan(f*x)*tan(e) + 6*C*c*d*tan(f*x)^3*tan(e) + 3*B*d^2*tan(f*x)^3*tan
(e) - 6*C*c*d*tan(f*x)^2*tan(e)^2 - 3*B*d^2*tan(f*x)^2*tan(e)^2 + 6*C*c*d*t
an(f*x)*tan(e)^3 + 3*B*d^2*tan(f*x)*tan(e)^3 - 2*C*d^2*tan(f*x)^3 - 9*B*c^2
*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + t
an(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) - 18*A*c
*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
 tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 18*C
*c*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 9*
B*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)*tan(e) + 1
2*C*c^2*tan(f*x)^2*tan(e) + 24*B*c*d*tan(f*x)^2*tan(e) + 12*A*d^2*tan(f*x)^
2*tan(e) - 18*C*d^2*tan(f*x)^2*tan(e) + 12*C*c^2*tan(f*x)*tan(e)^2 + 24*B*c
*d*tan(f*x)*tan(e)^2 + 12*A*d^2*tan(f*x)*tan(e)^2 - 18*C*d^2*tan(f*x)*tan(e
)^2 - 2*C*d^2*tan(e)^3 - 6*A*c^2*f*x + 6*C*c^2*f*x + 12*B*c*d*f*x + 6*A*d^2
*f*x - 6*C*d^2*f*x - 6*C*c*d*tan(f*x)^2 - 3*B*d^2*tan(f*x)^2 + 6*C*c*d*tan(
f*x)*tan(e) + 3*B*d^2*tan(f*x)*tan(e) - 6*C*c*d*tan(e)^2 - 3*B*d^2*tan(e)^2
 + 3*B*c^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) + 6*A*c*d*log(
4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1)) - 6*C*c*d*log(4*(tan(f*x)^4*t
an(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*
x)*tan(e) + 1)/(tan(e)^2 + 1)) - 3*B*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/
(tan(e)^2 + 1)) - 6*C*c^2*tan(f*x) - 12*B*c*d*tan(f*x) - 6*A*d^2*tan(f*x) +
 6*C*d^2*tan(f*x) - 6*C*c^2*tan(e) - 12*B*c*d*tan(e) - 6*A*d^2*tan(e) + 6*C
*d^2*tan(e) - 6*C*c*d - 3*B*d^2)/(f*tan(f*x)^3*tan(e)^3 - 3*f*tan(f*x)^2*ta
n(e)^2 + 3*f*tan(f*x)*tan(e) - f)
```

Mupad [B]
time = 8.81, size = 141, normalized size = 1.08

$$\frac{\tan(e+f\,x)^2\,\left(\frac{B\,d^2}{2}+C\,c\,d\right)}{f}-x\,\left(A\,d^2-A\,c^2+C\,c^2-C\,d^2+2\,B\,c\,d\right)+\frac{\tan(e+f\,x)\,\left(A\,d^2+C\,c^2-C\,d^2+2\,B\,c\,d\right)}{f}+\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{B\,c^2}{2}-\frac{B\,d^2}{2}+A\,c\,d-C\,c\,d\right)}{f}+\frac{C\,d^2\,\tan(e+f\,x)^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

```
[Out] (tan(e + f*x)^2*((B*d^2)/2 + C*c*d))/f - x*(A*d^2 - A*c^2 + C*c^2 - C*d^2 +
 2*B*c*d) + (tan(e + f*x)*(A*d^2 + C*c^2 - C*d^2 + 2*B*c*d))/f + (log(tan(e
 + f*x)^2 + 1)*((B*c^2)/2 - (B*d^2)/2 + A*c*d - C*c*d))/f + (C*d^2*tan(e +
f*x)^3)/(3*f)
```

3.61 $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=254

$$-\frac{(a(c^2C+2Bcd-Cd^2-A(c^2-d^2))-b(2c(A-C)d+B(c^2-d^2)))\,x}{a^2+b^2}-\frac{(a(Bc^2-2cCd-Bd^2)+b(c^2C+$$

```
[Out] -(a*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b*(2*c*(A-C)*d+B*(c^2-d^2)))*x/(a^2+b
^2)-(a*(B*c^2-B*d^2-2*C*c*d)+b*(2*B*c*d+C*c^2-C*d^2)+A*(2*a*c*d-b*(c^2-d^2)
))*ln(cos(f*x+e))/(a^2+b^2)/f+(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^2*ln(a+b*tan(f
*x+e))/b^3/(a^2+b^2)/f+d*(B*b*d-C*a*d+C*b*c)*tan(f*x+e)/b^2/f+1/2*C*(c+d*ta
n(f*x+e))^2/b/f
```

Rubi [A]
time = 0.55, antiderivative size = 252, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3707, 3698, 31, 3556}

$$-\frac{\log(\cos(e+fx))\left(2aAcd+aB(c^2-d^2)-2acCd-Ab(c^2-d^2)+b(2Bcd+c^2C-Cd^2)\right)}{f\left(a^2+b^2\right)}-\frac{x(a(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)-b(2cd(A-C)+B(c^2-d^2)))}{a^2+b^2}+\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\log(a+b\tan(e+fx))}{b^3f\left(a^2+b^2\right)}+\frac{d\tan(e+fx)(-aCd+bBd+bcC)}{b^2f}+\frac{C(c+d\tan(e+fx))^2}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x]),x]
```

```
[Out] -(((a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b*(2*c*(A - C)*d + B*(c^2
 - d^2)))*x)/(a^2 + b^2)) - ((2*a*A*c*d - 2*a*c*C*d - A*b*(c^2 - d^2) + a*B
*(c^2 - d^2) + b*(c^2*C + 2*B*c*d - C*d^2))*Log[Cos[e + f*x]])/((a^2 + b^2)
*f) + ((A*b^2 - a*(b*B - a*C))*(b*c - a*d)^2*Log[a + b*Tan[e + f*x]])/(b^3*
(a^2 + b^2)*f) + (d*(b*c*C + b*B*d - a*C*d)*Tan[e + f*x])/(b^2*f) + (C*(c +
 d*Tan[e + f*x])^2)/(2*b*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
```

```
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \frac{C(c+d\tan(e+fx))^2}{2bf} + \frac{\int (c+d\tan(e+fx)}{\ldots}$$

$$= \frac{d(bcC+bBd-aCd)\tan(e+fx)}{b^2f} + C(\ldots$$

$$= -\frac{(a(c^2C+2Bcd-Cd^2-A(c^2-d^2)) \ldots}{a^2+b\ldots}$$

$$= -\frac{(a(c^2C+2Bcd-Cd^2-A(c^2-d^2)) \ldots}{a^2+b\ldots}$$

$$= -\frac{(a(c^2C+2Bcd-Cd^2-A(c^2-d^2)) \ldots}{a^2+b\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.96, size = 190, normalized size = 0.75

$$\frac{\frac{b(-iA+B+iC)(c+id)^2\log(i-\tan(e+fx))}{a+ib}+\frac{b(iA+B-iC)(c-id)^2\log(i+\tan(e+fx))}{a-ib}+\frac{2(Ab^2+a(-bB+aC))(bc-ad)^2\log(a+b\tan(e+fx))}{b^2(a^2+b^2)}+\frac{2d(bcC+bBd-aCd)\tan(e+fx)}{b}+C(c+d\tan(e+fx))^2}{2bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x]),x]
```

```
[Out] ((b*((-I)*A + B + I*C)*(c + I*d)^2*Log[I - Tan[e + f*x]])/(a + I*b) + (b*(I
*A + B - I*C)*(c - I*d)^2*Log[I + Tan[e + f*x]])/(a - I*b) + (2*(A*b^2 + a*
(-(b*B) + a*C))*(b*c - a*d)^2*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)) +
(2*d*(b*c*C + b*B*d - a*C*d)*Tan[e + f*x])/b + C*(c + d*Tan[e + f*x])^2)/(2
*b*f)
```

Maple [A]
time = 0.25, size = 317, normalized size = 1.25

method	result
derivativedivides	$\frac{\frac{d\left(\frac{Cbd\left(\tan^2(fx+e)\right)}{2}+b\tan(fx+e)Bd-\tan(fx+e)Cad+2Cbc\tan(fx+e)\right)}{b^2}+\frac{\left(Aa^2d^2b^2-2Aab^3cd+Ab^4c^2-Ba^3d^2b+2Ba^2cdb^2-Ba\ldots\right.}{b^3\left(a^2\ldots\right.}}{\ldots}$
default	$\frac{\frac{d\left(\frac{Cbd\left(\tan^2(fx+e)\right)}{2}+b\tan(fx+e)Bd-\tan(fx+e)Cad+2Cbc\tan(fx+e)\right)}{b^2}+\frac{\left(Aa^2d^2b^2-2Aab^3cd+Ab^4c^2-Ba^3d^2b+2Ba^2cdb^2-Ba\ldots\right.}{b^3\left(a^2\ldots\right.}}{\ldots}$
norman	$\frac{(Aac^2-Aad^2+2Abcd-2Bacd+Bbc^2-Bbd^2-Cac^2+Cad^2-2Cbcd)x}{a^2+b^2}+\frac{d(Bbd-aCd+2Cbc)\tan(fx+e)}{b^2f}+\frac{Cd^2(\tan\ldots}{2\ldots}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(d/b^2*(1/2*C*b*d*tan(f*x+e)^2+b*tan(f*x+e)*B*d-tan(f*x+e)*C*a*d+2*C*b*
c*tan(f*x+e))+1/b^3*(A*a^2*b^2*d^2-2*A*a*b^3*c*d+A*b^4*c^2-B*a^3*b*d^2+2*B*
a^2*b^2*c*d-B*a*b^3*c^2+C*a^4*d^2-2*C*a^3*b*c*d+C*a^2*b^2*c^2)/(a^2+b^2)*ln
(a+b*tan(f*x+e))+1/(a^2+b^2)*(1/2*(2*A*a*c*d-A*b*c^2+A*b*d^2+B*a*c^2-B*a*d^
2+2*B*b*c*d-2*C*a*c*d+C*b*c^2-C*b*d^2)*ln(1+tan(f*x+e)^2)+(A*a*c^2-A*a*d^2+
2*A*b*c*d-2*B*a*c*d+B*b*c^2-B*b*d^2-C*a*c^2+C*a*d^2-2*C*b*c*d)*arctan(tan(f
*x+e))))
```

Maxima [A]
time = 0.51, size = 295, normalized size = 1.16

$$\frac{\frac{2\,\left(\left(\left(A-C\right)a+Bb\right)c^2-2\,\left(Ba-\left(A-C\right)b\right)cd-\left(\left(A-C\right)a+Bb\right)d^2\right)\left(fx+e\right)}{a^2+b^2}+\frac{2\,\left(\left(Ca^2b^2-Bab^3+Ab^4\right)c^2-2\,\left(Ca^3b-Ba^2b^2+Aab^3\right)cd+\left(Ca^4-Ba^3b+Aa^2b^2\right)d^2\right)\log\left(b\tan\left(fx+e\right)+a\right)}{a^2b^3+b^5}+\frac{\left(\left(Ba-\left(A-C\right)b\right)c^2+2\,\left(\left(A-C\right)a+Bb\right)cd-\left(Ba-\left(A-C\right)b\right)d^2\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{a^2+b^2}+\frac{Cbd^2\tan\left(fx+e\right)^2+2\,\left(2\,Cbcd-\left(Ca-Bb\right)d^2\right)\tan\left(fx+e\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a + B*b)*c^2 - 2*(B*a - (A - C)*b)*c*d - ((A - C)*a + B*b)
*d^2)*(f*x + e)/(a^2 + b^2) + 2*((C*a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C*a
^3*b - B*a^2*b^2 + A*a*b^3)*c*d + (C*a^4 - B*a^3*b + A*a^2*b^2)*d^2)*log(b*
tan(f*x + e) + a)/(a^2*b^3 + b^5) + ((B*a - (A - C)*b)*c^2 + 2*((A - C)*a +
 B*b)*c*d - (B*a - (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + (C
*b*d^2*tan(f*x + e)^2 + 2*(2*C*b*c*d - (C*a - B*b)*d^2)*tan(f*x + e))/b^2)/
f
```

Fricas [A]
time = 4.50, size = 403, normalized size = 1.59

$$\frac{\left(Ca^2b^2+Cb^4\right)d^2\tan\left(fx+e\right)^2+2\left(\left(\left(A-C\right)ab^3+Bb^4\right)c^2-2\left(Bab^3-\left(A-C\right)b^4\right)cd-\left(\left(A-C\right)ab^3+Bb^4\right)d^2\right)fx+\left(\left(Ca^2b^2-Bab^3+Ab^4\right)c^2-2\left(Ca^3b-Ba^2b^2+Aab^3\right)cd+\left(Ca^4-Ba^3b+Aa^2b^2\right)d^2\right)\log\left(\frac{b^2\tan\left(fx+e\right)^2+2ab\tan\left(fx+e\right)+a^2}{\tan\left(fx+e\right)^2+1}\right)-\left(\left(Ca^2b^2+Cb^4\right)c^2-2\left(Ca^3b-Ba^2b^2+Cab^3-Bb^4\right)cd+\left(Ca^4-Ba^3b+Aa^2b^2-Bab^3+\left(A-C\right)b^4\right)d^2\right)\log\left(\frac{1}{\tan\left(fx+e\right)^2+1}\right)+2\left(2\left(Ca^2b^2+Cb^4\right)cd-\left(Ca^3b-Ba^2b^2+Cab^3-Bb^4\right)d^2\right)\tan\left(fx+e\right)}{2\left(a^2b^3+b^5\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] 1/2*((C*a^2*b^2 + C*b^4)*d^2*tan(f*x + e)^2 + 2*(((A - C)*a*b^3 + B*b^4)*c^
2 - 2*(B*a*b^3 - (A - C)*b^4)*c*d - ((A - C)*a*b^3 + B*b^4)*d^2)*f*x + ((C*
a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C*a^3*b - B*a^2*b^2 + A*a*b^3)*c*d + (C
*a^4 - B*a^3*b + A*a^2*b^2)*d^2)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x +
e) + a^2)/(tan(f*x + e)^2 + 1)) - ((C*a^2*b^2 + C*b^4)*c^2 - 2*(C*a^3*b - B
```

```
*a^2*b^2 + C*a*b^3 - B*b^4)*c*d + (C*a^4 - B*a^3*b + A*a^2*b^2 - B*a*b^3 +
(A - C)*b^4)*d^2)*log(1/(tan(f*x + e)^2 + 1)) + 2*(2*(C*a^2*b^2 + C*b^4)*c*
d - (C*a^3*b - B*a^2*b^2 + C*a*b^3 - B*b^4)*d^2)*tan(f*x + e))/((a^2*b^3 +
b^5)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.64, size = 4444, normalized size = 17.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(
a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c**2*x + A*c*d*log(tan(e + f*x)**2 + 1)/f
 - A*d**2*x + A*d**2*tan(e + f*x)/f + B*c**2*log(tan(e + f*x)**2 + 1)/(2*f)
 - 2*B*c*d*x + 2*B*c*d*tan(e + f*x)/f - B*d**2*log(tan(e + f*x)**2 + 1)/(2*
f) + B*d**2*tan(e + f*x)**2/(2*f) - C*c**2*x + C*c**2*tan(e + f*x)/f - C*c*
d*log(tan(e + f*x)**2 + 1)/f + C*c*d*tan(e + f*x)**2/f + C*d**2*x + C*d**2*
tan(e + f*x)**3/(3*f) - C*d**2*tan(e + f*x)/f)/a, Eq(b, 0)), (I*A*c**2*f*x*
tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + A*c**2*f*x/(2*b*f*tan(e + f*x
) - 2*I*b*f) + I*A*c**2/(2*b*f*tan(e + f*x) - 2*I*b*f) + 2*A*c*d*f*x*tan(e
+ f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - 2*I*A*c*d*f*x/(2*b*f*tan(e + f*x) -
 2*I*b*f) - 2*A*c*d/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*A*d**2*f*x*tan(e + f
*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + A*d**2*f*x/(2*b*f*tan(e + f*x) - 2*I*b
*f) + A*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*
I*b*f) - I*A*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - 2*I*b*f) -
 I*A*d**2/(2*b*f*tan(e + f*x) - 2*I*b*f) + B*c**2*f*x*tan(e + f*x)/(2*b*f*t
an(e + f*x) - 2*I*b*f) - I*B*c**2*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) - B*c*
*2/(2*b*f*tan(e + f*x) - 2*I*b*f) + 2*I*B*c*d*f*x*tan(e + f*x)/(2*b*f*tan(e
 + f*x) - 2*I*b*f) + 2*B*c*d*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + 2*B*c*d*l
og(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - 2*I*B
*c*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - 2*I*b*f) - 2*I*B*c*d/(2
*b*f*tan(e + f*x) - 2*I*b*f) - 3*B*d**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x
) - 2*I*b*f) + 3*I*B*d**2*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*B*d**2*log
(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + B*d**2*
log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - 2*I*b*f) + 2*B*d**2*tan(e +
f*x)**2/(2*b*f*tan(e + f*x) - 2*I*b*f) + 3*B*d**2/(2*b*f*tan(e + f*x) - 2*I
*b*f) + I*C*c**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + C*c**2*f
*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + C*c**2*log(tan(e + f*x)**2 + 1)*tan(e +
 f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*C*c**2*log(tan(e + f*x)**2 + 1)/(2
*b*f*tan(e + f*x) - 2*I*b*f) - I*C*c**2/(2*b*f*tan(e + f*x) - 2*I*b*f) - 6*
C*c*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + 6*I*C*c*d*f*x/(2*b*
f*tan(e + f*x) - 2*I*b*f) + 2*I*C*c*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)
```

```
/(2*b*f*tan(e + f*x) - 2*I*b*f) + 2*C*c*d*log(tan(e + f*x)**2 + 1)/(2*b*f*t
an(e + f*x) - 2*I*b*f) + 4*C*c*d*tan(e + f*x)**2/(2*b*f*tan(e + f*x) - 2*I*
b*f) + 6*C*c*d/(2*b*f*tan(e + f*x) - 2*I*b*f) - 3*I*C*d**2*f*x*tan(e + f*x)
/(2*b*f*tan(e + f*x) - 2*I*b*f) - 3*C*d**2*f*x/(2*b*f*tan(e + f*x) - 2*I*b*
f) - 2*C*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2
*I*b*f) + 2*I*C*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - 2*I*b*f
) + C*d**2*tan(e + f*x)**3/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*C*d**2*tan(e
+ f*x)**2/(2*b*f*tan(e + f*x) - 2*I*b*f) + 3*I*C*d**2/(2*b*f*tan(e + f*x) -
 2*I*b*f), Eq(a, -I*b)), (-I*A*c**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) +
2*I*b*f) + A*c**2*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*A*c**2/(2*b*f*tan(
e + f*x) + 2*I*b*f) + 2*A*c*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*
f) + 2*I*A*c*d*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - 2*A*c*d/(2*b*f*tan(e +
f*x) + 2*I*b*f) - I*A*d**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f)
+ A*d**2*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) + A*d**2*log(tan(e + f*x)**2 +
1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*A*d**2*log(tan(e + f*x)*
*2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*A*d**2/(2*b*f*tan(e + f*x) + 2*I
*b*f) + B*c**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*B*c**2*f
*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - B*c**2/(2*b*f*tan(e + f*x) + 2*I*b*f) -
 2*I*B*c*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*B*c*d*f*x/(2
*b*f*tan(e + f*x) + 2*I*b*f) + 2*B*c*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x
)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2*I*B*c*d*log(tan(e + f*x)**2 + 1)/(2*b*
f*tan(e + f*x) + 2*I*b*f) + 2*I*B*c*d/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*B*
d**2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) - 3*I*B*d**2*f*x/(2*b*
f*tan(e + f*x) + 2*I*b*f) - I*B*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/
(2*b*f*tan(e + f*x) + 2*I*b*f) + B*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan
(e + f*x) + 2*I*b*f) + 2*B*d**2*tan(e + f*x)**2/(2*b*f*tan(e + f*x) + 2*I*b
*f) + 3*B*d**2/(2*b*f*tan(e + f*x) + 2*I*b*f) - I*C*c**2*f*x*tan(e + f*x)/(
2*b*f*tan(e + f*x) + 2*I*b*f) + C*c**2*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) +
 C*c**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f
) + I*C*c**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + I*C*
c**2/(2*b*f*tan(e + f*x) + 2*I*b*f) - 6*C*c*d*f*x*tan(e + f*x)/(2*b*f*tan(e
 + f*x) + 2*I*b*f) - 6*I*C*c*d*f*x/(2*b*f*tan(e + f*x) + 2*I*b*f) - 2*I*C*c
*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 2
*C*c*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) + 2*I*b*f) + 4*C*c*d*ta
n(e + f*x)**2/(2*b*f*tan(e + f*x) + 2*I*b*f) + 6*C*c*d/(2*b*f*tan(e + f*x)
+ 2*I*b*f) + 3*I*C*d**2*f*x*tan(e + f*x)/(2*b*f...
```

Giac [A]
time = 0.85, size = 338, normalized size = 1.33

$$\frac{\frac{2\left(Aac^2-Cac^2+Bbc^2-2\,Bacd+2\,Abcd-2\,Cbcd-Aad^2+Cad^2-Bbd^2\right)(fx+e)}{a^2+b^2}+\frac{\left(Bac^2-Abc^2+Cbc^2+2\,Aacd-2\,Cacd+2\,Bbcd-Bad^2+Abd^2-Cbd^2\right)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}+\frac{2\left(Ca^2b^2c^2-Bab^3c^2+Ab^4c^2-2\,Ca^3bcd+2\,Ba^2b^2cd-2\,Aab^3cd+Ca^4d^2-Ba^3bd^2+Aa^2b^2d^2\right)\log(|b\tan(fx+e)+a|)}{a^2b^3+b^5}+\frac{Cbd^2\tan(fx+e)^2+4\,Cbcd\tan(fx+e)-2\,Cad^2\tan(fx+e)+2\,Bbd^2\tan(fx+e)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^2 - C*a*c^2 + B*b*c^2 - 2*B*a*c*d + 2*A*b*c*d - 2*C*b*c*d - A
*a*d^2 + C*a*d^2 - B*b*d^2)*(f*x + e)/(a^2 + b^2) + (B*a*c^2 - A*b*c^2 + C*
b*c^2 + 2*A*a*c*d - 2*C*a*c*d + 2*B*b*c*d - B*a*d^2 + A*b*d^2 - C*b*d^2)*lo
g(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 2*(C*a^2*b^2*c^2 - B*a*b^3*c^2 + A*b^4*
c^2 - 2*C*a^3*b*c*d + 2*B*a^2*b^2*c*d - 2*A*a*b^3*c*d + C*a^4*d^2 - B*a^3*b
*d^2 + A*a^2*b^2*d^2)*log(abs(b*tan(f*x + e) + a))/(a^2*b^3 + b^5) + (C*b*d
^2*tan(f*x + e)^2 + 4*C*b*c*d*tan(f*x + e) - 2*C*a*d^2*tan(f*x + e) + 2*B*b
*d^2*tan(f*x + e))/b^2)/f
```

Mupad [B]
time = 11.28, size = 325, normalized size = 1.28

$$\frac{\tan(e+f\,x)\,\left(\frac{B\,d^2+2\,C\,c\,d}{b}-\frac{C\,a\,d^2}{b^2}\right)}{f}+\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(b^2\,\left(C\,a^2\,c^2+2\,B\,a^2\,c\,d+A\,a^2\,d^2\right)-b\,\left(B\,a^3\,d^2+2\,C\,c\,a^3\,d\right)-b^3\,\left(B\,a\,c^2+2\,A\,a\,d\,c\right)+A\,b^4\,c^2+C\,a^4\,d^2\right)}{f\,\left(a^2\,b^3+b^5\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,d^2-A\,c^2+B\,c^2\,1\mathrm{i}-B\,d^2\,1\mathrm{i}+C\,c^2-C\,d^2+A\,c\,d\,2\mathrm{i}+2\,B\,c\,d-C\,c\,d\,2\mathrm{i}\right)}{2\,f\,\left(b+a\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B\,c^2-B\,d^2+2\,A\,c\,d-2\,C\,c\,d-A\,c^2\,1\mathrm{i}+A\,d^2\,1\mathrm{i}+C\,c^2\,1\mathrm{i}-C\,d^2\,1\mathrm{i}+B\,c\,d\,2\mathrm{i}\right)}{2\,f\,\left(a+b\,1\mathrm{i}\right)}+\frac{C\,d^2\tan(e+f\,x)^2}{2\,b\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b
*tan(e + f*x)),x)
```

```
[Out] (tan(e + f*x)*((B*d^2 + 2*C*c*d)/b - (C*a*d^2)/b^2))/f + (log(a + b*tan(e +
 f*x))*(b^2*(A*a^2*d^2 + C*a^2*c^2 + 2*B*a^2*c*d) - b*(B*a^3*d^2 + 2*C*a^3*
c*d) - b^3*(B*a*c^2 + 2*A*a*c*d) + A*b^4*c^2 + C*a^4*d^2))/(f*(b^5 + a^2*b^
3)) + (log(tan(e + f*x) + 1i)*(A*d^2 - A*c^2 + B*c^2*1i - B*d^2*1i + C*c^2
- C*d^2 + A*c*d*2i + 2*B*c*d - C*c*d*2i))/(2*f*(a*1i + b)) + (log(tan(e + f
*x) - 1i)*(A*d^2*1i - A*c^2*1i + B*c^2 - B*d^2 + C*c^2*1i - C*d^2*1i + 2*A*
c*d + B*c*d*2i - 2*C*c*d))/(2*f*(a + b*1i)) + (C*d^2*tan(e + f*x)^2)/(2*b*f
)
```

3.62 $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=415

$$-\frac{(a^2(c^2C+2Bcd-Cd^2-A(c^2-d^2))-b^2(c^2C+2Bcd-Cd^2-A(c^2-d^2))-2ab(2c(A-C)d+B(c^2-}{(a^2+b^2)^2}$$

```
[Out] -(a^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2
))-2*a*b*(2*c*(A-C)*d+B*(c^2-d^2)))*x/(a^2+b^2)^2-(2*a*b*(c^2*C+2*B*c*d-C*d
^2-A*(c^2-d^2))+a^2*(2*c*(A-C)*d+B*(c^2-d^2))-b^2*(2*c*(A-C)*d+B*(c^2-d^2))
)*ln(cos(f*x+e))/(a^2+b^2)^2/f-(-a*d+b*c)*(a^3*b*B*d-2*a^4*C*d-b^4*(2*A*d+B
*c)-a*b^3*(2*A*c-3*B*d-2*C*c)+a^2*b^2*(B*c-4*C*d))*ln(a+b*tan(f*x+e))/b^3/(
a^2+b^2)^2/f+(A*b^2-B*a*b+2*C*a^2+C*b^2)*d^2*tan(f*x+e)/b^2/(a^2+b^2)/f-(A*
b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^2/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.73, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3726, 3718, 3707, 3698, 31, 3556}

$$-\frac{\log(\cos(e+fx))\left(a^2(2cd(A-C)+B(c^2-d^2))+2ab(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)-b^2(2cd(A-C)+B(c^2-d^2))\right)}{f\left(a^2+b^2\right)^2}-\frac{x(a^2(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)-2ab(2cd(A-C)+B(c^2-d^2))-b^2(-A(c^2-d^2)+2Bcd+c^2C-Cd^2))}{\left(a^2+b^2\right)^2}-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{d^2\tan(e+fx)\left(2a^2C-abB+Ab^2+b^2C\right)}{b^2f\left(a^2+b^2\right)}-\frac{(bc-ad)\left(-2a^4Cd+a^3bBd+a^2b^2(Bc-4Cd)-ab^3(2Ac-3Bd-2cC)-b^4(2Ad+Bc)\right)\log(a+b\tan(e+fx))}{b^3f\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x])^2,x]
```

```
[Out] -(((a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) - 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x)/(a^2 + b
^2)^2) - ((2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + a^2*(2*c*(A -
C)*d + B*(c^2 - d^2)) - b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*
x]])/((a^2 + b^2)^2*f) - ((b*c - a*d)*(a^3*b*B*d - 2*a^4*C*d - b^4*(B*c + 2
*A*d) - a*b^3*(2*A*c - 2*c*C - 3*B*d) + a^2*b^2*(B*c - 4*C*d))*Log[a + b*Ta
n[e + f*x]])/(b^3*(a^2 + b^2)^2*f) + ((A*b^2 - a*b*B + 2*a^2*C + b^2*C)*d^2
*Tan[e + f*x])/(b^2*(a^2 + b^2)*f) - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e
+ f*x])^2)/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+}{b\,(a^2+b^2)\,f(a+b\tan(e+fx))}$$

$$= \frac{(Ab^2-abB+2a^2C+b^2C)\,d^2\tan(e+}{b^2\,(a^2+b^2)\,f}$$

$$= -\frac{(a^2(c^2C+2Bcd-Cd^2-A(c^2-d^2)}{}$$

$$= -\frac{(a^2(c^2C+2Bcd-Cd^2-A(c^2-d^2)}{}$$

$$= -\frac{(a^2(c^2C+2Bcd-Cd^2-A(c^2-d^2)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.39, size = 2640, normalized size = 6.36

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((-I)*(-2*a^6*A*b^6*c^2 + (2*I)*a^5*A*b^7*c^2 - 2*a^4*A*b^8*c^2 + (2*I)*a^3
*A*b^9*c^2 + a^7*b^5*B*c^2 - I*a^6*b^6*B*c^2 - a^3*b^9*B*c^2 + I*a^2*b^10*B
*c^2 + 2*a^6*b^6*c^2*C - (2*I)*a^5*b^7*c^2*C + 2*a^4*b^8*c^2*C - (2*I)*a^3*
b^9*c^2*C + 2*a^7*A*b^5*c*d - (2*I)*a^6*A*b^6*c*d - 2*a^3*A*b^9*c*d + (2*I)
*a^2*A*b^10*c*d + 4*a^6*b^6*B*c*d - (4*I)*a^5*b^7*B*c*d + 4*a^4*b^8*B*c*d -
 (4*I)*a^3*b^9*B*c*d - 2*a^9*b^3*c*C*d + (2*I)*a^8*b^4*c*C*d - 8*a^7*b^5*c*
C*d + (8*I)*a^6*b^6*c*C*d - 6*a^5*b^7*c*C*d + (6*I)*a^4*b^8*c*C*d + 2*a^6*A
*b^6*d^2 - (2*I)*a^5*A*b^7*d^2 + 2*a^4*A*b^8*d^2 - (2*I)*a^3*A*b^9*d^2 - a^
9*b^3*B*d^2 + I*a^8*b^4*B*d^2 - 4*a^7*b^5*B*d^2 + (4*I)*a^6*b^6*B*d^2 - 3*a
^5*b^7*B*d^2 + (3*I)*a^4*b^8*B*d^2 + 2*a^10*b^2*C*d^2 - (2*I)*a^9*b^3*C*d^2
 + 6*a^8*b^4*C*d^2 - (6*I)*a^7*b^5*C*d^2 + 4*a^6*b^6*C*d^2 - (4*I)*a^5*b^7*
C*d^2)*(e + f*x)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^2
)/(a^2*(a - I*b)^4*(a + I*b)^3*b^5*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a
 + b*Tan[e + f*x])^2) - (I*(2*a*A*b^4*c^2 - a^2*b^3*B*c^2 + b^5*B*c^2 - 2*a
*b^4*c^2*C - 2*a^2*A*b^3*c*d + 2*A*b^5*c*d - 4*a*b^4*B*c*d + 2*a^4*b*c*C*d
+ 6*a^2*b^3*c*C*d - 2*a*A*b^4*d^2 + a^4*b*B*d^2 + 3*a^2*b^3*B*d^2 - 2*a^5*C
*d^2 - 4*a^3*b^2*C*d^2)*ArcTan[Tan[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*
x])^2*(c + d*Tan[e + f*x])^2)/(b^3*(a^2 + b^2)^2*f*(c*Cos[e + f*x] + d*Sin[
e + f*x])^2*(a + b*Tan[e + f*x])^2) + ((-2*b*c*C*d - b*B*d^2 + 2*a*C*d^2)*L
```

```
og[Cos[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^2
)/(b^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^2) + ((2*
a*A*b^4*c^2 - a^2*b^3*B*c^2 + b^5*B*c^2 - 2*a*b^4*c^2*C - 2*a^2*A*b^3*c*d +
 2*A*b^5*c*d - 4*a*b^4*B*c*d + 2*a^4*b*c*C*d + 6*a^2*b^3*c*C*d - 2*a*A*b^4*
d^2 + a^4*b*B*d^2 + 3*a^2*b^3*B*d^2 - 2*a^5*C*d^2 - 4*a^3*b^2*C*d^2)*Log[(a
*Cos[e + f*x] + b*Sin[e + f*x])^2]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c +
 d*Tan[e + f*x])^2)/(2*b^3*(a^2 + b^2)^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x]
)^2*(a + b*Tan[e + f*x])^2) + (Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x
])*(a^5*b*C*d^2 + 2*a^3*b^3*C*d^2 + a*b^5*C*d^2 + a^4*A*b^2*c^2*(e + f*x) -
 a^2*A*b^4*c^2*(e + f*x) + 2*a^3*b^3*B*c^2*(e + f*x) - a^4*b^2*c^2*C*(e + f
*x) + a^2*b^4*c^2*C*(e + f*x) + 4*a^3*A*b^3*c*d*(e + f*x) - 2*a^4*b^2*B*c*d
*(e + f*x) + 2*a^2*b^4*B*c*d*(e + f*x) - 4*a^3*b^3*c*C*d*(e + f*x) - a^4*A*
b^2*d^2*(e + f*x) + a^2*A*b^4*d^2*(e + f*x) - 2*a^3*b^3*B*d^2*(e + f*x) + a
^4*b^2*C*d^2*(e + f*x) - a^2*b^4*C*d^2*(e + f*x) - a^5*b*C*d^2*Cos[2*(e + f
*x)] - 2*a^3*b^3*C*d^2*Cos[2*(e + f*x)] - a*b^5*C*d^2*Cos[2*(e + f*x)] + a^
4*A*b^2*c^2*(e + f*x)*Cos[2*(e + f*x)] - a^2*A*b^4*c^2*(e + f*x)*Cos[2*(e +
 f*x)] + 2*a^3*b^3*B*c^2*(e + f*x)*Cos[2*(e + f*x)] - a^4*b^2*c^2*C*(e + f*
x)*Cos[2*(e + f*x)] + a^2*b^4*c^2*C*(e + f*x)*Cos[2*(e + f*x)] + 4*a^3*A*b^
3*c*d*(e + f*x)*Cos[2*(e + f*x)] - 2*a^4*b^2*B*c*d*(e + f*x)*Cos[2*(e + f*x
)] + 2*a^2*b^4*B*c*d*(e + f*x)*Cos[2*(e + f*x)] - 4*a^3*b^3*c*C*d*(e + f*x)
*Cos[2*(e + f*x)] - a^4*A*b^2*d^2*(e + f*x)*Cos[2*(e + f*x)] + a^2*A*b^4*d^
2*(e + f*x)*Cos[2*(e + f*x)] - 2*a^3*b^3*B*d^2*(e + f*x)*Cos[2*(e + f*x)] +
 a^4*b^2*C*d^2*(e + f*x)*Cos[2*(e + f*x)] - a^2*b^4*C*d^2*(e + f*x)*Cos[2*(
e + f*x)] + a^2*A*b^4*c^2*Sin[2*(e + f*x)] + A*b^6*c^2*Sin[2*(e + f*x)] - a
^3*b^3*B*c^2*Sin[2*(e + f*x)] - a*b^5*B*c^2*Sin[2*(e + f*x)] + a^4*b^2*c^2*
C*Sin[2*(e + f*x)] + a^2*b^4*c^2*C*Sin[2*(e + f*x)] - 2*a^3*A*b^3*c*d*Sin[2
*(e + f*x)] - 2*a*A*b^5*c*d*Sin[2*(e + f*x)] + 2*a^4*b^2*B*c*d*Sin[2*(e + f
*x)] + 2*a^2*b^4*B*c*d*Sin[2*(e + f*x)] - 2*a^5*b*c*C*d*Sin[2*(e + f*x)] -
2*a^3*b^3*c*C*d*Sin[2*(e + f*x)] + a^4*A*b^2*d^2*Sin[2*(e + f*x)] + a^2*A*b
^4*d^2*Sin[2*(e + f*x)] - a^5*b*B*d^2*Sin[2*(e + f*x)] - a^3*b^3*B*d^2*Sin[
2*(e + f*x)] + 2*a^6*C*d^2*Sin[2*(e + f*x)] + 3*a^4*b^2*C*d^2*Sin[2*(e + f*
x)] + a^2*b^4*C*d^2*Sin[2*(e + f*x)] + a^3*A*b^3*c^2*(e + f*x)*Sin[2*(e + f
*x)] - a*A*b^5*c^2*(e + f*x)*Sin[2*(e + f*x)] + 2*a^2*b^4*B*c^2*(e + f*x)*S
in[2*(e + f*x)] - a^3*b^3*c^2*C*(e + f*x)*Sin[2*(e + f*x)] + a*b^5*c^2*C*(e
 + f*x)*Sin[2*(e + f*x)] + 4*a^2*A*b^4*c*d*(e + f*x)*Sin[2*(e + f*x)] - 2*a
^3*b^3*B*c*d*(e + f*x)*Sin[2*(e + f*x)] + 2*a*b^5*B*c*d*(e + f*x)*Sin[2*(e
+ f*x)] - 4*a^2*b^4*c*C*d*(e + f*x)*Sin[2*(e + f*x)] - a^3*A*b^3*d^2*(e + f
*x)*Sin[2*(e + f*x)] + a*A*b^5*d^2*(e + f*x)*Sin[2*(e + f*x)] - 2*a^2*b^4*B
*d^2*(e + f*x)*Sin[2*(e + f*x)] + a^3*b^3*C*d^2*(e + f*x)*Sin[2*(e + f*x)]
- a*b^5*C*d^2*(e + f*x)*Sin[2*(e + f*x)])*(c + d*Tan[e + f*x])^2)/(2*a*(a -
 I*b)^2*(a + I*b)^2*b^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e
+ f*x])^2)
```

Maple [A]

time = 0.37, size = 552, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{C\,d^2\tan(fx+e)}{b^2}-\frac{A\,a^2d^2b^2-2Aa\,b^3cd+A\,b^4c^2-B\,a^3d^2b+2B\,a^2cd\,b^2-Ba\,b^3c^2+a^4C\,d^2-2C\,a^3cdb+C\,a^2c^2b^2}{b^3\left(a^2+b^2\right)(a+b\tan(fx+e))}+\frac{\left(-2A\,a^2b^3cd+2Aa\ldots\right.}{\ldots}}{\ldots}$
default	$\frac{\frac{C\,d^2\tan(fx+e)}{b^2}-\frac{A\,a^2d^2b^2-2Aa\,b^3cd+A\,b^4c^2-B\,a^3d^2b+2B\,a^2cd\,b^2-Ba\,b^3c^2+a^4C\,d^2-2C\,a^3cdb+C\,a^2c^2b^2}{b^3\left(a^2+b^2\right)(a+b\tan(fx+e))}+\frac{\left(-2A\,a^2b^3cd+2Aa\ldots\right.}{\ldots}}{\ldots}$
norman	$\frac{a\left(A\,a^2c^2-A\,a^2d^2+4Aabcd-A\,b^2c^2+A\,b^2d^2-2B\,a^2cd+2Bab\,c^2-2Bab\,d^2+2B\,b^2cd-C\,a^2c^2+a^2C\,d^2-4Cabcd+C\,b^2c^2-C\,b^2d^2\right)x}{a^4+2a^2b^2+b^4}+\ldots$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(C*d^2/b^2*tan(f*x+e)-1/b^3*(A*a^2*b^2*d^2-2*A*a*b^3*c*d+A*b^4*c^2-B*a^
3*b*d^2+2*B*a^2*b^2*c*d-B*a*b^3*c^2+C*a^4*d^2-2*C*a^3*b*c*d+C*a^2*b^2*c^2)/
(a^2+b^2)/(a+b*tan(f*x+e))+(-2*A*a^2*b^3*c*d+2*A*a*b^4*c^2-2*A*a*b^4*d^2+2*
A*b^5*c*d+B*a^4*b*d^2-B*a^2*b^3*c^2+3*B*a^2*b^3*d^2-4*B*a*b^4*c*d+B*b^5*c^2
-2*C*a^5*d^2+2*C*a^4*b*c*d-4*C*a^3*b^2*d^2+6*C*a^2*b^3*c*d-2*C*a*b^4*c^2)/b
^3/(a^2+b^2)^2*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^2*(1/2*(2*A*a^2*c*d-2*A*a*b*c
^2+2*A*a*b*d^2-2*A*b^2*c*d+B*a^2*c^2-B*a^2*d^2+4*B*a*b*c*d-B*b^2*c^2+B*b^2*
d^2-2*C*a^2*c*d+2*C*a*b*c^2-2*C*a*b*d^2+2*C*b^2*c*d)*ln(1+tan(f*x+e)^2)+(A*
a^2*c^2-A*a^2*d^2+4*A*a*b*c*d-A*b^2*c^2+A*b^2*d^2-2*B*a^2*c*d+2*B*a*b*c^2-2
*B*a*b*d^2+2*B*b^2*c*d-C*a^2*c^2+C*a^2*d^2-4*C*a*b*c*d+C*b^2*c^2-C*b^2*d^2)
*arctan(tan(f*x+e))))
```

Maxima [A]

time = 0.55, size = 501, normalized size = 1.21

$$\frac{\frac{2\,Cd^2\tan(fx+e)}{b^2}+\frac{2\left(\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)c^2-2\left(Ba^2-2\left(A-C\right)ab-Bb^2\right)cd-\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)d^2\right)(fx+e)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(\left(Ba^2b^3-2\left(A-C\right)ab^4-Bb^5\right)c^2-2\left(Ca^4b-\left(A-3\,C\right)a^2b^3-2\,Bab^4+Ab^5\right)cd+\left(2\,Ca^5-Ba^4b+4\,Ca^3b^2-3\,Ba^2b^3+2\,Aab^4\right)d^2\right)\log(b\tan(fx+e)+a)}{a^4b^3+2\,a^2b^5+b^7}+\frac{\left(\left(Ba^2-2\left(A-C\right)ab-Bb^2\right)c^2+2\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)cd-\left(Ba^2-2\left(A-C\right)ab-Bb^2\right)d^2\right)\log\left(\tan(fx+e)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(\left(Ca^2b^2-Bab^3+Ab^4\right)c^2-2\left(Ca^3b-Ba^2b^2+Aab^3\right)cd+\left(Ca^4-Ba^3b+Aa^2b^2\right)d^2\right)}{a^3b^3+ab^5+\left(a^2b^4+b^6\right)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*d^2*tan(f*x + e)/b^2 + 2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^
2 - 2*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 + 2*B*a*b - (A - C
)*b^2)*d^2)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a^2*b^3 - 2*(A - C)*a
*b^4 - B*b^5)*c^2 - 2*(C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*b^4 + A*b^5)*c*d
 + (2*C*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*d^2)*log(b*t
an(f*x + e) + a)/(a^4*b^3 + 2*a^2*b^5 + b^7) + ((B*a^2 - 2*(A - C)*a*b - B*
b^2)*c^2 + 2*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d - (B*a^2 - 2*(A - C)
```

```
*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*((C*
a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C*a^3*b - B*a^2*b^2 + A*a*b^3)*c*d + (C
*a^4 - B*a^3*b + A*a^2*b^2)*d^2)/(a^3*b^3 + a*b^5 + (a^2*b^4 + b^6)*tan(f*x
 + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 973 vs. $2(418) = 836$.
time = 5.96, size = 973, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*d^2*tan(f*x + e)^2 - 2*(C*a^2*b^4
- B*a*b^5 + A*b^6)*c^2 + 4*(C*a^3*b^3 - B*a^2*b^4 + A*a*b^5)*c*d - 2*(C*a^4
*b^2 - B*a^3*b^3 + A*a^2*b^4)*d^2 + 2*(((A - C)*a^3*b^3 + 2*B*a^2*b^4 - (A
- C)*a*b^5)*c^2 - 2*(B*a^3*b^3 - 2*(A - C)*a^2*b^4 - B*a*b^5)*c*d - ((A - C
)*a^3*b^3 + 2*B*a^2*b^4 - (A - C)*a*b^5)*d^2)*f*x - ((B*a^3*b^3 - 2*(A - C)
*a^2*b^4 - B*a*b^5)*c^2 - 2*(C*a^5*b - (A - 3*C)*a^3*b^3 - 2*B*a^2*b^4 + A*
a*b^5)*c*d + (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 3*B*a^3*b^3 + 2*A*a^2*b^4)*
d^2 + ((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^2 - 2*(C*a^4*b^2 - (A - 3*C)
*a^2*b^4 - 2*B*a*b^5 + A*b^6)*c*d + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 -
3*B*a^2*b^4 + 2*A*a*b^5)*d^2)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b
*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (2*(C*a^5*b + 2*C*a^3*b^3 + C*
a*b^5)*c*d - (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 2*B*a^3*b^3 + 2*C*a^2*b^4 -
 B*a*b^5)*d^2 + (2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*c*d - (2*C*a^5*b - B*a
^4*b^2 + 4*C*a^3*b^3 - 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*d^2)*tan(f*x + e))*
log(1/(tan(f*x + e)^2 + 1)) + 2*((C*a^3*b^3 - B*a^2*b^4 + A*a*b^5)*c^2 - 2*
(C*a^4*b^2 - B*a^3*b^3 + A*a^2*b^4)*c*d + (2*C*a^5*b - B*a^4*b^2 + (A + 2*C
)*a^3*b^3 + C*a*b^5)*d^2 + (((A - C)*a^2*b^4 + 2*B*a*b^5 - (A - C)*b^6)*c^2
 - 2*(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c*d - ((A - C)*a^2*b^4 + 2*B*a*b
^5 - (A - C)*b^6)*d^2)*f*x)*tan(f*x + e))/((a^4*b^4 + 2*a^2*b^6 + b^8)*f*ta
n(f*x + e) + (a^5*b^3 + 2*a^3*b^5 + a*b^7)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.50, size = 16225, normalized size = 39.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2,
Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c**2*x + A*c*d*log(tan(e + f*x)**2 + 1
)/f - A*d**2*x + A*d**2*tan(e + f*x)/f + B*c**2*log(tan(e + f*x)**2 + 1)/(2
*f) - 2*B*c*d*x + 2*B*c*d*tan(e + f*x)/f - B*d**2*log(tan(e + f*x)**2 + 1)/
(2*f) + B*d**2*tan(e + f*x)**2/(2*f) - C*c**2*x + C*c**2*tan(e + f*x)/f - C
*c*d*log(tan(e + f*x)**2 + 1)/f + C*c*d*tan(e + f*x)**2/f + C*d**2*x + C*d*
*2*tan(e + f*x)**3/(3*f) - C*d**2*tan(e + f*x)/f)/a**2, Eq(b, 0)), (-A*c**2
*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) -
4*b**2*f) + 2*I*A*c**2*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**
2*f*tan(e + f*x) - 4*b**2*f) + A*c**2*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b
**2*f*tan(e + f*x) - 4*b**2*f) - A*c**2*tan(e + f*x)/(4*b**2*f*tan(e + f*x)
**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c**2/(4*b**2*f*tan(e + f*
x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c*d*f*x*tan(e + f*x)**2
/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 4*A*c*d*
f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b*
*2*f) - 2*I*A*c*d*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) -
 4*b**2*f) + 2*I*A*c*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*
tan(e + f*x) - 4*b**2*f) + A*d**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x
)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*A*d**2*f*x*tan(e + f*x)/(4
*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - A*d**2*f*x/
(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*A*d**2*
tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f
) + 2*I*A*d**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2
*f) + I*B*c**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*t
an(e + f*x) - 4*b**2*f) + 2*B*c**2*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)*
*2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - I*B*c**2*f*x/(4*b**2*f*tan(e + f
*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*B*c**2*tan(e + f*x)/(4*b**
2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*B*c*d*f*x*tan
(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f
) - 4*I*B*c*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e
 + f*x) - 4*b**2*f) - 2*B*c*d*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*ta
n(e + f*x) - 4*b**2*f) - 6*B*c*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8
*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 4*I*B*c*d/(4*b**2*f*tan(e + f*x)**2 -
8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*I*B*d**2*f*x*tan(e + f*x)**2/(4*b**
2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 6*B*d**2*f*x*ta
n(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f)
- 3*I*B*d**2*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b*
*2*f) + 2*B*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*tan(e +
 f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 4*I*B*d**2*log(tan(e + f*x
)**2 + 1)*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x)
- 4*b**2*f) - 2*B*d**2*log(tan(e + f*x)**2 + 1)/(4*b**2*f*tan(e + f*x)**2 -
 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 5*I*B*d**2*tan(e + f*x)/(4*b**2*f*ta
n(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 4*B*d**2/(4*b**2*f*ta
n(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + C*c**2*f*x*tan(e + f*
x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I
```

```
*C*c**2*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x
) - 4*b**2*f) - C*c**2*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f
*x) - 4*b**2*f) - 3*C*c**2*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**
2*f*tan(e + f*x) - 4*b**2*f) + 2*I*C*c**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b
**2*f*tan(e + f*x) - 4*b**2*f) + 6*I*C*c*d*f*x*tan(e + f*x)**2/(4*b**2*f*ta
n(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 12*C*c*d*f*x*tan(e +
f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 6*I*
C*c*d*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) +
 4*C*c*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2
 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 8*I*C*c*d*log(tan(e + f*x)**2 + 1)
*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*
f) - 4*C*c*d*log(tan(e + f*x)**2 + 1)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*
f*tan(e + f*x) - 4*b**2*f) - 10*I*C*c*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)
**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 8*C*c*d/(4*b**2*f*tan(e + f*x)*
*2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 9*C*d**2*f*x*tan(e + f*x)**2/(4*
b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 18*I*C*d**2*
f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b*
*2*f) + 9*C*d**2*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) -
4*b**2*f) + 4*I*C*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*t
an(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 8*C*d**2*log(tan(e +
 f*x)**2 + 1)*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f
*x) - 4*b**2*f) - 4*I*C*d**2*log(tan(e + f*x)**...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(418) = 836.
time = 0.92, size = 912, normalized size = 2.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*d^2*tan(f*x + e)/b^2 + 2*(A*a^2*c^2 - C*a^2*c^2 + 2*B*a*b*c^2 - A*
b^2*c^2 + C*b^2*c^2 - 2*B*a^2*c*d + 4*A*a*b*c*d - 4*C*a*b*c*d + 2*B*b^2*c*d
 - A*a^2*d^2 + C*a^2*d^2 - 2*B*a*b*d^2 + A*b^2*d^2 - C*b^2*d^2)*(f*x + e)/(
a^4 + 2*a^2*b^2 + b^4) + (B*a^2*c^2 - 2*A*a*b*c^2 + 2*C*a*b*c^2 - B*b^2*c^2
 + 2*A*a^2*c*d - 2*C*a^2*c*d + 4*B*a*b*c*d - 2*A*b^2*c*d + 2*C*b^2*c*d - B*
a^2*d^2 + 2*A*a*b*d^2 - 2*C*a*b*d^2 + B*b^2*d^2)*log(tan(f*x + e)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - 2*(B*a^2*b^3*c^2 - 2*A*a*b^4*c^2 + 2*C*a*b^4*c^2 -
B*b^5*c^2 - 2*C*a^4*b*c*d + 2*A*a^2*b^3*c*d - 6*C*a^2*b^3*c*d + 4*B*a*b^4*c
*d - 2*A*b^5*c*d + 2*C*a^5*d^2 - B*a^4*b*d^2 + 4*C*a^3*b^2*d^2 - 3*B*a^2*b^
3*d^2 + 2*A*a*b^4*d^2)*log(abs(b*tan(f*x + e) + a))/(a^4*b^3 + 2*a^2*b^5 +
b^7) + 2*(B*a^2*b^4*c^2*tan(f*x + e) - 2*A*a*b^5*c^2*tan(f*x + e) + 2*C*a*b
^5*c^2*tan(f*x + e) - B*b^6*c^2*tan(f*x + e) - 2*C*a^4*b^2*c*d*tan(f*x + e)
```

```
 + 2*A*a^2*b^4*c*d*tan(f*x + e) - 6*C*a^2*b^4*c*d*tan(f*x + e) + 4*B*a*b^5*
c*d*tan(f*x + e) - 2*A*b^6*c*d*tan(f*x + e) + 2*C*a^5*b*d^2*tan(f*x + e) -
B*a^4*b^2*d^2*tan(f*x + e) + 4*C*a^3*b^3*d^2*tan(f*x + e) - 3*B*a^2*b^4*d^2
*tan(f*x + e) + 2*A*a*b^5*d^2*tan(f*x + e) - C*a^4*b^2*c^2 + 2*B*a^3*b^3*c^
2 - 3*A*a^2*b^4*c^2 + C*a^2*b^4*c^2 - A*b^6*c^2 - 2*B*a^4*b^2*c*d + 4*A*a^3
*b^3*c*d - 4*C*a^3*b^3*c*d + 2*B*a^2*b^4*c*d + C*a^6*d^2 - A*a^4*b^2*d^2 +
3*C*a^4*b^2*d^2 - 2*B*a^3*b^3*d^2 + A*a^2*b^4*d^2)/((a^4*b^3 + 2*a^2*b^5 +
b^7)*(b*tan(f*x + e) + a)))/f
```

Mupad [B]
time = 34.03, size = 2500, normalized size = 6.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b
*tan(e + f*x))^2,x)

[Out] (log((2*C^2*a^5*d^4 + 4*C^2*a^3*b^2*d^4 - 2*C^2*a^5*c^2*d^2 - A*B*b^5*c^4 -
 2*A*C*a^5*d^4 + B*C*b^5*c^4 - A^2*a*b^4*c^4 - A^2*a*b^4*d^4 + B^2*a*b^4*c^
4 + B^2*a*b^4*d^4 - C^2*a*b^4*c^4 + 2*A^2*b^5*c*d^3 - 2*A^2*b^5*c^3*d + C^2
*a*b^4*d^4 + 2*B^2*b^5*c^3*d - 4*C^2*a^3*b^2*c^2*d^2 + A*B*a^2*b^3*c^4 + 3*
A*B*a^2*b^3*d^4 - 4*A*C*a^3*b^2*d^4 - B*C*a^2*b^3*c^4 + 5*A*B*b^5*c^2*d^2 +
 2*A*C*a^5*c^2*d^2 - 3*B*C*a^2*b^3*d^4 - B*C*b^5*c^2*d^2 + 2*B^2*a^4*b*c*d^
3 - 2*C^2*a^4*b*c*d^3 + 2*C^2*a^4*b*c^3*d + 6*A^2*a*b^4*c^2*d^2 - 2*A^2*a^2
*b^3*c*d^3 + 2*A^2*a^2*b^3*c^3*d - 6*B^2*a*b^4*c^2*d^2 + 6*B^2*a^2*b^3*c*d^
3 - 2*B^2*a^2*b^3*c^3*d + 4*C^2*a*b^4*c^2*d^2 - 6*C^2*a^2*b^3*c*d^3 + 6*C^2
*a^2*b^3*c^3*d + A*B*a^4*b*d^4 + 2*A*C*a*b^4*c^4 - B*C*a^4*b*d^4 - 2*A*C*b^
5*c*d^3 + 2*A*C*b^5*c^3*d - 4*B*C*a^5*c*d^3 - 8*A*B*a*b^4*c*d^3 + 8*A*B*a*b
^4*c^3*d + 2*A*C*a^4*b*c*d^3 - 2*A*C*a^4*b*c^3*d + 4*B*C*a*b^4*c*d^3 - 8*B*
C*a*b^4*c^3*d - A*B*a^4*b*c^2*d^2 - 10*A*C*a*b^4*c^2*d^2 + 8*A*C*a^2*b^3*c*
d^3 - 8*A*C*a^2*b^3*c^3*d - 8*B*C*a^3*b^2*c*d^3 + 5*B*C*a^4*b*c^2*d^2 - 8*A
*B*a^2*b^3*c^2*d^2 + 4*A*C*a^3*b^2*c^2*d^2 + 16*B*C*a^2*b^3*c^2*d^2)/(b^2*(
a^2 + b^2)^2) + ((c*1i + d)^2*((tan(e + f*x)*(3*B*b^5*c^2 - 5*B*b^5*d^2 - 4
*C*a^5*d^2 + 6*A*b^5*c*d - 10*C*b^5*c*d + 4*A*a*b^4*c^2 - 4*A*a*b^4*d^2 + 2
*B*a^4*b*d^2 - 4*C*a*b^4*c^2 + 8*C*a*b^4*d^2 - B*a^2*b^3*c^2 + B*a^2*b^3*d^
2 - 8*B*a*b^4*c*d + 4*C*a^4*b*c*d - 2*A*a^2*b^3*c*d + 2*C*a^2*b^3*c*d))/(b^
2*(a^2 + b^2)) - (A*b^2*d^2 - A*b^2*c^2 - 8*C*a^2*d^2 + C*b^2*c^2 - C*b^2*d
^2 + 4*B*a*b*d^2 + 2*B*b^2*c*d + 8*C*a*b*c*d)/b + (b*(c*1i + d)^2*(4*a*b -
a^2*tan(e + f*x) + 3*b^2*tan(e + f*x))*(A*1i + B - C*1i))/(a*1i + b)^2)*(A*
1i + B - C*1i))/(2*(a*1i + b)^2) + (tan(e + f*x)*(A^2*b^5*c^4 + A^2*b^5*d^4
 + B^2*b^5*d^4 + C^2*b^5*c^4 + C^2*b^5*d^4 + B^2*a^2*b^3*c^4 + 3*B^2*a^2*b^
3*d^4 - 2*A^2*b^5*c^2*d^2 + 3*B^2*b^5*c^2*d^2 + 2*C^2*b^5*c^2*d^2 - 2*A*C*b
^5*c^4 - 2*A*C*b^5*d^4 - 2*B*C*a^5*d^4 + B^2*a^4*b*d^4 - 4*C^2*a^5*c*d^3 +
4*A^2*a^2*b^3*c^2*d^2 - 4*B^2*a^2*b^3*c^2*d^2 + 12*C^2*a^2*b^3*c^2*d^2 - 4*
```

B*C*a^3*b^2*d^4 + 2*B*C*a^5*c^2*d^2 + 4*A^2*a*b^4*c*d^3 - 4*A^2*a*b^4*c^3*d
 - 4*B^2*a*b^4*c*d^3 + 4*B^2*a*b^4*c^3*d - 4*C^2*a*b^4*c^3*d - B^2*a^4*b*c^
2*d^2 - 8*C^2*a^3*b^2*c*d^3 + 4*C^2*a^4*b*c^2*d^2 - 2*A*B*a*b^4*c^4 - 2*A*B
*a*b^4*d^4 + 2*B*C*a*b^4*c^4 + 2*A*B*b^5*c*d^3 - 4*A*B*b^5*c^3*d + 4*A*C*a^
5*c*d^3 + 2*B*C*b^5*c^3*d - 2*A*B*a^4*b*c*d^3 - 4*A*C*a*b^4*c*d^3 + 8*A*C*a
*b^4*c^3*d + 4*B*C*a^4*b*c*d^3 - 2*B*C*a^4*b*c^3*d + 12*A*B*a*b^4*c^2*d^2 -
 8*A*B*a^2*b^3*c*d^3 + 4*A*B*a^2*b^3*c^3*d + 8*A*C*a^3*b^2*c*d^3 - 4*A*C*a^
4*b*c^2*d^2 - 10*B*C*a*b^4*c^2*d^2 + 12*B*C*a^2*b^3*c*d^3 - 8*B*C*a^2*b^3*c
^3*d - 16*A*C*a^2*b^3*c^2*d^2 + 4*B*C*a^3*b^2*c^2*d^2))/(b^2*(a^2 + b^2)^2)
)*(A*d^2*1i - A*c^2*1i - B*c^2 + B*d^2 + C*c^2*1i - C*d^2*1i - 2*A*c*d + B*
c*d*2i + 2*C*c*d))/(2*f*(a*b*2i - a^2 + b^2)) - (log(a + b*tan(e + f*x))*(b
^3*(B*a^2*c^2 - 3*B*a^2*d^2 + 2*A*a^2*c*d - 6*C*a^2*c*d) - b^5*(B*c^2 + 2*A
*c*d) - b*(B*a^4*d^2 + 2*C*a^4*c*d) + b^4*(2*A*a*d^2 - 2*A*a*c^2 + 2*C*a*c^
2 + 4*B*a*c*d) + 2*C*a^5*d^2 + 4*C*a^3*b^2*d^2))/(f*(b^7 + 2*a^2*b^5 + a^4*
b^3)) + (log((2*C^2*a^5*d^4 + 4*C^2*a^3*b^2*d^4 - 2*C^2*a^5*c^2*d^2 - A*B*b
^5*c^4 - 2*A*C*a^5*d^4 + B*C*b^5*c^4 - A^2*a*b^4*c^4 - A^2*a*b^4*d^4 + B^2*
a*b^4*c^4 + B^2*a*b^4*d^4 - C^2*a*b^4*c^4 + 2*A^2*b^5*c*d^3 - 2*A^2*b^5*c^3
*d + C^2*a*b^4*d^4 + 2*B^2*b^5*c^3*d - 4*C^2*a^3*b^2*c^2*d^2 + A*B*a^2*b^3*
c^4 + 3*A*B*a^2*b^3*d^4 - 4*A*C*a^3*b^2*d^4 - B*C*a^2*b^3*c^4 + 5*A*B*b^5*c
^2*d^2 + 2*A*C*a^5*c^2*d^2 - 3*B*C*a^2*b^3*d^4 - B*C*b^5*c^2*d^2 + 2*B^2*a^
4*b*c*d^3 - 2*C^2*a^4*b*c*d^3 + 2*C^2*a^4*b*c^3*d + 6*A^2*a*b^4*c^2*d^2 - 2
*A^2*a^2*b^3*c*d^3 + 2*A^2*a^2*b^3*c^3*d - 6*B^2*a*b^4*c^2*d^2 + 6*B^2*a^2*
b^3*c*d^3 - 2*B^2*a^2*b^3*c^3*d + 4*C^2*a*b^4*c^2*d^2 - 6*C^2*a^2*b^3*c*d^3
 + 6*C^2*a^2*b^3*c^3*d + A*B*a^4*b*d^4 + 2*A*C*a*b^4*c^4 - B*C*a^4*b*d^4 -
2*A*C*b^5*c*d^3 + 2*A*C*b^5*c^3*d - 4*B*C*a^5*c*d^3 - 8*A*B*a*b^4*c*d^3 + 8
*A*B*a*b^4*c^3*d + 2*A*C*a^4*b*c*d^3 - 2*A*C*a^4*b*c^3*d + 4*B*C*a*b^4*c*d^
3 - 8*B*C*a*b^4*c^3*d - A*B*a^4*b*c^2*d^2 - 10*A*C*a*b^4*c^2*d^2 + 8*A*C*a^
2*b^3*c*d^3 - 8*A*C*a^2*b^3*c^3*d - 8*B*C*a^3*b^2*c*d^3 + 5*B*C*a^4*b*c^2*d
^2 - 8*A*B*a^2*b^3*c^2*d^2 + 4*A*C*a^3*b^2*c^2*d^2 + 16*B*C*a^2*b^3*c^2*d^2
)/(b^2*(a^2 + b^2)^2) + ((c*1i - d)^2*((A*b^2*d^2 - A*b^2*c^2 - 8*C*a^2*d^2
 + C*b^2*c^2 - C*b^2*d^2 + 4*B*a*b*d^2 + 2*B*b^2*c*d + 8*C*a*b*c*d)/b - (ta
n(e + f*x)*(3*B*b^5*c^2 - 5*B*b^5*d^2 - 4*C*a^5*d^2 + 6*A*b^5*c*d - 10*C*b^
5*c*d + 4*A*a*b^4*c^2 - 4*A*a*b^4*d^2 + 2*B*a^4*b*d^2 - 4*C*a*b^4*c^2 + 8*C
*a*b^4*d^2 - B*a^2*b^3*c^2 + B*a^2*b^3*d^2 - 8*B*a*b^4*c*d + 4*C*a^4*b*c*d
- 2*A*a^2*b^3*c*d + 2*C*a^2*b^3*c*d))/(b^2*(a^2 + b^2)) + (b*(c*1i - d)^2*(
4*a*b - a^2*tan(e + f*x) + 3*b^2*tan(e + f*x))*(A + B*1i - C)*1i)/(a*1i - b
)^2)*(A + B*1i - C)*1i)/(2*(a*1i - b)^2) + (tan(e + f*x)*(A^2*b^5*c^4 + A^2
*b^5*d^4 + B^2*b^5*d^4 + C^2*b^5*c^4 + C^2*b^5*d^4 + B^2*a^2*b^3*c^4 + 3*B^
2*a^2*b^3*d^4 - 2*A^2*b^5*c^2*d^2 + 3*B^2*b^5*c^2*d^2 + 2*C^2*b^5*c^2*d^2 -
 2*A*C*b^5*c^4 - 2*A*C*b^5*d^4 - 2*B*C*a^5*d^4 ...

3.63 $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=597

$$-\frac{(a^3(c^2C+2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C+2Bcd-Cd^2-A(c^2-d^2))-3a^2b(2c(A-C)d+B(}{(a^2+b^2)^3}$$

```
[Out] -(a^3*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-3*a*b^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2
-d^2))-3*a^2*b*(2*c*(A-C)*d+B*(c^2-d^2))+b^3*(2*c*(A-C)*d+B*(c^2-d^2)))*x/(
a^2+b^2)^3-(3*a^2*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b^3*(c^2*C+2*B*c*d-C*
d^2-A*(c^2-d^2))+a^3*(2*c*(A-C)*d+B*(c^2-d^2))-3*a*b^2*(2*c*(A-C)*d+B*(c^2-
d^2)))*ln(cos(f*x+e))/(a^2+b^2)^3/f+(a^6*C*d^2+3*a^4*b^2*C*d^2-3*a^2*b^4*(c
^2*C+2*B*c*d-2*C*d^2-A*(c^2-d^2))+b^6*(c*(2*B*d+C*c)-A*(c^2-d^2))-a^3*b^3*(
2*c*(A-C)*d+B*(c^2-d^2))+3*a*b^5*(2*c*(A-C)*d+B*(c^2-d^2)))*ln(a+b*tan(f*x+
e))/b^3/(a^2+b^2)^3/f-(-a*d+b*c)*(a^4*C*d+b^4*(A*d+B*c)+2*a*b^3*(A*c-B*d-C*
c)-a^2*b^2*(B*c+(A-3*C)*d))/b^3/(a^2+b^2)^2/f/(a+b*tan(f*x+e))-1/2*(A*b^2-a
*(B*b-C*a))*(c+d*tan(f*x+e))^2/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2
```

Rubi [A]
time = 0.88, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3726, 3716, 3707, 3698, 31, 3556}

$$-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^2}{2bf(a^2+b^2)(a+b\tan(e+fx))^2}-\frac{(bc-ad)(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc))}{b^3f(a^2+b^2)^2(a+b\tan(e+fx))}-\frac{\log(\cos(e+fx))(a^3(2cd(A-C)+B(c^2-d^2))+3a^2b(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)-3ab^2(2cd(A-C)+B(c^2-d^2))-b^3(-A(c^2-d^2)+2Bcd+c^2C-Cd^2))}{f(a^2+b^2)^3}-\frac{x(a^3(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)-3a^2b(2cd(A-C)+B(c^2-d^2))-3ab^2(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)+b^3(2cd(A-C)+B(c^2-d^2)))}{(a^2+b^2)^3}+\frac{(a^6Cd^2+3a^4b^2Cd^2-a^3b^3(2cd(A-C)+B(c^2-d^2))-3a^2b^4(-A(c^2-d^2)+2Bcd+c^2C-2Cd^2)+3ab^5(2cd(A-C)+B(c^2-d^2))+b^6(c(2Bd+cC)-A(c^2-d^2)))\log(a+b\tan(e+fx))}{b^3f(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x])^3,x]
```

```
[Out] -(((a^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C + 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) - 3*a^2*b*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^3*
(2*c*(A - C)*d + B*(c^2 - d^2)))*x)/(a^2 + b^2)^3) - ((3*a^2*b*(c^2*C + 2*B
*c*d - C*d^2 - A*(c^2 - d^2)) - b^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2
)) + a^3*(2*c*(A - C)*d + B*(c^2 - d^2)) - 3*a*b^2*(2*c*(A - C)*d + B*(c^2
- d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)^3*f) + ((a^6*C*d^2 + 3*a^4*b^2*C*d
^2 - 3*a^2*b^4*(c^2*C + 2*B*c*d - 2*C*d^2 - A*(c^2 - d^2)) + b^6*(c*(c*C +
2*B*d) - A*(c^2 - d^2)) - a^3*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)) + 3*a*b^5
*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[a + b*Tan[e + f*x]])/(b^3*(a^2 + b^2)
^3*f) - ((b*c - a*d)*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d)
 - a^2*b^2*(B*c + (A - 3*C)*d)))/(b^3*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
 - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^2)/(2*b*(a^2 + b^2)*f*(a +
 b*Tan[e + f*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))}$$

$$= -\frac{(bc-ad)\,(a^4Cd+b^4(Bc+Ad)+2a}{b^3\,(a^2+b^2)^2}$$

$$= -\frac{(a^3(c^2C+2Bcd-Cd^2-A(c^2-d^2)}{}$$

$$= -\frac{(a^3(c^2C+2Bcd-Cd^2-A(c^2-d^2)}{}$$

$$= -\frac{(a^3(c^2C+2Bcd-Cd^2-A(c^2-d^2)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.51, size = 2499, normalized size = 4.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((-(A*b^4*c^2) + a*b^3*B*c^2 - a^2*b^2*c^2*C + 2*a*A*b^3*c*d - 2*a^2*b^2*B*
c*d + 2*a^3*b*c*C*d - a^2*A*b^2*d^2 + a^3*b*B*d^2 - a^4*C*d^2)*Sec[e + f*x]
*(a*Cos[e + f*x] + b*Sin[e + f*x])*(c + d*Tan[e + f*x])^2)/(2*(a - I*b)^2*(
a + I*b)^2*b*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)
+ ((a^3*A*c^2 - 3*a*A*b^2*c^2 + 3*a^2*b*B*c^2 - b^3*B*c^2 - a^3*c^2*C + 3*a
*b^2*c^2*C + 6*a^2*A*b*c*d - 2*A*b^3*c*d - 2*a^3*B*c*d + 6*a*b^2*B*c*d - 6*
a^2*b*c*C*d + 2*b^3*c*C*d - a^3*A*d^2 + 3*a*A*b^2*d^2 - 3*a^2*b*B*d^2 + b^3
*B*d^2 + a^3*C*d^2 - 3*a*b^2*C*d^2)*(e + f*x)*Sec[e + f*x]*(a*Cos[e + f*x]
+ b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2)/((a - I*b)^3*(a + I*b)^3*f*(c*C
os[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3) + (((3*I)*a^9*A*b^6
*c^2 + 3*a^8*A*b^7*c^2 + (5*I)*a^7*A*b^8*c^2 + 5*a^6*A*b^9*c^2 + I*a^5*A*b^
10*c^2 + a^4*A*b^11*c^2 - I*a^3*A*b^12*c^2 - a^2*A*b^13*c^2 - I*a^10*b^5*B*
c^2 - a^9*b^6*B*c^2 + I*a^8*b^7*B*c^2 + a^7*b^8*B*c^2 + (5*I)*a^6*b^9*B*c^2
 + 5*a^5*b^10*B*c^2 + (3*I)*a^4*b^11*B*c^2 + 3*a^3*b^12*B*c^2 - (3*I)*a^9*b
```

```
^6*c^2*C - 3*a^8*b^7*c^2*C - (5*I)*a^7*b^8*c^2*C - 5*a^6*b^9*c^2*C - I*a^5*
b^10*c^2*C - a^4*b^11*c^2*C + I*a^3*b^12*c^2*C + a^2*b^13*c^2*C - (2*I)*a^1
0*A*b^5*c*d - 2*a^9*A*b^6*c*d + (2*I)*a^8*A*b^7*c*d + 2*a^7*A*b^8*c*d + (10
*I)*a^6*A*b^9*c*d + 10*a^5*A*b^10*c*d + (6*I)*a^4*A*b^11*c*d + 6*a^3*A*b^12
*c*d - (6*I)*a^9*b^6*B*c*d - 6*a^8*b^7*B*c*d - (10*I)*a^7*b^8*B*c*d - 10*a^
6*b^9*B*c*d - (2*I)*a^5*b^10*B*c*d - 2*a^4*b^11*B*c*d + (2*I)*a^3*b^12*B*c*
d + 2*a^2*b^13*B*c*d + (2*I)*a^10*b^5*c*C*d + 2*a^9*b^6*c*C*d - (2*I)*a^8*b
^7*c*C*d - 2*a^7*b^8*c*C*d - (10*I)*a^6*b^9*c*C*d - 10*a^5*b^10*c*C*d - (6*
I)*a^4*b^11*c*C*d - 6*a^3*b^12*c*C*d - (3*I)*a^9*A*b^6*d^2 - 3*a^8*A*b^7*d^
2 - (5*I)*a^7*A*b^8*d^2 - 5*a^6*A*b^9*d^2 - I*a^5*A*b^10*d^2 - a^4*A*b^11*d
^2 + I*a^3*A*b^12*d^2 + a^2*A*b^13*d^2 + I*a^10*b^5*B*d^2 + a^9*b^6*B*d^2 -
 I*a^8*b^7*B*d^2 - a^7*b^8*B*d^2 - (5*I)*a^6*b^9*B*d^2 - 5*a^5*b^10*B*d^2 -
 (3*I)*a^4*b^11*B*d^2 - 3*a^3*b^12*B*d^2 + I*a^13*b^2*C*d^2 + a^12*b^3*C*d^
2 + (5*I)*a^11*b^4*C*d^2 + 5*a^10*b^5*C*d^2 + (13*I)*a^9*b^6*C*d^2 + 13*a^8
*b^7*C*d^2 + (15*I)*a^7*b^8*C*d^2 + 15*a^6*b^9*C*d^2 + (6*I)*a^5*b^10*C*d^2
 + 6*a^4*b^11*C*d^2)*(e + f*x)*Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x
])^3*(c + d*Tan[e + f*x])^2)/(a^2*(a - I*b)^6*(a + I*b)^5*b^5*f*(c*Cos[e +
f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3) - (I*(3*a^2*A*b^4*c^2 - A*
b^6*c^2 - a^3*b^3*B*c^2 + 3*a*b^5*B*c^2 - 3*a^2*b^4*c^2*C + b^6*c^2*C - 2*a
^3*A*b^3*c*d + 6*a*A*b^5*c*d - 6*a^2*b^4*B*c*d + 2*b^6*B*c*d + 2*a^3*b^3*c*
C*d - 6*a*b^5*c*C*d - 3*a^2*A*b^4*d^2 + A*b^6*d^2 + a^3*b^3*B*d^2 - 3*a*b^5
*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*C*d^2 + 6*a^2*b^4*C*d^2)*ArcTan[Tan[e + f*x]
]*Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2)/
(b^3*(a^2 + b^2)^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x
])^3) - (C*d^2*Log[Cos[e + f*x]]*Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f
*x])^3*(c + d*Tan[e + f*x])^2)/(b^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(
a + b*Tan[e + f*x])^3) + ((3*a^2*A*b^4*c^2 - A*b^6*c^2 - a^3*b^3*B*c^2 + 3*
a*b^5*B*c^2 - 3*a^2*b^4*c^2*C + b^6*c^2*C - 2*a^3*A*b^3*c*d + 6*a*A*b^5*c*d
 - 6*a^2*b^4*B*c*d + 2*b^6*B*c*d + 2*a^3*b^3*c*C*d - 6*a*b^5*c*C*d - 3*a^2*
A*b^4*d^2 + A*b^6*d^2 + a^3*b^3*B*d^2 - 3*a*b^5*B*d^2 + a^6*C*d^2 + 3*a^4*b
^2*C*d^2 + 6*a^2*b^4*C*d^2)*Log[(a*Cos[e + f*x] + b*Sin[e + f*x])^2]*Sec[e
+ f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2)/(2*b^3*(
a^2 + b^2)^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)
+ (Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(3*a*A*b^4*c^2*Sin[e +
f*x] - 2*a^2*b^3*B*c^2*Sin[e + f*x] + b^5*B*c^2*Sin[e + f*x] + a^3*b^2*c^2*
C*Sin[e + f*x] - 2*a*b^4*c^2*C*Sin[e + f*x] - 4*a^2*A*b^3*c*d*Sin[e + f*x]
+ 2*A*b^5*c*d*Sin[e + f*x] + 2*a^3*b^2*B*c*d*Sin[e + f*x] - 4*a*b^4*B*c*d*S
in[e + f*x] + 6*a^2*b^3*c*C*d*Sin[e + f*x] + a^3*A*b^2*d^2*Sin[e + f*x] - 2
*a*A*b^4*d^2*Sin[e + f*x] + 3*a^2*b^3*B*d^2*Sin[e + f*x] - a^5*C*d^2*Sin[e
+ f*x] - 4*a^3*b^2*C*d^2*Sin[e + f*x])*(c + d*Tan[e + f*x])^2)/(a*(a - I*b)
^2*(a + I*b)^2*b^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x
])^3)
```

Maple [A]

time = 0.58, size = 865, normalized size = 1.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^3*(1/2*(2*A*a^3*c*d-3*A*a^2*b*c^2+3*A*a^2*b*d^2-6*A*a*b^2*
c*d+A*b^3*c^2-A*b^3*d^2+B*a^3*c^2-B*a^3*d^2+6*B*a^2*b*c*d-3*B*a*b^2*c^2+3*B
*a*b^2*d^2-2*B*b^3*c*d-2*C*a^3*c*d+3*C*a^2*b*c^2-3*C*a^2*b*d^2+6*C*a*b^2*c*
d-C*b^3*c^2+C*b^3*d^2)*ln(1+tan(f*x+e)^2)+(A*a^3*c^2-A*a^3*d^2+6*A*a^2*b*c*
d-3*A*a*b^2*c^2+3*A*a*b^2*d^2-2*A*b^3*c*d-2*B*a^3*c*d+3*B*a^2*b*c^2-3*B*a^2
*b*d^2+6*B*a*b^2*c*d-B*b^3*c^2+B*b^3*d^2-C*a^3*c^2+C*a^3*d^2-6*C*a^2*b*c*d+
3*C*a*b^2*c^2-3*C*a*b^2*d^2+2*C*b^3*c*d)*arctan(tan(f*x+e)))-1/2*(A*a^2*b^2
*d^2-2*A*a*b^3*c*d+A*b^4*c^2-B*a^3*b*d^2+2*B*a^2*b^2*c*d-B*a*b^3*c^2+C*a^4*
d^2-2*C*a^3*b*c*d+C*a^2*b^2*c^2)/b^3/(a^2+b^2)/(a+b*tan(f*x+e))^2-(-2*A*a^2
*b^3*c*d+2*A*a*b^4*c^2-2*A*a*b^4*d^2+2*A*b^5*c*d+B*a^4*b*d^2-B*a^2*b^3*c^2+
3*B*a^2*b^3*d^2-4*B*a*b^4*c*d+B*b^5*c^2-2*C*a^5*d^2+2*C*a^4*b*c*d-4*C*a^3*b
^2*d^2+6*C*a^2*b^3*c*d-2*C*a*b^4*c^2)/b^3/(a^2+b^2)^2/(a+b*tan(f*x+e))+1/(a
^2+b^2)^3*(-2*A*a^3*b^3*c*d+3*A*a^2*b^4*c^2-3*A*a^2*b^4*d^2+6*A*a*b^5*c*d-A
*b^6*c^2+A*b^6*d^2-B*a^3*b^3*c^2+B*a^3*b^3*d^2-6*B*a^2*b^4*c*d+3*B*a*b^5*c^
2-3*B*a*b^5*d^2+2*B*b^6*c*d+C*a^6*d^2+3*C*a^4*b^2*d^2+2*C*a^3*b^3*c*d-3*C*a
^2*b^4*c^2+6*C*a^2*b^4*d^2-6*C*a*b^5*c*d+C*b^6*c^2)/b^3*ln(a+b*tan(f*x+e)))
```

Maxima [A]
time = 0.55, size = 845, normalized size = 1.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c^2 - 2*(B*a^3
- 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c*d - ((A - C)*a^3 + 3*B*a^2*b
 - 3*(A - C)*a*b^2 - B*b^3)*d^2)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) - 2*((B*a^3*b^3 - 3*(A - C)*a^2*b^4 - 3*B*a*b^5 + (A - C)*b^6)*c^2 + 2*
((A - C)*a^3*b^3 + 3*B*a^2*b^4 - 3*(A - C)*a*b^5 - B*b^6)*c*d - (C*a^6 + 3*
C*a^4*b^2 + B*a^3*b^3 - 3*(A - 2*C)*a^2*b^4 - 3*B*a*b^5 + A*b^6)*d^2)*log(b
*tan(f*x + e) + a)/(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) + ((B*a^3 - 3*(A
 - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c^2 + 2*((A - C)*a^3 + 3*B*a^2*b - 3
*(A - C)*a*b^2 - B*b^3)*c*d - (B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C
)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (
(C*a^4*b^2 - 3*B*a^3*b^3 + (5*A - 3*C)*a^2*b^4 + B*a*b^5 + A*b^6)*c^2 + 2*(
C*a^5*b + B*a^4*b^2 - (3*A - 5*C)*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5)*c*d - (3
*C*a^6 - B*a^5*b - (A - 7*C)*a^4*b^2 - 5*B*a^3*b^3 + 3*A*a^2*b^4)*d^2 - 2*(
(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^2 - 2*(C*a^4*b^2 - (A - 3*C)*a^2*b^
4 - 2*B*a*b^5 + A*b^6)*c*d + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^2
```

```
*b^4 + 2*A*a*b^5)*d^2)*tan(f*x + e))/(a^6*b^3 + 2*a^4*b^5 + a^2*b^7 + (a^4*
b^5 + 2*a^2*b^7 + b^9)*tan(f*x + e)^2 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*tan
(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1711 vs. $2(600) = 1200$.
time = 3.95, size = 1711, normalized size = 2.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((3*C*a^4*b^4 - 5*B*a^3*b^5 + (7*A - 3*C)*a^2*b^6 + B*a*b^7 + A*b^8)*c
^2 - 2*(C*a^5*b^3 - 3*B*a^4*b^4 + 5*(A - C)*a^3*b^5 + 3*B*a^2*b^6 - A*a*b^7
)*c*d - (C*a^6*b^2 + B*a^5*b^3 - (3*A - 7*C)*a^4*b^4 - 5*B*a^3*b^5 + 3*A*a^
2*b^6)*d^2 - 2*(((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*a^2*
b^6)*c^2 - 2*(B*a^5*b^3 - 3*(A - C)*a^4*b^4 - 3*B*a^3*b^5 + (A - C)*a^2*b^6
)*c*d - ((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*a^2*b^6)*d^2
)*f*x - ((C*a^4*b^4 - 3*B*a^3*b^5 + 5*(A - C)*a^2*b^6 + 3*B*a*b^7 - A*b^8)*
c^2 + 2*(C*a^5*b^3 + B*a^4*b^4 - (3*A - 7*C)*a^3*b^5 - 5*B*a^2*b^6 + 3*A*a*
b^7)*c*d - (3*C*a^6*b^2 - B*a^5*b^3 - (A - 9*C)*a^4*b^4 - 7*B*a^3*b^5 + 5*A
*a^2*b^6)*d^2 + 2*(((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8
)*c^2 - 2*(B*a^3*b^5 - 3*(A - C)*a^2*b^6 - 3*B*a*b^7 + (A - C)*b^8)*c*d - (
(A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8)*d^2)*f*x)*tan(f*x
+ e)^2 + ((B*a^5*b^3 - 3*(A - C)*a^4*b^4 - 3*B*a^3*b^5 + (A - C)*a^2*b^6)*c
^2 + 2*((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*a^2*b^6)*c*d
- (C*a^8 + 3*C*a^6*b^2 + B*a^5*b^3 - 3*(A - 2*C)*a^4*b^4 - 3*B*a^3*b^5 + A*
a^2*b^6)*d^2 + ((B*a^3*b^5 - 3*(A - C)*a^2*b^6 - 3*B*a*b^7 + (A - C)*b^8)*c
^2 + 2*((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8)*c*d - (C*a
^6*b^2 + 3*C*a^4*b^4 + B*a^3*b^5 - 3*(A - 2*C)*a^2*b^6 - 3*B*a*b^7 + A*b^8)
*d^2)*tan(f*x + e)^2 + 2*((B*a^4*b^4 - 3*(A - C)*a^3*b^5 - 3*B*a^2*b^6 + (A
 - C)*a*b^7)*c^2 + 2*((A - C)*a^4*b^4 + 3*B*a^3*b^5 - 3*(A - C)*a^2*b^6 - B
*a*b^7)*c*d - (C*a^7*b + 3*C*a^5*b^3 + B*a^4*b^4 - 3*(A - 2*C)*a^3*b^5 - 3*
B*a^2*b^6 + A*a*b^7)*d^2)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan
(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) + ((C*a^6*b^2 + 3*C*a^4*b^4 + 3*C*a^
2*b^6 + C*b^8)*d^2*tan(f*x + e)^2 + 2*(C*a^7*b + 3*C*a^5*b^3 + 3*C*a^3*b^5
+ C*a*b^7)*d^2*tan(f*x + e) + (C*a^8 + 3*C*a^6*b^2 + 3*C*a^4*b^4 + C*a^2*b^
6)*d^2)*log(1/(tan(f*x + e)^2 + 1)) - 2*((C*a^5*b^3 - 2*B*a^4*b^4 + 3*(A -
C)*a^3*b^5 + 3*B*a^2*b^6 - (3*A - 2*C)*a*b^7 - B*b^8)*c^2 + 2*(B*a^5*b^3 -
(2*A - 3*C)*a^4*b^4 - 3*B*a^3*b^5 + 3*(A - C)*a^2*b^6 + 2*B*a*b^7 - A*b^8)*
c*d - (C*a^7*b - (A - 3*C)*a^5*b^3 - 3*B*a^4*b^4 + (3*A - 4*C)*a^3*b^5 + 3*
B*a^2*b^6 - 2*A*a*b^7)*d^2 + 2*(((A - C)*a^4*b^4 + 3*B*a^3*b^5 - 3*(A - C)*
a^2*b^6 - B*a*b^7)*c^2 - 2*(B*a^4*b^4 - 3*(A - C)*a^3*b^5 - 3*B*a^2*b^6 + (
```

```
A - C)*a*b^7)*c*d - ((A - C)*a^4*b^4 + 3*B*a^3*b^5 - 3*(A - C)*a^2*b^6 - B*
a*b^7)*d^2)*f*x)*tan(f*x + e))/((a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*f*
tan(f*x + e)^2 + 2*(a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*f*tan(f*x + e
) + (a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. 2(600) = 1200.
time = 1.20, size = 1714, normalized size = 2.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c^2 - C*a^3*c^2 + 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*c
^2 - B*b^3*c^2 - 2*B*a^3*c*d + 6*A*a^2*b*c*d - 6*C*a^2*b*c*d + 6*B*a*b^2*c*
d - 2*A*b^3*c*d + 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 - 3*B*a^2*b*d^2 + 3*A
*a*b^2*d^2 - 3*C*a*b^2*d^2 + B*b^3*d^2)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6) + (B*a^3*c^2 - 3*A*a^2*b*c^2 + 3*C*a^2*b*c^2 - 3*B*a*b^2*c^2 + A
*b^3*c^2 - C*b^3*c^2 + 2*A*a^3*c*d - 2*C*a^3*c*d + 6*B*a^2*b*c*d - 6*A*a*b^
2*c*d + 6*C*a*b^2*c*d - 2*B*b^3*c*d - B*a^3*d^2 + 3*A*a^2*b*d^2 - 3*C*a^2*b
*d^2 + 3*B*a*b^2*d^2 - A*b^3*d^2 + C*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b^3*c^2 - 3*A*a^2*b^4*c^2 + 3*C*a
^2*b^4*c^2 - 3*B*a*b^5*c^2 + A*b^6*c^2 - C*b^6*c^2 + 2*A*a^3*b^3*c*d - 2*C*
a^3*b^3*c*d + 6*B*a^2*b^4*c*d - 6*A*a*b^5*c*d + 6*C*a*b^5*c*d - 2*B*b^6*c*d
 - C*a^6*d^2 - 3*C*a^4*b^2*d^2 - B*a^3*b^3*d^2 + 3*A*a^2*b^4*d^2 - 6*C*a^2*
b^4*d^2 + 3*B*a*b^5*d^2 - A*b^6*d^2)*log(abs(b*tan(f*x + e) + a))/(a^6*b^3
+ 3*a^4*b^5 + 3*a^2*b^7 + b^9) + (3*B*a^3*b^4*c^2*tan(f*x + e)^2 - 9*A*a^2*
b^5*c^2*tan(f*x + e)^2 + 9*C*a^2*b^5*c^2*tan(f*x + e)^2 - 9*B*a*b^6*c^2*tan
(f*x + e)^2 + 3*A*b^7*c^2*tan(f*x + e)^2 - 3*C*b^7*c^2*tan(f*x + e)^2 + 6*A
*a^3*b^4*c*d*tan(f*x + e)^2 - 6*C*a^3*b^4*c*d*tan(f*x + e)^2 + 18*B*a^2*b^5
*c*d*tan(f*x + e)^2 - 18*A*a*b^6*c*d*tan(f*x + e)^2 + 18*C*a*b^6*c*d*tan(f*
x + e)^2 - 6*B*b^7*c*d*tan(f*x + e)^2 - 3*C*a^6*b*d^2*tan(f*x + e)^2 - 9*C*
```

```
a^4*b^3*d^2*tan(f*x + e)^2 - 3*B*a^3*b^4*d^2*tan(f*x + e)^2 + 9*A*a^2*b^5*d
^2*tan(f*x + e)^2 - 18*C*a^2*b^5*d^2*tan(f*x + e)^2 + 9*B*a*b^6*d^2*tan(f*x
 + e)^2 - 3*A*b^7*d^2*tan(f*x + e)^2 + 8*B*a^4*b^3*c^2*tan(f*x + e) - 22*A*
a^3*b^4*c^2*tan(f*x + e) + 22*C*a^3*b^4*c^2*tan(f*x + e) - 18*B*a^2*b^5*c^2
*tan(f*x + e) + 2*A*a*b^6*c^2*tan(f*x + e) - 2*C*a*b^6*c^2*tan(f*x + e) - 2
*B*b^7*c^2*tan(f*x + e) - 4*C*a^6*b*c*d*tan(f*x + e) + 16*A*a^4*b^3*c*d*tan
(f*x + e) - 28*C*a^4*b^3*c*d*tan(f*x + e) + 44*B*a^3*b^4*c*d*tan(f*x + e) -
 36*A*a^2*b^5*c*d*tan(f*x + e) + 24*C*a^2*b^5*c*d*tan(f*x + e) - 4*B*a*b^6*
c*d*tan(f*x + e) - 4*A*b^7*c*d*tan(f*x + e) - 2*C*a^7*d^2*tan(f*x + e) - 2*
B*a^6*b*d^2*tan(f*x + e) - 6*C*a^5*b^2*d^2*tan(f*x + e) - 14*B*a^4*b^3*d^2*
tan(f*x + e) + 22*A*a^3*b^4*d^2*tan(f*x + e) - 28*C*a^3*b^4*d^2*tan(f*x + e
) + 12*B*a^2*b^5*d^2*tan(f*x + e) - 2*A*a*b^6*d^2*tan(f*x + e) - C*a^6*b*c^
2 + 6*B*a^5*b^2*c^2 - 14*A*a^4*b^3*c^2 + 11*C*a^4*b^3*c^2 - 7*B*a^3*b^4*c^2
 - 3*A*a^2*b^5*c^2 - B*a*b^6*c^2 - A*b^7*c^2 - 2*C*a^7*c*d - 2*B*a^6*b*c*d
+ 12*A*a^5*b^2*c*d - 18*C*a^5*b^2*c*d + 22*B*a^4*b^3*c*d - 14*A*a^3*b^4*c*d
 + 8*C*a^3*b^4*c*d - 2*A*a*b^6*c*d - B*a^7*d^2 - A*a^6*b*d^2 + C*a^6*b*d^2
- 9*B*a^5*b^2*d^2 + 11*A*a^4*b^3*d^2 - 11*C*a^4*b^3*d^2 + 4*B*a^3*b^4*d^2)/
((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(f*x + e) + a)^2))/f
```

Mupad [B]
time = 29.28, size = 807, normalized size = 1.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b
*tan(e + f*x))^3,x)
```

```
[Out] - (log(a + b*tan(e + f*x))*((a^2*(b^4*(3*A*d^2 - 3*A*c^2 + 3*C*c^2 - 6*C*d^
2 + 6*B*c*d) + 3*C*b^4*d^2) - b^6*(A*d^2 - A*c^2 + C*c^2 + 2*B*c*d) + C*b^6
*d^2 - a*b^5*(3*B*c^2 - 3*B*d^2 + 6*A*c*d - 6*C*c*d) + a^3*b^3*(B*c^2 - B*d
^2 + 2*A*c*d - 2*C*c*d))/(b^9 + 3*a^2*b^7 + 3*a^4*b^5 + a^6*b^3) - (C*d^2)/
b^3))/f - ((A*b^6*c^2 - 3*C*a^6*d^2 + B*a*b^5*c^2 + B*a^5*b*d^2 + 5*A*a^2*b
^4*c^2 - 3*A*a^2*b^4*d^2 + A*a^4*b^2*d^2 - 3*B*a^3*b^3*c^2 + 5*B*a^3*b^3*d^
2 - 3*C*a^2*b^4*c^2 + C*a^4*b^2*c^2 - 7*C*a^4*b^2*d^2 + 2*A*a*b^5*c*d + 2*C
*a^5*b*c*d - 6*A*a^3*b^3*c*d - 6*B*a^2*b^4*c*d + 2*B*a^4*b^2*c*d + 10*C*a^3
*b^3*c*d)/(2*b^3*(a^4 + b^4 + 2*a^2*b^2)) + (tan(e + f*x)*(B*b^5*c^2 - 2*C*
a^5*d^2 + 2*A*b^5*c*d + 2*A*a*b^4*c^2 - 2*A*a*b^4*d^2 + B*a^4*b*d^2 - 2*C*a
*b^4*c^2 - B*a^2*b^3*c^2 + 3*B*a^2*b^3*d^2 - 4*C*a^3*b^2*d^2 - 4*B*a*b^4*c*
d + 2*C*a^4*b*c*d - 2*A*a^2*b^3*c*d + 6*C*a^2*b^3*c*d))/(b^2*(a^4 + b^4 + 2
*a^2*b^2)))/(f*(a^2 + b^2*tan(e + f*x)^2 + 2*a*b*tan(e + f*x))) - (log(tan(
e + f*x) - 1i)*(A*d^2*1i - A*c^2*1i + B*c^2 - B*d^2 + C*c^2*1i - C*d^2*1i +
 2*A*c*d + B*c*d*2i - 2*C*c*d))/(2*f*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) -
 (log(tan(e + f*x) + 1i)*(A*d^2 - A*c^2 + B*c^2*1i - B*d^2*1i + C*c^2 - C*d
^2 + A*c*d*2i + 2*B*c*d - C*c*d*2i))/(2*f*(a*b^2*3i - 3*a^2*b - a^3*1i + b^
3))
```

3.64 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\,(A+B\tan(e+$

Optimal. Leaf size=603

$$\left(a^2\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3\right)+b^2\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\right)-2a$$

```
[Out] (a^2*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)+b^2*(c^3*C+3*B*c^2*d
-3*c*C*d^2-B*d^3-A*(c^3-3*c*d^2))-2*a*b*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2
)))*x+(2*a*b*(c^3*C+3*B*c^2*d-3*c*C*d^2-B*d^3-A*(c^3-3*c*d^2))-a^2*((A-C)*d
*(3*c^2-d^2)+B*(c^3-3*c*d^2))+b^2*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2)))*ln
(cos(f*x+e))/f-d*(2*a*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-a^2*(2*c*(A-C)*d+
B*(c^2-d^2))+b^2*(2*c*(A-C)*d+B*(c^2-d^2)))*tan(f*x+e)/f+1/2*(2*a*b*(A*c-B*
d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*d))*(c+d*tan(f*x+e))^2/f+1/3*(a^2*B
-b^2*B+2*a*b*(A-C))*(c+d*tan(f*x+e))^3/f+1/60*(5*a^2*C*d^2-6*a*b*d*(-5*B*d+
C*c)+b^2*(c^2*C-3*B*c*d+15*(A-C)*d^2))*(c+d*tan(f*x+e))^4/d^3/f-1/15*b*(-3*
B*b*d-C*a*d+C*b*c)*tan(f*x+e)*(c+d*tan(f*x+e))^4/d^2/f+1/6*C*(a+b*tan(f*x+e
))^2*(c+d*tan(f*x+e))^4/d/f
```

Rubi [A]
time = 1.08, antiderivative size = 603, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3711, 3609, 3606, 3556}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] (a^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + b^2*(c^3
*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)*d*
(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x + ((2*a*b*(c^3*C + 3*B*c^2*d - 3*c*C*
d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - a^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 -
3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Log[Cos[e +
f*x]])/f - (d*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(
A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Tan[e + f*
x])/f + ((2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A -
 C)*d))*(c + d*Tan[e + f*x])^2)/(2*f) + ((a^2*B - b^2*B + 2*a*b*(A - C))*(c
 + d*Tan[e + f*x])^3)/(3*f) + ((5*a^2*C*d^2 - 6*a*b*d*(c*C - 5*B*d) + b^2*(
c^2*C - 3*B*c*d + 15*(A - C)*d^2))*(c + d*Tan[e + f*x])^4)/(60*d^3*f) - (b*
(b*c*C - 3*b*B*d - a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(15*d^2*f) +
 (C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^4)/(6*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{C(a+b\tan(e+f}{}$$

$$=-\frac{b(bcC-3bBd-}{}$$

$$=\frac{(5a^2Cd^2-6abd(c}{}$$

$$=\frac{(a^2B-b^2B+2ab}{}$$

$$=\frac{(2ab(Ac-cC-B}{}$$

$$=\left(a^2\left(Ac^3-c^3C-\right.\right.$$

$$=\left(a^2\left(Ac^3-c^3C-\right.\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.41, size = 419, normalized size = 0.69

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}+\frac{-\frac{2b(bcC-3bBd-aCd)\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{-\frac{\left(5a^2Cd^2-6abd(cC-5Bd)+b^2\left(c^2C-3Bcd+15(A-C)d^2\right)\right)(c+d\tan(e+fx))^4}{2df}+\frac{5\left(3d\left(2ab(Ac-cC+Bd)+a^2(Bc-(A-C)d)-b^2(Bc-(A-C)d)\right)\left((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6cd^2\tan(e+fx)+d^3\tan^2(e+fx)\right)+\left(a^2B-b^2B+2ab(A-C)\right)d\left(3i(c+id)^4\log(i-\tan(e+fx))-3i(c-id)^4\log(i+\tan(e+fx))-6d^2\left(6c^2-d^2\right)\tan(e+fx)-12cd^3\tan^2(e+fx)-2d^4\tan^3(e+fx)\right)\right)}{f}}{5d}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^4)/(6*d*f) + ((-2*b*(b*c*C -
 3*b*B*d - a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(5*d*f) - (-1/2*((5*
a^2*C*d^2 - 6*a*b*d*(c*C - 5*B*d) + b^2*(c^2*C - 3*B*c*d + 15*(A - C)*d^2))
*(c + d*Tan[e + f*x])^4)/(d*f) + (5*(3*d*(2*a*b*(A*c - c*C + B*d) + a^2*(B*
c - (A - C)*d) - b^2*(B*c - (A - C)*d))*((I*c - d)^3*Log[I - Tan[e + f*x]]
- (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*
x]^2) + (a^2*B - b^2*B + 2*a*b*(A - C))*d*((3*I)*(c + I*d)^4*Log[I - Tan[e
+ f*x]] - (3*I)*(c - I*d)^4*Log[I + Tan[e + f*x]] - 6*d^2*(6*c^2 - d^2)*Tan
[e + f*x] - 12*c*d^3*Tan[e + f*x]^2 - 2*d^4*Tan[e + f*x]^3)))/f)/(5*d))/(6*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. 2(593) = 1186.
time = 0.36, size = 1239, normalized size = 2.05

method	result
norman	$(A\,a^2c^3 - 3A\,a^2c\,d^2 - 6Aab\,c^2d + 2Aab\,d^3 - A\,b^2c^3 + 3A\,b^2c\,d^2 - 3B\,a^2c^2d + B\,a^2d^3 - 2$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(3*A*a*b*c*d^2*tan(f*x+e)^2+3*B*a*b*c^2*d*tan(f*x+e)^2+A*b^2*c*d^2*tan(
f*x+e)^3+B*b^2*c^2*d*tan(f*x+e)^3+C*a^2*c*d^2*tan(f*x+e)^3+C*a*b*c^3*tan(f*
x+e)^2+3/2*B*a^2*c*d^2*tan(f*x+e)^2-B*a*b*d^3*tan(f*x+e)^2-3/2*B*b^2*c*d^2*
tan(f*x+e)^2+3/2*C*a^2*c^2*d*tan(f*x+e)^2-3/2*C*b^2*c^2*d*tan(f*x+e)^2+3/5*
C*b^2*c*d^2*tan(f*x+e)^5+1/2*B*a*b*d^3*tan(f*x+e)^4+3/4*B*b^2*c*d^2*tan(f*x
+e)^4+3/4*C*b^2*c^2*d*tan(f*x+e)^4+2/3*A*a*b*d^3*tan(f*x+e)^3-2/3*C*a*b*d^3
*tan(f*x+e)^3-C*b^2*c*d^2*tan(f*x+e)^3+3/2*A*b^2*c^2*d*tan(f*x+e)^2+3*B*a^2
*c^2*d*tan(f*x+e)+2*B*a*b*c^3*tan(f*x+e)-3*B*b^2*c^2*d*tan(f*x+e)-3*C*a^2*c
*d^2*tan(f*x+e)+2*C*a*b*d^3*tan(f*x+e)+3*C*b^2*c*d^2*tan(f*x+e)+3*A*a^2*c*d
^2*tan(f*x+e)-2*A*a*b*d^3*tan(f*x+e)-3*A*b^2*c*d^2*tan(f*x+e)+2/5*C*a*b*d^3
*tan(f*x+e)^5+1/2*(3*A*a^2*c^2*d-A*a^2*d^3+2*A*a*b*c^3-6*A*a*b*c*d^2-3*A*b^
2*c^2*d+A*b^2*d^3+B*a^2*c^3-3*B*a^2*c*d^2-6*B*a*b*c^2*d+2*B*a*b*d^3-B*b^2*c
^3+3*B*b^2*c*d^2-3*C*a^2*c^2*d+C*a^2*d^3-2*C*a*b*c^3+6*C*a*b*c*d^2+3*C*b^2*
c^2*d-C*b^2*d^3)*ln(1+tan(f*x+e)^2)+(A*a^2*c^3-3*A*a^2*c*d^2-6*A*a*b*c^2*d+
2*A*a*b*d^3-A*b^2*c^3+3*A*b^2*c*d^2-3*B*a^2*c^2*d+B*a^2*d^3-2*B*a*b*c^3+6*B
*a*b*c*d^2+3*B*b^2*c^2*d-B*b^2*d^3-C*a^2*c^3+3*C*a^2*c*d^2+6*C*a*b*c^2*d-2*
C*a*b*d^3+C*b^2*c^3-3*C*b^2*c*d^2)*arctan(tan(f*x+e))+1/2*C*b^2*d^3*tan(f*x
+e)^2+1/2*A*a^2*d^3*tan(f*x+e)^2-1/2*A*b^2*d^3*tan(f*x+e)^2+1/2*B*b^2*c^3*t
an(f*x+e)^2-B*a^2*d^3*tan(f*x+e)+B*b^2*d^3*tan(f*x+e)+C*a^2*c^3*tan(f*x+e)-
C*b^2*c^3*tan(f*x+e)+A*b^2*c^3*tan(f*x+e)+1/5*B*b^2*d^3*tan(f*x+e)^5+1/6*C*
b^2*d^3*tan(f*x+e)^6+1/4*A*b^2*d^3*tan(f*x+e)^4+1/4*C*a^2*d^3*tan(f*x+e)^4-
1/4*C*b^2*d^3*tan(f*x+e)^4+1/3*B*a^2*d^3*tan(f*x+e)^3-1/3*B*b^2*d^3*tan(f*x
+e)^3+1/3*C*b^2*c^3*tan(f*x+e)^3-1/2*C*a^2*d^3*tan(f*x+e)^2-3*C*a*b*c*d^2*t
an(f*x+e)^2+3/2*C*a*b*c*d^2*tan(f*x+e)^4+2*B*a*b*c*d^2*tan(f*x+e)^3+2*C*a*b
*c^2*d*tan(f*x+e)^3+6*A*a*b*c^2*d*tan(f*x+e)-6*B*a*b*c*d^2*tan(f*x+e)-6*C*a
*b*c^2*d*tan(f*x+e))
```

Maxima [A]
time = 0.52, size = 688, normalized size = 1.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] 1/60*(10*C*b^2*d^3*tan(f*x + e)^6 + 12*(3*C*b^2*c*d^2 + (2*C*a*b + B*b^2)*d
^3)*tan(f*x + e)^5 + 15*(3*C*b^2*c^2*d + 3*(2*C*a*b + B*b^2)*c*d^2 + (C*a^2
 + 2*B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e)^4 + 20*(C*b^2*c^3 + 3*(2*C*a*b
+ B*b^2)*c^2*d + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A -
C)*a*b - B*b^2)*d^3)*tan(f*x + e)^3 + 30*((2*C*a*b + B*b^2)*c^3 + 3*(C*a^2
+ 2*B*a*b + (A - C)*b^2)*c^2*d + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^2 +
((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*tan(f*x + e)^2 + 60*(((A - C)*a^
2 - 2*B*a*b - (A - C)*b^2)*c^3 - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d -
3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A - C)*a*b - B*
b^2)*d^3)*(f*x + e) + 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3 + 3*((A - C)*
a^2 - 2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^
2 - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)^2 + 1) + 60
*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c
^2*d + 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 + 2*(A - C)*a
*b - B*b^2)*d^3)*tan(f*x + e))/f
```

Fricas [**A**]
time = 2.45, size = 686, normalized size = 1.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*b^2*d^3*tan(f*x + e)^6 + 12*(3*C*b^2*c*d^2 + (2*C*a*b + B*b^2)*d
^3)*tan(f*x + e)^5 + 15*(3*C*b^2*c^2*d + 3*(2*C*a*b + B*b^2)*c*d^2 + (C*a^2
 + 2*B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e)^4 + 20*(C*b^2*c^3 + 3*(2*C*a*b
+ B*b^2)*c^2*d + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A -
C)*a*b - B*b^2)*d^3)*tan(f*x + e)^3 + 60*(((A - C)*a^2 - 2*B*a*b - (A - C)*
b^2)*c^3 - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d - 3*((A - C)*a^2 - 2*B*a
*b - (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^3)*f*x + 30*((2
*C*a*b + B*b^2)*c^3 + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2*d + 3*(B*a^2 +
2*(A - C)*a*b - B*b^2)*c*d^2 + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*t
an(f*x + e)^2 - 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3 + 3*((A - C)*a^2 -
2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^2 - ((
A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 60*(
(C*a^2 + 2*B*a*b + (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2
*d + 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 + 2*(A - C)*a*b
 - B*b^2)*d^3)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(547) = 1094$.
time = 0.49, size = 1819, normalized size = 3.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Piecewise((A*a**2*c**3*x + 3*A*a**2*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) -
 3*A*a**2*c*d**2*x + 3*A*a**2*c*d**2*tan(e + f*x)/f - A*a**2*d**3*log(tan(e
 + f*x)**2 + 1)/(2*f) + A*a**2*d**3*tan(e + f*x)**2/(2*f) + A*a*b*c**3*log(
tan(e + f*x)**2 + 1)/f - 6*A*a*b*c**2*d*x + 6*A*a*b*c**2*d*tan(e + f*x)/f -
 3*A*a*b*c*d**2*log(tan(e + f*x)**2 + 1)/f + 3*A*a*b*c*d**2*tan(e + f*x)**2
/f + 2*A*a*b*d**3*x + 2*A*a*b*d**3*tan(e + f*x)**3/(3*f) - 2*A*a*b*d**3*tan
(e + f*x)/f - A*b**2*c**3*x + A*b**2*c**3*tan(e + f*x)/f - 3*A*b**2*c**2*d*
log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*b**2*c**2*d*tan(e + f*x)**2/(2*f) + 3*
A*b**2*c*d**2*x + A*b**2*c*d**2*tan(e + f*x)**3/f - 3*A*b**2*c*d**2*tan(e +
 f*x)/f + A*b**2*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**2*d**3*tan(e +
f*x)**4/(4*f) - A*b**2*d**3*tan(e + f*x)**2/(2*f) + B*a**2*c**3*log(tan(e +
 f*x)**2 + 1)/(2*f) - 3*B*a**2*c**2*d*x + 3*B*a**2*c**2*d*tan(e + f*x)/f -
3*B*a**2*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a**2*c*d**2*tan(e + f*
x)**2/(2*f) + B*a**2*d**3*x + B*a**2*d**3*tan(e + f*x)**3/(3*f) - B*a**2*d*
*3*tan(e + f*x)/f - 2*B*a*b*c**3*x + 2*B*a*b*c**3*tan(e + f*x)/f - 3*B*a*b*
c**2*d*log(tan(e + f*x)**2 + 1)/f + 3*B*a*b*c**2*d*tan(e + f*x)**2/f + 6*B*
a*b*c*d**2*x + 2*B*a*b*c*d**2*tan(e + f*x)**3/f - 6*B*a*b*c*d**2*tan(e + f*
x)/f + B*a*b*d**3*log(tan(e + f*x)**2 + 1)/f + B*a*b*d**3*tan(e + f*x)**4/(
2*f) - B*a*b*d**3*tan(e + f*x)**2/f - B*b**2*c**3*log(tan(e + f*x)**2 + 1)/
(2*f) + B*b**2*c**3*tan(e + f*x)**2/(2*f) + 3*B*b**2*c**2*d*x + B*b**2*c**2
*d*tan(e + f*x)**3/f - 3*B*b**2*c**2*d*tan(e + f*x)/f + 3*B*b**2*c*d**2*log
(tan(e + f*x)**2 + 1)/(2*f) + 3*B*b**2*c*d**2*tan(e + f*x)**4/(4*f) - 3*B*b
**2*c*d**2*tan(e + f*x)**2/(2*f) - B*b**2*d**3*x + B*b**2*d**3*tan(e + f*x)
**5/(5*f) - B*b**2*d**3*tan(e + f*x)**3/(3*f) + B*b**2*d**3*tan(e + f*x)/f
- C*a**2*c**3*x + C*a**2*c**3*tan(e + f*x)/f - 3*C*a**2*c**2*d*log(tan(e +
f*x)**2 + 1)/(2*f) + 3*C*a**2*c**2*d*tan(e + f*x)**2/(2*f) + 3*C*a**2*c*d**
2*x + C*a**2*c*d**2*tan(e + f*x)**3/f - 3*C*a**2*c*d**2*tan(e + f*x)/f + C*
a**2*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*a**2*d**3*tan(e + f*x)**4/(4*f
) - C*a**2*d**3*tan(e + f*x)**2/(2*f) - C*a*b*c**3*log(tan(e + f*x)**2 + 1)
/f + C*a*b*c**3*tan(e + f*x)**2/f + 6*C*a*b*c**2*d*x + 2*C*a*b*c**2*d*tan(e
 + f*x)**3/f - 6*C*a*b*c**2*d*tan(e + f*x)/f + 3*C*a*b*c*d**2*log(tan(e + f
*x)**2 + 1)/f + 3*C*a*b*c*d**2*tan(e + f*x)**4/(2*f) - 3*C*a*b*c*d**2*tan(e
 + f*x)**2/f - 2*C*a*b*d**3*x + 2*C*a*b*d**3*tan(e + f*x)**5/(5*f) - 2*C*a*
b*d**3*tan(e + f*x)**3/(3*f) + 2*C*a*b*d**3*tan(e + f*x)/f + C*b**2*c**3*x
+ C*b**2*c**3*tan(e + f*x)**3/(3*f) - C*b**2*c**3*tan(e + f*x)/f + 3*C*b**2
```

```
*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*b**2*c**2*d*tan(e + f*x)**4/(4
*f) - 3*C*b**2*c**2*d*tan(e + f*x)**2/(2*f) - 3*C*b**2*c*d**2*x + 3*C*b**2*
c*d**2*tan(e + f*x)**5/(5*f) - C*b**2*c*d**2*tan(e + f*x)**3/f + 3*C*b**2*c
*d**2*tan(e + f*x)/f - C*b**2*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**2*
d**3*tan(e + f*x)**6/(6*f) - C*b**2*d**3*tan(e + f*x)**4/(4*f) + C*b**2*d**
3*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(a + b*tan(e))**2*(c + d*tan(e))**3*
(A + B*tan(e) + C*tan(e)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24014 vs. $2(602) = 1204$.
time = 20.71, size = 24014, normalized size = 39.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] 1/60*(60*A*a^2*c^3*f*x*tan(f*x)^6*tan(e)^6 - 60*C*a^2*c^3*f*x*tan(f*x)^6*ta
n(e)^6 - 120*B*a*b*c^3*f*x*tan(f*x)^6*tan(e)^6 - 60*A*b^2*c^3*f*x*tan(f*x)^
6*tan(e)^6 + 60*C*b^2*c^3*f*x*tan(f*x)^6*tan(e)^6 - 180*B*a^2*c^2*d*f*x*tan
(f*x)^6*tan(e)^6 - 360*A*a*b*c^2*d*f*x*tan(f*x)^6*tan(e)^6 + 360*C*a*b*c^2*
d*f*x*tan(f*x)^6*tan(e)^6 + 180*B*b^2*c^2*d*f*x*tan(f*x)^6*tan(e)^6 - 180*A
*a^2*c*d^2*f*x*tan(f*x)^6*tan(e)^6 + 180*C*a^2*c*d^2*f*x*tan(f*x)^6*tan(e)^
6 + 360*B*a*b*c*d^2*f*x*tan(f*x)^6*tan(e)^6 + 180*A*b^2*c*d^2*f*x*tan(f*x)^
6*tan(e)^6 - 180*C*b^2*c*d^2*f*x*tan(f*x)^6*tan(e)^6 + 60*B*a^2*d^3*f*x*tan
(f*x)^6*tan(e)^6 + 120*A*a*b*d^3*f*x*tan(f*x)^6*tan(e)^6 - 120*C*a*b*d^3*f*
x*tan(f*x)^6*tan(e)^6 - 60*B*b^2*d^3*f*x*tan(f*x)^6*tan(e)^6 - 30*B*a^2*c^3
*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + t
an(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 60
*A*a*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan
(e)^6 + 60*C*a*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)^6*tan(e)^6 + 30*B*b^2*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1))*tan(f*x)^6*tan(e)^6 - 90*A*a^2*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*t
an(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1
)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 90*C*a^2*c^2*d*log(4*(tan(f*x)^4*ta
n(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x
)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 180*B*a*b*c^2*d*log(4*(
tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 90*A*b^2*c
^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6
```

```
- 90*C*b^2*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x
)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)
^6*tan(e)^6 + 90*B*a^2*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
1))*tan(f*x)^6*tan(e)^6 + 180*A*a*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 180*C*a*b*c*d^2*log(4*(tan(f*x)^4*ta
n(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x
)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 90*B*b^2*c*d^2*log(4*(t
an(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 30*A*a^2*d^
3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +
tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 3
0*C*a^2*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^6*ta
n(e)^6 - 60*B*a*b*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ta
n(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan
(f*x)^6*tan(e)^6 - 30*A*b^2*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)^6*tan(e)^6 + 30*C*b^2*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*ta
n(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 360*A*a^2*c^3*f*x*tan(f*x)^5*tan(e)^
5 + 360*C*a^2*c^3*f*x*tan(f*x)^5*tan(e)^5 + 720*B*a*b*c^3*f*x*tan(f*x)^5*ta
n(e)^5 + 360*A*b^2*c^3*f*x*tan(f*x)^5*tan(e)^5 - 360*C*b^2*c^3*f*x*tan(f*x)
^5*tan(e)^5 + 1080*B*a^2*c^2*d*f*x*tan(f*x)^5*tan(e)^5 + 2160*A*a*b*c^2*d*f
*x*tan(f*x)^5*tan(e)^5 - 2160*C*a*b*c^2*d*f*x*tan(f*x)^5*tan(e)^5 - 1080*B*
b^2*c^2*d*f*x*tan(f*x)^5*tan(e)^5 + 1080*A*a^2*c*d^2*f*x*tan(f*x)^5*tan(e)^
5 - 1080*C*a^2*c*d^2*f*x*tan(f*x)^5*tan(e)^5 - 2160*B*a*b*c*d^2*f*x*tan(f*x
)^5*tan(e)^5 - 1080*A*b^2*c*d^2*f*x*tan(f*x)^5*tan(e)^5 + 1080*C*b^2*c*d^2*
f*x*tan(f*x)^5*tan(e)^5 - 360*B*a^2*d^3*f*x*tan(f*x)^5*tan(e)^5 - 720*A*a*b
*d^3*f*x*tan(f*x)^5*tan(e)^5 + 720*C*a*b*d^3*f*x*tan(f*x)^5*tan(e)^5 + 360*
B*b^2*d^3*f*x*tan(f*x)^5*tan(e)^5 + 60*C*a*b*c^3*tan(f*x)^6*tan(e)^6 + 30*B
*b^2*c^3*tan(f*x)^6*tan(e)^6 + 90*C*a^2*c^2*d*tan(f*x)^6*tan(e)^6 + 180*B*a
*b*c^2*d*tan(f*x)^6*tan(e)^6 + 90*A*b^2*c^2*d*tan(f*x)^6*tan(e)^6 - 135*C*b
^2*c^2*d*tan(f*x)^6*tan(e)^6 + 90*B*a^2*c*d^2*tan(f*x)^6*tan(e)^6 + 180*A*a
*b*c*d^2*tan(f*x)^6*tan(e)^6 - 270*C*a*b*c*d^2*tan(f*x)^6*tan(e)^6 - 135*B*
b^2*c*d^2*tan(f*x)^6*tan(e)^6 + 30*A*a^2*d^3*tan(f*x)^6*tan(e)^6 - 45*C*a^2
*d^3*tan(f*x)^6*tan(e)^6 - 90*B*a*b*d^3*tan(f*x)^6*tan(e)^6 - 45*A*b^2*d^3*
tan(f*x)^6*tan(e)^6 + 55*C*b^2*d^3*tan(f*x)^6*tan(e)^6 + 180*B*a^2*c^3*log(
4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + ...
```

Mupad [B]
time = 9.31, size = 891, normalized size = 1.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2),x)
```

```
[Out] x*(A*a^2*c^3 - A*b^2*c^3 + B*a^2*d^3 - C*a^2*c^3 - B*b^2*d^3 + C*b^2*c^3 +
2*A*a*b*d^3 - 2*B*a*b*c^3 - 2*C*a*b*d^3 - 3*A*a^2*c*d^2 + 3*A*b^2*c*d^2 - 3
*B*a^2*c^2*d + 3*B*b^2*c^2*d + 3*C*a^2*c*d^2 - 3*C*b^2*c*d^2 - 6*A*a*b*c^2*
d + 6*B*a*b*c*d^2 + 6*C*a*b*c^2*d) - (tan(e + f*x)*(B*a^2*d^3 - A*b^2*c^3 -
 b*d^2*(B*b*d + 2*C*a*d + 3*C*b*c) - C*a^2*c^3 + C*b^2*c^3 + 2*A*a*b*d^3 -
2*B*a*b*c^3 - 3*A*a^2*c*d^2 + 3*A*b^2*c*d^2 - 3*B*a^2*c^2*d + 3*B*b^2*c^2*d
 + 3*C*a^2*c*d^2 - 6*A*a*b*c^2*d + 6*B*a*b*c*d^2 + 6*C*a*b*c^2*d))/f - (log
(tan(e + f*x)^2 + 1)*((A*a^2*d^3)/2 - (B*a^2*c^3)/2 - (A*b^2*d^3)/2 + (B*b^
2*c^3)/2 - (C*a^2*d^3)/2 + (C*b^2*d^3)/2 - A*a*b*c^3 - B*a*b*d^3 + C*a*b*c^
3 - (3*A*a^2*c^2*d)/2 + (3*A*b^2*c^2*d)/2 + (3*B*a^2*c*d^2)/2 - (3*B*b^2*c*
d^2)/2 + (3*C*a^2*c^2*d)/2 - (3*C*b^2*c^2*d)/2 + 3*A*a*b*c*d^2 + 3*B*a*b*c^
2*d - 3*C*a*b*c*d^2))/f + (tan(e + f*x)^4*((A*b^2*d^3)/4 + (C*a^2*d^3)/4 -
(C*b^2*d^3)/4 + (B*a*b*d^3)/2 + (3*B*b^2*c*d^2)/4 + (3*C*b^2*c^2*d)/4 + (3*
C*a*b*c*d^2)/2))/f + (tan(e + f*x)^3*((B*a^2*d^3)/3 - (b*d^2*(B*b*d + 2*C*a
*d + 3*C*b*c))/3 + (C*b^2*c^3)/3 + (2*A*a*b*d^3)/3 + A*b^2*c*d^2 + B*b^2*c^
2*d + C*a^2*c*d^2 + 2*B*a*b*c*d^2 + 2*C*a*b*c^2*d))/f + (tan(e + f*x)^2*((A
*a^2*d^3)/2 - (A*b^2*d^3)/2 + (B*b^2*c^3)/2 - (C*a^2*d^3)/2 + (C*b^2*d^3)/2
 - B*a*b*d^3 + C*a*b*c^3 + (3*A*b^2*c^2*d)/2 + (3*B*a^2*c*d^2)/2 - (3*B*b^2
*c*d^2)/2 + (3*C*a^2*c^2*d)/2 - (3*C*b^2*c^2*d)/2 + 3*A*a*b*c*d^2 + 3*B*a*b
*c^2*d - 3*C*a*b*c*d^2))/f + (b*d^2*tan(e + f*x)^5*(B*b*d + 2*C*a*d + 3*C*b
*c))/(5*f) + (C*b^2*d^3*tan(e + f*x)^6)/(6*f)
```

3.65 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3 (A+B\tan(e+f$

Optimal. Leaf size=389

$$\left(a\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3\right)-b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)x-\frac{(A(bc^3+3a}{}$$

```
[Out] (a*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)-b*((A-C)*d*(3*c^2-d^2)
+B*(c^3-3*c*d^2)))*x-(A*(3*a*c^2*d-a*d^3+b*c^3-3*b*c*d^2)-b*(3*B*c^2*d-B*d^
3+C*c^3-3*C*c*d^2)+a*(B*c^3-3*B*c*d^2-3*C*c^2*d+C*d^3))*ln(cos(f*x+e))/f+d*
(a*(B*c^2-B*d^2-2*C*c*d)-b*(2*B*c*d+C*c^2-C*d^2)+A*(2*a*c*d+b*(c^2-d^2)))*t
an(f*x+e)/f+1/2*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*(c+d*tan(f*x+e))^2/f+
1/3*(A*b+B*a-C*b)*(c+d*tan(f*x+e))^3/f-1/20*(-5*B*b*d-5*C*a*d+C*b*c)*(c+d*t
an(f*x+e))^4/d^2/f+1/5*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^4/d/f
```

Rubi [A]
time = 0.52, antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3718, 3711, 3609, 3606, 3556}

$$\frac{d\tan(e+fx)\left(2aAcd+aB(c^2-d^2)-2acCd+Ab(c^2-d^2)-b(2Bcd+c^2C-Cd^2)\right)}{f}-\frac{\log(\cos(e+fx))\left(A(3ac^2d-ad^3+bc^3-3bcd^2)+a(Bc^3-3Bcd^2-3c^2Cd+Cd^3)-b(3Bc^2d-Bd^3+c^3C-3cCd^2)\right)}{f}-x\left(-a\left(Ac^3-3Acd^2-3Bc^2d+Bd^3-c^3C+3cCd^2\right)+bd(A-C)\left(3c^2-d^2\right)+bB\left(c^3-3cd^2\right)\right)+\frac{(aB+Ab-bC)(c+d\tan(e+fx))^3}{3f}+\frac{(c+d\tan(e+fx))^2(aAd+aBc-aCd+Abc-bBd-bcC)}{2f}-\frac{(-5aCd-5bBd+bcC)(c+d\tan(e+fx))^4}{20d^2f}+\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```

```
[Out] -((b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^3*C - 3*B
*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3))*x) - ((A*(b*c^3 + 3*a*c^2*d - 3*b*
c*d^2 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c
^2*C*d - 3*B*c*d^2 + C*d^3))*Log[Cos[e + f*x]])/f + (d*(2*a*A*c*d - 2*a*c*C
*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C + 2*B*c*d - C*d^2))*Tan[e
 + f*x])/f + ((A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*(c + d*Tan[e
+ f*x])^2)/(2*f) + ((A*b + a*B - b*C)*(c + d*Tan[e + f*x])^3)/(3*f) - ((b*c
*C - 5*b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^4)/(20*d^2*f) + (b*C*Tan[e + f
*x]*(c + d*Tan[e + f*x])^4)/(5*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{bC\tan(e+fx)(c+}{5d}$$

$$= -\frac{(bcC-5bBd-5}{}$$

$$= \frac{(Ab+aB-bC)(c}{3}$$

$$= \frac{(Abc+aBc-bcC}{}$$

$$= -\big(b(A-C)d\big(3c^2$$

$$= -\big(b(A-C)d\big(3c^2$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.26, size = 297, normalized size = 0.76

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\frac{(bcC-5bBd-5aCd)(c+d\tan(e+fx))^4}{4df}+\frac{5(3(Abc+aBc-bcC-aAd+bBd+aCd)((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6cd^2\tan(e+fx)+d^3\tan^2(e+fx))+(Ab+aB-bC)(3i(c+id)^4\log(i-\tan(e+fx))-3i(c-id)^4\log(i+\tan(e+fx))-6d^2(6c^2-d^2)\tan(e+fx)-12cd^3\tan^2(e+fx)-2d^4\tan^3(e+fx)))}{6f}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] (b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(5*d*f) - (((b*c*C - 5*b*B*d - 5*
a*C*d)*(c + d*Tan[e + f*x])^4)/(4*d*f) + (5*(3*(A*b*c + a*B*c - b*c*C - a*A
*d + b*B*d + a*C*d)*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[I
+ Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2) + (A*b + a*B -
 b*C)*((3*I)*(c + I*d)^4*Log[I - Tan[e + f*x]] - (3*I)*(c - I*d)^4*Log[I +
Tan[e + f*x]] - 6*d^2*(6*c^2 - d^2)*Tan[e + f*x] - 12*c*d^3*Tan[e + f*x]^2
- 2*d^4*Tan[e + f*x]^3)))/(6*f))/(5*d)
```

Maple [A]
time = 0.16, size = 639, normalized size = 1.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*((A*a*c^3-3*A*a*c*d^2-3*A*b*c^2*d+A*b*d^3-3*B*a*c^2*d+B*a*d^3-B*b*c^3+3
*B*b*c*d^2-C*a*c^3+3*C*a*c*d^2+3*C*b*c^2*d-C*b*d^3)*arctan(tan(f*x+e))+1/2*
(3*A*a*c^2*d-A*a*d^3+A*b*c^3-3*A*b*c*d^2+B*a*c^3-3*B*a*c*d^2-3*B*b*c^2*d+B*
b*d^3-3*C*a*c^2*d+C*a*d^3-C*b*c^3+3*C*b*c*d^2)*ln(1+tan(f*x+e)^2)+1/5*C*b*d
^3*tan(f*x+e)^5-1/3*C*b*d^3*tan(f*x+e)^3+1/4*B*b*d^3*tan(f*x+e)^4+1/4*C*a*d
^3*tan(f*x+e)^4+1/3*A*b*d^3*tan(f*x+e)^3+1/2*A*a*d^3*tan(f*x+e)^2-1/2*B*b*d
^3*tan(f*x+e)^2-1/2*C*a*d^3*tan(f*x+e)^2+1/3*B*a*d^3*tan(f*x+e)^3+B*b*c^3*t
an(f*x+e)+C*a*c^3*tan(f*x+e)+C*b*d^3*tan(f*x+e)-A*b*d^3*tan(f*x+e)-B*a*d^3*
tan(f*x+e)+1/2*C*b*c^3*tan(f*x+e)^2+3*B*a*c^2*d*tan(f*x+e)-3*B*b*c*d^2*tan(
f*x+e)-3*C*a*c*d^2*tan(f*x+e)+C*b*c^2*d*tan(f*x+e)^3+B*b*c*d^2*tan(f*x+e)^3
+C*a*c*d^2*tan(f*x+e)^3-3*C*b*c^2*d*tan(f*x+e)+3*A*a*c*d^2*tan(f*x+e)+3*A*b
*c^2*d*tan(f*x+e)+3/4*C*b*c*d^2*tan(f*x+e)^4+3/2*A*b*c*d^2*tan(f*x+e)^2+3/2
*B*a*c*d^2*tan(f*x+e)^2+3/2*B*b*c^2*d*tan(f*x+e)^2+3/2*C*a*c^2*d*tan(f*x+e)
^2-3/2*C*b*c*d^2*tan(f*x+e)^2)
```

Maxima [A]
time = 0.52, size = 394, normalized size = 1.01

$$\frac{12\,Cbd^3\tan(fx+e)^5+15\,(3\,Cbcd^2+(Ca+Bb)d^3)\tan(fx+e)^4+20\,(3\,Cbc^2d+3\,(Ca+Bb)cd^2+(Ba+(A-C)b)d^3)\tan(fx+e)^3+30\,(Cbc^3+3\,(Ca+Bb)c^2d+3\,(Ba+(A-C)b)cd^2+((A-C)a-Bb)d^3)\tan(fx+e)^2+60\,(((A-C)a-Bb)c^3-3\,(Ba+(A-C)b)c^2d-3\,((A-C)a-Bb)cd^2+(Ba+(A-C)b)d^3)(fx+e)+30\,((Ba+(A-C)b)c^3+3\,((A-C)a-Bb)c^2d-3\,(Ba+(A-C)b)cd^2-((A-C)a-Bb)d^3)\log(\tan(fx+e)^2+1)+60\,((Ca+Bb)c^3+3\,(Ba+(A-C)b)c^2d+3\,((A-C)a-Bb)cd^2-(Ba+(A-C)b)d^3)\tan(fx+e)}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] 1/60*(12*C*b*d^3*tan(f*x + e)^5 + 15*(3*C*b*c*d^2 + (C*a + B*b)*d^3)*tan(f*
x + e)^4 + 20*(3*C*b*c^2*d + 3*(C*a + B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*t
an(f*x + e)^3 + 30*(C*b*c^3 + 3*(C*a + B*b)*c^2*d + 3*(B*a + (A - C)*b)*c*d
^2 + ((A - C)*a - B*b)*d^3)*tan(f*x + e)^2 + 60*(((A - C)*a - B*b)*c^3 - 3*
(B*a + (A - C)*b)*c^2*d - 3*((A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b)*d^3
)*(f*x + e) + 30*((B*a + (A - C)*b)*c^3 + 3*((A - C)*a - B*b)*c^2*d - 3*(B*
a + (A - C)*b)*c*d^2 - ((A - C)*a - B*b)*d^3)*log(tan(f*x + e)^2 + 1) + 60*
((C*a + B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d + 3*((A - C)*a - B*b)*c*d^2 -
(B*a + (A - C)*b)*d^3)*tan(f*x + e))/f
```

Fricas [A]
time = 1.59, size = 392, normalized size = 1.01

$$\frac{12\,Cbd^3\tan(fx+e)^5+15\,(3\,Cbcd^2+(Ca+Bb)d^3)\tan(fx+e)^4+20\,(3\,Cbc^2d+3\,(Ca+Bb)cd^2+(Ba+(A-C)b)d^3)\tan(fx+e)^3+60\,(((A-C)a-Bb)c^3-3\,(Ba+(A-C)b)c^2d-3\,((A-C)a-Bb)cd^2+(Ba+(A-C)b)d^3)fx+30\,(Cbc^3+3\,(Ca+Bb)c^2d+3\,(Ba+(A-C)b)cd^2+((A-C)a-Bb)d^3)\tan(fx+e)^2-30\,((Ba+(A-C)b)c^3+3\,((A-C)a-Bb)c^2d-3\,(Ba+(A-C)b)cd^2-((A-C)a-Bb)d^3)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+60\,((Ca+Bb)c^3+3\,(Ba+(A-C)b)c^2d+3\,((A-C)a-Bb)cd^2-(Ba+(A-C)b)d^3)\tan(fx+e)}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*b*d^3*tan(f*x + e)^5 + 15*(3*C*b*c*d^2 + (C*a + B*b)*d^3)*tan(f*
x + e)^4 + 20*(3*C*b*c^2*d + 3*(C*a + B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*t
an(f*x + e)^3 + 60*(((A - C)*a - B*b)*c^3 - 3*(B*a + (A - C)*b)*c^2*d - 3*(
(A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*f*x + 30*(C*b*c^3 + 3*(C*a
+ B*b)*c^2*d + 3*(B*a + (A - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*tan(f*x +
 e)^2 - 30*((B*a + (A - C)*b)*c^3 + 3*((A - C)*a - B*b)*c^2*d - 3*(B*a + (A
 - C)*b)*c*d^2 - ((A - C)*a - B*b)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 60*((
C*a + B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d + 3*((A - C)*a - B*b)*c*d^2 - (B
*a + (A - C)*b)*d^3)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(379) = 758.
time = 0.35, size = 1001, normalized size = 2.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Piecewise((A*a*c**3*x + 3*A*a*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*a
*c*d**2*x + 3*A*a*c*d**2*tan(e + f*x)/f - A*a*d**3*log(tan(e + f*x)**2 + 1)
/(2*f) + A*a*d**3*tan(e + f*x)**2/(2*f) + A*b*c**3*log(tan(e + f*x)**2 + 1)
/(2*f) - 3*A*b*c**2*d*x + 3*A*b*c**2*d*tan(e + f*x)/f - 3*A*b*c*d**2*log(ta
```

```
n(e + f*x)**2 + 1)/(2*f) + 3*A*b*c*d**2*tan(e + f*x)**2/(2*f) + A*b*d**3*x
+ A*b*d**3*tan(e + f*x)**3/(3*f) - A*b*d**3*tan(e + f*x)/f + B*a*c**3*log(t
an(e + f*x)**2 + 1)/(2*f) - 3*B*a*c**2*d*x + 3*B*a*c**2*d*tan(e + f*x)/f -
3*B*a*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*c*d**2*tan(e + f*x)**2/
(2*f) + B*a*d**3*x + B*a*d**3*tan(e + f*x)**3/(3*f) - B*a*d**3*tan(e + f*x)
/f - B*b*c**3*x + B*b*c**3*tan(e + f*x)/f - 3*B*b*c**2*d*log(tan(e + f*x)**
2 + 1)/(2*f) + 3*B*b*c**2*d*tan(e + f*x)**2/(2*f) + 3*B*b*c*d**2*x + B*b*c*
d**2*tan(e + f*x)**3/f - 3*B*b*c*d**2*tan(e + f*x)/f + B*b*d**3*log(tan(e +
 f*x)**2 + 1)/(2*f) + B*b*d**3*tan(e + f*x)**4/(4*f) - B*b*d**3*tan(e + f*x
)**2/(2*f) - C*a*c**3*x + C*a*c**3*tan(e + f*x)/f - 3*C*a*c**2*d*log(tan(e
+ f*x)**2 + 1)/(2*f) + 3*C*a*c**2*d*tan(e + f*x)**2/(2*f) + 3*C*a*c*d**2*x
+ C*a*c*d**2*tan(e + f*x)**3/f - 3*C*a*c*d**2*tan(e + f*x)/f + C*a*d**3*log
(tan(e + f*x)**2 + 1)/(2*f) + C*a*d**3*tan(e + f*x)**4/(4*f) - C*a*d**3*tan
(e + f*x)**2/(2*f) - C*b*c**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*c**3*tan
(e + f*x)**2/(2*f) + 3*C*b*c**2*d*x + C*b*c**2*d*tan(e + f*x)**3/f - 3*C*b*
c**2*d*tan(e + f*x)/f + 3*C*b*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*b
*c*d**2*tan(e + f*x)**4/(4*f) - 3*C*b*c*d**2*tan(e + f*x)**2/(2*f) - C*b*d*
*3*x + C*b*d**3*tan(e + f*x)**5/(5*f) - C*b*d**3*tan(e + f*x)**3/(3*f) + C*
b*d**3*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))*(c + d*tan(e))**3*(A +
B*tan(e) + C*tan(e)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 11805 vs. $2(388) = 776$.
time = 8.91, size = 11805, normalized size = 30.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] 1/60*(60*A*a*c^3*f*x*tan(f*x)^5*tan(e)^5 - 60*C*a*c^3*f*x*tan(f*x)^5*tan(e)
^5 - 60*B*b*c^3*f*x*tan(f*x)^5*tan(e)^5 - 180*B*a*c^2*d*f*x*tan(f*x)^5*tan(
e)^5 - 180*A*b*c^2*d*f*x*tan(f*x)^5*tan(e)^5 + 180*C*b*c^2*d*f*x*tan(f*x)^5
*tan(e)^5 - 180*A*a*c*d^2*f*x*tan(f*x)^5*tan(e)^5 + 180*C*a*c*d^2*f*x*tan(f
*x)^5*tan(e)^5 + 180*B*b*c*d^2*f*x*tan(f*x)^5*tan(e)^5 + 60*B*a*d^3*f*x*tan
(f*x)^5*tan(e)^5 + 60*A*b*d^3*f*x*tan(f*x)^5*tan(e)^5 - 60*C*b*d^3*f*x*tan(
f*x)^5*tan(e)^5 - 30*B*a*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
1))*tan(f*x)^5*tan(e)^5 - 30*A*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x
)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan
(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 30*C*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2
*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 90*A*a*c^2*d*log(4*(tan(f*x)^4*ta
n(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x
```

)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*C*a*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*B*b*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*B*a*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*A*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 90*C*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 30*A*a*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 30*C*a*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 30*B*b*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 300*A*a*c^3*f*x*tan(f*x)^4*tan(e)^4 + 300*C*a*c^3*f*x*tan(f*x)^4*tan(e)^4 + 300*B*b*c^3*f*x*tan(f*x)^4*tan(e)^4 + 900*B*a*c^2*d*f*x*tan(f*x)^4*tan(e)^4 + 900*A*b*c^2*d*f*x*tan(f*x)^4*tan(e)^4 - 900*C*b*c^2*d*f*x*tan(f*x)^4*tan(e)^4 + 900*A*a*c*d^2*f*x*tan(f*x)^4*tan(e)^4 - 900*C*a*c*d^2*f*x*tan(f*x)^4*tan(e)^4 - 900*B*b*c*d^2*f*x*tan(f*x)^4*tan(e)^4 - 300*B*a*d^3*f*x*tan(f*x)^4*tan(e)^4 - 300*A*b*d^3*f*x*tan(f*x)^4*tan(e)^4 + 300*C*b*d^3*f*x*tan(f*x)^4*tan(e)^4 + 30*C*b*c^3*tan(f*x)^5*tan(e)^5 + 90*C*a*c^2*d*tan(f*x)^5*tan(e)^5 + 90*B*b*c^2*d*tan(f*x)^5*tan(e)^5 + 90*B*a*c*d^2*tan(f*x)^5*tan(e)^5 + 90*A*b*c*d^2*tan(f*x)^5*tan(e)^5 - 135*C*b*c*d^2*tan(f*x)^5*tan(e)^5 + 30*A*a*d^3*tan(f*x)^5*tan(e)^5 - 45*C*a*d^3*tan(f*x)^5*tan(e)^5 - 45*B*b*d^3*tan(f*x)^5*tan(e)^5 + 150*B*a*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 150*A*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 150*C*b*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 450*A*a*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 450*C*a*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 450*B*b*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 450*B*a*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 450*A*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*ta

```
n(e)^4 + 450*C*b*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)^4*tan(e)^4 - 150*A*a*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*t
an(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
 + 1))*tan(f*x)^4*tan(e)^4 + 150*C*a*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan
(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/
(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 150*B*b*d^3*log(4*(tan(f*x)^4*tan(e)^
2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 +...
```

Mupad [B]
time = 9.04, size = 478, normalized size = 1.23

$$x\,\left(A\,a\,c^3+A\,b\,d^3+B\,a\,d^3-B\,b\,c^3-C\,a\,c^3-C\,b\,d^3-3\,A\,a\,c\,d^2-3\,A\,b\,c^2\,d-3\,B\,a\,c^2\,d+3\,B\,b\,c\,d^2+3\,C\,a\,c\,d^2+3\,C\,b\,c^2\,d\right)+\frac{{\mathrm{tan}\left(e+f\,x\right)}^4\,\left(\frac{B\,b\,d^3}{4}+\frac{C\,a\,d^3}{4}+\frac{3\,C\,b\,c\,d^2}{4}\right)}{f}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^3\,\left(\frac{A\,b\,d^3}{3}+\frac{B\,a\,d^3}{3}-\frac{C\,b\,d^3}{3}+B\,b\,c\,d^2+C\,a\,c\,d^2+C\,b\,c^2\,d\right)}{f}+\frac{{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(\frac{A\,a\,d^3}{2}-\frac{B\,b\,d^3}{2}-\frac{C\,a\,d^3}{2}+\frac{C\,b\,c^3}{2}+\frac{3\,A\,b\,c\,d^2}{2}+\frac{3\,B\,a\,c\,d^2}{2}+\frac{3\,B\,b\,c^2\,d}{2}+\frac{3\,C\,a\,c^2\,d}{2}-\frac{3\,C\,b\,c\,d^2}{2}\right)}{f}-\frac{\ln\left({\mathrm{tan}\left(e+f\,x\right)}^2+1\right)\,\left(\frac{A\,a\,d^3}{2}-\frac{A\,b\,c^3}{2}-\frac{B\,a\,c^3}{2}-\frac{B\,b\,d^3}{2}-\frac{C\,a\,d^3}{2}+\frac{C\,b\,c^3}{2}-\frac{3\,A\,a\,c^2\,d}{2}+\frac{3\,A\,b\,c\,d^2}{2}+\frac{3\,B\,a\,c\,d^2}{2}+\frac{3\,B\,b\,c^2\,d}{2}+\frac{3\,C\,a\,c^2\,d}{2}-\frac{3\,C\,b\,c\,d^2}{2}\right)}{f}+\frac{\mathrm{tan}\left(e+f\,x\right)\,\left(B\,b\,c^3-B\,a\,d^3-A\,b\,d^3+C\,a\,c^3+C\,b\,d^3+3\,A\,a\,c\,d^2+3\,A\,b\,c^2\,d+3\,B\,a\,c^2\,d-3\,B\,b\,c\,d^2-3\,C\,a\,c\,d^2-3\,C\,b\,c^2\,d\right)}{f}+\frac{C\,b\,d^3\,{\mathrm{tan}\left(e+f\,x\right)}^5}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan
(e + f*x)^2),x)
```

```
[Out] x*(A*a*c^3 + A*b*d^3 + B*a*d^3 - B*b*c^3 - C*a*c^3 - C*b*d^3 - 3*A*a*c*d^2
- 3*A*b*c^2*d - 3*B*a*c^2*d + 3*B*b*c*d^2 + 3*C*a*c*d^2 + 3*C*b*c^2*d) + (t
an(e + f*x)^4*((B*b*d^3)/4 + (C*a*d^3)/4 + (3*C*b*c*d^2)/4))/f + (tan(e + f
*x)^3*((A*b*d^3)/3 + (B*a*d^3)/3 - (C*b*d^3)/3 + B*b*c*d^2 + C*a*c*d^2 + C*
b*c^2*d))/f + (tan(e + f*x)^2*((A*a*d^3)/2 - (B*b*d^3)/2 - (C*a*d^3)/2 + (C
*b*c^3)/2 + (3*A*b*c*d^2)/2 + (3*B*a*c*d^2)/2 + (3*B*b*c^2*d)/2 + (3*C*a*c^
2*d)/2 - (3*C*b*c*d^2)/2))/f - (log(tan(e + f*x)^2 + 1)*((A*a*d^3)/2 - (A*b
*c^3)/2 - (B*a*c^3)/2 - (B*b*d^3)/2 - (C*a*d^3)/2 + (C*b*c^3)/2 - (3*A*a*c^
2*d)/2 + (3*A*b*c*d^2)/2 + (3*B*a*c*d^2)/2 + (3*B*b*c^2*d)/2 + (3*C*a*c^2*d
)/2 - (3*C*b*c*d^2)/2))/f + (tan(e + f*x)*(B*b*c^3 - B*a*d^3 - A*b*d^3 + C*
a*c^3 + C*b*d^3 + 3*A*a*c*d^2 + 3*A*b*c^2*d + 3*B*a*c^2*d - 3*B*b*c*d^2 - 3
*C*a*c*d^2 - 3*C*b*c^2*d))/f + (C*b*d^3*tan(e + f*x)^5)/(5*f)
```

3.66 $\int (c+d\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+$

Optimal. Leaf size=191

$$-\left(\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\right)x\right)-\frac{\left((A-C)d(3c^2-d^2)+B(c^3-3cd^2)\right)\log(\cos(e+f$$

```
[Out] -(c^3*C+3*B*c^2*d-3*c*C*d^2-B*d^3-A*(c^3-3*c*d^2))*x-((A-C)*d*(3*c^2-d^2)+B
*(c^3-3*c*d^2))*ln(cos(f*x+e))/f+d*(2*c*(A-C)*d+B*(c^2-d^2))*tan(f*x+e)/f+1
/2*(B*c+(A-C)*d)*(c+d*tan(f*x+e))^2/f+1/3*B*(c+d*tan(f*x+e))^3/f+1/4*C*(c+d
*tan(f*x+e))^4/d/f
```

Rubi [A]
time = 0.17, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3711, 3609, 3606, 3556}

$$\frac{d\tan(e+fx)\left(2cd(A-C)+B(c^2-d^2)\right)}{f}-\frac{\left(d(A-C)\left(3c^2-d^2\right)+B(c^3-3cd^2)\right)\log(\cos(e+fx))}{f}-x\left(-A\left(c^3-3cd^2\right)+3Bc^2d-Bd^3+c^3C-3cCd^2\right)+\frac{(d(A-C)+Bc)(c+d\tan(e+fx))^2}{2f}+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -((c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2))*x) - (((A - C
)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2))*Log[Cos[e + f*x]])/f + (d*(2*c*(A -
C)*d + B*(c^2 - d^2))*Tan[e + f*x])/f + ((B*c + (A - C)*d)*(c + d*Tan[e + f
*x])^2)/(2*f) + (B*(c + d*Tan[e + f*x])^3)/(3*f) + (C*(c + d*Tan[e + f*x])^
4)/(4*d*f)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C(c+d\tan(e+fx))^4}{4df}+\int (A-C+B$$
$$= \frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e$$
$$= \frac{(Bc+(A-C)d)(c+d\tan(e+fx))^2}{2f}+$$
$$= -\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3\right.$$
$$= -\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.61, size = 212, normalized size = 1.11

$$\frac{3C(c+d\tan(e+fx))^4-6(Bc+(-A+C)d)\left((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6cd^2\tan(e+fx)+d^3\tan^2(e+fx)\right)+2B\left(-3i(c+id)^4\log(i-\tan(e+fx))+3i(c-id)^4\log(i+\tan(e+fx))-6d^2(-6c^2+d^2)\tan(e+fx)+12cd^3\tan^2(e+fx)+2d^4\tan^3(e+fx)\right)}{12df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (3*C*(c + d*Tan[e + f*x])^4 - 6*(B*c + (-A + C)*d)*((I*c - d)^3*Log[I - Tan
[e + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3
*Tan[e + f*x]^2) + 2*B*((-3*I)*(c + I*d)^4*Log[I - Tan[e + f*x]] + (3*I)*(c
 - I*d)^4*Log[I + Tan[e + f*x]] - 6*d^2*(-6*c^2 + d^2)*Tan[e + f*x] + 12*c*
d^3*Tan[e + f*x]^2 + 2*d^4*Tan[e + f*x]^3))/(12*d*f)
```

Maple [A]
time = 0.10, size = 265, normalized size = 1.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/f*(1/4*C*d^3*tan(f*x+e)^4+1/3*B*d^3*tan(f*x+e)^3+C*c*d^2*tan(f*x+e)^3+1/2
*A*d^3*tan(f*x+e)^2+3/2*B*c*d^2*tan(f*x+e)^2+3/2*C*c^2*d*tan(f*x+e)^2-1/2*C
*d^3*tan(f*x+e)^2+3*A*c*d^2*tan(f*x+e)+3*B*c^2*d*tan(f*x+e)-B*d^3*tan(f*x+e
)+c^3*C*tan(f*x+e)-3*c*C*d^2*tan(f*x+e)+1/2*(3*A*c^2*d-A*d^3+B*c^3-3*B*c*d^
2-3*C*c^2*d+C*d^3)*ln(1+tan(f*x+e)^2)+(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^
3+3*C*c*d^2)*arctan(tan(f*x+e)))
```

Maxima [A]
time = 0.52, size = 208, normalized size = 1.09

$$\frac{3\,Cd^3\tan\left(fx+e\right)^4+4\,(3\,Ccd^2+Bd^3)\tan\left(fx+e\right)^3+6\,(3\,Cc^2d+3\,Bcd^2+(A-C)d^3)\tan\left(fx+e\right)^2+12\,((A-C)c^3-3\,Bc^2d-3\,(A-C)cd^2+Bd^3)(fx+e)+6\,(Bc^3+3\,(A-C)c^2d-3\,Bcd^2-(A-C)d^3)\log\left(\tan\left(fx+e\right)^2+1\right)+12\,(Cc^3+3\,Bc^2d+3\,(A-C)cd^2-Bd^3)\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
maxima")
```

```
[Out] 1/12*(3*C*d^3*tan(f*x + e)^4 + 4*(3*C*c*d^2 + B*d^3)*tan(f*x + e)^3 + 6*(3*
C*c^2*d + 3*B*c*d^2 + (A - C)*d^3)*tan(f*x + e)^2 + 12*((A - C)*c^3 - 3*B*c
^2*d - 3*(A - C)*c*d^2 + B*d^3)*(f*x + e) + 6*(B*c^3 + 3*(A - C)*c^2*d - 3*
B*c*d^2 - (A - C)*d^3)*log(tan(f*x + e)^2 + 1) + 12*(C*c^3 + 3*B*c^2*d + 3*
(A - C)*c*d^2 - B*d^3)*tan(f*x + e))/f
```

Fricas [A]
time = 0.99, size = 206, normalized size = 1.08

$$\frac{3\,Cd^3\tan\left(fx+e\right)^4+4\,(3\,Ccd^2+Bd^3)\tan\left(fx+e\right)^3+12\,((A-C)c^3-3\,Bc^2d-3\,(A-C)cd^2+Bd^3)fx+6\,(3\,Cc^2d+3\,Bcd^2+(A-C)d^3)\tan\left(fx+e\right)^2-6\,(Bc^3+3\,(A-C)c^2d-3\,Bcd^2-(A-C)d^3)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+12\,(Cc^3+3\,Bc^2d+3\,(A-C)cd^2-Bd^3)\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
fricas")
```

```
[Out] 1/12*(3*C*d^3*tan(f*x + e)^4 + 4*(3*C*c*d^2 + B*d^3)*tan(f*x + e)^3 + 12*((
A - C)*c^3 - 3*B*c^2*d - 3*(A - C)*c*d^2 + B*d^3)*f*x + 6*(3*C*c^2*d + 3*B*
c*d^2 + (A - C)*d^3)*tan(f*x + e)^2 - 6*(B*c^3 + 3*(A - C)*c^2*d - 3*B*c*d^
2 - (A - C)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 12*(C*c^3 + 3*B*c^2*d + 3*(A
 - C)*c*d^2 - B*d^3)*tan(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(163) = 326$.
time = 0.18, size = 410, normalized size = 2.15

$$\begin{cases} Ac^3x+\frac{3Ac^2d\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-3Acd^2x+\frac{3Acd^2\tan\left(e+fx\right)}{f}-\frac{Ad^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Ad^3\tan^2\left(e+fx\right)}{2f}+\frac{Bc^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-3Bc^2dx+\frac{3Bc^2d\tan\left(e+fx\right)}{f}-\frac{3Bcd^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{3Bcd^2\tan^2\left(e+fx\right)}{2f}+Bd^3x+\frac{Bd^3\tan^3\left(e+fx\right)}{3f}-\frac{Bd^3\tan\left(e+fx\right)}{f}-Cc^3x+\frac{Cc^3\tan\left(e+fx\right)}{f}-\frac{3Cc^2d\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{3Cc^2d\tan^2\left(e+fx\right)}{2f}+3Ccd^2x+\frac{Ccd^2\tan^3\left(e+fx\right)}{f}-\frac{3Ccd^2\tan\left(e+fx\right)}{f}+\frac{Cd^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Cd^3\tan^4\left(e+fx\right)}{4f}-\frac{Cd^3\tan^2\left(e+fx\right)}{2f} & \text{for}\: f \neq 0 \\ x\left(c+d\tan\left(e\right)\right)^3\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Piecewise((A*c**3*x + 3*A*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*c*d**
2*x + 3*A*c*d**2*tan(e + f*x)/f - A*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + A
*d**3*tan(e + f*x)**2/(2*f) + B*c**3*log(tan(e + f*x)**2 + 1)/(2*f) - 3*B*c
**2*d*x + 3*B*c**2*d*tan(e + f*x)/f - 3*B*c*d**2*log(tan(e + f*x)**2 + 1)/(
2*f) + 3*B*c*d**2*tan(e + f*x)**2/(2*f) + B*d**3*x + B*d**3*tan(e + f*x)**3
/(3*f) - B*d**3*tan(e + f*x)/f - C*c**3*x + C*c**3*tan(e + f*x)/f - 3*C*c**
2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*c**2*d*tan(e + f*x)**2/(2*f) + 3*C
*c*d**2*x + C*c*d**2*tan(e + f*x)**3/f - 3*C*c*d**2*tan(e + f*x)/f + C*d**3
*log(tan(e + f*x)**2 + 1)/(2*f) + C*d**3*tan(e + f*x)**4/(4*f) - C*d**3*tan
(e + f*x)**2/(2*f), Ne(f, 0)), (x*(c + d*tan(e))**3*(A + B*tan(e) + C*tan(e
)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4300 vs. $2(190) = 380$.
time = 2.89, size = 4300, normalized size = 22.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] 1/12*(12*A*c^3*f*x*tan(f*x)^4*tan(e)^4 - 12*C*c^3*f*x*tan(f*x)^4*tan(e)^4 -
 36*B*c^2*d*f*x*tan(f*x)^4*tan(e)^4 - 36*A*c*d^2*f*x*tan(f*x)^4*tan(e)^4 +
36*C*c*d^2*f*x*tan(f*x)^4*tan(e)^4 + 12*B*d^3*f*x*tan(f*x)^4*tan(e)^4 - 6*B
*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4
- 18*A*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*
tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^4*t
an(e)^4 + 18*C*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan
(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(
f*x)^4*tan(e)^4 + 18*B*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(
e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 +
1))*tan(f*x)^4*tan(e)^4 + 6*A*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3
*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)
^2 + 1))*tan(f*x)^4*tan(e)^4 - 6*C*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(t
an(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 48*A*c^3*f*x*tan(f*x)^3*tan(e)^3 + 48*C
*c^3*f*x*tan(f*x)^3*tan(e)^3 + 144*B*c^2*d*f*x*tan(f*x)^3*tan(e)^3 + 144*A*
c*d^2*f*x*tan(f*x)^3*tan(e)^3 - 144*C*c*d^2*f*x*tan(f*x)^3*tan(e)^3 - 48*B*
d^3*f*x*tan(f*x)^3*tan(e)^3 + 18*C*c^2*d*tan(f*x)^4*tan(e)^4 + 18*B*c*d^2*t
an(f*x)^4*tan(e)^4 + 6*A*d^3*tan(f*x)^4*tan(e)^4 - 9*C*d^3*tan(f*x)^4*tan(e
)^4 + 24*B*c^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^
2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3
*tan(e)^3 + 72*A*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + t
```

an(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*ta
n(f*x)^3*tan(e)^3 - 72*C*c^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*ta
n(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2
+ 1))*tan(f*x)^3*tan(e)^3 - 72*B*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f
*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(t
an(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 24*A*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2
*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) +
1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 24*C*d^3*log(4*(tan(f*x)^4*tan(e)
^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*ta
n(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 12*C*c^3*tan(f*x)^4*tan(e)^
3 - 36*B*c^2*d*tan(f*x)^4*tan(e)^3 - 36*A*c*d^2*tan(f*x)^4*tan(e)^3 + 36*C*
c*d^2*tan(f*x)^4*tan(e)^3 + 12*B*d^3*tan(f*x)^4*tan(e)^3 - 12*C*c^3*tan(f*x
)^3*tan(e)^4 - 36*B*c^2*d*tan(f*x)^3*tan(e)^4 - 36*A*c*d^2*tan(f*x)^3*tan(e
)^4 + 36*C*c*d^2*tan(f*x)^3*tan(e)^4 + 12*B*d^3*tan(f*x)^3*tan(e)^4 + 72*A*
c^3*f*x*tan(f*x)^2*tan(e)^2 - 72*C*c^3*f*x*tan(f*x)^2*tan(e)^2 - 216*B*c^2*
d*f*x*tan(f*x)^2*tan(e)^2 - 216*A*c*d^2*f*x*tan(f*x)^2*tan(e)^2 + 216*C*c*d
^2*f*x*tan(f*x)^2*tan(e)^2 + 72*B*d^3*f*x*tan(f*x)^2*tan(e)^2 + 18*C*c^2*d*
tan(f*x)^4*tan(e)^2 + 18*B*c*d^2*tan(f*x)^4*tan(e)^2 + 6*A*d^3*tan(f*x)^4*t
an(e)^2 - 6*C*d^3*tan(f*x)^4*tan(e)^2 - 36*C*c^2*d*tan(f*x)^3*tan(e)^3 - 36
*B*c*d^2*tan(f*x)^3*tan(e)^3 - 12*A*d^3*tan(f*x)^3*tan(e)^3 + 24*C*d^3*tan(
f*x)^3*tan(e)^3 + 18*C*c^2*d*tan(f*x)^2*tan(e)^4 + 18*B*c*d^2*tan(f*x)^2*ta
n(e)^4 + 6*A*d^3*tan(f*x)^2*tan(e)^4 - 6*C*d^3*tan(f*x)^2*tan(e)^4 - 12*C*c
*d^2*tan(f*x)^4*tan(e) - 4*B*d^3*tan(f*x)^4*tan(e) - 36*B*c^3*log(4*(tan(f*
x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*
tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 108*A*c^2*d*log(
4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 108*C*c
^2*d*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2
+ 108*B*c*d^2*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(f*x)^2
*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f*x)^2*
tan(e)^2 + 36*A*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e) + tan(
f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))*tan(f
*x)^2*tan(e)^2 - 36*C*d^3*log(4*(tan(f*x)^4*tan(e)^2 - 2*tan(f*x)^3*tan(e)
+ tan(f*x)^2*tan(e)^2 + tan(f*x)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(e)^2 + 1))
*tan(f*x)^2*tan(e)^2 + 36*C*c^3*tan(f*x)^3*tan(e)^2 + 108*B*c^2*d*tan(f*x)^
3*tan(e)^2 + 108*A*c*d^2*tan(f*x)^3*tan(e)^2 - 144*C*c*d^2*tan(f*x)^3*tan(e
)^2 - 48*B*d^3*tan(f*x)^3*tan(e)^2 + 36*C*c^3*tan(f*x)^2*tan(e)^3 + 108*B*c
^2*d*tan(f*x)^2*tan(e)^3 + 108*A*c*d^2*tan(f*x)^2*tan(e)^3 - 144*C*c*d^2*ta
n(f*x)^2*tan(e)^3 - 48*B*d^3*tan(f*x)^2*tan(e)^3 - 12*C*c*d^2*tan(f*x)*tan(
e)^4 - 4*B*d^3*tan(f*x)*tan(e)^4 + 3*C*d^3*tan(f*x)^4 - 48*A*c^3*f*x*tan(f*
x)*tan(e) + 48*C*c^3*f*x*tan(f*x)*tan(e) + 144*B*c^2*d*f*x*tan(f*x)*tan(e)
+ 144*A*c*d^2*f*x*tan(f*x)*tan(e) - 144*C*c*d^2*f*x*tan(f*x)*tan(e) - 48*B*
d^3*f*x*tan(f*x)*tan(e) - 36*C*c^2*d*tan(f*x)^3...

Mupad [B]
time = 8.79, size = 221, normalized size = 1.16

$$x\,\left(A\,c^3+B\,d^3-C\,c^3-3\,A\,c\,d^2-3\,B\,c^2\,d+3\,C\,c\,d^2\right)+\frac{\tan(e+f\,x)\,\left(C\,c^3-B\,d^3+3\,A\,c\,d^2+3\,B\,c^2\,d-3\,C\,c\,d^2\right)}{f}+\frac{\tan(e+f\,x)^3\,\left(\frac{B\,d^3}{3}+C\,c\,d^2\right)}{f}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{A\,d^3}{2}-\frac{B\,c^3}{2}-\frac{C\,d^3}{2}-\frac{3\,A\,c^2\,d}{2}+\frac{3\,B\,c\,d^2}{2}+\frac{3\,C\,c^2\,d}{2}\right)}{f}+\frac{\tan(e+f\,x)^2\,\left(\frac{A\,d^3}{2}-\frac{C\,d^3}{2}+\frac{3\,B\,c\,d^2}{2}+\frac{3\,C\,c^2\,d}{2}\right)}{f}+\frac{C\,d^3\,\tan(e+f\,x)^4}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

```
[Out] x*(A*c^3 + B*d^3 - C*c^3 - 3*A*c*d^2 - 3*B*c^2*d + 3*C*c*d^2) + (tan(e + f*
x)*(C*c^3 - B*d^3 + 3*A*c*d^2 + 3*B*c^2*d - 3*C*c*d^2))/f + (tan(e + f*x)^3
*((B*d^3)/3 + C*c*d^2))/f - (log(tan(e + f*x)^2 + 1)*((A*d^3)/2 - (B*c^3)/2
 - (C*d^3)/2 - (3*A*c^2*d)/2 + (3*B*c*d^2)/2 + (3*C*c^2*d)/2))/f + (tan(e +
 f*x)^2*((A*d^3)/2 - (C*d^3)/2 + (3*B*c*d^2)/2 + (3*C*c^2*d)/2))/f + (C*d^3
*tan(e + f*x)^4)/(4*f)
```

3.67 $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=363

$$-\frac{(a(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3-3cd^2))-b((A-C)d(3c^2-d^2)+B(c^3-3cd^2)))\,x}{a^2+b^2}-\frac{(b(c^3C+$$

```
[Out] -(a*(c^3*C+3*B*c^2*d-3*c*C*d^2-B*d^3-A*(c^3-3*c*d^2))-b*((A-C)*d*(3*c^2-d^2
)+B*(c^3-3*c*d^2)))*x/(a^2+b^2)-(b*(3*B*c^2*d-B*d^3+C*c^3-3*C*c*d^2)+a*(B*c
^3-3*B*c*d^2-3*C*c^2*d+C*d^3)+A*(a*d*(3*c^2-d^2)-b*(c^3-3*c*d^2)))*ln(cos(f
*x+e))/(a^2+b^2)/f+(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^3*ln(a+b*tan(f*x+e))/b^4/
(a^2+b^2)/f+d*(b^2*d*(B*c+(A-C)*d)+(-a*d+b*c)*(B*b*d-C*a*d+C*b*c))*tan(f*x+
e)/b^3/f+1/2*(B*b*d-C*a*d+C*b*c)*(c+d*tan(f*x+e))^2/b^2/f+1/3*C*(c+d*tan(f*
x+e))^3/b/f
```

Rubi [A]
time = 1.04, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3707, 3698, 31, 3556}

$$-\frac{\log(\cos(e+fx))\,(A(ad(3c^2-d^2)-b(c^3-3cd^2))+a(Bc^3-3Bcd^2-3c^2Cd+Cd^3)+b(3Bc^2d-Bd^3+c^3C-3cCd^2))}{f\,(a^2+b^2)}-\frac{x(a(-A(c^3-3cd^2)+3Bc^2d-Bd^3+c^3C-3cCd^2)-b(d(A-C)\,(3c^2-d^2)+B(c^3-3cd^2)))}{a^2+b^2}+\frac{(bc-ad)^3\,(Ab^2-a(bB-aC))\log(a+b\tan(e+fx))}{b^4f\,(a^2+b^2)}+\frac{d\tan(e+fx)\,((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc))}{b^3f}+\frac{(-aCd+bBd+bcC)(c+d\tan(e+fx))^2}{2b^2f}+\frac{C(c+d\tan(e+fx))^3}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x]),x]
```

```
[Out] -(((a*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - b*((A -
 C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)) - ((b*(c^3*C + 3*
B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c^2*C*d - 3*B*c*d^2 + C*d^3) +
A*(a*d*(3*c^2 - d^2) - b*(c^3 - 3*c*d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)*
f) + ((A*b^2 - a*(b*B - a*C))*(b*c - a*d)^3*Log[a + b*Tan[e + f*x]])/(b^4*(
a^2 + b^2)*f) + (d*(b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(b*c*C + b*B*d -
a*C*d))*Tan[e + f*x])/(b^3*f) + ((b*c*C + b*B*d - a*C*d)*(c + d*Tan[e + f*x
])^2)/(2*b^2*f) + (C*(c + d*Tan[e + f*x])^3)/(3*b*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \frac{C(c+d\tan(e+fx))^3}{3bf} + \frac{\int \frac{(c+d\tan(e+f}{}}{}$$

$$= \frac{(bcC+bBd-aCd)(c+d\tan(e+fx)}{2b^2 f}$$

$$= \frac{d(b^2d(Bc+(A-C)d)+(bc-ad)(bc}{b^3 f}$$

$$= -\frac{(a(c^3C+3Bc^2d-3cCd^2-Bd^3-A}{}$$

$$= -\frac{(a(c^3C+3Bc^2d-3cCd^2-Bd^3-A}{}$$

$$= -\frac{(a(c^3C+3Bc^2d-3cCd^2-Bd^3-A}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.03, size = 255, normalized size = 0.70

$$\frac{\frac{3b^2(-iA+B+iC)(c+id)^3\log(i-\tan(e+fx))}{a+ib}-\frac{3b^2(A-iB-C)(ic+d)^3\log(i+\tan(e+fx))}{a-ib}+\frac{6(Ab^2+a(-bB+aC))(bc-ad)^3\log(a+b\tan(e+fx))}{b^2(a^2+b^2)}+6bd^2(Bc+(A-C)d)\tan(e+fx)+\frac{6d(bc-ad)(bcC+bBd-aCd)\tan(e+fx)}{b}+3(bcC+bBd-aCd)(c+d\tan(e+fx))^2+2bC(c+d\tan(e+fx))^3}{6b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x]),x]
```

```
[Out] ((3*b^2*((-I)*A + B + I*C)*(c + I*d)^3*Log[I - Tan[e + f*x]])/(a + I*b) - (
3*b^2*(A - I*B - C)*(I*c + d)^3*Log[I + Tan[e + f*x]])/(a - I*b) + (6*(A*b^
2 + a*(-(b*B) + a*C))*(b*c - a*d)^3*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^
2)) + 6*b*d^2*(B*c + (A - C)*d)*Tan[e + f*x] + (6*d*(b*c - a*d)*(b*c*C + b*
B*d - a*C*d)*Tan[e + f*x])/b + 3*(b*c*C + b*B*d - a*C*d)*(c + d*Tan[e + f*x
])^2 + 2*b*C*(c + d*Tan[e + f*x])^3)/(6*b^2*f)
```

Maple [A]
time = 0.34, size = 542, normalized size = 1.49

method	result
norman	$\frac{(Aa\,c^3-3Aac\,d^2+3Ab\,c^2d-Ab\,d^3-3Ba\,c^2d+Ba\,d^3+Bb\,c^3-3Bbc\,d^2-Ca\,c^3+3Cac\,d^2-3Cb\,c^2d+Cb\,d^3)x}{a^2+b^2}+\frac{(A\,b^2d^2-}{}$
derivativedivides	$\frac{\frac{\frac{(3Aa\,c^2d-Aa\,d^3-Ab\,c^3+3Abc\,d^2+Ba\,c^3-3Bac\,d^2+3Bb\,c^2d-Bb\,d^3-3Ca\,c^2d+Ca\,d^3+Cb\,c^3-3Cbc\,d^2)\ln(1+\tan^2(fx+e))}{2}+(Aa\,c^3}{a^2+b^2}}{}$

default	$\frac{\frac{\left(3Aa\,c^2d-Aa\,d^3-Ab\,c^3+3Abc\,d^2+Ba\,c^3-3Bac\,d^2+3Bb\,c^2d-Bb\,d^3-3Ca\,c^2d+Ca\,d^3+Cb\,c^3-3Cbc\,d^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(Aa\,c^3-\right.}{a^2+b^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)*(1/2*(3*A*a*c^2*d-A*a*d^3-A*b*c^3+3*A*b*c*d^2+B*a*c^3-3*B*
a*c*d^2+3*B*b*c^2*d-B*b*d^3-3*C*a*c^2*d+C*a*d^3+C*b*c^3-3*C*b*c*d^2)*ln(1+t
an(f*x+e)^2)+(A*a*c^3-3*A*a*c*d^2+3*A*b*c^2*d-A*b*d^3-3*B*a*c^2*d+B*a*d^3+B
*b*c^3-3*B*b*c*d^2-C*a*c^3+3*C*a*c*d^2-3*C*b*c^2*d+C*b*d^3)*arctan(tan(f*x+
e)))+d/b^3*(1/3*C*b^2*d^2*tan(f*x+e)^3+1/2*B*b^2*d^2*tan(f*x+e)^2-1/2*C*a*b
*d^2*tan(f*x+e)^2+3/2*C*b^2*c*d*tan(f*x+e)^2+A*b^2*d^2*tan(f*x+e)-B*a*b*d^2
*tan(f*x+e)+3*B*b^2*c*d*tan(f*x+e)+a^2*C*d^2*tan(f*x+e)-3*C*a*b*c*d*tan(f*x
+e)+3*C*b^2*c^2*tan(f*x+e)-C*b^2*d^2*tan(f*x+e))+(-A*a^3*b^2*d^3+3*A*a^2*b^
3*c*d^2-3*A*a*b^4*c^2*d+A*b^5*c^3+B*a^4*b*d^3-3*B*a^3*b^2*c*d^2+3*B*a^2*b^3
*c^2*d-B*a*b^4*c^3-C*a^5*d^3+3*C*a^4*b*c*d^2-3*C*a^3*b^2*c^2*d+C*a^2*b^3*c^
3)/b^4/(a^2+b^2)*ln(a+b*tan(f*x+e)))
```

Maxima [A]

time = 0.52, size = 442, normalized size = 1.22

$$\frac{\frac{6\left(\left((A-C)a+Bb\right)c^3-3\left(Ba-(A-C)b\right)c^2d-3\left((A-C)a+Bb\right)cd^2+\left(Ba-(A-C)b\right)d^3\right)(fx+e)}{a^2+b^2}+\frac{6\left(\left(Ca^2b^3-Bab^4+Ab^5\right)c^3-3\left(Ca^3b^2-Ba^2b^3+Aab^4\right)c^2d+3\left(Ca^4b-Ba^3b^2+Aa^2b^3\right)cd^2-\left(Ca^5-Ba^4b+Aa^3b^2\right)d^3\right)\log\left(b\tan(fx+e)+a\right)}{a^2b^4+b^6}+\frac{3\left(\left(Ba-(A-C)b\right)c^3+3\left((A-C)a+Bb\right)c^2d-3\left(Ba-(A-C)b\right)cd^2-\left((A-C)a+Bb\right)d^3\right)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}+\frac{2Cb^2d^3\tan(fx+e)^3+3\left(3Cb^2cd^2-(Cab-Bb^2)d^3\right)\tan(fx+e)^2+6\left(3Cb^2c^2d-3\left(Cab-Bb^2\right)cd^2+\left(Ca^2-Bab+(A-C)b^2\right)d^3\right)\tan(fx+e)}{b^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(((A - C)*a + B*b)*c^3 - 3*(B*a - (A - C)*b)*c^2*d - 3*((A - C)*a +
B*b)*c*d^2 + (B*a - (A - C)*b)*d^3)*(f*x + e)/(a^2 + b^2) + 6*((C*a^2*b^3 -
 B*a*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c^2*d + 3*(C*a^
4*b - B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a^4*b + A*a^3*b^2)*d^3)*log
(b*tan(f*x + e) + a)/(a^2*b^4 + b^6) + 3*((B*a - (A - C)*b)*c^3 + 3*((A - C
)*a + B*b)*c^2*d - 3*(B*a - (A - C)*b)*c*d^2 - ((A - C)*a + B*b)*d^3)*log(t
an(f*x + e)^2 + 1)/(a^2 + b^2) + (2*C*b^2*d^3*tan(f*x + e)^3 + 3*(3*C*b^2*c
*d^2 - (C*a*b - B*b^2)*d^3)*tan(f*x + e)^2 + 6*(3*C*b^2*c^2*d - 3*(C*a*b -
B*b^2)*c*d^2 + (C*a^2 - B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e))/b^3)/f
```

Fricas [A]

time = 2.58, size = 630, normalized size = 1.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(C*a^2*b^3 + C*b^5)*d^3*tan(f*x + e)^3 + 6*(((A - C)*a*b^4 + B*b^5)*
c^3 - 3*(B*a*b^4 - (A - C)*b^5)*c^2*d - 3*((A - C)*a*b^4 + B*b^5)*c*d^2 + (
B*a*b^4 - (A - C)*b^5)*d^3)*f*x + 3*(3*(C*a^2*b^3 + C*b^5)*c*d^2 - (C*a^3*b
^2 - B*a^2*b^3 + C*a*b^4 - B*b^5)*d^3)*tan(f*x + e)^2 + 3*((C*a^2*b^3 - B*a
*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c^2*d + 3*(C*a^4*b
- B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a^4*b + A*a^3*b^2)*d^3)*log((b^
2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - 3*((C*
a^2*b^3 + C*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + C*a*b^4 - B*b^5)*c^2*d +
3*(C*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + (A - C)*b^5)*c*d^2 - (C*a^5
- B*a^4*b + A*a^3*b^2 + (A - C)*a*b^4 + B*b^5)*d^3)*log(1/(tan(f*x + e)^2 +
 1)) + 6*(3*(C*a^2*b^3 + C*b^5)*c^2*d - 3*(C*a^3*b^2 - B*a^2*b^3 + C*a*b^4
- B*b^5)*c*d^2 + (C*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + (A - C)*b^5)*
d^3)*tan(f*x + e))/((a^2*b^4 + b^6)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 25.44, size = 7096, normalized size = 19.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e)),x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(
a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c**3*x + 3*A*c**2*d*log(tan(e + f*x)**2 +
 1)/(2*f) - 3*A*c*d**2*x + 3*A*c*d**2*tan(e + f*x)/f - A*d**3*log(tan(e + f
*x)**2 + 1)/(2*f) + A*d**3*tan(e + f*x)**2/(2*f) + B*c**3*log(tan(e + f*x)*
*2 + 1)/(2*f) - 3*B*c**2*d*x + 3*B*c**2*d*tan(e + f*x)/f - 3*B*c*d**2*log(t
an(e + f*x)**2 + 1)/(2*f) + 3*B*c*d**2*tan(e + f*x)**2/(2*f) + B*d**3*x + B
*d**3*tan(e + f*x)**3/(3*f) - B*d**3*tan(e + f*x)/f - C*c**3*x + C*c**3*tan
(e + f*x)/f - 3*C*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*c**2*d*tan(e
+ f*x)**2/(2*f) + 3*C*c*d**2*x + C*c*d**2*tan(e + f*x)**3/f - 3*C*c*d**2*ta
n(e + f*x)/f + C*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*d**3*tan(e + f*x)*
*4/(4*f) - C*d**3*tan(e + f*x)**2/(2*f))/a, Eq(b, 0)), (3*I*A*c**3*f*x*tan(
e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 3*A*c**3*f*x/(6*b*f*tan(e + f*x)
- 6*I*b*f) + 3*I*A*c**3/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*A*c**2*d*f*x*tan
(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 9*I*A*c**2*d*f*x/(6*b*f*tan(e +
f*x) - 6*I*b*f) - 9*A*c**2*d/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*I*A*c*d**2*
f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*A*c*d**2*f*x/(6*b*f*tan
(e + f*x) - 6*I*b*f) + 9*A*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*
b*f*tan(e + f*x) - 6*I*b*f) - 9*I*A*c*d**2*log(tan(e + f*x)**2 + 1)/(6*b*f*
```

tan(e + f*x) - 6*I*b*f) - 9*I*A*c*d**2/(6*b*f*tan(e + f*x) - 6*I*b*f) - 9*A
*d**3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*I*A*d**3*f*x/(6*b
*f*tan(e + f*x) - 6*I*b*f) + 3*I*A*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*
x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 3*A*d**3*log(tan(e + f*x)**2 + 1)/(6*b*
f*tan(e + f*x) - 6*I*b*f) + 6*A*d**3*tan(e + f*x)**2/(6*b*f*tan(e + f*x) -
6*I*b*f) + 9*A*d**3/(6*b*f*tan(e + f*x) - 6*I*b*f) + 3*B*c**3*f*x*tan(e + f
*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 3*I*B*c**3*f*x/(6*b*f*tan(e + f*x) - 6
*I*b*f) - 3*B*c**3/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*I*B*c**2*d*f*x*tan(e
+ f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*B*c**2*d*f*x/(6*b*f*tan(e + f*x)
- 6*I*b*f) + 9*B*c**2*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e
+ f*x) - 6*I*b*f) - 9*I*B*c**2*d*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*
x) - 6*I*b*f) - 9*I*B*c**2*d/(6*b*f*tan(e + f*x) - 6*I*b*f) - 27*B*c*d**2*f
*x*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 27*I*B*c*d**2*f*x/(6*b*f*t
an(e + f*x) - 6*I*b*f) + 9*I*B*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)
/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*B*c*d**2*log(tan(e + f*x)**2 + 1)/(6*b*
f*tan(e + f*x) - 6*I*b*f) + 18*B*c*d**2*tan(e + f*x)**2/(6*b*f*tan(e + f*x)
 - 6*I*b*f) + 27*B*c*d**2/(6*b*f*tan(e + f*x) - 6*I*b*f) - 9*I*B*d**3*f*x*t
an(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 9*B*d**3*f*x/(6*b*f*tan(e + f*
x) - 6*I*b*f) - 6*B*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e
 + f*x) - 6*I*b*f) + 6*I*B*d**3*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x
) - 6*I*b*f) + 3*B*d**3*tan(e + f*x)**3/(6*b*f*tan(e + f*x) - 6*I*b*f) + 3*
I*B*d**3*tan(e + f*x)**2/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*I*B*d**3/(6*b*f
*tan(e + f*x) - 6*I*b*f) + 3*I*C*c**3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x)
- 6*I*b*f) + 3*C*c**3*f*x/(6*b*f*tan(e + f*x) - 6*I*b*f) + 3*C*c**3*log(tan
(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 3*I*C*c**3*
log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 3*I*C*c**3/(6*b*f
*tan(e + f*x) - 6*I*b*f) - 27*C*c**2*d*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x)
 - 6*I*b*f) + 27*I*C*c**2*d*f*x/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*I*C*c**2
*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9
*C*c**2*d*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 18*C*c*
*2*d*tan(e + f*x)**2/(6*b*f*tan(e + f*x) - 6*I*b*f) + 27*C*c**2*d/(6*b*f*ta
n(e + f*x) - 6*I*b*f) - 27*I*C*c*d**2*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x)
- 6*I*b*f) - 27*C*c*d**2*f*x/(6*b*f*tan(e + f*x) - 6*I*b*f) - 18*C*c*d**2*l
og(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 18*I*
C*c*d**2*log(tan(e + f*x)**2 + 1)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*C*c*d*
*2*tan(e + f*x)**3/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*I*C*c*d**2*tan(e + f*
x)**2/(6*b*f*tan(e + f*x) - 6*I*b*f) + 27*I*C*c*d**2/(6*b*f*tan(e + f*x) -
6*I*b*f) + 15*C*d**3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 15*I
*C*d**3*f*x/(6*b*f*tan(e + f*x) - 6*I*b*f) - 6*I*C*d**3*log(tan(e + f*x)**2
 + 1)*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 6*C*d**3*log(tan(e + f*
x)**2 + 1)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 2*C*d**3*tan(e + f*x)**4/(6*b*f
*tan(e + f*x) - 6*I*b*f) + I*C*d**3*tan(e + f*x)**3/(6*b*f*tan(e + f*x) - 6
*I*b*f) - 9*C*d**3*tan(e + f*x)**2/(6*b*f*tan(e + f*x) - 6*I*b*f) - 15*C*d*
*3/(6*b*f*tan(e + f*x) - 6*I*b*f), Eq(a, -I*b)), (-3*I*A*c**3*f*x*tan(e + f
*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 3*A*c**3*f*x/(6*b*f*tan(e + f*x) + 6*I

```
*b*f) - 3*I*A*c**3/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*A*c**2*d*f*x*tan(e +
f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*I*A*c**2*d*f*x/(6*b*f*tan(e + f*x)
+ 6*I*b*f) - 9*A*c**2*d/(6*b*f*tan(e + f*x) + 6*I*b*f) - 9*I*A*c*d**2*f*x*t
an(e + f*x)/(6*b*f*tan(e + f*x) + 6*I*b*f) + 9*...
```

Giac [A]
time = 1.10, size = 573, normalized size = 1.58

$$\frac{\frac{6\,(Aac^3-Cac^3+Bbc^3-3\,Bac^2d+3\,Abc^2d-3\,Cbc^2d-3\,Aacd^2+3\,Cacd^2-3\,Bbcd^2+Bad^3-Abd^3+Cbd^3)(fx+e)}{a^2+b^2}+\frac{3\,(Bac^3-Abc^3+Cbc^3+3\,Aac^2d-3\,Cac^2d+3\,Bbc^2d-3\,Bacd^2+3\,Abcd^2-3\,Cbcd^2-Aad^3+Cad^3-Bbd^3)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}+\frac{6\,(Ca^2b^3c^3-Bab^4c^3+Ab^5c^3-3\,Ca^3b^2c^2d+3\,Ba^2b^3c^2d-3\,Aab^4c^2d+3\,Ca^4bcd^2-3\,Ba^3b^2cd^2+3\,Aa^2b^3cd^2-Ca^5d^3+Ba^4bd^3-Aa^3b^2d^3)\log(|b\tan(fx+e)+a|)}{a^2b^4+b^6}+\frac{2\,Cb^2d^3\tan(fx+e)^3+9\,Cb^2cd^2\tan(fx+e)^2-3\,Cabd^3\tan(fx+e)^2+3\,Bb^2d^3\tan(fx+e)^2+18\,Cb^2c^2d\tan(fx+e)-18\,Cabcd^2\tan(fx+e)+18\,Bb^2cd^2\tan(fx+e)+6\,Ca^2d^3\tan(fx+e)-6\,Babd^3\tan(fx+e)+6\,Ab^2d^3\tan(fx+e)-6\,Cb^2d^3\tan(fx+e)}{b^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(A*a*c^3 - C*a*c^3 + B*b*c^3 - 3*B*a*c^2*d + 3*A*b*c^2*d - 3*C*b*c^2
*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 - 3*B*b*c*d^2 + B*a*d^3 - A*b*d^3 + C*b*d^3)
*(f*x + e)/(a^2 + b^2) + 3*(B*a*c^3 - A*b*c^3 + C*b*c^3 + 3*A*a*c^2*d - 3*C
*a*c^2*d + 3*B*b*c^2*d - 3*B*a*c*d^2 + 3*A*b*c*d^2 - 3*C*b*c*d^2 - A*a*d^3
+ C*a*d^3 - B*b*d^3)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 6*(C*a^2*b^3*c^3
 - B*a*b^4*c^3 + A*b^5*c^3 - 3*C*a^3*b^2*c^2*d + 3*B*a^2*b^3*c^2*d - 3*A*a*
b^4*c^2*d + 3*C*a^4*b*c*d^2 - 3*B*a^3*b^2*c*d^2 + 3*A*a^2*b^3*c*d^2 - C*a^5
*d^3 + B*a^4*b*d^3 - A*a^3*b^2*d^3)*log(abs(b*tan(f*x + e) + a))/(a^2*b^4 +
 b^6) + (2*C*b^2*d^3*tan(f*x + e)^3 + 9*C*b^2*c*d^2*tan(f*x + e)^2 - 3*C*a*
b*d^3*tan(f*x + e)^2 + 3*B*b^2*d^3*tan(f*x + e)^2 + 18*C*b^2*c^2*d*tan(f*x
+ e) - 18*C*a*b*c*d^2*tan(f*x + e) + 18*B*b^2*c*d^2*tan(f*x + e) + 6*C*a^2*
d^3*tan(f*x + e) - 6*B*a*b*d^3*tan(f*x + e) + 6*A*b^2*d^3*tan(f*x + e) - 6*
C*b^2*d^3*tan(f*x + e))/b^3)/f
```

Mupad [B]
time = 13.00, size = 508, normalized size = 1.40

$$\frac{\tan(e+f\,x)^2\,\left(\frac{B\,d^3+3\,C\,c\,d^2}{2\,b}-\frac{C\,a\,d^3}{2\,b^2}\right)}{f}-\frac{\tan(e+f\,x)\,\left(\frac{a\,\left(\frac{B\,d^3+3\,C\,c\,d^2}{b}-\frac{C\,a\,d^3}{b^2}\right)}{b}-\frac{3\,C\,c^2\,d+3\,B\,c\,d^2+A\,d^3}{b}+\frac{C\,d^3}{b}\right)}{f}-\frac{\ln(a+b\tan(e+f\,x))\,\left(b^4\,(B\,a\,c^3+3\,A\,a\,d\,c^2)-b^3\,(C\,a^2\,c^3+3\,B\,a^2\,c^2\,d+3\,A\,a^2\,c\,d^2)+b^2\,(3\,C\,a^3\,c^2\,d+3\,B\,a^3\,c\,d^2+A\,a^3\,d^3)-b\,(B\,a^4\,d^3+3\,C\,c\,a^4\,d^2)-A\,b^5\,c^3+C\,a^5\,d^3\right)}{f\,(a^2\,b^4+b^6)}-\frac{\ln(\tan(e+f\,x)+1\mathrm{i})\,(A\,c^3+A\,d^3\,1\mathrm{i}-B\,c^3\,1\mathrm{i}+B\,d^3-C\,c^3-C\,d^3\,1\mathrm{i}-3\,A\,c\,d^2-A\,c^2\,d\,3\mathrm{i}+B\,c\,d^2\,3\mathrm{i}-3\,B\,c^2\,d+3\,C\,c\,d^2+C\,c^2\,d\,3\mathrm{i})}{2\,f\,(b+a\,1\mathrm{i})}-\frac{\ln(\tan(e+f\,x)-\mathrm{i})\,(A\,d^3-B\,c^3-C\,d^3-3\,A\,c^2\,d+3\,B\,c\,d^2+3\,C\,c^2\,d+A\,c^3\,1\mathrm{i}+B\,d^3\,1\mathrm{i}-C\,c^3\,1\mathrm{i}-A\,c\,d^2\,3\mathrm{i}-B\,c^2\,d\,3\mathrm{i}+C\,c\,d^2\,3\mathrm{i})}{2\,f\,(a+b\,1\mathrm{i})}+\frac{C\,d^3\tan(e+f\,x)^3}{3\,b\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b
*tan(e + f*x)),x)
```

```
[Out] (tan(e + f*x)^2*((B*d^3 + 3*C*c*d^2)/(2*b) - (C*a*d^3)/(2*b^2)))/f - (tan(e
 + f*x)*((a*((B*d^3 + 3*C*c*d^2)/b - (C*a*d^3)/b^2))/b - (A*d^3 + 3*B*c*d^2
 + 3*C*c^2*d)/b + (C*d^3)/b))/f - (log(a + b*tan(e + f*x))*(b^4*(B*a*c^3 +
3*A*a*c^2*d) - b^3*(C*a^2*c^3 + 3*A*a^2*c*d^2 + 3*B*a^2*c^2*d) + b^2*(A*a^3
*d^3 + 3*B*a^3*c*d^2 + 3*C*a^3*c^2*d) - b*(B*a^4*d^3 + 3*C*a^4*c*d^2) - A*b
^5*c^3 + C*a^5*d^3))/(f*(b^6 + a^2*b^4)) - (log(tan(e + f*x) + 1i)*(A*c^3 +
 A*d^3*1i - B*c^3*1i + B*d^3 - C*c^3 - C*d^3*1i - 3*A*c*d^2 - A*c^2*d*3i +
B*c*d^2*3i - 3*B*c^2*d + 3*C*c*d^2 + C*c^2*d*3i))/(2*f*(a*1i + b)) - (log(t
an(e + f*x) - 1i)*(A*c^3*1i + A*d^3 - B*c^3 + B*d^3*1i - C*c^3*1i - C*d^3 -
 A*c*d^2*3i - 3*A*c^2*d + 3*B*c*d^2 - B*c^2*d*3i + C*c*d^2*3i + 3*C*c^2*d))
/(2*f*(a + b*1i)) + (C*d^3*tan(e + f*x)^3)/(3*b*f)
```

3.68 $\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=574

$$-\frac{(b^2(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3)+a^2(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3-3cd^2))-2a}{(a^2+b^2)^2}$$

```
[Out] -(b^2*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)+a^2*(c^3*C+3*B*c^2*
d-3*c*C*d^2-B*d^3-A*(c^3-3*c*d^2))-2*a*b*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^
2)))*x/(a^2+b^2)^2+(2*a*b*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)
-a^2*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2))+b^2*((A-C)*d*(3*c^2-d^2)+B*(c^3-
3*c*d^2)))*ln(cos(f*x+e))/(a^2+b^2)^2/f-(-a*d+b*c)^2*(2*a^3*b*B*d-3*a^4*C*d
-b^4*(3*A*d+B*c)-2*a*b^3*(A*c-2*B*d-C*c)+a^2*b^2*(B*c-(A+5*C)*d))*ln(a+b*ta
n(f*x+e))/b^4/(a^2+b^2)^2/f-d^2*(3*a^3*C*d-A*b^2*(-a*d+b*c)-b^3*(B*d+2*C*c)
-a^2*b*(2*B*d+3*C*c)+a*b^2*(B*c+2*C*d))*tan(f*x+e)/b^3/(a^2+b^2)/f+1/2*(2*A
*b^2-2*B*a*b+3*C*a^2+C*b^2)*d*(c+d*tan(f*x+e))^2/b^2/(a^2+b^2)/f-(A*b^2-a*(
B*b-C*a))*(c+d*tan(f*x+e))^3/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]

time = 1.52, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3726, 3728, 3718, 3707, 3698, 31, 3556}

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b
*Tan[e + f*x])^2,x]
```

```
[Out] -(((b^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + a^2*(
c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)
*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)^2) + ((2*a*b*(A*c^3 -
 c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) - a^2*((A - C)*d*(3*c^2
 - d^2) + B*(c^3 - 3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*
d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)^2*f) - ((b*c - a*d)^2*(2*a^3*b*B*d -
 3*a^4*C*d - b^4*(B*c + 3*A*d) - 2*a*b^3*(A*c - c*C - 2*B*d) + a^2*b^2*(B*c
 - (A + 5*C)*d))*Log[a + b*Tan[e + f*x]])/(b^4*(a^2 + b^2)^2*f) - (d^2*(3*a
^3*C*d - A*b^2*(b*c - a*d) - b^3*(2*c*C + B*d) - a^2*b*(3*c*C + 2*B*d) + a*
b^2*(B*c + 2*C*d))*Tan[e + f*x])/(b^3*(a^2 + b^2)*f) + ((2*A*b^2 - 2*a*b*B
+ 3*a^2*C + b^2*C)*d*(c + d*Tan[e + f*x])^2)/(2*b^2*(a^2 + b^2)*f) - ((A*b^
2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^3)/(b*(a^2 + b^2)*f*(a + b*Tan[e +
f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
```

```
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+f}{b\,(a^2+b^2)\,f(a+b\tan(e+fx))}$$

$$= \frac{(2Ab^2-2abB+3a^2C+b^2C)\,d(c+d\,\mathrm{ta}}{2b^2\,(a^2+b^2)\,f}$$

$$= -\frac{d^2(3a^3Cd-Ab^2(bc-ad)-b^3(2cC+}{}$$

$$= -\frac{(b^2(Ac^3-c^3C-3Bc^2d-3Acd^2+3c}{}$$

$$= -\frac{(b^2(Ac^3-c^3C-3Bc^2d-3Acd^2+3c}{}$$

$$= -\frac{(b^2(Ac^3-c^3C-3Bc^2d-3Acd^2+3c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.75, size = 2467, normalized size = 4.30

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((a^2*A*c^3 - A*b^2*c^3 + 2*a*b*B*c^3 - a^2*c^3*C + b^2*c^3*C + 6*a*A*b*c^2
*d - 3*a^2*B*c^2*d + 3*b^2*B*c^2*d - 6*a*b*c^2*C*d - 3*a^2*A*c*d^2 + 3*A*b^
2*c*d^2 - 6*a*b*B*c*d^2 + 3*a^2*c*C*d^2 - 3*b^2*c*C*d^2 - 2*a*A*b*d^3 + a^2
*B*d^3 - b^2*B*d^3 + 2*a*b*C*d^3)*(e + f*x)*Cos[e + f*x]*(a*Cos[e + f*x] +
b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)/((a - I*b)^2*(a + I*b)^2*f*(c*Cos
[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2) - (I*(-2*a^6*A*b^8*c^
3 + (2*I)*a^5*A*b^9*c^3 - 2*a^4*A*b^10*c^3 + (2*I)*a^3*A*b^11*c^3 + a^7*b^7
*B*c^3 - I*a^6*b^8*B*c^3 - a^3*b^11*B*c^3 + I*a^2*b^12*B*c^3 + 2*a^6*b^8*c^
3*C - (2*I)*a^5*b^9*c^3*C + 2*a^4*b^10*c^3*C - (2*I)*a^3*b^11*c^3*C + 3*a^7
*A*b^7*c^2*d - (3*I)*a^6*A*b^8*c^2*d - 3*a^3*A*b^11*c^2*d + (3*I)*a^2*A*b^1
2*c^2*d + 6*a^6*b^8*B*c^2*d - (6*I)*a^5*b^9*B*c^2*d + 6*a^4*b^10*B*c^2*d -
(6*I)*a^3*b^11*B*c^2*d - 3*a^9*b^5*c^2*C*d + (3*I)*a^8*b^6*c^2*C*d - 12*a^7
*b^7*c^2*C*d + (12*I)*a^6*b^8*c^2*C*d - 9*a^5*b^9*c^2*C*d + (9*I)*a^4*b^10*
c^2*C*d + 6*a^6*A*b^8*c*d^2 - (6*I)*a^5*A*b^9*c*d^2 + 6*a^4*A*b^10*c*d^2 -
(6*I)*a^3*A*b^11*c*d^2 - 3*a^9*b^5*B*c*d^2 + (3*I)*a^8*b^6*B*c*d^2 - 12*a^7
*b^7*B*c*d^2 + (12*I)*a^6*b^8*B*c*d^2 - 9*a^5*b^9*B*c*d^2 + (9*I)*a^4*b^10*
B*c*d^2 + 6*a^10*b^4*c*C*d^2 - (6*I)*a^9*b^5*c*C*d^2 + 18*a^8*b^6*c*C*d^2 -
 (18*I)*a^7*b^7*c*C*d^2 + 12*a^6*b^8*c*C*d^2 - (12*I)*a^5*b^9*c*C*d^2 - a^9
*A*b^5*d^3 + I*a^8*A*b^6*d^3 - 4*a^7*A*b^7*d^3 + (4*I)*a^6*A*b^8*d^3 - 3*a^
5*A*b^9*d^3 + (3*I)*a^4*A*b^10*d^3 + 2*a^10*b^4*B*d^3 - (2*I)*a^9*b^5*B*d^3
 + 6*a^8*b^6*B*d^3 - (6*I)*a^7*b^7*B*d^3 + 4*a^6*b^8*B*d^3 - (4*I)*a^5*b^9*
B*d^3 - 3*a^11*b^3*C*d^3 + (3*I)*a^10*b^4*C*d^3 - 8*a^9*b^5*C*d^3 + (8*I)*a
^8*b^6*C*d^3 - 5*a^7*b^7*C*d^3 + (5*I)*a^6*b^8*C*d^3)*(e + f*x)*Cos[e + f*x
]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(a^2*(a - I*b
)^4*(a + I*b)^3*b^7*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*
x])^2) - (I*(2*a*A*b^5*c^3 - a^2*b^4*B*c^3 + b^6*B*c^3 - 2*a*b^5*c^3*C - 3*
a^2*A*b^4*c^2*d + 3*A*b^6*c^2*d - 6*a*b^5*B*c^2*d + 3*a^4*b^2*c^2*C*d + 9*a
^2*b^4*c^2*C*d - 6*a*A*b^5*c*d^2 + 3*a^4*b^2*B*c*d^2 + 9*a^2*b^4*B*c*d^2 -
6*a^5*b*c*C*d^2 - 12*a^3*b^3*c*C*d^2 + a^4*A*b^2*d^3 + 3*a^2*A*b^4*d^3 - 2*
a^5*b*B*d^3 - 4*a^3*b^3*B*d^3 + 3*a^6*C*d^3 + 5*a^4*b^2*C*d^3)*ArcTan[Tan[e
 + f*x]]*Cos[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*
x])^3)/(b^4*(a^2 + b^2)^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[
e + f*x])^2) + ((-3*b^2*c^2*C*d - 3*b^2*B*c*d^2 + 6*a*b*c*C*d^2 - A*b^2*d^3
 + 2*a*b*B*d^3 - 3*a^2*C*d^3 + b^2*C*d^3)*Cos[e + f*x]*Log[Cos[e + f*x]]*(a
*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(b^4*f*(c*Cos[e +
 f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2) + ((2*a*A*b^5*c^3 - a^2*b
^4*B*c^3 + b^6*B*c^3 - 2*a*b^5*c^3*C - 3*a^2*A*b^4*c^2*d + 3*A*b^6*c^2*d -
6*a*b^5*B*c^2*d + 3*a^4*b^2*c^2*C*d + 9*a^2*b^4*c^2*C*d - 6*a*A*b^5*c*d^2 +
 3*a^4*b^2*B*c*d^2 + 9*a^2*b^4*B*c*d^2 - 6*a^5*b*c*C*d^2 - 12*a^3*b^3*c*C*d
^2 + a^4*A*b^2*d^3 + 3*a^2*A*b^4*d^3 - 2*a^5*b*B*d^3 - 4*a^3*b^3*B*d^3 + 3*
a^6*C*d^3 + 5*a^4*b^2*C*d^3)*Cos[e + f*x]*Log[(a*Cos[e + f*x] + b*Sin[e + f
*x])^2]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(2*b^4*
(a^2 + b^2)^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)
 + (C*d^3*Sec[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f
*x])^3)/(2*b^2*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2
```

```
) + ((a*Cos[e + f*x] + b*Sin[e + f*x])^2*(3*b*c*C*d^2*Sin[e + f*x] + b*B*d^
3*Sin[e + f*x] - 2*a*C*d^3*Sin[e + f*x])*(c + d*Tan[e + f*x])^3)/(b^3*f*(c*
Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2) + (Cos[e + f*x]*(a
*Cos[e + f*x] + b*Sin[e + f*x])*(A*b^5*c^3*Sin[e + f*x] - a*b^4*B*c^3*Sin[e
 + f*x] + a^2*b^3*c^3*C*Sin[e + f*x] - 3*a*A*b^4*c^2*d*Sin[e + f*x] + 3*a^2
*b^3*B*c^2*d*Sin[e + f*x] - 3*a^3*b^2*c^2*C*d*Sin[e + f*x] + 3*a^2*A*b^3*c*
d^2*Sin[e + f*x] - 3*a^3*b^2*B*c*d^2*Sin[e + f*x] + 3*a^4*b*c*C*d^2*Sin[e +
 f*x] - a^3*A*b^2*d^3*Sin[e + f*x] + a^4*b*B*d^3*Sin[e + f*x] - a^5*C*d^3*S
in[e + f*x])*(c + d*Tan[e + f*x])^3)/(a*(a - I*b)*(a + I*b)*b^3*f*(c*Cos[e
+ f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)
```

Maple [A]
time = 0.62, size = 829, normalized size = 1.44 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(d^2/b^3*(1/2*C*d*tan(f*x+e)^2*b+b*tan(f*x+e)*B*d-2*tan(f*x+e)*C*a*d+3*
C*b*c*tan(f*x+e))-(-A*a^3*b^2*d^3+3*A*a^2*b^3*c*d^2-3*A*a*b^4*c^2*d+A*b^5*c
^3+B*a^4*b*d^3-3*B*a^3*b^2*c*d^2+3*B*a^2*b^3*c^2*d-B*a*b^4*c^3-C*a^5*d^3+3*
C*a^4*b*c*d^2-3*C*a^3*b^2*c^2*d+C*a^2*b^3*c^3)/b^4/(a^2+b^2)/(a+b*tan(f*x+e
))+1/b^4*(A*a^4*b^2*d^3-3*A*a^2*b^4*c^2*d+3*A*a^2*b^4*d^3+2*A*a*b^5*c^3-6*A
*a*b^5*c*d^2+3*A*b^6*c^2*d-2*B*a^5*b*d^3+3*B*a^4*b^2*c*d^2-4*B*a^3*b^3*d^3-
B*a^2*b^4*c^3+9*B*a^2*b^4*c*d^2-6*B*a*b^5*c^2*d+B*b^6*c^3+3*C*a^6*d^3-6*C*a
^5*b*c*d^2+3*C*a^4*b^2*c^2*d+5*C*a^4*b^2*d^3-12*C*a^3*b^3*c*d^2+9*C*a^2*b^4
*c^2*d-2*C*a*b^5*c^3)/(a^2+b^2)^2*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^2*(1/2*(3*
A*a^2*c^2*d-A*a^2*d^3-2*A*a*b*c^3+6*A*a*b*c*d^2-3*A*b^2*c^2*d+A*b^2*d^3+B*a
^2*c^3-3*B*a^2*c*d^2+6*B*a*b*c^2*d-2*B*a*b*d^3-B*b^2*c^3+3*B*b^2*c*d^2-3*C*
a^2*c^2*d+C*a^2*d^3+2*C*a*b*c^3-6*C*a*b*c*d^2+3*C*b^2*c^2*d-C*b^2*d^3)*ln(1
+tan(f*x+e)^2)+(A*a^2*c^3-3*A*a^2*c*d^2+6*A*a*b*c^2*d-2*A*a*b*d^3-A*b^2*c^3
+3*A*b^2*c*d^2-3*B*a^2*c^2*d+B*a^2*d^3+2*B*a*b*c^3-6*B*a*b*c*d^2+3*B*b^2*c^
2*d-B*b^2*d^3-C*a^2*c^3+3*C*a^2*c*d^2-6*C*a*b*c^2*d+2*C*a*b*d^3+C*b^2*c^3-3
*C*b^2*c*d^2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.54, size = 691, normalized size = 1.20

$$\frac{\frac{2\,\left(\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)c^3-3\,\left(Ba^2-2\,\left(A-C\right)ab-Bb^2\right)c^2d-3\,\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)cd^2+\left(Ba^2-2\,\left(A-C\right)ab-Bb^2\right)d^3\right)\left(fx+e\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,\left(\left(Ba^2b^4-2\,\left(A-C\right)ab^5-Bb^6\right)c^3-3\,\left(Ca^4b^2-\left(A-3\,C\right)a^2b^4-2\,Bab^5+Ab^6\right)c^2d+3\,\left(2\,Ca^5b-Ba^4b^2+4\,Ca^3b^3-3\,Ba^2b^4+2\,Aab^5\right)cd^2-\left(3\,Ca^6-2\,Ba^5b+\left(A+5\,C\right)a^4b^2-4\,Ba^3b^3+3\,Aa^2b^4\right)d^3\right)\log\left(b\tan\left(fx+e\right)+a\right)}{a^4b^4+2\,a^2b^6+b^8}+\frac{\left(\left(Ba^2-2\,\left(A-C\right)ab-Bb^2\right)c^3+3\,\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)c^2d-3\,\left(Ba^2-2\,\left(A-C\right)ab-Bb^2\right)cd^2-\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)d^3\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,\left(\left(Ca^2b^3-Bab^4+Ab^5\right)c^3-3\,\left(Ca^3b^2-Ba^2b^3+Aab^4\right)c^2d+3\,\left(Ca^4b-Ba^3b^2+Aa^2b^3\right)cd^2-\left(Ca^5-Ba^4b+Aa^3b^2\right)d^3\right)}{a^3b^4+ab^6+\left(a^2b^5+b^7\right)\tan\left(fx+e\right)}+\frac{Cbd^3\tan\left(fx+e\right)^2+2\,\left(3\,Cbcd^2-\left(2\,Ca-Bb\right)d^3\right)\tan\left(fx+e\right)}{b^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^3 - 3*(B*a^2 - 2*(A - C)*a*
b - B*b^2)*c^2*d - 3*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d^2 + (B*a^2 -
```

```
2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a^2*
b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^3 - 3*(C*a^4*b^2 - (A - 3*C)*a^2*b^4 - 2*B
*a*b^5 + A*b^6)*c^2*d + 3*(2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^2*b^
4 + 2*A*a*b^5)*c*d^2 - (3*C*a^6 - 2*B*a^5*b + (A + 5*C)*a^4*b^2 - 4*B*a^3*b
^3 + 3*A*a^2*b^4)*d^3)*log(b*tan(f*x + e) + a)/(a^4*b^4 + 2*a^2*b^6 + b^8)
+ ((B*a^2 - 2*(A - C)*a*b - B*b^2)*c^3 + 3*((A - C)*a^2 + 2*B*a*b - (A - C)
*b^2)*c^2*d - 3*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d^2 - ((A - C)*a^2 + 2*B*
a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2
*((C*a^2*b^3 - B*a*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c
^2*d + 3*(C*a^4*b - B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a^4*b + A*a^3
*b^2)*d^3)/(a^3*b^4 + a*b^6 + (a^2*b^5 + b^7)*tan(f*x + e)) + (C*b*d^3*tan(
f*x + e)^2 + 2*(3*C*b*c*d^2 - (2*C*a - B*b)*d^3)*tan(f*x + e))/b^3)/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1522 vs. $2(577) = 1154$.
time = 4.13, size = 1522, normalized size = 2.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((C*a^4*b^3 + 2*C*a^2*b^5 + C*b^7)*d^3*tan(f*x + e)^3 - 2*(C*a^2*b^5 -
B*a*b^6 + A*b^7)*c^3 + 6*(C*a^3*b^4 - B*a^2*b^5 + A*a*b^6)*c^2*d - 6*(C*a^4
*b^3 - B*a^3*b^4 + A*a^2*b^5)*c*d^2 + (3*C*a^5*b^2 - 2*B*a^4*b^3 + 2*(A + C
)*a^3*b^4 + C*a*b^6)*d^3 + 2*(((A - C)*a^3*b^4 + 2*B*a^2*b^5 - (A - C)*a*b^
6)*c^3 - 3*(B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c^2*d - 3*((A - C)*a^3
*b^4 + 2*B*a^2*b^5 - (A - C)*a*b^6)*c*d^2 + (B*a^3*b^4 - 2*(A - C)*a^2*b^5
- B*a*b^6)*d^3)*f*x + (6*(C*a^4*b^3 + 2*C*a^2*b^5 + C*b^7)*c*d^2 - (3*C*a^5
*b^2 - 2*B*a^4*b^3 + 6*C*a^3*b^4 - 4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^7)*d^3)*
tan(f*x + e)^2 - ((B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c^3 - 3*(C*a^5*
b^2 - (A - 3*C)*a^3*b^4 - 2*B*a^2*b^5 + A*a*b^6)*c^2*d + 3*(2*C*a^6*b - B*a
^5*b^2 + 4*C*a^4*b^3 - 3*B*a^3*b^4 + 2*A*a^2*b^5)*c*d^2 - (3*C*a^7 - 2*B*a^
6*b + (A + 5*C)*a^5*b^2 - 4*B*a^4*b^3 + 3*A*a^3*b^4)*d^3 + ((B*a^2*b^5 - 2*
(A - C)*a*b^6 - B*b^7)*c^3 - 3*(C*a^4*b^3 - (A - 3*C)*a^2*b^5 - 2*B*a*b^6 +
 A*b^7)*c^2*d + 3*(2*C*a^5*b^2 - B*a^4*b^3 + 4*C*a^3*b^4 - 3*B*a^2*b^5 + 2*
A*a*b^6)*c*d^2 - (3*C*a^6*b - 2*B*a^5*b^2 + (A + 5*C)*a^4*b^3 - 4*B*a^3*b^4
 + 3*A*a^2*b^5)*d^3)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x
+ e) + a^2)/(tan(f*x + e)^2 + 1)) - (3*(C*a^5*b^2 + 2*C*a^3*b^4 + C*a*b^6)*
c^2*d - 3*(2*C*a^6*b - B*a^5*b^2 + 4*C*a^4*b^3 - 2*B*a^3*b^4 + 2*C*a^2*b^5
- B*a*b^6)*c*d^2 + (3*C*a^7 - 2*B*a^6*b + (A + 5*C)*a^5*b^2 - 4*B*a^4*b^3 +
 (2*A + C)*a^3*b^4 - 2*B*a^2*b^5 + (A - C)*a*b^6)*d^3 + (3*(C*a^4*b^3 + 2*C
*a^2*b^5 + C*b^7)*c^2*d - 3*(2*C*a^5*b^2 - B*a^4*b^3 + 4*C*a^3*b^4 - 2*B*a^
2*b^5 + 2*C*a*b^6 - B*b^7)*c*d^2 + (3*C*a^6*b - 2*B*a^5*b^2 + (A + 5*C)*a^4
```

```
*b^3 - 4*B*a^3*b^4 + (2*A + C)*a^2*b^5 - 2*B*a*b^6 + (A - C)*b^7)*d^3)*tan(
f*x + e))*log(1/(tan(f*x + e)^2 + 1)) + (2*(C*a^3*b^4 - B*a^2*b^5 + A*a*b^6
)*c^3 - 6*(C*a^4*b^3 - B*a^3*b^4 + A*a^2*b^5)*c^2*d + 6*(2*C*a^5*b^2 - B*a^
4*b^3 + (A + 2*C)*a^3*b^4 + C*a*b^6)*c*d^2 - (6*C*a^6*b - 4*B*a^5*b^2 + (2*
A + 7*C)*a^4*b^3 - 4*B*a^3*b^4 + 2*C*a^2*b^5 - 2*B*a*b^6 - C*b^7)*d^3 + 2*(
((A - C)*a^2*b^5 + 2*B*a*b^6 - (A - C)*b^7)*c^3 - 3*(B*a^2*b^5 - 2*(A - C)*
a*b^6 - B*b^7)*c^2*d - 3*((A - C)*a^2*b^5 + 2*B*a*b^6 - (A - C)*b^7)*c*d^2
+ (B*a^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)*d^3)*f*x)*tan(f*x + e))/((a^4*b^5 +
 2*a^2*b^7 + b^9)*f*tan(f*x + e) + (a^5*b^4 + 2*a^3*b^6 + a*b^8)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 33.12, size = 24300, normalized size = 42.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(c + d*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2,
Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-A*c**3*f*x*tan(e + f*x)**2/(4*b**2*f*tan
(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c**3*f*x*tan(e +
 f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + A*c
**3*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - A
*c**3*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*
b**2*f) + 2*I*A*c**3/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) -
4*b**2*f) + 3*I*A*c**2*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*
I*b**2*f*tan(e + f*x) - 4*b**2*f) + 6*A*c**2*d*f*x*tan(e + f*x)/(4*b**2*f*t
an(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*I*A*c**2*d*f*x/(4*
b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*I*A*c**2*d
*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*
f) + 3*A*c*d**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*
tan(e + f*x) - 4*b**2*f) - 6*I*A*c*d**2*f*x*tan(e + f*x)/(4*b**2*f*tan(e +
f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*A*c*d**2*f*x/(4*b**2*f*ta
n(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 9*A*c*d**2*tan(e + f*
x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 6*I*A*
c*d**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*
I*A*d**3*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e +
 f*x) - 4*b**2*f) + 6*A*d**3*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8
*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*I*A*d**3*f*x/(4*b**2*f*tan(e + f*x)*
*2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*A*d**3*log(tan(e + f*x)**2 + 1
)*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b
**2*f) - 4*I*A*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*b**2*f*tan(e +
 f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*A*d**3*log(tan(e + f*x)*
*2 + 1)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 5
```

```
*I*A*d**3*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x)
- 4*b**2*f) - 4*A*d**3/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x)
- 4*b**2*f) + I*B*c**3*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*
b**2*f*tan(e + f*x) - 4*b**2*f) + 2*B*c**3*f*x*tan(e + f*x)/(4*b**2*f*tan(e
 + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - I*B*c**3*f*x/(4*b**2*f*t
an(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*B*c**3*tan(e + f*x
)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*B*c**
2*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x)
 - 4*b**2*f) - 6*I*B*c**2*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*
I*b**2*f*tan(e + f*x) - 4*b**2*f) - 3*B*c**2*d*f*x/(4*b**2*f*tan(e + f*x)**
2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 9*B*c**2*d*tan(e + f*x)/(4*b**2*f
*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 6*I*B*c**2*d/(4*b*
*2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 9*I*B*c*d**2*f
*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*
b**2*f) + 18*B*c*d**2*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2
*f*tan(e + f*x) - 4*b**2*f) - 9*I*B*c*d**2*f*x/(4*b**2*f*tan(e + f*x)**2 -
8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 6*B*c*d**2*log(tan(e + f*x)**2 + 1)*t
an(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2
*f) - 12*I*B*c*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*b**2*f*tan(e +
 f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 6*B*c*d**2*log(tan(e + f*x
)**2 + 1)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) -
 15*I*B*c*d**2*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e +
f*x) - 4*b**2*f) - 12*B*c*d**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e
 + f*x) - 4*b**2*f) - 9*B*d**3*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**
2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 18*I*B*d**3*f*x*tan(e + f*x)/(4*b
**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 9*B*d**3*f*x/
(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 4*I*B*d**
3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*
b**2*f*tan(e + f*x) - 4*b**2*f) + 8*B*d**3*log(tan(e + f*x)**2 + 1)*tan(e +
 f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 4*I
*B*d**3*log(tan(e + f*x)**2 + 1)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan
(e + f*x) - 4*b**2*f) + 4*B*d**3*tan(e + f*x)**3/(4*b**2*f*tan(e + f*x)**2
- 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 19*B*d**3*tan(e + f*x)/(4*b**2*f*ta
n(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 14*I*B*d**3/(4*b**2*f
*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + C*c**3*f*x*tan(e +
 f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) -
2*I*C*c**3*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e +
f*x) - 4*b**2*f) - C*c**3*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e
+ f*x) - 4*b**2*f) - 3*C*c**3*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*
b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*C*c**3/(4*b**2*f*tan(e + f*x)**2 - 8*
I*b**2*f*tan(e + f*x) - 4*b**2*f) + 9*I*C*c**2*...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1357 vs. $2(577) = 1154$.

time = 1.24, size = 1357, normalized size = 2.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c^3 - C*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 - 3*B*a
^2*c^2*d + 6*A*a*b*c^2*d - 6*C*a*b*c^2*d + 3*B*b^2*c^2*d - 3*A*a^2*c*d^2 +
3*C*a^2*c*d^2 - 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 + B*a^2*d^3 -
 2*A*a*b*d^3 + 2*C*a*b*d^3 - B*b^2*d^3)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) +
 (B*a^2*c^3 - 2*A*a*b*c^3 + 2*C*a*b*c^3 - B*b^2*c^3 + 3*A*a^2*c^2*d - 3*C*a
^2*c^2*d + 6*B*a*b*c^2*d - 3*A*b^2*c^2*d + 3*C*b^2*c^2*d - 3*B*a^2*c*d^2 +
6*A*a*b*c*d^2 - 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 - A*a^2*d^3 + C*a^2*d^3 - 2*B
*a*b*d^3 + A*b^2*d^3 - C*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2
+ b^4) - 2*(B*a^2*b^4*c^3 - 2*A*a*b^5*c^3 + 2*C*a*b^5*c^3 - B*b^6*c^3 - 3*C
*a^4*b^2*c^2*d + 3*A*a^2*b^4*c^2*d - 9*C*a^2*b^4*c^2*d + 6*B*a*b^5*c^2*d -
3*A*b^6*c^2*d + 6*C*a^5*b*c*d^2 - 3*B*a^4*b^2*c*d^2 + 12*C*a^3*b^3*c*d^2 -
9*B*a^2*b^4*c*d^2 + 6*A*a*b^5*c*d^2 - 3*C*a^6*d^3 + 2*B*a^5*b*d^3 - A*a^4*b
^2*d^3 - 5*C*a^4*b^2*d^3 + 4*B*a^3*b^3*d^3 - 3*A*a^2*b^4*d^3)*log(abs(b*tan
(f*x + e) + a))/(a^4*b^4 + 2*a^2*b^6 + b^8) + 2*(B*a^2*b^5*c^3*tan(f*x + e)
 - 2*A*a*b^6*c^3*tan(f*x + e) + 2*C*a*b^6*c^3*tan(f*x + e) - B*b^7*c^3*tan(
f*x + e) - 3*C*a^4*b^3*c^2*d*tan(f*x + e) + 3*A*a^2*b^5*c^2*d*tan(f*x + e)
- 9*C*a^2*b^5*c^2*d*tan(f*x + e) + 6*B*a*b^6*c^2*d*tan(f*x + e) - 3*A*b^7*c
^2*d*tan(f*x + e) + 6*C*a^5*b^2*c*d^2*tan(f*x + e) - 3*B*a^4*b^3*c*d^2*tan(
f*x + e) + 12*C*a^3*b^4*c*d^2*tan(f*x + e) - 9*B*a^2*b^5*c*d^2*tan(f*x + e)
 + 6*A*a*b^6*c*d^2*tan(f*x + e) - 3*C*a^6*b*d^3*tan(f*x + e) + 2*B*a^5*b^2*
d^3*tan(f*x + e) - A*a^4*b^3*d^3*tan(f*x + e) - 5*C*a^4*b^3*d^3*tan(f*x + e
) + 4*B*a^3*b^4*d^3*tan(f*x + e) - 3*A*a^2*b^5*d^3*tan(f*x + e) - C*a^4*b^3
*c^3 + 2*B*a^3*b^4*c^3 - 3*A*a^2*b^5*c^3 + C*a^2*b^5*c^3 - A*b^7*c^3 - 3*B*
a^4*b^3*c^2*d + 6*A*a^3*b^4*c^2*d - 6*C*a^3*b^4*c^2*d + 3*B*a^2*b^5*c^2*d +
 3*C*a^6*b*c*d^2 - 3*A*a^4*b^3*c*d^2 + 9*C*a^4*b^3*c*d^2 - 6*B*a^3*b^4*c*d^
2 + 3*A*a^2*b^5*c*d^2 - 2*C*a^7*d^3 + B*a^6*b*d^3 - 4*C*a^5*b^2*d^3 + 3*B*a
^4*b^3*d^3 - 2*A*a^3*b^4*d^3)/((a^4*b^4 + 2*a^2*b^6 + b^8)*(b*tan(f*x + e)
+ a)) + (C*b^2*d^3*tan(f*x + e)^2 + 6*C*b^2*c*d^2*tan(f*x + e) - 4*C*a*b*d^
3*tan(f*x + e) + 2*B*b^2*d^3*tan(f*x + e))/b^4)/f
```

Mupad [B]
time = 15.70, size = 701, normalized size = 1.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b
*tan(e + f*x))^2,x)
```

```
[Out] (tan(e + f*x)*((B*d^3 + 3*C*c*d^2)/b^2 - (2*C*a*d^3)/b^3))/f - (log(tan(e +
 f*x) + 1i)*(A*c^3*1i - A*d^3 + B*c^3 + B*d^3*1i - C*c^3*1i + C*d^3 - A*c*d
^2*3i + 3*A*c^2*d - 3*B*c*d^2 - B*c^2*d*3i + C*c*d^2*3i - 3*C*c^2*d))/(2*f*
(a*b*2i - a^2 + b^2)) + (log(a + b*tan(e + f*x))*(b^4*(3*A*a^2*d^3 - B*a^2*
c^3 - 3*A*a^2*c^2*d + 9*B*a^2*c*d^2 + 9*C*a^2*c^2*d) - b^5*(2*C*a*c^3 - 2*A
*a*c^3 + 6*A*a*c*d^2 + 6*B*a*c^2*d) - b^3*(4*B*a^3*d^3 + 12*C*a^3*c*d^2) +
b^6*(B*c^3 + 3*A*c^2*d) - b*(2*B*a^5*d^3 + 6*C*a^5*c*d^2) + b^2*(A*a^4*d^3
+ 5*C*a^4*d^3 + 3*B*a^4*c*d^2 + 3*C*a^4*c^2*d) + 3*C*a^6*d^3))/(f*(b^8 + 2*
a^2*b^6 + a^4*b^4)) - (log(tan(e + f*x) - 1i)*(A*c^3 - A*d^3*1i + B*c^3*1i
+ B*d^3 - C*c^3 + C*d^3*1i - 3*A*c*d^2 + A*c^2*d*3i - B*c*d^2*3i - 3*B*c^2*
d + 3*C*c*d^2 - C*c^2*d*3i))/(2*f*(2*a*b - a^2*1i + b^2*1i)) - (A*b^5*c^3 -
 C*a^5*d^3 - B*a*b^4*c^3 + B*a^4*b*d^3 - A*a^3*b^2*d^3 + C*a^2*b^3*c^3 + 3*
A*a^2*b^3*c*d^2 + 3*B*a^2*b^3*c^2*d - 3*B*a^3*b^2*c*d^2 - 3*C*a^3*b^2*c^2*d
 - 3*A*a*b^4*c^2*d + 3*C*a^4*b*c*d^2)/(b*f*(a*b^3 + b^4*tan(e + f*x))*(a^2
+ b^2)) + (C*d^3*tan(e + f*x)^2)/(2*b^2*f)
```

3.69 $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=798

$$-\frac{(3ab^2(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3)+a^3(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3-3cd^2))-}{(a^2+b^2)^3}$$

[Out] -(3*a*b^2*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)+a^3*(c^3*C+3*B*c^2*d-3*c*C*d^2-B*d^3-A*(c^3-3*c*d^2))-3*a^2*b*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2))+b^3*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2)))*x/(a^2+b^2)^3-(b^3*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)+3*a^2*b*(c^3*C+3*B*c^2*d-3*c*C*d^2-B*d^3-A*(c^3-3*c*d^2))+a^3*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2))-3*a*b^2*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2)))*ln(cos(f*x+e))/(a^2+b^2)^3/f-(-a*d+b*c)*(a^5*b*B*d^2-3*a^6*C*d^2+a^4*b^2*d*(B*c-9*C*d)+a^3*b^3*B*(c^2+3*d^2)-b^6*(c*(3*B*d+C*c)-A*(c^2-3*d^2))-a*b^5*(8*c*(A-C)*d+3*B*(c^2-2*d^2))+a^2*b^4*(3*c^2*C+6*B*c*d-10*C*d^2-A*(3*c^2-d^2)))*ln(a+b*tan(f*x+e))/b^4/(a^2+b^2)^3/f-d^2*(a^3*b*B*d-3*a^4*C*d-a*b^3*(2*A*c-3*B*d-2*C*c)+a^2*b^2*(B*c-6*C*d)-b^4*(B*c+(2*A+C)*d))*tan(f*x+e)/b^3/(a^2+b^2)^2/f+1/2*(a^3*b*B*d-3*a^4*C*d-b^4*(3*A*d+2*B*c)-a*b^3*(4*A*c-5*B*d-4*C*c)+a^2*b^2*(2*B*c+(A-7*C)*d))*(c+d*tan(f*x+e))^2/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))-1/2*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^3/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2

Rubi [A]
time = 1.85, antiderivative size = 798, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3726, 3718, 3707, 3698, 31, 3556}

Antiderivative was successfully verified.

[In] Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*Tan[e + f*x])^3,x]

[Out] -(((3*a*b^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + a^3*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 3*a^2*b*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)/(a^2 + b^2)^3) - ((b^3*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + 3*a^2*b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) + a^3*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)) - 3*a*b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)^3*f) - ((b*c - a*d)*(a^5*b*B*d^2 - 3*a^6*C*d^2 + a^4*b^2*d*(B*c - 9*C*d) + a^3*b^3*B*(c^2 + 3*d^2) - b^6*(c*(c*C + 3*B*d) - A*(c^2 - 3*d^2)) - a*b^5*(8*c*(A - C)*d + 3*B*(c^2 - 2*d^2)) + a^2*b^4*(3*c^2*C + 6*B*c*d - 10*C*d^2 - A*(3*c^2 - d^2)))*Log[a + b*Tan[e + f*x]])/(b^4*(a^2 +

```
 b^2)^3*f) - (d^2*(a^3*b*B*d - 3*a^4*C*d - a*b^3*(2*A*c - 2*c*C - 3*B*d) +
a^2*b^2*(B*c - 6*C*d) - b^4*(B*c + (2*A + C)*d))*Tan[e + f*x])/(b^3*(a^2 +
b^2)^2*f) + ((a^3*b*B*d - 3*a^4*C*d - b^4*(2*B*c + 3*A*d) - a*b^3*(4*A*c -
4*c*C - 5*B*d) + a^2*b^2*(2*B*c + (A - 7*C)*d))*(c + d*Tan[e + f*x])^2)/(2*
b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c + d
*Tan[e + f*x])^3)/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+f}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

$$= \frac{(a^3bBd-3a^4Cd-b^4(2Bc+3Ad)-ab}{2b}$$

$$= -\frac{d^2(a^3bBd-3a^4Cd-ab^3(2Ac-2cC}{}$$

$$= -\frac{(3ab^2(Ac^3-c^3C-3Bc^2d-3Acd^2+}{}$$

$$= -\frac{(3ab^2(Ac^3-c^3C-3Bc^2d-3Acd^2+}{}$$

$$= -\frac{(3ab^2(Ac^3-c^3C-3Bc^2d-3Acd^2+}{}$$

Mathematica [A]
time = 13.87, size = 1451, normalized size = 1.82

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((3*a*b^2*(-(A*c^3) + c^3*C + 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 - B*d^3) +
a^3*(-(c^3*C) - 3*B*c^2*d + 3*c*C*d^2 + B*d^3 + A*(c^3 - 3*c*d^2)) + b^3*((
A - C)*d*(-3*c^2 + d^2) - B*(c^3 - 3*c*d^2)) + 3*a^2*b*(-((A - C)*d*(-3*c^2
 + d^2)) + B*(c^3 - 3*c*d^2)))*(e + f*x)*(a*Cos[e + f*x] + b*Sin[e + f*x])^
3*(c + d*Tan[e + f*x])^3)/((a^2 + b^2)^3*f*(c*Cos[e + f*x] + d*Sin[e + f*x]
```

```
)^3*(a + b*Tan[e + f*x])^3) - (d^2*(3*b*c*C + b*B*d - 3*a*C*d)*Log[1 - Tan[
(e + f*x)/2]^2]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^3)
/(b^4*f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^3) + ((3*a
^2*b*(-(A*c^3) + c^3*C + 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 - B*d^3) + b^3*(
-(c^3*C) - 3*B*c^2*d + 3*c*C*d^2 + B*d^3 + A*(c^3 - 3*c*d^2)) + a^3*(-((A -
 C)*d*(-3*c^2 + d^2)) + B*(c^3 - 3*c*d^2)) - 3*a*b^2*(-((A - C)*d*(-3*c^2 +
 d^2)) + B*(c^3 - 3*c*d^2)))*Log[1 + Tan[(e + f*x)/2]^2]*(a*Cos[e + f*x] +
b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^3)/((a^2 + b^2)^3*f*(c*Cos[e + f*x]
+ d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^3) - ((b*c - a*d)*(a^5*b*B*d^2 - 3
*a^6*C*d^2 + a^4*b^2*d*(B*c - 9*C*d) + a^3*b^3*B*(c^2 + 3*d^2) + b^6*(-(c*(
c*C + 3*B*d)) + A*(c^2 - 3*d^2)) + a*b^5*(8*c*(-A + C)*d - 3*B*(c^2 - 2*d^2
)) + a^2*b^4*(3*c^2*C + 6*B*c*d - 10*C*d^2 + A*(-3*c^2 + d^2)))*Log[-2*b*Ta
n[(e + f*x)/2] + a*(-1 + Tan[(e + f*x)/2]^2)]*(a*Cos[e + f*x] + b*Sin[e + f
*x])^3*(c + d*Tan[e + f*x])^3)/(b^4*(a^2 + b^2)^3*f*(c*Cos[e + f*x] + d*Sin
[e + f*x])^3*(a + b*Tan[e + f*x])^3) - (2*C*d^3*(a*Cos[e + f*x] + b*Sin[e +
 f*x])^3*Tan[(e + f*x)/2]*(c + d*Tan[e + f*x])^3)/(b^3*f*(c*Cos[e + f*x] +
d*Sin[e + f*x])^3*(-1 + Tan[(e + f*x)/2]^2)*(a + b*Tan[e + f*x])^3) + (2*(A
*b^2 + a*(-(b*B) + a*C))*(-(b*c) + a*d)^3*(a*Cos[e + f*x] + b*Sin[e + f*x])
^3*(a + 2*b*Tan[(e + f*x)/2])*(c + d*Tan[e + f*x])^3)/(a^3*b^2*(a^2 + b^2)*
f*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + 2*b*Tan[(e + f*x)/2] - a*Tan[(e
+ f*x)/2]^2)^2*(a + b*Tan[e + f*x])^3) - (2*(b*c - a*d)^2*(a*Cos[e + f*x] +
 b*Sin[e + f*x])^3*(A*b^6*c + 2*a^6*C*d*Tan[(e + f*x)/2] - a*b^5*(B*c + A*(
d - c*Tan[(e + f*x)/2])) - a^5*b*(B*d*Tan[(e + f*x)/2] + C*(d - c*Tan[(e +
f*x)/2])) + a^4*b^2*(c*(C - 2*B*Tan[(e + f*x)/2]) + d*(B + 4*C*Tan[(e + f*x
)/2])) + a^2*b^4*(c*C + B*d + A*(c + 2*d*Tan[(e + f*x)/2])) - a^3*b^3*(A*d
+ C*d - 3*A*c*Tan[(e + f*x)/2] + c*C*Tan[(e + f*x)/2] + B*(c + 3*d*Tan[(e +
 f*x)/2])))*(c + d*Tan[e + f*x])^3)/(a^3*b^3*(a^2 + b^2)^2*f*(c*Cos[e + f*x
] + d*Sin[e + f*x])^3*(-2*b*Tan[(e + f*x)/2] + a*(-1 + Tan[(e + f*x)/2]^2))
*(a + b*Tan[e + f*x])^3)
```

Maple [A]
time = 0.90, size = 1271, normalized size = 1.59 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(C*d^3/b^3*tan(f*x+e)+1/(a^2+b^2)^3*(1/2*(3*A*a^3*c^2*d-A*a^3*d^3-3*A*a
^2*b*c^3+9*A*a^2*b*c*d^2-9*A*a*b^2*c^2*d+3*A*a*b^2*d^3+A*b^3*c^3-3*A*b^3*c*
d^2+B*a^3*c^3-3*B*a^3*c*d^2+9*B*a^2*b*c^2*d-3*B*a^2*b*d^3-3*B*a*b^2*c^3+9*B
*a*b^2*c*d^2-3*B*b^3*c^2*d+B*b^3*d^3-3*C*a^3*c^2*d+C*a^3*d^3+3*C*a^2*b*c^3-
9*C*a^2*b*c*d^2+9*C*a*b^2*c^2*d-3*C*a*b^2*d^3-C*b^3*c^3+3*C*b^3*c*d^2)*ln(1
+tan(f*x+e)^2)+(A*a^3*c^3-3*A*a^3*c*d^2+9*A*a^2*b*c^2*d-3*A*a^2*b*d^3-3*A*a
*b^2*c^3+9*A*a*b^2*c*d^2-3*A*b^3*c^2*d+A*b^3*d^3-3*B*a^3*c^2*d+B*a^3*d^3+3*
B*a^2*b*c^3-9*B*a^2*b*c*d^2+9*B*a*b^2*c^2*d-3*B*a*b^2*d^3-B*b^3*c^3+3*B*b^3
*c*d^2-C*a^3*c^3+3*C*a^3*c*d^2-9*C*a^2*b*c^2*d+3*C*a^2*b*d^3+3*C*a*b^2*c^3-
```

```
9*C*a*b^2*c*d^2+3*C*b^3*c^2*d-C*b^3*d^3)*arctan(tan(f*x+e)))-1/2*(-A*a^3*b^
2*d^3+3*A*a^2*b^3*c*d^2-3*A*a*b^4*c^2*d+A*b^5*c^3+B*a^4*b*d^3-3*B*a^3*b^2*c
*d^2+3*B*a^2*b^3*c^2*d-B*a*b^4*c^3-C*a^5*d^3+3*C*a^4*b*c*d^2-3*C*a^3*b^2*c^
2*d+C*a^2*b^3*c^3)/b^4/(a^2+b^2)/(a+b*tan(f*x+e))^2-1/b^4*(A*a^4*b^2*d^3-3*
A*a^2*b^4*c^2*d+3*A*a^2*b^4*d^3+2*A*a*b^5*c^3-6*A*a*b^5*c*d^2+3*A*b^6*c^2*d
-2*B*a^5*b*d^3+3*B*a^4*b^2*c*d^2-4*B*a^3*b^3*d^3-B*a^2*b^4*c^3+9*B*a^2*b^4*
c*d^2-6*B*a*b^5*c^2*d+B*b^6*c^3+3*C*a^6*d^3-6*C*a^5*b*c*d^2+3*C*a^4*b^2*c^2
*d+5*C*a^4*b^2*d^3-12*C*a^3*b^3*c*d^2+9*C*a^2*b^4*c^2*d-2*C*a*b^5*c^3)/(a^2
+b^2)^2/(a+b*tan(f*x+e))+(-3*A*a^3*b^4*c^2*d+A*a^3*b^4*d^3+3*A*a^2*b^5*c^3-
9*A*a^2*b^5*c*d^2+9*A*a*b^6*c^2*d-3*A*a*b^6*d^3-A*b^7*c^3+3*A*b^7*c*d^2+B*a
^6*b*d^3+3*B*a^4*b^3*d^3-B*a^3*b^4*c^3+3*B*a^3*b^4*c*d^2-9*B*a^2*b^5*c^2*d+
6*B*a^2*b^5*d^3+3*B*a*b^6*c^3-9*B*a*b^6*c*d^2+3*B*b^7*c^2*d-3*C*a^7*d^3+3*C
*a^6*b*c*d^2-9*C*a^5*b^2*d^3+9*C*a^4*b^3*c*d^2+3*C*a^3*b^4*c^2*d-10*C*a^3*b
^4*d^3-3*C*a^2*b^5*c^3+18*C*a^2*b^5*c*d^2-9*C*a*b^6*c^2*d+C*b^7*c^3)/b^4/(a
^2+b^2)^3*ln(a+b*tan(f*x+e)))
```

Maxima [A]
time = 0.57, size = 1126, normalized size = 1.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*d^3*tan(f*x + e)/b^3 + 2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b
^2 - B*b^3)*c^3 - 3*(B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c^2
*d - 3*((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c*d^2 + (B*a^3 -
 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d^3)*(f*x + e)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) - 2*((B*a^3*b^4 - 3*(A - C)*a^2*b^5 - 3*B*a*b^6 + (A -
C)*b^7)*c^3 + 3*((A - C)*a^3*b^4 + 3*B*a^2*b^5 - 3*(A - C)*a*b^6 - B*b^7)*c
^2*d - 3*(C*a^6*b + 3*C*a^4*b^3 + B*a^3*b^4 - 3*(A - 2*C)*a^2*b^5 - 3*B*a*b
^6 + A*b^7)*c*d^2 + (3*C*a^7 - B*a^6*b + 9*C*a^5*b^2 - 3*B*a^4*b^3 - (A - 1
0*C)*a^3*b^4 - 6*B*a^2*b^5 + 3*A*a*b^6)*d^3)*log(b*tan(f*x + e) + a)/(a^6*b
^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10) + ((B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2
+ (A - C)*b^3)*c^3 + 3*((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*
c^2*d - 3*(B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c*d^2 - ((A -
 C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*d^3)*log(tan(f*x + e)^2 + 1)
/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((C*a^4*b^3 - 3*B*a^3*b^4 + (5*A - 3
*C)*a^2*b^5 + B*a*b^6 + A*b^7)*c^3 + 3*(C*a^5*b^2 + B*a^4*b^3 - (3*A - 5*C)
*a^3*b^4 - 3*B*a^2*b^5 + A*a*b^6)*c^2*d - 3*(3*C*a^6*b - B*a^5*b^2 - (A - 7
*C)*a^4*b^3 - 5*B*a^3*b^4 + 3*A*a^2*b^5)*c*d^2 + (5*C*a^7 - 3*B*a^6*b + (A
+ 9*C)*a^5*b^2 - 7*B*a^4*b^3 + 5*A*a^3*b^4)*d^3 - 2*((B*a^2*b^5 - 2*(A - C)
*a*b^6 - B*b^7)*c^3 - 3*(C*a^4*b^3 - (A - 3*C)*a^2*b^5 - 2*B*a*b^6 + A*b^7)
*c^2*d + 3*(2*C*a^5*b^2 - B*a^4*b^3 + 4*C*a^3*b^4 - 3*B*a^2*b^5 + 2*A*a*b^6
```

```
)*c*d^2 - (3*C*a^6*b - 2*B*a^5*b^2 + (A + 5*C)*a^4*b^3 - 4*B*a^3*b^4 + 3*A*
a^2*b^5)*d^3)*tan(f*x + e))/(a^6*b^4 + 2*a^4*b^6 + a^2*b^8 + (a^4*b^6 + 2*a
^2*b^8 + b^10)*tan(f*x + e)^2 + 2*(a^5*b^5 + 2*a^3*b^7 + a*b^9)*tan(f*x + e
)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2562 vs. $2(801) = 1602$.
time = 4.55, size = 2562, normalized size = 3.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^6*b^3 + 3*C*a^4*b^5 + 3*C*a^2*b^7 + C*b^9)*d^3*tan(f*x + e)^3 -
 (3*C*a^4*b^5 - 5*B*a^3*b^6 + (7*A - 3*C)*a^2*b^7 + B*a*b^8 + A*b^9)*c^3 +
3*(C*a^5*b^4 - 3*B*a^4*b^5 + 5*(A - C)*a^3*b^6 + 3*B*a^2*b^7 - A*a*b^8)*c^2
*d + 3*(C*a^6*b^3 + B*a^5*b^4 - (3*A - 7*C)*a^4*b^5 - 5*B*a^3*b^6 + 3*A*a^2
*b^7)*c*d^2 - (3*C*a^7*b^2 - B*a^6*b^3 - (A - 9*C)*a^5*b^4 - 7*B*a^4*b^5 +
5*A*a^3*b^6)*d^3 + 2*(((A - C)*a^5*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*b^6 -
B*a^2*b^7)*c^3 - 3*(B*a^5*b^4 - 3*(A - C)*a^4*b^5 - 3*B*a^3*b^6 + (A - C)*a
^2*b^7)*c^2*d - 3*((A - C)*a^5*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*b^6 - B*a^
2*b^7)*c*d^2 + (B*a^5*b^4 - 3*(A - C)*a^4*b^5 - 3*B*a^3*b^6 + (A - C)*a^2*b
^7)*d^3)*f*x + ((C*a^4*b^5 - 3*B*a^3*b^6 + 5*(A - C)*a^2*b^7 + 3*B*a*b^8 -
A*b^9)*c^3 + 3*(C*a^5*b^4 + B*a^4*b^5 - (3*A - 7*C)*a^3*b^6 - 5*B*a^2*b^7 +
 3*A*a*b^8)*c^2*d - 3*(3*C*a^6*b^3 - B*a^5*b^4 - (A - 9*C)*a^4*b^5 - 7*B*a^
3*b^6 + 5*A*a^2*b^7)*c*d^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 + (A + 23*C)*a^5*b^
4 - 9*B*a^4*b^5 + (7*A + 12*C)*a^3*b^6 + 4*C*a*b^8)*d^3 + 2*(((A - C)*a^3*b
^6 + 3*B*a^2*b^7 - 3*(A - C)*a*b^8 - B*b^9)*c^3 - 3*(B*a^3*b^6 - 3*(A - C)*
a^2*b^7 - 3*B*a*b^8 + (A - C)*b^9)*c^2*d - 3*((A - C)*a^3*b^6 + 3*B*a^2*b^7
 - 3*(A - C)*a*b^8 - B*b^9)*c*d^2 + (B*a^3*b^6 - 3*(A - C)*a^2*b^7 - 3*B*a*
b^8 + (A - C)*b^9)*d^3)*f*x)*tan(f*x + e)^2 - ((B*a^5*b^4 - 3*(A - C)*a^4*b
^5 - 3*B*a^3*b^6 + (A - C)*a^2*b^7)*c^3 + 3*((A - C)*a^5*b^4 + 3*B*a^4*b^5
 - 3*(A - C)*a^3*b^6 - B*a^2*b^7)*c^2*d - 3*(C*a^8*b + 3*C*a^6*b^3 + B*a^5*b
^4 - 3*(A - 2*C)*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*c*d^2 + (3*C*a^9 - B*a^
8*b + 9*C*a^7*b^2 - 3*B*a^6*b^3 - (A - 10*C)*a^5*b^4 - 6*B*a^4*b^5 + 3*A*a^
3*b^6)*d^3 + ((B*a^3*b^6 - 3*(A - C)*a^2*b^7 - 3*B*a*b^8 + (A - C)*b^9)*c^3
 + 3*((A - C)*a^3*b^6 + 3*B*a^2*b^7 - 3*(A - C)*a*b^8 - B*b^9)*c^2*d - 3*(C
*a^6*b^3 + 3*C*a^4*b^5 + B*a^3*b^6 - 3*(A - 2*C)*a^2*b^7 - 3*B*a*b^8 + A*b^
9)*c*d^2 + (3*C*a^7*b^2 - B*a^6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 - (A - 10*C
)*a^3*b^6 - 6*B*a^2*b^7 + 3*A*a*b^8)*d^3)*tan(f*x + e)^2 + 2*((B*a^4*b^5 -
3*(A - C)*a^3*b^6 - 3*B*a^2*b^7 + (A - C)*a*b^8)*c^3 + 3*((A - C)*a^4*b^5 +
 3*B*a^3*b^6 - 3*(A - C)*a^2*b^7 - B*a*b^8)*c^2*d - 3*(C*a^7*b^2 + 3*C*a^5*
b^4 + B*a^4*b^5 - 3*(A - 2*C)*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*c*d^2 + (3*C
```

```
*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 - (A - 10*C)*a^4*b^5 - 6*B*a
^3*b^6 + 3*A*a^2*b^7)*d^3)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*ta
n(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (3*(C*a^8*b + 3*C*a^6*b^3 + 3*C*a
^4*b^5 + C*a^2*b^7)*c*d^2 - (3*C*a^9 - B*a^8*b + 9*C*a^7*b^2 - 3*B*a^6*b^3
+ 9*C*a^5*b^4 - 3*B*a^4*b^5 + 3*C*a^3*b^6 - B*a^2*b^7)*d^3 + (3*(C*a^6*b^3
+ 3*C*a^4*b^5 + 3*C*a^2*b^7 + C*b^9)*c*d^2 - (3*C*a^7*b^2 - B*a^6*b^3 + 9*C
*a^5*b^4 - 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 + 3*C*a*b^8 - B*b^9)*d^3
)*tan(f*x + e)^2 + 2*(3*(C*a^7*b^2 + 3*C*a^5*b^4 + 3*C*a^3*b^6 + C*a*b^8)*c
*d^2 - (3*C*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 + 9*C*a^4*b^5 - 3
*B*a^3*b^6 + 3*C*a^2*b^7 - B*a*b^8)*d^3)*tan(f*x + e))*log(1/(tan(f*x + e)^
2 + 1)) + 2*((C*a^5*b^4 - 2*B*a^4*b^5 + 3*(A - C)*a^3*b^6 + 3*B*a^2*b^7 - (
3*A - 2*C)*a*b^8 - B*b^9)*c^3 + 3*(B*a^5*b^4 - (2*A - 3*C)*a^4*b^5 - 3*B*a^
3*b^6 + 3*(A - C)*a^2*b^7 + 2*B*a*b^8 - A*b^9)*c^2*d - 3*(C*a^7*b^2 - (A -
3*C)*a^5*b^4 - 3*B*a^4*b^5 + (3*A - 4*C)*a^3*b^6 + 3*B*a^2*b^7 - 2*A*a*b^8)
*c*d^2 + (3*C*a^8*b - B*a^7*b^2 + 6*C*a^6*b^3 - 3*B*a^5*b^4 + (3*A - 2*C)*a
^4*b^5 + 4*B*a^3*b^6 - (3*A - C)*a^2*b^7)*d^3 + 2*(((A - C)*a^4*b^5 + 3*B*a
^3*b^6 - 3*(A - C)*a^2*b^7 - B*a*b^8)*c^3 - 3*(B*a^4*b^5 - 3*(A - C)*a^3*b^
6 - 3*B*a^2*b^7 + (A - C)*a*b^8)*c^2*d - 3*((A - C)*a^4*b^5 + 3*B*a^3*b^6 -
 3*(A - C)*a^2*b^7 - B*a*b^8)*c*d^2 + (B*a^4*b^5 - 3*(A - C)*a^3*b^6 - 3*B*
a^2*b^7 + (A - C)*a*b^8)*d^3)*f*x)*tan(f*x + e))/((a^6*b^6 + 3*a^4*b^8 + 3*
a^2*b^10 + b^12)*f*tan(f*x + e)^2 + 2*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*b^9 + a*
b^11)*f*tan(f*x + e) + (a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x
+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2505 vs. 2(801) = 1602.
time = 1.50, size = 2505, normalized size = 3.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*d^3*tan(f*x + e)/b^3 + 2*(A*a^3*c^3 - C*a^3*c^3 + 3*B*a^2*b*c^3 -
3*A*a*b^2*c^3 + 3*C*a*b^2*c^3 - B*b^3*c^3 - 3*B*a^3*c^2*d + 9*A*a^2*b*c^2*d
 - 9*C*a^2*b*c^2*d + 9*B*a*b^2*c^2*d - 3*A*b^3*c^2*d + 3*C*b^3*c^2*d - 3*A*
a^3*c*d^2 + 3*C*a^3*c*d^2 - 9*B*a^2*b*c*d^2 + 9*A*a*b^2*c*d^2 - 9*C*a*b^2*c
*d^2 + 3*B*b^3*c*d^2 + B*a^3*d^3 - 3*A*a^2*b*d^3 + 3*C*a^2*b*d^3 - 3*B*a*b^
2*d^3 + A*b^3*d^3 - C*b^3*d^3)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (B*a^3*c^3 - 3*A*a^2*b*c^3 + 3*C*a^2*b*c^3 - 3*B*a*b^2*c^3 + A*b^3*c^3
- C*b^3*c^3 + 3*A*a^3*c^2*d - 3*C*a^3*c^2*d + 9*B*a^2*b*c^2*d - 9*A*a*b^2*c
^2*d + 9*C*a*b^2*c^2*d - 3*B*b^3*c^2*d - 3*B*a^3*c*d^2 + 9*A*a^2*b*c*d^2 -
9*C*a^2*b*c*d^2 + 9*B*a*b^2*c*d^2 - 3*A*b^3*c*d^2 + 3*C*b^3*c*d^2 - A*a^3*d
^3 + C*a^3*d^3 - 3*B*a^2*b*d^3 + 3*A*a*b^2*d^3 - 3*C*a*b^2*d^3 + B*b^3*d^3)
*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b^4
*c^3 - 3*A*a^2*b^5*c^3 + 3*C*a^2*b^5*c^3 - 3*B*a*b^6*c^3 + A*b^7*c^3 - C*b^
7*c^3 + 3*A*a^3*b^4*c^2*d - 3*C*a^3*b^4*c^2*d + 9*B*a^2*b^5*c^2*d - 9*A*a*b
^6*c^2*d + 9*C*a*b^6*c^2*d - 3*B*b^7*c^2*d - 3*C*a^6*b*c*d^2 - 9*C*a^4*b^3*
c*d^2 - 3*B*a^3*b^4*c*d^2 + 9*A*a^2*b^5*c*d^2 - 18*C*a^2*b^5*c*d^2 + 9*B*a*
b^6*c*d^2 - 3*A*b^7*c*d^2 + 3*C*a^7*d^3 - B*a^6*b*d^3 + 9*C*a^5*b^2*d^3 - 3
*B*a^4*b^3*d^3 - A*a^3*b^4*d^3 + 10*C*a^3*b^4*d^3 - 6*B*a^2*b^5*d^3 + 3*A*a
*b^6*d^3)*log(abs(b*tan(f*x + e) + a))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b
^10) + (3*B*a^3*b^6*c^3*tan(f*x + e)^2 - 9*A*a^2*b^7*c^3*tan(f*x + e)^2 + 9
*C*a^2*b^7*c^3*tan(f*x + e)^2 - 9*B*a*b^8*c^3*tan(f*x + e)^2 + 3*A*b^9*c^3*
tan(f*x + e)^2 - 3*C*b^9*c^3*tan(f*x + e)^2 + 9*A*a^3*b^6*c^2*d*tan(f*x + e
)^2 - 9*C*a^3*b^6*c^2*d*tan(f*x + e)^2 + 27*B*a^2*b^7*c^2*d*tan(f*x + e)^2
- 27*A*a*b^8*c^2*d*tan(f*x + e)^2 + 27*C*a*b^8*c^2*d*tan(f*x + e)^2 - 9*B*b
^9*c^2*d*tan(f*x + e)^2 - 9*C*a^6*b^3*c*d^2*tan(f*x + e)^2 - 27*C*a^4*b^5*c
*d^2*tan(f*x + e)^2 - 9*B*a^3*b^6*c*d^2*tan(f*x + e)^2 + 27*A*a^2*b^7*c*d^2
*tan(f*x + e)^2 - 54*C*a^2*b^7*c*d^2*tan(f*x + e)^2 + 27*B*a*b^8*c*d^2*tan(
f*x + e)^2 - 9*A*b^9*c*d^2*tan(f*x + e)^2 + 9*C*a^7*b^2*d^3*tan(f*x + e)^2
- 3*B*a^6*b^3*d^3*tan(f*x + e)^2 + 27*C*a^5*b^4*d^3*tan(f*x + e)^2 - 9*B*a^
4*b^5*d^3*tan(f*x + e)^2 - 3*A*a^3*b^6*d^3*tan(f*x + e)^2 + 30*C*a^3*b^6*d^
3*tan(f*x + e)^2 - 18*B*a^2*b^7*d^3*tan(f*x + e)^2 + 9*A*a*b^8*d^3*tan(f*x
+ e)^2 + 8*B*a^4*b^5*c^3*tan(f*x + e) - 22*A*a^3*b^6*c^3*tan(f*x + e) + 22*
C*a^3*b^6*c^3*tan(f*x + e) - 18*B*a^2*b^7*c^3*tan(f*x + e) + 2*A*a*b^8*c^3*
tan(f*x + e) - 2*C*a*b^8*c^3*tan(f*x + e) - 2*B*b^9*c^3*tan(f*x + e) - 6*C*
a^6*b^3*c^2*d*tan(f*x + e) + 24*A*a^4*b^5*c^2*d*tan(f*x + e) - 42*C*a^4*b^5
*c^2*d*tan(f*x + e) + 66*B*a^3*b^6*c^2*d*tan(f*x + e) - 54*A*a^2*b^7*c^2*d*
tan(f*x + e) + 36*C*a^2*b^7*c^2*d*tan(f*x + e) - 6*B*a*b^8*c^2*d*tan(f*x +
e) - 6*A*b^9*c^2*d*tan(f*x + e) - 6*C*a^7*b^2*c*d^2*tan(f*x + e) - 6*B*a^6*
b^3*c*d^2*tan(f*x + e) - 18*C*a^5*b^4*c*d^2*tan(f*x + e) - 42*B*a^4*b^5*c*d
^2*tan(f*x + e) + 66*A*a^3*b^6*c*d^2*tan(f*x + e) - 84*C*a^3*b^6*c*d^2*tan(
f*x + e) + 36*B*a^2*b^7*c*d^2*tan(f*x + e) - 6*A*a*b^8*c*d^2*tan(f*x + e) +
 12*C*a^8*b*d^3*tan(f*x + e) - 2*B*a^7*b^2*d^3*tan(f*x + e) - 2*A*a^6*b^3*d
^3*tan(f*x + e) + 38*C*a^6*b^3*d^3*tan(f*x + e) - 6*B*a^5*b^4*d^3*tan(f*x +
 e) - 14*A*a^4*b^5*d^3*tan(f*x + e) + 50*C*a^4*b^5*d^3*tan(f*x + e) - 28*B*
a^3*b^6*d^3*tan(f*x + e) + 12*A*a^2*b^7*d^3*tan(f*x + e) - C*a^6*b^3*c^3 +
```

```
6*B*a^5*b^4*c^3 - 14*A*a^4*b^5*c^3 + 11*C*a^4*b^5*c^3 - 7*B*a^3*b^6*c^3 - 3
*A*a^2*b^7*c^3 - B*a*b^8*c^3 - A*b^9*c^3 - 3*C*a^7*b^2*c^2*d - 3*B*a^6*b^3*
c^2*d + 18*A*a^5*b^4*c^2*d - 27*C*a^5*b^4*c^2*d + 33*B*a^4*b^5*c^2*d - 21*A
*a^3*b^6*c^2*d + 12*C*a^3*b^6*c^2*d - 3*A*a*b^8*c^2*d - 3*B*a^7*b^2*c*d^2 -
 3*A*a^6*b^3*c*d^2 + 3*C*a^6*b^3*c*d^2 - 27*B*a^5*b^4*c*d^2 + 33*A*a^4*b^5*
c*d^2 - 33*C*a^4*b^5*c*d^2 + 12*B*a^3*b^6*c*d^2 + 4*C*a^9*d^3 - A*a^7*b^2*d
^3 + 13*C*a^7*b^2*d^3 + B*a^6*b^3*d^3 - 9*A*a^5*b^4*d^3 + 21*C*a^5*b^4*d^3
- 11*B*a^4*b^5*d^3 + 4*A*a^3*b^6*d^3)/((a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b
^10)*(b*tan(f*x + e) + a)^2))/f
```

Mupad [B]
time = 19.24, size = 1172, normalized size = 1.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b
*tan(e + f*x))^3,x)
```

```
[Out] (log(tan(e + f*x) + 1i)*(A*c^3 + A*d^3*1i - B*c^3*1i + B*d^3 - C*c^3 - C*d^
3*1i - 3*A*c*d^2 - A*c^2*d*3i + B*c*d^2*3i - 3*B*c^2*d + 3*C*c*d^2 + C*c^2*
d*3i))/(2*f*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) - ((tan(e + f*x)*(B*b^6*c^
3 + 3*C*a^6*d^3 + 2*A*a*b^5*c^3 - 2*B*a^5*b*d^3 - 2*C*a*b^5*c^3 + 3*A*b^6*c
^2*d + 3*A*a^2*b^4*d^3 + A*a^4*b^2*d^3 - B*a^2*b^4*c^3 - 4*B*a^3*b^3*d^3 +
5*C*a^4*b^2*d^3 - 3*A*a^2*b^4*c^2*d + 9*B*a^2*b^4*c*d^2 + 3*B*a^4*b^2*c*d^2
 + 9*C*a^2*b^4*c^2*d - 12*C*a^3*b^3*c*d^2 + 3*C*a^4*b^2*c^2*d - 6*A*a*b^5*c
*d^2 - 6*B*a*b^5*c^2*d - 6*C*a^5*b*c*d^2))/(a^4 + b^4 + 2*a^2*b^2) + (A*b^7
*c^3 + 5*C*a^7*d^3 + B*a*b^6*c^3 - 3*B*a^6*b*d^3 + 5*A*a^2*b^5*c^3 + 5*A*a^
3*b^4*d^3 + A*a^5*b^2*d^3 - 3*B*a^3*b^4*c^3 - 7*B*a^4*b^3*d^3 - 3*C*a^2*b^5
*c^3 + C*a^4*b^3*c^3 + 9*C*a^5*b^2*d^3 - 9*A*a^2*b^5*c*d^2 - 9*A*a^3*b^4*c^
2*d + 3*A*a^4*b^3*c*d^2 - 9*B*a^2*b^5*c^2*d + 15*B*a^3*b^4*c*d^2 + 3*B*a^4*
b^3*c^2*d + 3*B*a^5*b^2*c*d^2 + 15*C*a^3*b^4*c^2*d - 21*C*a^4*b^3*c*d^2 + 3
*C*a^5*b^2*c^2*d + 3*A*a*b^6*c^2*d - 9*C*a^6*b*c*d^2)/(2*b*(a^4 + b^4 + 2*a
^2*b^2)))/(f*(a^2*b^3 + b^5*tan(e + f*x)^2 + 2*a*b^4*tan(e + f*x))) + (log(
a + b*tan(e + f*x))*(b^3*(3*B*a^4*d^3 + 9*C*a^4*c*d^2) - b^6*(3*A*a*d^3 - 3
*B*a*c^3 - 9*A*a*c^2*d + 9*B*a*c*d^2 + 9*C*a*c^2*d) + b^5*(3*A*a^2*c^3 + 6*
B*a^2*d^3 - 3*C*a^2*c^3 - 9*A*a^2*c*d^2 - 9*B*a^2*c^2*d + 18*C*a^2*c*d^2) +
 b^4*(A*a^3*d^3 - B*a^3*c^3 - 10*C*a^3*d^3 - 3*A*a^3*c^2*d + 3*B*a^3*c*d^2
+ 3*C*a^3*c^2*d) + b*(B*a^6*d^3 + 3*C*a^6*c*d^2) + b^7*(C*c^3 - A*c^3 + 3*A
*c*d^2 + 3*B*c^2*d) - 3*C*a^7*d^3 - 9*C*a^5*b^2*d^3))/(f*(b^10 + 3*a^2*b^8
+ 3*a^4*b^6 + a^6*b^4)) + (log(tan(e + f*x) - 1i)*(A*c^3*1i + A*d^3 - B*c^3
 + B*d^3*1i - C*c^3*1i - C*d^3 - A*c*d^2*3i - 3*A*c^2*d + 3*B*c*d^2 - B*c^2
*d*3i + C*c*d^2*3i + 3*C*c^2*d))/(2*f*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i))
+ (C*d^3*tan(e + f*x))/(b^3*f)
```

3.70 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=337

$$\frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)-3a^2b(Bc-(A-C)d)+b^3(Bc-(A-C)d))\,x}{c^2+d^2}-\frac{(3a^2b(Ac-$$

```
[Out] (a^3*(A*c+B*d-C*c)-3*a*b^2*(A*c+B*d-C*c)-3*a^2*b*(B*c-(A-C)*d)+b^3*(B*c-(A-
C)*d))*x/(c^2+d^2)-(3*a^2*b*(A*c+B*d-C*c)-b^3*(A*c+B*d-C*c)+a^3*(B*c-(A-C)*
d)-3*a*b^2*(B*c-(A-C)*d))*ln(cos(f*x+e))/(c^2+d^2)/f-(-a*d+b*c)^3*(A*d^2-B*
c*d+C*c^2)*ln(c+d*tan(f*x+e))/d^4/(c^2+d^2)/f+b*(b*(A*b+B*a-C*b)*d^2+(-a*d+
b*c)*(-B*b*d-C*a*d+C*b*c))*tan(f*x+e)/d^3/f-1/2*(-B*b*d-C*a*d+C*b*c)*(a+b*t
an(f*x+e))^2/d^2/f+1/3*C*(a+b*tan(f*x+e))^3/d/f
```

Rubi [A]

time = 1.01, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3707, 3698, 31, 3556}

$$-\frac{\log(\cos(e+fx))\left(a^3(Bc-d(A-C))+3a^2b(Ac+Bd-cC)-3ab^2(Bc-d(A-C))-b^3(Ac+Bd-cC)\right)}{f\left(c^2+d^2\right)}+\frac{x\left(a^3(Ac+Bd-cC)-3a^2b(Bc-d(A-C))-3ab^2(Ac+Bd-cC)+b^3(Bc-d(A-C))\right)}{c^2+d^2}-\frac{(bc-ad)^3\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d^4f\left(c^2+d^2\right)}+\frac{b\tan(e+fx)\left(bd^2(aB+Ab-bC)+(bc-ad)(-aCd-bBd+bcC)\right)}{d^3f}-\frac{(-aCd-bBd+bcC)(a+b\tan(e+fx))^2}{2d^2f}+\frac{C(a+b\tan(e+fx))^3}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x]),x]
```

```
[Out] ((a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c - c*C + B*d) - 3*a^2*b*(B*c - (A - C
)*d) + b^3*(B*c - (A - C)*d))*x)/(c^2 + d^2) - ((3*a^2*b*(A*c - c*C + B*d)
- b^3*(A*c - c*C + B*d) + a^3*(B*c - (A - C)*d) - 3*a*b^2*(B*c - (A - C)*d)
)*Log[Cos[e + f*x]])/((c^2 + d^2)*f) - ((b*c - a*d)^3*(c^2*C - B*c*d + A*d^
2)*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)*f) + (b*(b*(A*b + a*B - b*C)*d
^2 + (b*c - a*d)*(b*c*C - b*B*d - a*C*d))*Tan[e + f*x])/(d^3*f) - ((b*c*C -
 b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2)/(2*d^2*f) + (C*(a + b*Tan[e + f*x])
^3)/(3*d*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = \frac{C(a+b\tan(e+fx))^3}{3df} + \frac{\int \frac{(a+b\tan(e+f}{}}{}$$

$$= -\frac{(bcC-bBd-aCd)(a+b\tan(e+f}{2d^2f}$$

$$= \frac{b(b(Ab+aB-bC)d^2+(bc-ad)(bcC}{d^3f}$$

$$= \frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+}{}$$

$$= \frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+}{}$$

$$= \frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.86, size = 258, normalized size = 0.77

$$\frac{\frac{3(a+ib)^3(-iA+B+iC)d^2\log(i-\tan(e+fx))}{c+id}+\frac{3(a-ib)^3(iA+B-iC)d^2\log(i+\tan(e+fx))}{c-id}+\frac{6(-bc+ad)^3(c^2C-Bcd+Ad^2)\log(c+d\tan(e+fx))}{d^2(c^2+d^2)}+6b^2(Ab+aB-bC)d\tan(e+fx)-\frac{6b(bc-ad)(-bcC+bBd+aCd)\tan(e+fx)}{d}-3(bcC-bBd-aCd)(a+b\tan(e+fx))^2+2Cd(a+b\tan(e+fx))^3}{6d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x]),x]
```

```
[Out] ((3*(a + I*b)^3*((-I)*A + B + I*C)*d^2*Log[I - Tan[e + f*x]])/(c + I*d) + (
3*(a - I*b)^3*(I*A + B - I*C)*d^2*Log[I + Tan[e + f*x]])/(c - I*d) + (6*(-(
b*c) + a*d)^3*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 +
d^2)) + 6*b^2*(A*b + a*B - b*C)*d*Tan[e + f*x] - (6*b*(b*c - a*d)*(-(b*c*C)
 + b*B*d + a*C*d)*Tan[e + f*x])/d - 3*(b*c*C - b*B*d - a*C*d)*(a + b*Tan[e
+ f*x])^2 + 2*C*d*(a + b*Tan[e + f*x])^3)/(6*d^2*f)
```

Maple [A]
time = 0.43, size = 542, normalized size = 1.61

method	result
norman	$\frac{\left(A a^3 c+3A a^2 bd-3Aa b^2 c-A b^3 d+B a^3 d-3B a^2 bc-3Ba b^2 d+B b^3 c-C a^3 c-3C a^2 bd+3Ca b^2 c+C b^3 d\right)x}{c^2+d^2}+\frac{(A b^2 d^2+}{}$
derivativedivides	$\frac{\frac{b\left(\frac{C b^2 d^2\left(\tan^3(fx+e)\right)}{3}+\frac{B b^2 d^2\left(\tan^2(fx+e)\right)}{2}+\frac{3Cab d^2\left(\tan^2(fx+e)\right)}{2}-\frac{C b^2 cd\left(\tan^2(fx+e)\right)}{2}+A b^2 d^2\tan(fx+e)+3Bab d^2\tan(f\right.}{d^3}}{}$

default	$\frac{b\left(\frac{C\,b^{2} d^{2}\left(\tan^{3}(fx+e)\right)}{3}+\frac{B\,b^{2} d^{2}\left(\tan^{2}(fx+e)\right)}{2}+\frac{3Cab\,d^{2}\left(\tan^{2}(fx+e)\right)}{2}-\frac{C\,b^{2} cd\left(\tan^{2}(fx+e)\right)}{2}+A\,b^{2} d^{2}\tan(fx+e)+3Bab\,d^{2}\tan(fx-\right.}{d^{3}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(b/d^3*(1/3*C*b^2*d^2*tan(f*x+e)^3+1/2*B*b^2*d^2*tan(f*x+e)^2+3/2*C*a*b
*d^2*tan(f*x+e)^2-1/2*C*b^2*c*d*tan(f*x+e)^2+A*b^2*d^2*tan(f*x+e)+3*B*a*b*d
^2*tan(f*x+e)-B*b^2*c*d*tan(f*x+e)+3*a^2*C*d^2*tan(f*x+e)-3*C*a*b*c*d*tan(f
*x+e)+C*b^2*c^2*tan(f*x+e)-C*b^2*d^2*tan(f*x+e))+1/d^4*(A*a^3*d^5-3*A*a^2*b
*c*d^4+3*A*a*b^2*c^2*d^3-A*b^3*c^3*d^2-B*a^3*c*d^4+3*B*a^2*b*c^2*d^3-3*B*a*
b^2*c^3*d^2+B*b^3*c^4*d+C*a^3*c^2*d^3-3*C*a^2*b*c^3*d^2+3*C*a*b^2*c^4*d-C*b
^3*c^5)/(c^2+d^2)*ln(c+d*tan(f*x+e))+1/(c^2+d^2)*(1/2*(-A*a^3*d+3*A*a^2*b*c
+3*A*a*b^2*d-A*b^3*c+B*a^3*c+3*B*a^2*b*d-3*B*a*b^2*c-B*b^3*d+C*a^3*d-3*C*a^
2*b*c-3*C*a*b^2*d+C*b^3*c)*ln(1+tan(f*x+e)^2)+(A*a^3*c+3*A*a^2*b*d-3*A*a*b^
2*c-A*b^3*d+B*a^3*d-3*B*a^2*b*c-3*B*a*b^2*d+B*b^3*c-C*a^3*c-3*C*a^2*b*d+3*C
*a*b^2*c+C*b^3*d)*arctan(tan(f*x+e))))
```

Maxima [A]

time = 0.56, size = 451, normalized size = 1.34

$$\frac{\frac{6\left(\left(\left((A-C)a^{3}-3\,Ba^{2}b-3\,(A-C)ab^{2}+Bb^{3}\right)c+\left(Ba^{3}+3\,(A-C)a^{2}b-3\,Bab^{2}-(A-C)b^{3}\right)d\right)(fx+e)}{c^{2}+d^{2}}-\frac{6\left(Cb^{3}c^{5}-Aa^{3}d^{5}-\left(3\,Cab^{2}+Bb^{3}\right)c^{4}d+\left(3\,Ca^{2}b+3\,Bab^{2}+Ab^{3}\right)c^{3}d^{2}-\left(Ca^{3}+3\,Ba^{2}b+3\,Aab^{2}\right)c^{2}d^{3}+\left(Ba^{3}+3\,Aa^{2}b\right)cd^{4}\right)\log\left(d\tan(fx+e)+c\right)}{c^{2}d^{4}+d^{6}}+\frac{3\left(\left(Ba^{3}+3\,(A-C)a^{2}b-3\,Bab^{2}-(A-C)b^{3}\right)c-\left((A-C)a^{3}-3\,Ba^{2}b-3\,(A-C)ab^{2}+Bb^{3}\right)d\right)\log\left(\tan(fx+e)^{2}+1\right)}{c^{2}+d^{2}}+\frac{2\,Cb^{3}d^{2}\tan(fx+e)^{3}-3\left(Cb^{3}cd-\left(3\,Cab^{2}+Bb^{3}\right)d^{2}\right)\tan(fx+e)^{2}+6\left(Cb^{3}c^{2}-\left(3\,Cab^{2}+Bb^{3}\right)cd+\left(3\,Ca^{2}b+3\,Bab^{2}+(A-C)b^{3}\right)d^{2}\right)\tan(fx+e)}{d^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c + (B*a^3 + 3*
(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*(f*x + e)/(c^2 + d^2) - 6*(C*b^
3*c^5 - A*a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 + A*
b^3)*c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (B*a^3 + 3*A*a^2*b
)*c*d^4)*log(d*tan(f*x + e) + c)/(c^2*d^4 + d^6) + 3*((B*a^3 + 3*(A - C)*a^
2*b - 3*B*a*b^2 - (A - C)*b^3)*c - ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b
^2 + B*b^3)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) + (2*C*b^3*d^2*tan(f*x +
 e)^3 - 3*(C*b^3*c*d - (3*C*a*b^2 + B*b^3)*d^2)*tan(f*x + e)^2 + 6*(C*b^3*c
^2 - (3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*d^2)*t
an(f*x + e))/d^3)/f
```

Fricas [A]

time = 4.20, size = 634, normalized size = 1.88

$$2\left(Cb^{3}c^{2}d^{3}+Cb^{3}d^{5}\right)\tan\left(fx+e\right)^{3}+6\left(\left(\left((A-C)a^{3}-3\,Ba^{2}b-3\,(A-C)ab^{2}+Bb^{3}\right)cd^{4}+\left(Ba^{3}+3\,(A-C)a^{2}b-3\,Bab^{2}-(A-C)b^{3}\right)d^{5}\right)fx-3\left(Cb^{3}c^{3}d^{2}+Cb^{3}cd^{4}-\left(3\,Cab^{2}+Bb^{3}\right)c^{2}d^{3}-\left(3\,Cab^{2}+Bb^{3}\right)d^{5}\right)\tan\left(fx+e\right)^{2}-3\left(Cb^{3}c^{5}-Aa^{3}d^{5}-\left(3\,Cab^{2}\right.\right.$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(C*b^3*c^2*d^3 + C*b^3*d^5)*tan(f*x + e)^3 + 6*(((A - C)*a^3 - 3*B*a
^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^4 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^
2 - (A - C)*b^3)*d^5)*f*x - 3*(C*b^3*c^3*d^2 + C*b^3*c*d^4 - (3*C*a*b^2 + B
*b^3)*c^2*d^3 - (3*C*a*b^2 + B*b^3)*d^5)*tan(f*x + e)^2 - 3*(C*b^3*c^5 - A*
a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d
^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (B*a^3 + 3*A*a^2*b)*c*d^4)*l
og((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) +
3*(C*b^3*c^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*
c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (3*C*a^2*b + 3*B*a*b^2
+ (A - C)*b^3)*c*d^4 - (C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*d^5)*l
og(1/(tan(f*x + e)^2 + 1)) + 6*(C*b^3*c^4*d - (3*C*a*b^2 + B*b^3)*c^3*d^2 +
 (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 - (3*C*a*b^2 + B*b^3)*c*d^4 + (3*C
*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*d^5)*tan(f*x + e))/((c^2*d^4 + d^6)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 25.85, size = 7096, normalized size = 21.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(
c, 0) & Eq(d, 0) & Eq(f, 0)), (3*I*A*a**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f
*x) - 6*I*d*f) + 3*A*a**3*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) + 3*I*A*a**3/(
6*d*f*tan(e + f*x) - 6*I*d*f) + 9*A*a**2*b*f*x*tan(e + f*x)/(6*d*f*tan(e +
f*x) - 6*I*d*f) - 9*I*A*a**2*b*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*A*a**
2*b/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*I*A*a*b**2*f*x*tan(e + f*x)/(6*d*f*t
an(e + f*x) - 6*I*d*f) + 9*A*a*b**2*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*
A*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*
f) - 9*I*A*a*b**2*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) - 6*I*d*f) -
 9*I*A*a*b**2/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*A*b**3*f*x*tan(e + f*x)/(6
*d*f*tan(e + f*x) - 6*I*d*f) + 9*I*A*b**3*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f
) + 3*I*A*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) -
6*I*d*f) + 3*A*b**3*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) - 6*I*d*f)
 + 6*A*b**3*tan(e + f*x)**2/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*A*b**3/(6*d*
f*tan(e + f*x) - 6*I*d*f) + 3*B*a**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) -
 6*I*d*f) - 3*I*B*a**3*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) - 3*B*a**3/(6*d*f
*tan(e + f*x) - 6*I*d*f) + 9*I*B*a**2*b*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x
) - 6*I*d*f) + 9*B*a**2*b*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*B*a**2*b*l
```

og(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*I*B
*a**2*b*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*I*B*a**
2*b/(6*d*f*tan(e + f*x) - 6*I*d*f) - 27*B*a*b**2*f*x*tan(e + f*x)/(6*d*f*ta
n(e + f*x) - 6*I*d*f) + 27*I*B*a*b**2*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) +
9*I*B*a*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*
I*d*f) + 9*B*a*b**2*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) - 6*I*d*f)
 + 18*B*a*b**2*tan(e + f*x)**2/(6*d*f*tan(e + f*x) - 6*I*d*f) + 27*B*a*b**2
/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*I*B*b**3*f*x*tan(e + f*x)/(6*d*f*tan(e
+ f*x) - 6*I*d*f) - 9*B*b**3*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) - 6*B*b**3*
log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 6*I*
B*b**3*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 3*B*b**3*t
an(e + f*x)**3/(6*d*f*tan(e + f*x) - 6*I*d*f) + 3*I*B*b**3*tan(e + f*x)**2/
(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*I*B*b**3/(6*d*f*tan(e + f*x) - 6*I*d*f)
+ 3*I*C*a**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 3*C*a**3*f*x
/(6*d*f*tan(e + f*x) - 6*I*d*f) + 3*C*a**3*log(tan(e + f*x)**2 + 1)*tan(e +
 f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) - 3*I*C*a**3*log(tan(e + f*x)**2 + 1)/
(6*d*f*tan(e + f*x) - 6*I*d*f) - 3*I*C*a**3/(6*d*f*tan(e + f*x) - 6*I*d*f)
- 27*C*a**2*b*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 27*I*C*a**2
*b*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*I*C*a**2*b*log(tan(e + f*x)**2 +
1)*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*C*a**2*b*log(tan(e + f*x
)**2 + 1)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 18*C*a**2*b*tan(e + f*x)**2/(6*d
*f*tan(e + f*x) - 6*I*d*f) + 27*C*a**2*b/(6*d*f*tan(e + f*x) - 6*I*d*f) - 2
7*I*C*a*b**2*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) - 27*C*a*b**2*
f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) - 18*C*a*b**2*log(tan(e + f*x)**2 + 1)*t
an(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 18*I*C*a*b**2*log(tan(e + f*x)
2 + 1)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*C*a*b2*tan(e + f*x)**3/(6*d*f
*tan(e + f*x) - 6*I*d*f) + 9*I*C*a*b**2*tan(e + f*x)**2/(6*d*f*tan(e + f*x)
 - 6*I*d*f) + 27*I*C*a*b**2/(6*d*f*tan(e + f*x) - 6*I*d*f) + 15*C*b**3*f*x*
tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) - 15*I*C*b**3*f*x/(6*d*f*tan(e
+ f*x) - 6*I*d*f) - 6*I*C*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f
*tan(e + f*x) - 6*I*d*f) - 6*C*b**3*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e +
 f*x) - 6*I*d*f) + 2*C*b**3*tan(e + f*x)**4/(6*d*f*tan(e + f*x) - 6*I*d*f)
+ I*C*b**3*tan(e + f*x)**3/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*C*b**3*tan(e
+ f*x)**2/(6*d*f*tan(e + f*x) - 6*I*d*f) - 15*C*b**3/(6*d*f*tan(e + f*x) -
6*I*d*f), Eq(c, -I*d)), (-3*I*A*a**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) +
 6*I*d*f) + 3*A*a**3*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 3*I*A*a**3/(6*d*f
*tan(e + f*x) + 6*I*d*f) + 9*A*a**2*b*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x)
+ 6*I*d*f) + 9*I*A*a**2*b*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 9*A*a**2*b/(
6*d*f*tan(e + f*x) + 6*I*d*f) - 9*I*A*a*b**2*f*x*tan(e + f*x)/(6*d*f*tan(e
+ f*x) + 6*I*d*f) + 9*A*a*b**2*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*A*a*b
2*log(tan(e + f*x)2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d*f) +
9*I*A*a*b**2*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*I*
A*a*b**2/(6*d*f*tan(e + f*x) + 6*I*d*f) - 9*A*b**3*f*x*tan(e + f*x)/(6*d*f*
tan(e + f*x) + 6*I*d*f) - 9*I*A*b**3*f*x/(6*d*f*tan(e + f*x) + 6*I*d*f) - 3
*I*A*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*d

```
*f) + 3*A*b**3*log(tan(e + f*x)**2 + 1)/(6*d*f*tan(e + f*x) + 6*I*d*f) + 6*
A*b**3*tan(e + f*x)**2/(6*d*f*tan(e + f*x) + 6*I*d*f) + 9*A*b**3/(6*d*f*tan
(e + f*x) + 6*I*d*f) + 3*B*a**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) + 6*I*
d*f) + 3*I*B*a**3*f*x/(6*d*f*tan(e + f*x) + 6*I...
```

Giac [A]
time = 1.12, size = 573, normalized size = 1.70

$$\frac{\frac{6\left(Aa^3c - Ca^3c - 3Ba^2bc - 3Aab^2c + 3Cab^2c + Bb^3c + Ba^3d + 3Aa^2bd - 3Ca^2bd - 3Bab^2d - Ab^3d + Cb^3d\right)(fx+e)}{c^2+d^2} + \frac{3\left(Ba^3c + 3Aa^2bc - 3Ca^2bc - 3Bab^2c - Ab^3c + Cb^3c - Aa^3d + Ca^3d + 3Ba^2bd + 3Aab^2d - 3Cab^2d - Bb^3d\right)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2} - \frac{6\left(Cb^3c^5 - 3Cab^2c^4d - Bb^3c^4d + 3Ca^2bc^3d^2 + 3Bab^2c^3d^2 + Ab^3c^3d^2 - Ca^3c^2d^3 - 3Ba^2bc^2d^3 - 3Aab^2c^2d^3 + Ba^3cd^4 + 3Aa^2bcd^4 - Aa^3d^5\right)\log\left(|d\tan(fx+e)+c|\right)}{c^2d^4+d^6} + \frac{2Cb^3d^2\tan(fx+e)^3 - 3Cb^3cd\tan(fx+e)^2 + 9Cab^2d^2\tan(fx+e)^2 + 3Bb^3d^2\tan(fx+e)^2 + 6Cb^3c^2\tan(fx+e) - 18Cab^2cd\tan(fx+e) - 6Bb^3cd\tan(fx+e) + 18Ca^2bd^2\tan(fx+e) + 18Bab^2d^2\tan(fx+e) + 6Ab^3d^2\tan(fx+e) - 6Cb^3d^2\tan(fx+e)}{d^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(A*a^3*c - C*a^3*c - 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c + B*b^3
*c + B*a^3*d + 3*A*a^2*b*d - 3*C*a^2*b*d - 3*B*a*b^2*d - A*b^3*d + C*b^3*d)
*(f*x + e)/(c^2 + d^2) + 3*(B*a^3*c + 3*A*a^2*b*c - 3*C*a^2*b*c - 3*B*a*b^2
*c - A*b^3*c + C*b^3*c - A*a^3*d + C*a^3*d + 3*B*a^2*b*d + 3*A*a*b^2*d - 3*
C*a*b^2*d - B*b^3*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) - 6*(C*b^3*c^5 - 3
*C*a*b^2*c^4*d - B*b^3*c^4*d + 3*C*a^2*b*c^3*d^2 + 3*B*a*b^2*c^3*d^2 + A*b^
3*c^3*d^2 - C*a^3*c^2*d^3 - 3*B*a^2*b*c^2*d^3 - 3*A*a*b^2*c^2*d^3 + B*a^3*c
*d^4 + 3*A*a^2*b*c*d^4 - A*a^3*d^5)*log(abs(d*tan(f*x + e) + c))/(c^2*d^4 +
 d^6) + (2*C*b^3*d^2*tan(f*x + e)^3 - 3*C*b^3*c*d*tan(f*x + e)^2 + 9*C*a*b^
2*d^2*tan(f*x + e)^2 + 3*B*b^3*d^2*tan(f*x + e)^2 + 6*C*b^3*c^2*tan(f*x + e
) - 18*C*a*b^2*c*d*tan(f*x + e) - 6*B*b^3*c*d*tan(f*x + e) + 18*C*a^2*b*d^2
*tan(f*x + e) + 18*B*a*b^2*d^2*tan(f*x + e) + 6*A*b^3*d^2*tan(f*x + e) - 6*
C*b^3*d^2*tan(f*x + e))/d^3)/f
```

Mupad [B]
time = 13.39, size = 508, normalized size = 1.51

$$\frac{\tan(e+fx)^2\left(\frac{Bb^3+3Cab^2}{2d}-\frac{Cb^3c}{2d^2}\right)}{f}-\frac{\tan(e+fx)\left(\frac{c\left(\frac{Bb^3+3Cab^2}{d}-\frac{Cb^3c}{d^2}\right)}{d}-\frac{Ab^3+3Bab^2+3Ca^2b}{d}+\frac{Cb^3}{d}\right)}{f}-\frac{\ln\left(c+d\tan(e+fx)\right)\left(d^4\left(Ba^3c+3Aa^2bc\right)-d^3\left(Ca^3c^2+3Aab^2c^2+3Ba^2bc^2\right)+d^2\left(Ab^3c^3+3Bab^2c^3+3Ca^2bc^3\right)-d\left(Bb^3c^4+3Cab^2c^4\right)-Aa^3d^5+Cb^3c^5\right)}{f\left(c^2d^4+d^6\right)}-\frac{\ln\left(\tan(e+fx)+1i\right)\left(Aa^3+Ab^3\,1i-Ba^3\,1i+Bb^3-Ca^3-Cb^3\,1i-3Aab^2-Aa^2b\,3i+Bab^2\,3i-3Ba^2b+3Cab^2+Ca^2b\,3i\right)}{2f\left(c\,1i+d\right)}-\frac{\ln\left(\tan(e+fx)-1i\right)\left(Aa^3\,1i+Ab^3-Ba^3+Bb^3\,1i-Ca^3\,1i-Cb^3-Aab^2\,3i-3Aa^2b+3Bab^2-Ba^2b\,3i+Cab^2\,3i+3Ca^2b\right)}{2f\left(c+d\,1i\right)}+\frac{Cb^3\tan(e+fx)^3}{3df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x)),x)
```

```
[Out] (tan(e + f*x)^2*((B*b^3 + 3*C*a*b^2)/(2*d) - (C*b^3*c)/(2*d^2)))/f - (tan(e
 + f*x)*((c*((B*b^3 + 3*C*a*b^2)/d - (C*b^3*c)/d^2))/d - (A*b^3 + 3*B*a*b^2
 + 3*C*a^2*b)/d + (C*b^3)/d))/f - (log(c + d*tan(e + f*x))*(d^4*(B*a^3*c +
3*A*a^2*b*c) - d^3*(C*a^3*c^2 + 3*A*a*b^2*c^2 + 3*B*a^2*b*c^2) + d^2*(A*b^3
*c^3 + 3*B*a*b^2*c^3 + 3*C*a^2*b*c^3) - d*(B*b^3*c^4 + 3*C*a*b^2*c^4) - A*a
^3*d^5 + C*b^3*c^5))/(f*(d^6 + c^2*d^4)) - (log(tan(e + f*x) + 1i)*(A*a^3 +
 A*b^3*1i - B*a^3*1i + B*b^3 - C*a^3 - C*b^3*1i - 3*A*a*b^2 - A*a^2*b*3i +
B*a*b^2*3i - 3*B*a^2*b + 3*C*a*b^2 + C*a^2*b*3i))/(2*f*(c*1i + d)) - (log(t
an(e + f*x) - 1i)*(A*a^3*1i + A*b^3 - B*a^3 + B*b^3*1i - C*a^3*1i - C*b^3 -
 A*a*b^2*3i - 3*A*a^2*b + 3*B*a*b^2 - B*a^2*b*3i + C*a*b^2*3i + 3*C*a^2*b))
/(2*f*(c + d*1i)) + (C*b^3*tan(e + f*x)^3)/(3*d*f)
```

3.71 $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=236

$$\frac{(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)-2ab(Bc-(A-C)d))\,x}{c^2+d^2}-\frac{(2ab(Ac-cC+Bd)+a^2(Bc-(A-C}{(c^2}$$

[Out] (a^2*(A*c+B*d-C*c)-b^2*(A*c+B*d-C*c)-2*a*b*(B*c-(A-C)*d))*x/(c^2+d^2)-(2*a*b*(A*c+B*d-C*c)+a^2*(B*c-(A-C)*d)-b^2*(B*c-(A-C)*d))*ln(cos(f*x+e))/(c^2+d^2)/f+(-a*d+b*c)^2*(A*d^2-B*c*d+C*c^2)*ln(c+d*tan(f*x+e))/d^3/(c^2+d^2)/f-b*(-B*b*d-C*a*d+C*b*c)*tan(f*x+e)/d^2/f+1/2*C*(a+b*tan(f*x+e))^2/d/f

Rubi [A]
time = 0.52, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3707, 3698, 31, 3556}

$$-\frac{\log(\cos(e+fx))\left(a^2(Bc-d(A-C))+2ab(Ac+Bd-cC)-b^2(Bc-d(A-C))\right)}{f\left(c^2+d^2\right)}+\frac{x\left(a^2(Ac+Bd-cC)-2ab(Bc-d(A-C))-b^2(Ac+Bd-cC)\right)}{c^2+d^2}+\frac{(bc-ad)^2\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d^3f\left(c^2+d^2\right)}-\frac{b\tan(e+fx)(-aCd-bBd+bcC)}{d^2f}+\frac{C(a+b\tan(e+fx))^2}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((a^2*(A*c - c*C + B*d) - b^2*(A*c - c*C + B*d) - 2*a*b*(B*c - (A - C)*d))*x)/(c^2 + d^2) - ((2*a*b*(A*c - c*C + B*d) + a^2*(B*c - (A - C)*d) - b^2*(B*c - (A - C)*d))*Log[Cos[e + f*x]])/((c^2 + d^2)*f) + ((b*c - a*d)^2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)*f) - (b*(b*c*C - b*B*d - a*C*d)*Tan[e + f*x])/(d^2*f) + (C*(a + b*Tan[e + f*x])^2)/(2*d*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx = \frac{C(a+b\tan(e+fx))^2}{2df} + \frac{\int \frac{(a+b\tan(e+fx)}{}}{}$$

$$= -\frac{b(bcC-bBd-aCd)\tan(e+fx)}{d^2f} + \frac{C}{}$$

$$= \frac{(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd}{c^2+d^2}$$

$$= \frac{(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd}{c^2+d^2}$$

$$= \frac{(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd}{c^2+d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.92, size = 190, normalized size = 0.81

$$\frac{\frac{(a+ib)^2(-iA+B+iC)d\log(i-\tan(e+fx))}{c+id}+\frac{(a-ib)^2(iA+B-iC)d\log(i+\tan(e+fx))}{c-id}+\frac{2(bc-ad)^2(c^2C-Bcd+Ad^2)\log(c+d\tan(e+fx))}{d^2(c^2+d^2)}+\frac{2b(-bcC+bBd+aCd)\tan(e+fx)}{d}+C(a+b\tan(e+fx))^2}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x]),x]
```

```
[Out] (((a + I*b)^2*((-I)*A + B + I*C)*d*Log[I - Tan[e + f*x]])/(c + I*d) + ((a -
 I*b)^2*(I*A + B - I*C)*d*Log[I + Tan[e + f*x]])/(c - I*d) + (2*(b*c - a*d)
^2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)) + (2*
b*(-(b*c*C) + b*B*d + a*C*d)*Tan[e + f*x])/d + C*(a + b*Tan[e + f*x])^2)/(2
*d*f)
```

Maple [A]
time = 0.25, size = 317, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{b\left(\frac{Cbd\left(\tan^2(fx+e)\right)}{2}+b\tan(fx+e)Bd+2\tan(fx+e)Cad-Cbc\tan(fx+e)\right)}{d^2}+\frac{\left(A\,a^2d^4-2Aabc\,d^3+A\,b^2c^2d^2-B\,a^2c\,d^3+2Bab\,c^2d^2-B\right.}{d^3\left(c^2\right.}}{}$
default	$\frac{\frac{b\left(\frac{Cbd\left(\tan^2(fx+e)\right)}{2}+b\tan(fx+e)Bd+2\tan(fx+e)Cad-Cbc\tan(fx+e)\right)}{d^2}+\frac{\left(A\,a^2d^4-2Aabc\,d^3+A\,b^2c^2d^2-B\,a^2c\,d^3+2Bab\,c^2d^2-B\right.}{d^3\left(c^2\right.}}{}$
norman	$\frac{(A\,a^2c+2Aabd-A\,b^2c+B\,a^2d-2Babc-B\,b^2d-C\,a^2c-2Cabd+C\,b^2c)x}{c^2+d^2}+\frac{b(Bbd+2aCd-Cbc)\tan(fx+e)}{d^2f}+\frac{b^2C(\tan^2}{2a}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(b/d^2*(1/2*C*d*tan(f*x+e)^2*b+b*tan(f*x+e)*B*d+2*tan(f*x+e)*C*a*d-C*b*
c*tan(f*x+e))+1/d^3*(A*a^2*d^4-2*A*a*b*c*d^3+A*b^2*c^2*d^2-B*a^2*c*d^3+2*B*
a*b*c^2*d^2-B*b^2*c^3*d+C*a^2*c^2*d^2-2*C*a*b*c^3*d+C*b^2*c^4)/(c^2+d^2)*ln
(c+d*tan(f*x+e))+1/(c^2+d^2)*(1/2*(-A*a^2*d+2*A*a*b*c+A*b^2*d+B*a^2*c+2*B*a
*b*d-B*b^2*c+C*a^2*d-2*C*a*b*c-C*b^2*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c+2*A*a*b
*d-A*b^2*c+B*a^2*d-2*B*a*b*c-B*b^2*d-C*a^2*c-2*C*a*b*d+C*b^2*c)*arctan(tan(
f*x+e))))
```

Maxima [A]
time = 0.54, size = 299, normalized size = 1.27

$$\frac{\frac{2\,(((A-C)a^2-2\,Bab-(A-C)b^2)c+(Ba^2+2\,(A-C)ab-Bb^2)d)(fx+e)}{c^2+d^2}+\frac{2\,(Cb^2c^4+Aa^2d^4-(2\,Cab+Bb^2)c^3d+(Ca^2+2\,Bab+Ab^2)c^2d^2-(Ba^2+2\,Aab)cd^3)\log(d\tan(fx+e)+c)}{c^2d^3+d^5}+\frac{((Ba^2+2\,(A-C)ab-Bb^2)c-((A-C)a^2-2\,Bab-(A-C)b^2)d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}+\frac{Cb^2d\tan(fx+e)^2-2\,(Cb^2c-(2\,Cab+Bb^2)d)\tan(fx+e)}{d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b -
B*b^2)*d)*(f*x + e)/(c^2 + d^2) + 2*(C*b^2*c^4 + A*a^2*d^4 - (2*C*a*b + B*b
^2)*c^3*d + (C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - (B*a^2 + 2*A*a*b)*c*d^3)*lo
g(d*tan(f*x + e) + c)/(c^2*d^3 + d^5) + ((B*a^2 + 2*(A - C)*a*b - B*b^2)*c
- ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d
^2) + (C*b^2*d*tan(f*x + e)^2 - 2*(C*b^2*c - (2*C*a*b + B*b^2)*d)*tan(f*x +
 e))/d^2)/f
```

Fricas [A]
time = 6.80, size = 396, normalized size = 1.68

$$\frac{2\,(((A-C)a^2-2\,Bab-(A-C)b^2)cd^3+(Ba^2+2\,(A-C)ab-Bb^2)d^4)fx+(Cb^2c^2d^2+Cb^2d^4)\tan(fx+e)^2+(Cb^2c^4+Aa^2d^4-(2\,Cab+Bb^2)c^3d+(Ca^2+2\,Bab+Ab^2)c^2d^2-(Ba^2+2\,Aab)cd^3)\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-(Cb^2c^4-(2\,Cab+Bb^2)c^3d+(Ca^2+2\,Bab+Ab^2)c^2d^2-(2\,Cab+Bb^2)cd^3+(Ca^2+2\,Bab+(A-C)b^2)d^4)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)-2\,(Cb^2c^3d+Cb^2cd^3-(2\,Cab+Bb^2)c^2d^2-(2\,Cab+Bb^2)d^4)\tan(fx+e)}{2\,(c^2d^3+d^5)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^3 + (B*a^2 + 2*(A - C)*a*
b - B*b^2)*d^4)*f*x + (C*b^2*c^2*d^2 + C*b^2*d^4)*tan(f*x + e)^2 + (C*b^2*c
^4 + A*a^2*d^4 - (2*C*a*b + B*b^2)*c^3*d + (C*a^2 + 2*B*a*b + A*b^2)*c^2*d^
2 - (B*a^2 + 2*A*a*b)*c*d^3)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) +
 c^2)/(tan(f*x + e)^2 + 1)) - (C*b^2*c^4 - (2*C*a*b + B*b^2)*c^3*d + (C*a^2
```

+ 2*B*a*b + A*b^2)*c^2*d^2 - (2*C*a*b + B*b^2)*c*d^3 + (C*a^2 + 2*B*a*b + (A - C)*b^2)*d^4)*log(1/(tan(f*x + e)^2 + 1)) - 2*(C*b^2*c^3*d + C*b^2*c*d^3 - (2*C*a*b + B*b^2)*c^2*d^2 - (2*C*a*b + B*b^2)*d^4)*tan(f*x + e))/((c^2*d^3 + d^5)*f)

Sympy [C] Result contains complex when optimal does not.
time = 2.68, size = 4444, normalized size = 18.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), (I*A*a**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + A*a**2*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*A*a**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*A*a*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 2*I*A*a*b*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) - 2*A*a*b/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*A*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + A*b**2*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + A*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*A*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*A*b**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + B*a**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*B*a**2*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) - B*a**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*I*B*a*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*B*a*b*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*B*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 2*I*B*a*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 2*I*B*a*b/(2*d*f*tan(e + f*x) - 2*I*d*f) - 3*B*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*I*B*b**2*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*B*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + B*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*B*b**2*tan(e + f*x)**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*B*b**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*C*a**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*a**2*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C*a**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C*a**2/(2*d*f*tan(e + f*x) - 2*I*d*f) - 6*C*a*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 6*I*C*a*b*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*I*C*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*C*a*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 4*C*a*b*tan(e + f*x)**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 6*C*a*b/(2*d*f*tan(e + f*x) - 2*I*d*f) - 3*I*C*b**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 3*C*b**2*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) - 2*C*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*I*C*b**2*log(tan(e
```

```
+ f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*b**2*tan(e + f*x)**3/(2*d
*f*tan(e + f*x) - 2*I*d*f) + I*C*b**2*tan(e + f*x)**2/(2*d*f*tan(e + f*x) -
 2*I*d*f) + 3*I*C*b**2/(2*d*f*tan(e + f*x) - 2*I*d*f), Eq(c, -I*d)), (-I*A*
a**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + A*a**2*f*x/(2*d*f*ta
n(e + f*x) + 2*I*d*f) - I*A*a**2/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*A*a*b*f
*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*I*A*a*b*f*x/(2*d*f*tan(e
 + f*x) + 2*I*d*f) - 2*A*a*b/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*A*b**2*f*x*
tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + A*b**2*f*x/(2*d*f*tan(e + f*x
) + 2*I*d*f) + A*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e +
f*x) + 2*I*d*f) + I*A*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2
*I*d*f) + I*A*b**2/(2*d*f*tan(e + f*x) + 2*I*d*f) + B*a**2*f*x*tan(e + f*x)
/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*B*a**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*
f) - B*a**2/(2*d*f*tan(e + f*x) + 2*I*d*f) - 2*I*B*a*b*f*x*tan(e + f*x)/(2*
d*f*tan(e + f*x) + 2*I*d*f) + 2*B*a*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) +
2*B*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f
) + 2*I*B*a*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*I
*B*a*b/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*B*b**2*f*x*tan(e + f*x)/(2*d*f*ta
n(e + f*x) + 2*I*d*f) - 3*I*B*b**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*B
*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f)
+ B*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*B*b**2
*tan(e + f*x)**2/(2*d*f*tan(e + f*x) + 2*I*d*f) + 3*B*b**2/(2*d*f*tan(e + f
*x) + 2*I*d*f) - I*C*a**2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) +
 C*a**2*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*a**2*log(tan(e + f*x)**2 + 1
)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*C*a**2*log(tan(e + f*x)**
2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*C*a**2/(2*d*f*tan(e + f*x) + 2*I*
d*f) - 6*C*a*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) - 6*I*C*a*b*
f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - 2*I*C*a*b*log(tan(e + f*x)**2 + 1)*tan
(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*C*a*b*log(tan(e + f*x)**2 + 1)
/(2*d*f*tan(e + f*x) + 2*I*d*f) + 4*C*a*b*tan(e + f*x)**2/(2*d*f*tan(e + f*
x) + 2*I*d*f) + 6*C*a*b/(2*d*f*tan(e + f*x) + 2*I*d*f) + 3*I*C*b**2*f*x*tan
(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*C*b**2*f*x/(2*d*f*tan(e + f*x)
 + 2*I*d*f) - 2*C*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e +
 f*x) + 2*I*d*f) - 2*I*C*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x)
+ 2*I*d*f) + C*b**2*tan(e + f*x)**3/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*b*
*2*tan(e + f*x)**2/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*I*C*b**2/(2*d*f*tan(e
 + f*x) + 2*I*d*f), Eq(c, I*d)), ((A*a**2*x + A...
```

Giac [A]
time = 0.83, size = 338, normalized size = 1.43

$$\frac{\frac{2\,(Aa^2c-Ca^2c-2\,Babc-Ab^2c+Cb^2c+Ba^2d+2\,Aabd-2\,Cabd-Bb^2d)(fx+e)}{c^2+d^2}+\frac{(Ba^2c+2\,Aabc-2\,Cabc-Bb^2c-Aa^2d+Ca^2d+2\,Babd+Ab^2d-Cb^2d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}+\frac{2\,(Cb^2c^4-2\,Cabc^3d-Bb^2c^3d+Ca^2c^2d^2+2\,Babc^2d^2+Ab^2c^2d^2-Ba^2cd^3-2\,Aabcd^3+Aa^2d^4)\log(|d\tan(fx+e)+c|)}{c^2d^3+d^5}+\frac{Cb^2d\tan(fx+e)^2-2\,Cb^2c\tan(fx+e)+4\,Cabd\tan(fx+e)+2\,Bb^2d\tan(fx+e)}{d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c - C*a^2*c - 2*B*a*b*c - A*b^2*c + C*b^2*c + B*a^2*d + 2*A*a
*b*d - 2*C*a*b*d - B*b^2*d)*(f*x + e)/(c^2 + d^2) + (B*a^2*c + 2*A*a*b*c -
2*C*a*b*c - B*b^2*c - A*a^2*d + C*a^2*d + 2*B*a*b*d + A*b^2*d - C*b^2*d)*lo
g(tan(f*x + e)^2 + 1)/(c^2 + d^2) + 2*(C*b^2*c^4 - 2*C*a*b*c^3*d - B*b^2*c^
3*d + C*a^2*c^2*d^2 + 2*B*a*b*c^2*d^2 + A*b^2*c^2*d^2 - B*a^2*c*d^3 - 2*A*a
*b*c*d^3 + A*a^2*d^4)*log(abs(d*tan(f*x + e) + c))/(c^2*d^3 + d^5) + (C*b^2
*d*tan(f*x + e)^2 - 2*C*b^2*c*tan(f*x + e) + 4*C*a*b*d*tan(f*x + e) + 2*B*b
^2*d*tan(f*x + e))/d^2)/f
```

<u>**Mupad**</u> **[B]**
time = 11.20, size = 325, normalized size = 1.38

$$\frac{\tan(e+f\,x)\,\left(\frac{B\,b^2+2\,C\,a\,b}{d}-\frac{C\,b^2\,c}{d^2}\right)}{f}+\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(d^2\,\left(C\,a^2\,c^2+2\,B\,a\,b\,c^2+A\,b^2\,c^2\right)-d\,\left(B\,b^2\,c^3+2\,C\,a\,b\,c^3\right)-d^3\,\left(B\,c\,a^2+2\,A\,b\,c\,a\right)+A\,a^2\,d^4+C\,b^2\,c^4\right)}{f\,\left(c^2\,d^3+d^5\right)}+\frac{\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,\left(A\,b^2-A\,a^2+B\,a^2\,1{}\mathrm{i}-B\,b^2\,1{}\mathrm{i}+C\,a^2-C\,b^2+A\,a\,b\,2{}\mathrm{i}+2\,B\,a\,b-C\,a\,b\,2{}\mathrm{i}\right)}{2\,f\,\left(d+c\,1{}\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B\,a^2-B\,b^2+2\,A\,a\,b-2\,C\,a\,b-A\,a^2\,1{}\mathrm{i}+A\,b^2\,1{}\mathrm{i}+C\,a^2\,1{}\mathrm{i}-C\,b^2\,1{}\mathrm{i}+B\,a\,b\,2{}\mathrm{i}\right)}{2\,f\,\left(c+d\,1{}\mathrm{i}\right)}+\frac{C\,b^2\,\tan(e+f\,x)^2}{2\,d\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x)),x)
```

```
[Out] (tan(e + f*x)*((B*b^2 + 2*C*a*b)/d - (C*b^2*c)/d^2))/f + (log(c + d*tan(e +
 f*x))*(d^2*(A*b^2*c^2 + C*a^2*c^2 + 2*B*a*b*c^2) - d*(B*b^2*c^3 + 2*C*a*b*
c^3) - d^3*(B*a^2*c + 2*A*a*b*c) + A*a^2*d^4 + C*b^2*c^4))/(f*(d^5 + c^2*d^
3)) + (log(tan(e + f*x) + 1i)*(A*b^2 - A*a^2 + B*a^2*1i - B*b^2*1i + C*a^2
- C*b^2 + A*a*b*2i + 2*B*a*b - C*a*b*2i))/(2*f*(c*1i + d)) + (log(tan(e + f
*x) - 1i)*(A*b^2*1i - A*a^2*1i + B*a^2 - B*b^2 + C*a^2*1i - C*b^2*1i + 2*A*
a*b + B*a*b*2i - 2*C*a*b))/(2*f*(c + d*1i)) + (C*b^2*tan(e + f*x)^2)/(2*d*f
)
```

3.72 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=156

$$\frac{(a(Ac-cC+Bd)-b(Bc-(A-C)d))x}{c^2+d^2}-\frac{(Abc+aBc-bcC-aAd+bBd+aCd)\log(\cos(e+fx))}{(c^2+d^2)f}-\frac{(bc$$

```
[Out] (a*(A*c+B*d-C*c)-b*(B*c-(A-C)*d))*x/(c^2+d^2)-(-A*a*d+A*b*c+B*a*c+B*b*d+C*a
*d-C*b*c)*ln(cos(f*x+e))/(c^2+d^2)/f-(-a*d+b*c)*(A*d^2-B*c*d+C*c^2)*ln(c+d*
tan(f*x+e))/d^2/(c^2+d^2)/f+b*C*tan(f*x+e)/d/f
```

Rubi [A]
time = 0.23, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3718, 3707, 3698, 31, 3556}

$$-\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)\log(c+d\tan(e+fx))}{d^2f\left(c^2+d^2\right)}-\frac{\log(\cos(e+fx))(-aAd+aBc+aCd+Abc+bBd-bcC)}{f\left(c^2+d^2\right)}+\frac{x(a(Ac+Bd-cC)-b(Bc-d(A-C)))}{c^2+d^2}+\frac{bC\tan(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x]),x]
```

```
[Out] ((a*(A*c - c*C + B*d) - b*(B*c - (A - C)*d))*x)/(c^2 + d^2) - ((A*b*c + a*B
*c - b*c*C - a*A*d + b*B*d + a*C*d)*Log[Cos[e + f*x]])/((c^2 + d^2)*f) - ((
b*c - a*d)*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2
)*f) + (b*C*Tan[e + f*x])/(d*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
```

```
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = \frac{bC\tan(e+fx)}{df} - \frac{\int \frac{bcC-aAd-(Ab+aB-bC)d}{}}{}$$

$$= \frac{(a(Ac-cC+Bd)-b(Bc-(A-C)d)}{c^2+d^2}$$

$$= \frac{(a(Ac-cC+Bd)-b(Bc-(A-C)d)}{c^2+d^2}$$

$$= \frac{(a(Ac-cC+Bd)-b(Bc-(A-C)d)}{c^2+d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.72, size = 148, normalized size = 0.95

$$\frac{\frac{(-ia+b)(A+iB-C)\log(i-\tan(e+fx))}{c+id}+\frac{(ia+b)(A-iB-C)\log(i+\tan(e+fx))}{c-id}+\frac{2(-bc+ad)\left(c^2C-Bcd+Ad^2\right)\log(c+d\tan(e+fx))}{d^2(c^2+d^2)}+\frac{2bC\tan(e+fx)}{d}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x]),x]
```

```
[Out] ((((-I)*a + b)*(A + I*B - C)*Log[I - Tan[e + f*x]])/(c + I*d) + ((I*a + b)*
(A - I*B - C)*Log[I + Tan[e + f*x]])/(c - I*d) + (2*(-(b*c) + a*d)*(c^2*C -
```

```
B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)) + (2*b*C*Tan[e +
f*x])/d)/(2*f)
```

Maple [A]
time = 0.22, size = 173, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{Cb\tan(fx+e)}{d}+\frac{\left(Aa\,d^3-Abc\,d^2-Bac\,d^2+Bb\,c^2d+Ca\,c^2d-Cb\,c^3\right)\ln(c+d\tan(fx+e))}{d^2\left(c^2+d^2\right)}+\frac{(-Aad+Abc+Bac+Bbd+aCd-Cbc)\ln\left(1+\mathrm{t}\right.}{2}}{f}$
default	$\frac{\frac{Cb\tan(fx+e)}{d}+\frac{\left(Aa\,d^3-Abc\,d^2-Bac\,d^2+Bb\,c^2d+Ca\,c^2d-Cb\,c^3\right)\ln(c+d\tan(fx+e))}{d^2\left(c^2+d^2\right)}+\frac{(-Aad+Abc+Bac+Bbd+aCd-Cbc)\ln\left(1+\mathrm{t}\right.}{2}}{f}$
norman	$\frac{(Aac+Abd+Bad-Bbc-Cac-Cbd)x}{c^2+d^2}+\frac{bC\tan(fx+e)}{df}+\frac{(Aa\,d^3-Abc\,d^2-Bac\,d^2+Bb\,c^2d+Ca\,c^2d-Cb\,c^3)\ln(c+d\,\mathrm{ta}}{d^2f(c^2+d^2)}$
risch	$-\frac{2iBb\,c^2e}{df(c^2+d^2)}-\frac{2iCa\,c^2e}{df(c^2+d^2)}+\frac{2iCb\,c^3e}{d^2f(c^2+d^2)}-\frac{2idAae}{f(c^2+d^2)}+\frac{2iAbce}{f(c^2+d^2)}+\frac{2iBace}{f(c^2+d^2)}-\frac{2iBb\,c^2x}{d(c^2+d^2)}-\frac{2iCa\,c^2x}{d(c^2+d^2)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/f*(C*b/d*tan(f*x+e)+1/d^2*(A*a*d^3-A*b*c*d^2-B*a*c*d^2+B*b*c^2*d+C*a*c^2*
d-C*b*c^3)/(c^2+d^2)*ln(c+d*tan(f*x+e))+1/(c^2+d^2)*(1/2*(-A*a*d+A*b*c+B*a*
c+B*b*d+C*a*d-C*b*c)*ln(1+tan(f*x+e)^2)+(A*a*c+A*b*d+B*a*d-B*b*c-C*a*c-C*b*
d)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.55, size = 182, normalized size = 1.17

$$\frac{\frac{2\,Cb\tan\left(fx+e\right)}{d}+\frac{2\,(((A-C)a-Bb)c+(Ba+(A-C)b)d)(fx+e)}{c^2+d^2}-\frac{2\,\left(Cbc^3-Aad^3-(Ca+Bb)c^2d+(Ba+Ab)cd^2\right)\log\left(d\tan\left(fx+e\right)+c\right)}{c^2d^2+d^4}+\frac{((Ba+(A-C)b)c-((A-C)a-Bb)d)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^2+d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*b*tan(f*x + e)/d + 2*(((A - C)*a - B*b)*c + (B*a + (A - C)*b)*d)*(
f*x + e)/(c^2 + d^2) - 2*(C*b*c^3 - A*a*d^3 - (C*a + B*b)*c^2*d + (B*a + A*
b)*c*d^2)*log(d*tan(f*x + e) + c)/(c^2*d^2 + d^4) + ((B*a + (A - C)*b)*c -
((A - C)*a - B*b)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2))/f
```

Fricas [A]
time = 3.88, size = 217, normalized size = 1.39

$$\frac{2\,(((A-C)a-Bb)cd^2+(Ba+(A-C)b)d^3)fx-(Cbc^3-Aad^3-(Ca+Bb)c^2d+(Ba+Ab)cd^2)\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)+(Cbc^3+Cbcd^2-(Ca+Bb)c^2d-(Ca+Bb)d^3)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+2\,(Cbc^2d+Cbd^3)\tan\left(fx+e\right)}{2\,(c^2d^2+d^4)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*f*x - (C*b*c^3 - A
*a*d^3 - (C*a + B*b)*c^2*d + (B*a + A*b)*c*d^2)*log((d^2*tan(f*x + e)^2 + 2
*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) + (C*b*c^3 + C*b*c*d^2 - (C*
a + B*b)*c^2*d - (C*a + B*b)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 2*(C*b*c^2*
d + C*b*d^3)*tan(f*x + e))/((c^2*d^2 + d^4)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.02, size = 2387, normalized size = 15.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
),x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(c,
0) & Eq(d, 0) & Eq(f, 0)), (I*A*a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*
I*d*f) + A*a*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*A*a/(2*d*f*tan(e + f*x)
 - 2*I*d*f) + A*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*A*b*f
*x/(2*d*f*tan(e + f*x) - 2*I*d*f) - A*b/(2*d*f*tan(e + f*x) - 2*I*d*f) + B*
a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*B*a*f*x/(2*d*f*tan(e
+ f*x) - 2*I*d*f) - B*a/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*B*b*f*x*tan(e +
f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + B*b*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f
) + B*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f
) - I*B*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*B*b/(
2*d*f*tan(e + f*x) - 2*I*d*f) + I*C*a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x)
- 2*I*d*f) + C*a*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*a*log(tan(e + f*x)*
*2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C*a*log(tan(e + f*x
)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C*a/(2*d*f*tan(e + f*x) - 2*I*
d*f) - 3*C*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 3*I*C*b*f*x/
(2*d*f*tan(e + f*x) - 2*I*d*f) + I*C*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x
)/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(
e + f*x) - 2*I*d*f) + 2*C*b*tan(e + f*x)**2/(2*d*f*tan(e + f*x) - 2*I*d*f)
+ 3*C*b/(2*d*f*tan(e + f*x) - 2*I*d*f), Eq(c, -I*d)), (-I*A*a*f*x*tan(e + f
*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + A*a*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f)
 - I*A*a/(2*d*f*tan(e + f*x) + 2*I*d*f) + A*b*f*x*tan(e + f*x)/(2*d*f*tan(e
 + f*x) + 2*I*d*f) + I*A*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - A*b/(2*d*f*
tan(e + f*x) + 2*I*d*f) + B*a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*
f) + I*B*a*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - B*a/(2*d*f*tan(e + f*x) + 2
*I*d*f) - I*B*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + B*b*f*x/(
2*d*f*tan(e + f*x) + 2*I*d*f) + B*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(
```

```
2*d*f*tan(e + f*x) + 2*I*d*f) + I*B*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e
 + f*x) + 2*I*d*f) + I*B*b/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*a*f*x*tan(e
 + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*a*f*x/(2*d*f*tan(e + f*x) + 2*I*
d*f) + C*a*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*
d*f) + I*C*a*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*C*
a/(2*d*f*tan(e + f*x) + 2*I*d*f) - 3*C*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*
x) + 2*I*d*f) - 3*I*C*b*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*b*log(tan(
e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*b*log(tan(
e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + 2*C*b*tan(e + f*x)**2/(2*
d*f*tan(e + f*x) + 2*I*d*f) + 3*C*b/(2*d*f*tan(e + f*x) + 2*I*d*f), Eq(c, I
*d)), ((A*a*x + A*b*log(tan(e + f*x)**2 + 1)/(2*f) + B*a*log(tan(e + f*x)**
2 + 1)/(2*f) - B*b*x + B*b*tan(e + f*x)/f - C*a*x + C*a*tan(e + f*x)/f - C*
b*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*tan(e + f*x)**2/(2*f))/c, Eq(d, 0)),
 (x*(a + b*tan(e))*(A + B*tan(e) + C*tan(e)**2)/(c + d*tan(e)), Eq(f, 0)),
(2*A*a*c*d**2*f*x/(2*c**2*d**2*f + 2*d**4*f) + 2*A*a*d**3*log(c/d + tan(e +
 f*x))/(2*c**2*d**2*f + 2*d**4*f) - A*a*d**3*log(tan(e + f*x)**2 + 1)/(2*c*
*2*d**2*f + 2*d**4*f) - 2*A*b*c*d**2*log(c/d + tan(e + f*x))/(2*c**2*d**2*f
 + 2*d**4*f) + A*b*c*d**2*log(tan(e + f*x)**2 + 1)/(2*c**2*d**2*f + 2*d**4*
f) + 2*A*b*d**3*f*x/(2*c**2*d**2*f + 2*d**4*f) - 2*B*a*c*d**2*log(c/d + tan
(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) + B*a*c*d**2*log(tan(e + f*x)**2 + 1)
/(2*c**2*d**2*f + 2*d**4*f) + 2*B*a*d**3*f*x/(2*c**2*d**2*f + 2*d**4*f) + 2
*B*b*c**2*d*log(c/d + tan(e + f*x))/(2*c**2*d**2*f + 2*d**4*f) - 2*B*b*c*d*
*2*f*x/(2*c**2*d**2*f + 2*d**4*f) + B*b*d**3*log(tan(e + f*x)**2 + 1)/(2*c*
*2*d**2*f + 2*d**4*f) + 2*C*a*c**2*d*log(c/d + tan(e + f*x))/(2*c**2*d**2*f
 + 2*d**4*f) - 2*C*a*c*d**2*f*x/(2*c**2*d**2*f + 2*d**4*f) + C*a*d**3*log(t
an(e + f*x)**2 + 1)/(2*c**2*d**2*f + 2*d**4*f) - 2*C*b*c**3*log(c/d + tan(e
 + f*x))/(2*c**2*d**2*f + 2*d**4*f) + 2*C*b*c**2*d*tan(e + f*x)/(2*c**2*d**
2*f + 2*d**4*f) - C*b*c*d**2*log(tan(e + f*x)**2 + 1)/(2*c**2*d**2*f + 2*d*
*4*f) - 2*C*b*d**3*f*x/(2*c**2*d**2*f + 2*d**4*f) + 2*C*b*d**3*tan(e + f*x)
/(2*c**2*d**2*f + 2*d**4*f), True))
```

Giac [A]

time = 0.71, size = 186, normalized size = 1.19

$$\frac{\frac{2\,Cb\tan(fx+e)}{d}+\frac{2\,(Aac-Cac-Bbc+Bad+Abd-Cbd)(fx+e)}{c^2+d^2}+\frac{(Bac+Abc-Cbc-Aad+Cad+Bbd)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}-\frac{2\,(Cbc^3-Cac^2d-Bbc^2d+Bacd^2+Abcd^2-Aad^3)\log(|d\tan(fx+e)+c|)}{c^2d^2+d^4}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*b*tan(f*x + e)/d + 2*(A*a*c - C*a*c - B*b*c + B*a*d + A*b*d - C*b*
d)*(f*x + e)/(c^2 + d^2) + (B*a*c + A*b*c - C*b*c - A*a*d + C*a*d + B*b*d)*
log(tan(f*x + e)^2 + 1)/(c^2 + d^2) - 2*(C*b*c^3 - C*a*c^2*d - B*b*c^2*d +
B*a*c*d^2 + A*b*c*d^2 - A*a*d^3)*log(abs(d*tan(f*x + e) + c))/(c^2*d^2 + d^
4))/f
```

Mupad [B]
time = 10.25, size = 186, normalized size = 1.19

$$\frac{\ln\left(\tan(e+fx)-\mathrm{i}\right)\,\left(A\,b+B\,a-C\,b-A\,a\,1\mathrm{i}+B\,b\,1\mathrm{i}+C\,a\,1\mathrm{i}\right)}{2\,f\,\left(c+d\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(e+fx)+1\mathrm{i}\right)\,\left(B\,b+A\,b\,1\mathrm{i}+B\,a\,1\mathrm{i}-A\,a+C\,a-C\,b\,1\mathrm{i}\right)}{2\,f\,\left(d+c\,1\mathrm{i}\right)}-\frac{\ln\left(c+d\tan(e+fx)\right)\,\left(d^2\,\left(A\,b\,c+B\,a\,c\right)-d\,\left(B\,b\,c^2+C\,a\,c^2\right)-A\,a\,d^3+C\,b\,c^3\right)}{f\,\left(c^2\,d^2+d^4\right)}+\frac{C\,b\tan(e+fx)}{d\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*t
an(e + f*x)),x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(A*b - A*a*1i + B*a + B*b*1i + C*a*1i - C*b))/(2*f*
(c + d*1i)) + (log(tan(e + f*x) + 1i)*(A*b*1i - A*a + B*a*1i + B*b + C*a -
C*b*1i))/(2*f*(c*1i + d)) - (log(c + d*tan(e + f*x))*(d^2*(A*b*c + B*a*c) -
 d*(B*b*c^2 + C*a*c^2) - A*a*d^3 + C*b*c^3))/(f*(d^4 + c^2*d^2)) + (C*b*tan
(e + f*x))/(d*f)
```

3.73 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=99

$$\frac{(Ac-cC+Bd)x}{c^2+d^2}-\frac{(Bc-(A-C)d)\log(\cos(e+fx))}{(c^2+d^2)\,f}+\frac{(c^2C-Bcd+Ad^2)\log(c+d\tan(e+fx))}{d\,(c^2+d^2)\,f}$$

```
[Out] (A*c+B*d-C*c)*x/(c^2+d^2)-(B*c-(A-C)*d)*ln(cos(f*x+e))/(c^2+d^2)/f+(A*d^2-B
*c*d+C*c^2)*ln(c+d*tan(f*x+e))/d/(c^2+d^2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3707, 3698, 31, 3556}

$$\frac{(Ad^2-Bcd+c^2C)\log(c+d\tan(e+fx))}{df\,(c^2+d^2)}-\frac{(Bc-d(A-C))\log(\cos(e+fx))}{f\,(c^2+d^2)}+\frac{x(Ac+Bd-cC)}{c^2+d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((A*c - c*C + B*d)*x)/(c^2 + d^2) - ((B*c - (A - C)*d)*Log[Cos[e + f*x]])/(
(c^2 + d^2)*f) + ((c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d*(c^2
+ d^2)*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
```

```
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx = \frac{(Ac-cC+Bd)x}{c^2+d^2} - \frac{(-Bc+Ad-Cd)\int\tan(e+fx)\,dx}{c^2+d^2} + \frac{(c$$

$$= \frac{(Ac-cC+Bd)x}{c^2+d^2} - \frac{(Bc-(A-C)d)\log(\cos(e+fx))}{(c^2+d^2)\,f} + \frac{(c^2}{}$$

$$= \frac{(Ac-cC+Bd)x}{c^2+d^2} - \frac{(Bc-(A-C)d)\log(\cos(e+fx))}{(c^2+d^2)\,f} + \frac{(c^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 117, normalized size = 1.18

$$\frac{\frac{(-iA+B+iC)\log(i-\tan(e+fx))}{c+id} + \frac{(iA+B-iC)\log(i+\tan(e+fx))}{c-id} + \frac{2(c^2C-Bcd+Ad^2)\log(c+d\tan(e+fx))}{d(c^2+d^2)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x]),x]
```

```
[Out] ((((-I)*A + B + I*C)*Log[I - Tan[e + f*x]])/(c + I*d) + ((I*A + B - I*C)*Lo
g[I + Tan[e + f*x]])/(c - I*d) + (2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e
 + f*x]])/(d*(c^2 + d^2)))/(2*f)
```

Maple [A]
time = 0.19, size = 100, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{(-Ad+Bc+Cd)\ln\left(1+\tan^2(fx+e)\right)}{2}+(Ac+Bd-cC)\arctan(\tan(fx+e))}{c^2+d^2}+\frac{\left(A\,d^2-Bcd+c^2C\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)d}}{f}$
default	$\frac{\frac{\frac{(-Ad+Bc+Cd)\ln\left(1+\tan^2(fx+e)\right)}{2}+(Ac+Bd-cC)\arctan(\tan(fx+e))}{c^2+d^2}+\frac{\left(A\,d^2-Bcd+c^2C\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)d}}{f}$
norman	$\frac{(Ac+Bd-cC)x}{c^2+d^2}+\frac{\left(A\,d^2-Bcd+c^2C\right)\ln(c+d\tan(fx+e))}{d(c^2+d^2)f}-\frac{(Ad-Bc-Cd)\ln\left(1+\tan^2(fx+e)\right)}{2f(c^2+d^2)}$
risch	$\frac{ixB}{id-c}-\frac{xA}{id-c}+\frac{xC}{id-c}-\frac{2idAx}{c^2+d^2}-\frac{2idAe}{(c^2+d^2)f}+\frac{2iBcx}{c^2+d^2}+\frac{2iBce}{(c^2+d^2)f}-\frac{2ic^2Cx}{(c^2+d^2)d}-\frac{2ic^2Ce}{(c^2+d^2)df}+\frac{2iCx}{d}+\frac{2i}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(1/(c^2+d^2)*(1/2*(-A*d+B*c+C*d)*ln(1+tan(f*x+e)^2)+(A*c+B*d-C*c)*arcta
n(tan(f*x+e)))+(A*d^2-B*c*d+C*c^2)/(c^2+d^2)/d*ln(c+d*tan(f*x+e)))
```

Maxima [A]
time = 0.59, size = 109, normalized size = 1.10

$$\frac{\frac{2\,((A-C)c+Bd)(fx+e)}{c^2+d^2}+\frac{2\,(Cc^2-Bcd+Ad^2)\log(d\tan(fx+e)+c)}{c^2d+d^3}+\frac{(Bc-(A-C)d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*((A - C)*c + B*d)*(f*x + e)/(c^2 + d^2) + 2*(C*c^2 - B*c*d + A*d^2)*
log(d*tan(f*x + e) + c)/(c^2*d + d^3) + (B*c - (A - C)*d)*log(tan(f*x + e)^
2 + 1)/(c^2 + d^2))/f
```

Fricas [A]
time = 4.74, size = 122, normalized size = 1.23

$$\frac{2\,((A-C)cd+Bd^2)fx+(Cc^2-Bcd+Ad^2)\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-(Cc^2+Cd^2)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)}{2\,(c^2d+d^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*((A - C)*c*d + B*d^2)*f*x + (C*c^2 - B*c*d + A*d^2)*log((d^2*tan(f*x
 + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (C*c^2 + C*d^2)
*log(1/(tan(f*x + e)^2 + 1)))/((c^2*d + d^3)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 966, normalized size = 9.76

$$\begin{cases} \frac{\tilde{\infty}x\left(A+B\tan{(e)}+C\tan^2{(e)}\right)}{\tan{(e)}} & \text{for}\: c=0\wedge d=0\wedge f=0 \\ \frac{iAfx\tan{(e+fx)}}{2df\tan{(e+fx)}-2idf}+\frac{Afx}{2df\tan{(e+fx)}-2idf}+\frac{iA}{2df\tan{(e+fx)}-2idf}+\frac{Bfx\tan{(e+fx)}}{2df\tan{(e+fx)}-2idf}-\frac{iBfx}{2df\tan{(e+fx)}-2idf}-\frac{B}{2df\tan{(e+fx)}-2idf}+\frac{iCfx\tan{(e+fx)}}{2df\tan{(e+fx)}-2idf}+\frac{Cfx}{2df\tan{(e+fx)}-2idf}+\frac{C\log{\left(\tan^2{(e+fx)}+1\right)}\tan{(e+fx)}}{2df\tan{(e+fx)}-2idf}-\frac{iC\log{\left(\tan^2{(e+fx)}+1\right)}}{2df\tan{(e+fx)}-2idf}-\frac{iC}{2df\tan{(e+fx)}-2idf} & \text{for}\: c=-id \\ -\frac{iAfx\tan{(e+fx)}}{2df\tan{(e+fx)}+2idf}+\frac{Afx}{2df\tan{(e+fx)}+2idf}-\frac{iA}{2df\tan{(e+fx)}+2idf}+\frac{Bfx\tan{(e+fx)}}{2df\tan{(e+fx)}+2idf}+\frac{iBfx}{2df\tan{(e+fx)}+2idf}-\frac{B}{2df\tan{(e+fx)}+2idf}-\frac{iCfx\tan{(e+fx)}}{2df\tan{(e+fx)}+2idf}+\frac{Cfx}{2df\tan{(e+fx)}+2idf}+\frac{C\log{\left(\tan^2{(e+fx)}+1\right)}\tan{(e+fx)}}{2df\tan{(e+fx)}+2idf}+\frac{iC\log{\left(\tan^2{(e+fx)}+1\right)}}{2df\tan{(e+fx)}+2idf}+\frac{iC}{2df\tan{(e+fx)}+2idf} & \text{for}\: c=id \\ \frac{Ax+\frac{B\log{\left(\tan^2{(e+fx)}+1\right)}}{2f}-Cx+\frac{C\tan{(e+fx)}}{f}}{c} & \text{for}\: d=0 \\ \frac{x\left(A+B\tan{(e)}+C\tan^2{(e)}\right)}{c+d\tan{(e)}} & \text{for}\: f=0 \\ \frac{2Acdfx}{2c^2df+2d^3f}+\frac{2Ad^2\log{\left(\frac{c}{d}+\tan{(e+fx)}\right)}}{2c^2df+2d^3f}-\frac{Ad^2\log{\left(\tan^2{(e+fx)}+1\right)}}{2c^2df+2d^3f}-\frac{2Bcd\log{\left(\frac{c}{d}+\tan{(e+fx)}\right)}}{2c^2df+2d^3f}+\frac{Bcd\log{\left(\tan^2{(e+fx)}+1\right)}}{2c^2df+2d^3f}+\frac{2Bd^2fx}{2c^2df+2d^3f}+\frac{2Cc^2\log{\left(\frac{c}{d}+\tan{(e+fx)}\right)}}{2c^2df+2d^3f}-\frac{2Ccdfx}{2c^2df+2d^3f}+\frac{Cd^2\log{\left(\tan^2{(e+fx)}+1\right)}}{2c^2df+2d^3f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(c, 0) & Eq(d, 0) &
 Eq(f, 0)), (I*A*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + A*f*x/(2
*d*f*tan(e + f*x) - 2*I*d*f) + I*A/(2*d*f*tan(e + f*x) - 2*I*d*f) + B*f*x*t
an(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*B*f*x/(2*d*f*tan(e + f*x) -
2*I*d*f) - B/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*C*f*x*tan(e + f*x)/(2*d*f*t
an(e + f*x) - 2*I*d*f) + C*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*log(tan(e
 + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C*log(tan(e
 + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C/(2*d*f*tan(e + f*x) -
2*I*d*f), Eq(c, -I*d)), (-I*A*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*
f) + A*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*A/(2*d*f*tan(e + f*x) + 2*I*d
*f) + B*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*B*f*x/(2*d*f*ta
n(e + f*x) + 2*I*d*f) - B/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*C*f*x*tan(e +
f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f)
+ C*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) +
I*C*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*I*d*f) + I*C/(2*d*f*ta
n(e + f*x) + 2*I*d*f), Eq(c, I*d)), ((A*x + B*log(tan(e + f*x)**2 + 1)/(2*f
) - C*x + C*tan(e + f*x)/f)/c, Eq(d, 0)), (x*(A + B*tan(e) + C*tan(e)**2)/(
c + d*tan(e)), Eq(f, 0)), (2*A*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + 2*A*d**2*l
og(c/d + tan(e + f*x))/(2*c**2*d*f + 2*d**3*f) - A*d**2*log(tan(e + f*x)**2
 + 1)/(2*c**2*d*f + 2*d**3*f) - 2*B*c*d*log(c/d + tan(e + f*x))/(2*c**2*d*f
 + 2*d**3*f) + B*c*d*log(tan(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f) + 2*B
*d**2*f*x/(2*c**2*d*f + 2*d**3*f) + 2*C*c**2*log(c/d + tan(e + f*x))/(2*c**
2*d*f + 2*d**3*f) - 2*C*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + C*d**2*log(tan(e
+ f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f), True))
```

Giac [A]
time = 0.64, size = 109, normalized size = 1.10

$$\frac{\frac{2\,(Ac-Cc+Bd)(fx+e)}{c^2+d^2}+\frac{(Bc-Ad+Cd)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}+\frac{2\,(Cc^2-Bcd+Ad^2)\log(|d\tan(fx+e)+c|)}{c^2d+d^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(A*c - C*c + B*d)*(f*x + e)/(c^2 + d^2) + (B*c - A*d + C*d)*log(tan(
f*x + e)^2 + 1)/(c^2 + d^2) + 2*(C*c^2 - B*c*d + A*d^2)*log(abs(d*tan(f*x +
 e) + c))/(c^2*d + d^3))/f
```

Mupad [B]
time = 9.90, size = 109, normalized size = 1.10

$$\frac{\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,\left(C-A+B\,1{}\mathrm{i}\right)}{2\,f\,\left(d+c\,1{}\mathrm{i}\right)}+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B-A\,1{}\mathrm{i}+C\,1{}\mathrm{i}\right)}{2\,f\,\left(c+d\,1{}\mathrm{i}\right)}+\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(C\,c^2-B\,c\,d+A\,d^2\right)}{d\,f\,\left(c^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x)),x)
```

```
[Out] (log(tan(e + f*x) + 1i)*(B*1i - A + C))/(2*f*(c*1i + d)) + (log(tan(e + f*x
) - 1i)*(B - A*1i + C*1i))/(2*f*(c + d*1i)) + (log(c + d*tan(e + f*x))*(A*d
^2 + C*c^2 - B*c*d))/(d*f*(c^2 + d^2))
```

3.74 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=165

$$\frac{(a(Ac-cC+Bd)+b(Bc-(A-C)d))x}{(a^2+b^2)(c^2+d^2)}+\frac{(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)(bc-ad)f}-\frac{(c^2C-$$

```
[Out] (a*(A*c+B*d-C*c)+b*(B*c-(A-C)*d))*x/(a^2+b^2)/(c^2+d^2)+(A*b^2-a*(B*b-C*a))
*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)/(-a*d+b*c)/f-(A*d^2-B*c*d+C*c^2)*l
n(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)/(c^2+d^2)/f
```

Rubi [A]
time = 0.18, antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$,
Rules used = {3732, 3611}

$$\frac{x(a(Ac+Bd-cC)-bd(A-C)+bBc)}{(a^2+b^2)(c^2+d^2)}+\frac{(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)(bc-ad)}-\frac{(Ad^2-Bcd+c^2C)\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])),x]
```

```
[Out] ((b*B*c - b*(A - C)*d + a*(A*c - c*C + B*d))*x)/((a^2 + b^2)*(c^2 + d^2)) +
 ((A*b^2 - a*(b*B - a*C))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2
)*(b*c - a*d)*f) - ((c^2*C - B*c*d + A*d^2)*Log[c*Cos[e + f*x] + d*Sin[e +
f*x]])/((b*c - a*d)*(c^2 + d^2)*f)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx = \frac{(bBc-b(A-C)d+a(Ac-cC+Bd))x}{(a^2+b^2)\,(c^2+d^2)} + \frac{(Ab^2-a(bB-\ldots}{(a^2+\ldots}$$

$$= \frac{(bBc-b(A-C)d+a(Ac-cC+Bd))x}{(a^2+b^2)\,(c^2+d^2)} + \frac{(Ab^2-a(bB-\ldots}{\ldots}$$

Mathematica [A]

time = 0.97, size = 313, normalized size = 1.90

$$-\frac{\frac{\left(Abc-aBc-bcC+aAd+bBd-aCd+\frac{\sqrt{-b^2}(bBc+b(-A+C)d+a(Ac-cC+Bd))}{b}\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)}{(a^2+b^2)(c^2+d^2)}+\frac{2\left(Ab^2+a(-bB+aC)\right)\log(a+b\tan(e+fx))}{(a^2+b^2)(-bc+ad)}+\frac{\left(Abc-aBc-bcC+aAd+bBd-aCd+\frac{b(bBc+b(-A+C)d+a(Ac-cC+Bd))}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)}{(a^2+b^2)(c^2+d^2)}+\frac{2\left(c^2C-Bcd+Ad^2\right)\log(c+d\tan(e+fx))}{(bc-ad)(c^2+d^2)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])),x]
```

```
[Out] -1/2*(((A*b*c - a*B*c - b*c*C + a*A*d + b*B*d - a*C*d + (Sqrt[-b^2]*(b*B*c
+ b*(-A + C)*d + a*(A*c - c*C + B*d)))/b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]])
/((a^2 + b^2)*(c^2 + d^2)) + (2*(A*b^2 + a*(-(b*B) + a*C))*Log[a + b*Tan[e
+ f*x]])/((a^2 + b^2)*(-(b*c) + a*d)) + ((A*b*c - a*B*c - b*c*C + a*A*d + b
*B*d - a*C*d + (b*(b*B*c + b*(-A + C)*d + a*(A*c - c*C + B*d)))/Sqrt[-b^2])
*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) + (2*(c^2*C -
B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/f
```

Maple [A]

time = 0.34, size = 197, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{\left(A\,b^2-Bab+C\,a^2\right)\ln(a+b\tan(fx+e))}{(ad-bc)\left(a^2+b^2\right)}+\frac{\left(A\,d^2-Bcd+c^2C\right)\ln(c+d\tan(fx+e))}{(ad-bc)\left(c^2+d^2\right)}+\frac{\frac{(-Aad-Abc+Bac-Bbd+aCd+Cbc)\ln\left(1+\tan^2(\ldots\right.}{2}}{(\ldots}}{f}$
default	$\frac{-\frac{\left(A\,b^2-Bab+C\,a^2\right)\ln(a+b\tan(fx+e))}{(ad-bc)\left(a^2+b^2\right)}+\frac{\left(A\,d^2-Bcd+c^2C\right)\ln(c+d\tan(fx+e))}{(ad-bc)\left(c^2+d^2\right)}+\frac{\frac{(-Aad-Abc+Bac-Bbd+aCd+Cbc)\ln\left(1+\tan^2(\ldots\right.}{2}}{(\ldots}}{f}$
norman	$\frac{(Aac-Abd+Bad+Bbc-Cac+Cbd)x}{(a^2+b^2)(c^2+d^2)}+\frac{\left(A\,d^2-Bcd+c^2C\right)\ln(c+d\tan(fx+e))}{f(a\,c^2d+a\,d^3-b\,c^3-bc\,d^2)}-\frac{\left(A\,b^2-Bab+C\,a^2\right)\ln(a+b\tan(fx+\ldots}{(ad-bc)(a^2+b^2)f}$
risch	$\frac{2iC\,a^2x}{a^3d-a^2bc+a\,b^2d-b^3c}-\frac{xA}{iad+ibc-ac+bd}+\frac{xC}{iad+ibc-ac+bd}-\frac{2iBabe}{f(a^3d-a^2bc+a\,b^2d-b^3c)}-\frac{2ic^2Cx}{a\,c^2d+a\,d^3-b\,c^3-bc\,\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(A*b^2-B*a*b+C*a^2)/(a*d-b*c)/(a^2+b^2)*ln(a+b*tan(f*x+e))+(A*d^2-B*c
*d+C*c^2)/(a*d-b*c)/(c^2+d^2)*ln(c+d*tan(f*x+e))+1/(a^2+b^2)/(c^2+d^2)*(1/2
*(-A*a*d-A*b*c+B*a*c-B*b*d+C*a*d+C*b*c)*ln(1+tan(f*x+e)^2)+(A*a*c-A*b*d+B*a
*d+B*b*c-C*a*c+C*b*d)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.60, size = 247, normalized size = 1.50

$$\frac{\frac{2\,(((A-C)a+Bb)c+(Ba-(A-C)b)d)(fx+e)}{(a^2+b^2)c^2+(a^2+b^2)d^2}+\frac{2\,(Ca^2-Bab+Ab^2)\log(b\tan(fx+e)+a)}{(a^2b+b^3)c-(a^3+ab^2)d}-\frac{2\,(Cc^2-Bcd+Ad^2)\log(d\tan(fx+e)+c)}{bc^3-ac^2d+bcd^2-ad^3}+\frac{((Ba-(A-C)b)c-((A-C)a+Bb)d)\log\left(\tan(fx+e)^2+1\right)}{(a^2+b^2)c^2+(a^2+b^2)d^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a + B*b)*c + (B*a - (A - C)*b)*d)*(f*x + e)/((a^2 + b^2)*c
^2 + (a^2 + b^2)*d^2) + 2*(C*a^2 - B*a*b + A*b^2)*log(b*tan(f*x + e) + a)/(
(a^2*b + b^3)*c - (a^3 + a*b^2)*d) - 2*(C*c^2 - B*c*d + A*d^2)*log(d*tan(f*
x + e) + c)/(b*c^3 - a*c^2*d + b*c*d^2 - a*d^3) + ((B*a - (A - C)*b)*c - ((
A - C)*a + B*b)*d)*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*c^2 + (a^2 + b^2)*d
^2))/f
```

Fricas [A]
time = 10.94, size = 307, normalized size = 1.86

$$\frac{2\,(((A-C)ab+Bb^2)c^2-((A-C)a^2+(A-C)b^2)cd-(Ba^2-(A-C)ab)d^2)fx+((Ca^2-Bab+Ab^2)c^2+(Ca^2-Bab+Ab^2)d^2)\log\left(\frac{b^2\tan(fx+e)^2+2\,ab\tan(fx+e)+a^2}{\tan(fx+e)^2+1}\right)-((Ca^2+Cb^2)c^2-(Ba^2+Bb^2)cd+(Aa^2+Ab^2)d^2)\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)}{2\,((a^2b+b^3)c^3-(a^3+ab^2)c^2d+(a^2b+b^3)cd^2-(a^3+ab^2)d^3)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(((A - C)*a*b + B*b^2)*c^2 - ((A - C)*a^2 + (A - C)*b^2)*c*d - (B*a^
2 - (A - C)*a*b)*d^2)*f*x + ((C*a^2 - B*a*b + A*b^2)*c^2 + (C*a^2 - B*a*b +
 A*b^2)*d^2)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x +
 e)^2 + 1)) - ((C*a^2 + C*b^2)*c^2 - (B*a^2 + B*b^2)*c*d + (A*a^2 + A*b^2)*
d^2)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 +
1)))/(((a^2*b + b^3)*c^3 - (a^3 + a*b^2)*c^2*d + (a^2*b + b^3)*c*d^2 - (a^3
 + a*b^2)*d^3)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 43.03, size = 24052, normalized size = 145.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
),x)
```

```
[Out] Piecewise(((2*A*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + 2*A*d**2*log(c/d + tan(e
+ f*x))/(2*c**2*d*f + 2*d**3*f) - A*d**2*log(tan(e + f*x)**2 + 1)/(2*c**2*d
*f + 2*d**3*f) - 2*B*c*d*log(c/d + tan(e + f*x))/(2*c**2*d*f + 2*d**3*f) +
B*c*d*log(tan(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f) + 2*B*d**2*f*x/(2*c*
*2*d*f + 2*d**3*f) + 2*C*c**2*log(c/d + tan(e + f*x))/(2*c**2*d*f + 2*d**3*
f) - 2*C*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + C*d**2*log(tan(e + f*x)**2 + 1)/
(2*c**2*d*f + 2*d**3*f))/a, Eq(b, 0)), ((2*A*a*b*f*x/(2*a**2*b*f + 2*b**3*f
) + 2*A*b**2*log(a/b + tan(e + f*x))/(2*a**2*b*f + 2*b**3*f) - A*b**2*log(t
an(e + f*x)**2 + 1)/(2*a**2*b*f + 2*b**3*f) - 2*B*a*b*log(a/b + tan(e + f*x
))/(2*a**2*b*f + 2*b**3*f) + B*a*b*log(tan(e + f*x)**2 + 1)/(2*a**2*b*f + 2
*b**3*f) + 2*B*b**2*f*x/(2*a**2*b*f + 2*b**3*f) + 2*C*a**2*log(a/b + tan(e
+ f*x))/(2*a**2*b*f + 2*b**3*f) - 2*C*a*b*f*x/(2*a**2*b*f + 2*b**3*f) + C*b
**2*log(tan(e + f*x)**2 + 1)/(2*a**2*b*f + 2*b**3*f))/c, Eq(d, 0)), (I*A*c*
*2*f*x*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*
f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f
+ 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + A*c**2*f*x/(2*b*c**3*f*tan(e +
f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d*
*2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f
) + I*A*c**2/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e
 + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b
*d**3*f*tan(e + f*x) + 2*b*d**3*f) - 2*A*c*d*f*x*tan(e + f*x)/(2*b*c**3*f*t
an(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2
*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b
*d**3*f) + 2*I*A*c*d*f*x/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c*
*2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d*
*2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + I*A*d**2*f*x*tan(e + f*x)/
(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b
*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e
 + f*x) + 2*b*d**3*f) + A*d**2*f*x/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f
+ 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) -
2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - 2*A*d**2*log(c/d
 + tan(e + f*x))*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I
*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b
*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + 2*I*A*d**2*log(c/d +
tan(e + f*x))/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(
e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*
b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + A*d**2*log(tan(e + f*x)**2 + 1)*tan(e
 + f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*
x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3
*f*tan(e + f*x) + 2*b*d**3*f) - I*A*d**2*log(tan(e + f*x)**2 + 1)/(2*b*c**3
*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f
 + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) +
 2*b*d**3*f) + I*A*d**2/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**
2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**
2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + B*c**2*f*x*tan(e + f*x)/(2*
```

```
b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c*
*2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e +
f*x) + 2*b*d**3*f) - I*B*c**2*f*x/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f +
 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2
*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - B*c**2/(2*b*c**3*
f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f
+ 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) +
2*b*d**3*f) + 2*B*c*d*log(c/d + tan(e + f*x))*tan(e + f*x)/(2*b*c**3*f*tan(
e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*
c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d*
*3*f) - 2*I*B*c*d*log(c/d + tan(e + f*x))/(2*b*c**3*f*tan(e + f*x) - 2*I*b*
c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e +
f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - B*c*d*log
(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f
+ 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) -
2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + I*B*c*d*log(tan(
e + f*x)**2 + 1)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*t
an(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2
*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) - B*d**2*f*x*tan(e + f*x)/(2*b*c**3*
f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f
+ 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) +
2*b*d**3*f) + I*B*d**2*f*x/(2*b*c**3*f*tan(e + ...
```

Giac [A]
time = 0.76, size = 272, normalized size = 1.65

$$\frac{\frac{2\,(Aac-Cac+Bbc+Bad-Abd+Cbd)(fx+e)}{a^2c^2+b^2c^2+a^2d^2+b^2d^2}+\frac{(Bac-Abc+Cbc-Aad+Cad-Bbd)\log\left(\tan(fx+e)^2+1\right)}{a^2c^2+b^2c^2+a^2d^2+b^2d^2}+\frac{2\,(Ca^2b-Bab^2+Ab^3)\log(|b\tan(fx+e)+a|)}{a^2b^2c+b^4c-a^3bd-ab^3d}-\frac{2\,(Cc^2d-Bcd^2+Ad^3)\log(|d\tan(fx+e)+c|)}{bc^3d-ac^2d^2+bcd^3-ad^4}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c - C*a*c + B*b*c + B*a*d - A*b*d + C*b*d)*(f*x + e)/(a^2*c^2 +
 b^2*c^2 + a^2*d^2 + b^2*d^2) + (B*a*c - A*b*c + C*b*c - A*a*d + C*a*d - B*
b*d)*log(tan(f*x + e)^2 + 1)/(a^2*c^2 + b^2*c^2 + a^2*d^2 + b^2*d^2) + 2*(C
*a^2*b - B*a*b^2 + A*b^3)*log(abs(b*tan(f*x + e) + a))/(a^2*b^2*c + b^4*c -
 a^3*b*d - a*b^3*d) - 2*(C*c^2*d - B*c*d^2 + A*d^3)*log(abs(d*tan(f*x + e)
+ c))/(b*c^3*d - a*c^2*d^2 + b*c*d^3 - a*d^4))/f
```

Mupad [B]
time = 21.40, size = 196, normalized size = 1.19

$$\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(C\,c^2-B\,c\,d+A\,d^2\right)}{f\,\left(a\,d-b\,c\right)\,\left(c^2+d^2\right)}+\frac{\ln\left(\tan(e+f\,x)+1{}\mathrm{i}\right)\,\left(C-A+B\,1{}\mathrm{i}\right)}{2\,f\,\left(a\,c\,1{}\mathrm{i}+a\,d+b\,c-b\,d\,1{}\mathrm{i}\right)}-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(C\,a^2-B\,a\,b+A\,b^2\right)}{f\,\left(d\,a^3-c\,a^2\,b+d\,a\,b^2-c\,b^3\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A-C+B\,1{}\mathrm{i}\right)}{2\,f\,\left(a\,d-a\,c\,1{}\mathrm{i}+b\,c+b\,d\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*ta
n(e + f*x))),x)
```

```
[Out] (log(tan(e + f*x) + 1i)*(B*1i - A + C))/(2*f*(a*c*1i + a*d + b*c - b*d*1i))
 - (log(tan(e + f*x) - 1i)*(A + B*1i - C))/(2*f*(a*d - a*c*1i + b*c + b*d*1
i)) - (log(a + b*tan(e + f*x))*(A*b^2 + C*a^2 - B*a*b))/(f*(a^3*d - b^3*c -
 a^2*b*c + a*b^2*d)) + (log(c + d*tan(e + f*x))*(A*d^2 + C*c^2 - B*c*d))/(f
*(a*d - b*c)*(c^2 + d^2))
```

3.75 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=281

$$\frac{\left(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)+2ab(Bc-(A-C)d)\right)x}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)}+\frac{(2ab^3c(A-C)+2a^3bBd-a^4Cd+b^4(}{}$$

```
[Out] (a^2*(A*c+B*d-C*c)-b^2*(A*c+B*d-C*c)+2*a*b*(B*c-(A-C)*d))*x/(a^2+b^2)^2/(c^
2+d^2)+(2*a*b^3*c*(A-C)+2*a^3*b*B*d-a^4*C*d+b^4*(-A*d+B*c)-a^2*b^2*(3*A*d+B
*c-C*d))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^2/(-a*d+b*c)^2/f+d*(A*d^2-
B*c*d+C*c^2)*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^2/(c^2+d^2)/f+(-A*b^2
+a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))
```

Rubi [A]

time = 0.52, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3730, 3732, 3611}

$$\frac{x\left(a^2(Ac+Bd-cC)+2ab(Bc-d(A-C))-b^2(Ac+Bd-cC)\right)}{\left(a^2+b^2\right)^2\left(c^2+d^2\right)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}+\frac{\left(a^4(-C)d+2a^3bBd-a^2b^2(3Ad+Bc-Cd)+2ab^3c(A-C)+b^4(Bc-Ad)\right)\log(a\cos(e+fx)+b\sin(e+fx))}{f\left(a^2+b^2\right)^2(bc-ad)^2}+\frac{d\left(Ad^2-Bcd+c^2C\right)\log(c\cos(e+fx)+d\sin(e+fx))}{f\left(c^2+d^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])),x]
```

```
[Out] ((a^2*(A*c - c*C + B*d) - b^2*(A*c - c*C + B*d) + 2*a*b*(B*c - (A - C)*d))*
x)/((a^2 + b^2)^2*(c^2 + d^2)) + ((2*a*b^3*c*(A - C) + 2*a^3*b*B*d - a^4*C*
d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*d - C*d))*Log[a*Cos[e + f*x] + b*S
in[e + f*x]])/((a^2 + b^2)^2*(b*c - a*d)^2*f) + (d*(c^2*C - B*c*d + A*d^2)*
Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^2*(c^2 + d^2)*f) - (A*b^
2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
```

```
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx = -\frac{Ab^2-a(bB-aC)}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{-abc(A-C)+a}{}}{}$$

$$= \frac{(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)+2ab(Bc-(A}{(a^2+b^2)^2\,(c^2+d^2)}$$

$$= \frac{(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)+2ab(Bc-(A}{(a^2+b^2)^2\,(c^2+d^2)}$$

Mathematica [A]
time = 4.38, size = 543, normalized size = 1.93

$$\frac{-\frac{(bc-ad)\left(2aAbc-a^2Bc+b^2Bc-2abcC+a^2Ad-Ab^2d+2abBd-a^2Cd+b^2Cd+\frac{\sqrt{-b^2}\,(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)+2ab(Bc+(-A+C)d))}{b}\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)}{2(a^2+b^2)(c^2+d^2)}+\frac{(2ab^3c(-A+C)-2a^3bBd+a^4Cd+b^4(-Bc+Ad)+a^2b^2(Bc+3Ad-Cd))\log(a+b\tan(e+fx))}{(a^2+b^2)(-bc+ad)}-\frac{(bc-ad)\left(2aAbc-a^2Bc+b^2Bc-2abcC+a^2Ad-Ab^2d+2abBd-a^2Cd+b^2Cd+\frac{\sqrt{-b^2}\,(-a^2(Ac-cC+Bd)+b^2(Ac-cC+Bd)-2ab(Bc+(-A+C)d))}{b}\right)\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)}{2(a^2+b^2)(c^2+d^2)}+\frac{(a^2+b^2)d(c^2C-Bcd+Ad^2)\log(c+d\tan(e+fx))}{(bc-ad)(c^2+d^2)}-\frac{Ab^2}{a+b\tan(e+fx)}+\frac{a(bB-aC)}{a+b\tan(e+fx)}}{(a^2+b^2)\,(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])),x]
```

```
[Out] (-1/2*((b*c - a*d)*(2*a*A*b*c - a^2*B*c + b^2*B*c - 2*a*b*c*C + a^2*A*d - A
*b^2*d + 2*a*b*B*d - a^2*C*d + b^2*C*d + (Sqrt[-b^2]*(a^2*(A*c - c*C + B*d)
 - b^2*(A*c - c*C + B*d) + 2*a*b*(B*c + (-A + C)*d)))/b)*Log[Sqrt[-b^2] - b
*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) + ((2*a*b^3*c*(-A + C) - 2*a^3*b*
B*d + a^4*C*d + b^4*(-(B*c) + A*d) + a^2*b^2*(B*c + 3*A*d - C*d))*Log[a + b
*Tan[e + f*x]])/((a^2 + b^2)*(-(b*c) + a*d)) - ((b*c - a*d)*(2*a*A*b*c - a^
```

```
2*B*c + b^2*B*c - 2*a*b*c*C + a^2*A*d - A*b^2*d + 2*a*b*B*d - a^2*C*d + b^2
*C*d + (Sqrt[-b^2]*(-(a^2*(A*c - c*C + B*d)) + b^2*(A*c - c*C + B*d) - 2*a*
b*(B*c + (-A + C)*d)))/b)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/(2*(a^2 + b^2)*
(c^2 + d^2)) + ((a^2 + b^2)*d*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x
]])/((b*c - a*d)*(c^2 + d^2)) - (A*b^2)/(a + b*Tan[e + f*x]) + (a*(b*B - a*
C))/(a + b*Tan[e + f*x]))/((a^2 + b^2)*(b*c - a*d)*f)
```

Maple [A]
time = 0.84, size = 364, normalized size = 1.30

method	result
derivativedivides	$\frac{-\frac{\left(3A\,a^2b^2d-2Aa\,b^3c+A\,b^4d-2a^3bBd+B\,a^2b^2c-B\,b^4c+a^4Cd-C\,a^2b^2d+2Ca\,b^3c\right)\ln(a+b\tan(fx+e))}{(ad-bc)^2\left(a^2+b^2\right)^2}+\frac{A\,b^2-Bab+C\,a^2}{(ad-bc)\left(a^2+b^2\right)(a+b\tan(f\cdots}}{}$
default	$\frac{-\frac{\left(3A\,a^2b^2d-2Aa\,b^3c+A\,b^4d-2a^3bBd+B\,a^2b^2c-B\,b^4c+a^4Cd-C\,a^2b^2d+2Ca\,b^3c\right)\ln(a+b\tan(fx+e))}{(ad-bc)^2\left(a^2+b^2\right)^2}+\frac{A\,b^2-Bab+C\,a^2}{(ad-bc)\left(a^2+b^2\right)(a+b\tan(f\cdots}}{}$
norman	$\frac{\frac{a\left(A\,a^2c-2Aabd-A\,b^2c+B\,a^2d+2Babc-B\,b^2d-C\,a^2c+2Cabd+C\,b^2c\right)x}{\left(a^4+2a^2b^2+b^4\right)\left(c^2+d^2\right)}+\frac{A\,b^3-Ba\,b^2+C\,a^2b}{bf(ad-bc)\left(a^2+b^2\right)}+\frac{b\left(A\,a^2c-2Aabd-A\,b^2c+B\,a^2d+2Bab\cdots\right.}{\left(a^4+2a^2\cdots\right.}}{a+b\tan(fx+e)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(3*A*a^2*b^2*d-2*A*a*b^3*c+A*b^4*d-2*B*a^3*b*d+B*a^2*b^2*c-B*b^4*c+C*
a^4*d-C*a^2*b^2*d+2*C*a*b^3*c)/(a*d-b*c)^2/(a^2+b^2)^2*ln(a+b*tan(f*x+e))+(
A*b^2-B*a*b+C*a^2)/(a*d-b*c)/(a^2+b^2)/(a+b*tan(f*x+e))+(A*d^2-B*c*d+C*c^2)
*d/(a*d-b*c)^2/(c^2+d^2)*ln(c+d*tan(f*x+e))+1/(a^2+b^2)^2/(c^2+d^2)*(1/2*(-
A*a^2*d-2*A*a*b*c+A*b^2*d+B*a^2*c-2*B*a*b*d-B*b^2*c+C*a^2*d+2*C*a*b*c-C*b^2
*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c-2*A*a*b*d-A*b^2*c+B*a^2*d+2*B*a*b*c-B*b^2*d
-C*a^2*c+2*C*a*b*d+C*b^2*c)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.54, size = 525, normalized size = 1.87

$$\frac{\frac{2\left(\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)c+\left(Ba^2-2\left(A-C\right)ab-Bb^2\right)d\right)\left(fx+e\right)}{\left(a^4+2\,a^2b^2+b^4\right)c^2+\left(a^4+2\,a^2b^2+b^4\right)d^2}-\frac{2\left(\left(Ba^2b^2-2\left(A-C\right)ab^3-Bb^4\right)c+\left(Ca^4-2\,Ba^3b+\left(3\,A-C\right)a^2b^2+Ab^4\right)d\right)\log\left(b\tan\left(fx+e\right)+a\right)}{\left(a^4b^2+2\,a^2b^4+b^6\right)c^2-2\left(a^5b+2\,a^3b^3+ab^5\right)cd+\left(a^6+2\,a^4b^2+a^2b^4\right)d^2}+\frac{2\left(Cc^2d-Bcd^2+Ad^3\right)\log\left(d\tan\left(fx+e\right)+c\right)}{b^2c^4-2\,abc^3d-2\,abcd^3+a^2d^4+\left(a^2+b^2\right)c^2d^2}+\frac{\left(\left(Ba^2-2\left(A-C\right)ab-Bb^2\right)c-\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)d\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{\left(a^4+2\,a^2b^2+b^4\right)c^2+\left(a^4+2\,a^2b^2+b^4\right)d^2}-\frac{2\left(Ca^2-Bab+Ab^2\right)}{\left(a^3b+ab^3\right)c-\left(a^4+a^2b^2\right)d+\left(\left(a^2b^2+b^4\right)c-\left(a^3b+ab^3\right)d\right)\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c + (B*a^2 - 2*(A - C)*a*b -
B*b^2)*d)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*
```

```
d^2) - 2*((B*a^2*b^2 - 2*(A - C)*a*b^3 - B*b^4)*c + (C*a^4 - 2*B*a^3*b + (3
*A - C)*a^2*b^2 + A*b^4)*d)*log(b*tan(f*x + e) + a)/((a^4*b^2 + 2*a^2*b^4 +
 b^6)*c^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d + (a^6 + 2*a^4*b^2 + a^2*b^4)
*d^2) + 2*(C*c^2*d - B*c*d^2 + A*d^3)*log(d*tan(f*x + e) + c)/(b^2*c^4 - 2*
a*b*c^3*d - 2*a*b*c*d^3 + a^2*d^4 + (a^2 + b^2)*c^2*d^2) + ((B*a^2 - 2*(A -
 C)*a*b - B*b^2)*c - ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x +
 e)^2 + 1)/((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2) - 2*
(C*a^2 - B*a*b + A*b^2)/((a^3*b + a*b^3)*c - (a^4 + a^2*b^2)*d + ((a^2*b^2
+ b^4)*c - (a^3*b + a*b^3)*d)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(285) = 570$.
time = 11.32, size = 1355, normalized size = 4.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c^3 - 2*(C*a^3*b^2 - B*a^2*b^3 + A*a*
b^4)*c^2*d + 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c*d^2 - 2*(C*a^3*b^2 - B*a^2*b
^3 + A*a*b^4)*d^3 - 2*(((A - C)*a^3*b^2 + 2*B*a^2*b^3 - (A - C)*a*b^4)*c^3
- (2*(A - C)*a^4*b + 3*B*a^3*b^2 + B*a*b^4)*c^2*d + ((A - C)*a^5 + 3*(A - C
)*a^3*b^2 + 2*B*a^2*b^3)*c*d^2 + (B*a^5 - 2*(A - C)*a^4*b - B*a^3*b^2)*d^3)
*f*x + ((B*a^3*b^2 - 2*(A - C)*a^2*b^3 - B*a*b^4)*c^3 + (C*a^5 - 2*B*a^4*b
+ (3*A - C)*a^3*b^2 + A*a*b^4)*c^2*d + (B*a^3*b^2 - 2*(A - C)*a^2*b^3 - B*a
*b^4)*c*d^2 + (C*a^5 - 2*B*a^4*b + (3*A - C)*a^3*b^2 + A*a*b^4)*d^3 + ((B*a
^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c^3 + (C*a^4*b - 2*B*a^3*b^2 + (3*A - C)*
a^2*b^3 + A*b^5)*c^2*d + (B*a^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c*d^2 + (C*a
^4*b - 2*B*a^3*b^2 + (3*A - C)*a^2*b^3 + A*b^5)*d^3)*tan(f*x + e))*log((b^2
*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - ((C*a^5
 + 2*C*a^3*b^2 + C*a*b^4)*c^2*d - (B*a^5 + 2*B*a^3*b^2 + B*a*b^4)*c*d^2 + (
A*a^5 + 2*A*a^3*b^2 + A*a*b^4)*d^3 + ((C*a^4*b + 2*C*a^2*b^3 + C*b^5)*c^2*d
 - (B*a^4*b + 2*B*a^2*b^3 + B*b^5)*c*d^2 + (A*a^4*b + 2*A*a^2*b^3 + A*b^5)*
d^3)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan
(f*x + e)^2 + 1)) - 2*((C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c^3 - (C*a^4*b - B
*a^3*b^2 + A*a^2*b^3)*c^2*d + (C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c*d^2 - (C*
a^4*b - B*a^3*b^2 + A*a^2*b^3)*d^3 + (((A - C)*a^2*b^3 + 2*B*a*b^4 - (A - C
)*b^5)*c^3 - (2*(A - C)*a^3*b^2 + 3*B*a^2*b^3 + B*b^5)*c^2*d + ((A - C)*a^4
*b + 3*(A - C)*a^2*b^3 + 2*B*a*b^4)*c*d^2 + (B*a^4*b - 2*(A - C)*a^3*b^2 -
B*a^2*b^3)*d^3)*f*x)*tan(f*x + e))/(((a^4*b^3 + 2*a^2*b^5 + b^7)*c^4 - 2*(a
^5*b^2 + 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*c
^2*d^2 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c*d^3 + (a^6*b + 2*a^4*b^3 + a^2*b
^5)*d^4)*f*tan(f*x + e) + ((a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b + 2
```

```
*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c^2*d^2 -
 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c*d^3 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d^4)*f
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e)),x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(285) = 570.
time = 0.89, size = 846, normalized size = 3.01

$$\frac{\frac{2\,(Aa^2c-Ca^2c+2\,Babc-Ab^2c+Cb^2c+Ba^2d-2\,Aabd+2\,Cabd-Bb^2d)(fx+e)}{a^4c^2+2\,a^2b^2c^2+b^4c^2+a^4d^2+2\,a^2b^2d^2+b^4d^2}+\frac{(Ba^2c-2\,Aabc+2\,Cabc-Bb^2c-Aa^2d+Ca^2d-2\,Babd+Ab^2d-Cb^2d)\log\left(\tan(fx+e)^2+1\right)}{a^4c^2+2\,a^2b^2c^2+b^4c^2+a^4d^2+2\,a^2b^2d^2+b^4d^2}-\frac{2\,(Ba^2b^3c-2\,Aab^4c+2\,Cab^4c-Bb^5c+Ca^4bd-2\,Ba^3b^2d+3\,Aa^2b^3d-Ca^2b^3d+Ab^5d)\log(|b\tan(fx+e)+a|)}{a^4b^3c^2+2\,a^2b^5c^2+b^7c^2-2\,a^5b^2cd-4\,a^3b^4cd-2\,ab^6cd+a^6bd^2+2\,a^4b^3d^2+a^2b^5d^2}+\frac{2\,(Cc^2d^2-Bcd^3+Ad^4)\log(|d\tan(fx+e)+c|)}{b^2c^4d-2\,abc^3d^2+a^2c^2d^3+b^2c^2d^3-2\,abcd^4+a^2d^5}+\frac{2\,(Ba^2b^3c\tan(fx+e)-2\,Aab^4c\tan(fx+e)+2\,Cab^4c\tan(fx+e)-Bb^5c\tan(fx+e)+Ca^4bd\tan(fx+e)-2\,Ba^3b^2d\tan(fx+e)+3\,Aa^2b^3d\tan(fx+e)-Ca^2b^3d\tan(fx+e)+Ab^5d\tan(fx+e)-Ca^4bc+2\,Ba^3b^2c-3\,Aa^2b^3c+Ca^2b^3c-Ab^5c+2\,Ca^5d-3\,Ba^4bd+4\,Aa^3b^2d-Ba^2b^3d+2\,Aab^4d)}{(a^4b^2c^2+2\,a^2b^4c^2+b^6c^2-2\,a^5bcd-4\,a^3b^3cd-2\,ab^5cd+a^6d^2+2\,a^4b^2d^2+a^2b^4d^2)(b\tan(fx+e)+a)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c - C*a^2*c + 2*B*a*b*c - A*b^2*c + C*b^2*c + B*a^2*d - 2*A*a
*b*d + 2*C*a*b*d - B*b^2*d)*(f*x + e)/(a^4*c^2 + 2*a^2*b^2*c^2 + b^4*c^2 +
a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + (B*a^2*c - 2*A*a*b*c + 2*C*a*b*c - B*b
^2*c - A*a^2*d + C*a^2*d - 2*B*a*b*d + A*b^2*d - C*b^2*d)*log(tan(f*x + e)^
2 + 1)/(a^4*c^2 + 2*a^2*b^2*c^2 + b^4*c^2 + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d
^2) - 2*(B*a^2*b^3*c - 2*A*a*b^4*c + 2*C*a*b^4*c - B*b^5*c + C*a^4*b*d - 2*
B*a^3*b^2*d + 3*A*a^2*b^3*d - C*a^2*b^3*d + A*b^5*d)*log(abs(b*tan(f*x + e)
 + a))/(a^4*b^3*c^2 + 2*a^2*b^5*c^2 + b^7*c^2 - 2*a^5*b^2*c*d - 4*a^3*b^4*c
*d - 2*a*b^6*c*d + a^6*b*d^2 + 2*a^4*b^3*d^2 + a^2*b^5*d^2) + 2*(C*c^2*d^2
- B*c*d^3 + A*d^4)*log(abs(d*tan(f*x + e) + c))/(b^2*c^4*d - 2*a*b*c^3*d^2
+ a^2*c^2*d^3 + b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 2*(B*a^2*b^3*c*tan(f
*x + e) - 2*A*a*b^4*c*tan(f*x + e) + 2*C*a*b^4*c*tan(f*x + e) - B*b^5*c*tan
(f*x + e) + C*a^4*b*d*tan(f*x + e) - 2*B*a^3*b^2*d*tan(f*x + e) + 3*A*a^2*b
^3*d*tan(f*x + e) - C*a^2*b^3*d*tan(f*x + e) + A*b^5*d*tan(f*x + e) - C*a^4
*b*c + 2*B*a^3*b^2*c - 3*A*a^2*b^3*c + C*a^2*b^3*c - A*b^5*c + 2*C*a^5*d -
3*B*a^4*b*d + 4*A*a^3*b^2*d - B*a^2*b^3*d + 2*A*a*b^4*d)/((a^4*b^2*c^2 + 2*
a^2*b^4*c^2 + b^6*c^2 - 2*a^5*b*c*d - 4*a^3*b^3*c*d - 2*a*b^5*c*d + a^6*d^2
 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(b*tan(f*x + e) + a)))/f
```

Mupad [B]
time = 63.66, size = 393, normalized size = 1.40

$$\frac{\ln\left(\mathrm{tan}(e+f\,x)-\mathrm{i}\right)\,\left(B-A\,1\mathrm{i}+C\,1\mathrm{i}\right)}{2\,f\,\left(a^2\,c-b^2\,c-2\,a\,b\,d+a^2\,d\,1\mathrm{i}-b^2\,d\,1\mathrm{i}+a\,b\,c\,2\mathrm{i}\right)}-\frac{\ln\left(\mathrm{tan}(e+f\,x)+1\mathrm{i}\right)\,\left(A\,1\mathrm{i}+B-C\,1\mathrm{i}\right)}{2\,f\,\left(b^2\,c-a^2\,c+2\,a\,b\,d+a^2\,d\,1\mathrm{i}-b^2\,d\,1\mathrm{i}+a\,b\,c\,2\mathrm{i}\right)}-\frac{\ln\left(a+b\,\mathrm{tan}(e+f\,x)\right)\,\left(C\,d\,a^4-2\,B\,d\,a^3\,b+\left(3\,A\,d+B\,c-C\,d\right)\,a^2\,b^2+\left(2\,C\,c-2\,A\,c\right)\,a\,b^3+\left(A\,d-B\,c\right)\,b^4\right)}{f\,\left(a^6\,d^2-2\,a^5\,b\,c\,d+a^4\,b^2\,c^2+2\,a^4\,b^2\,d^2-4\,a^3\,b^3\,c\,d+2\,a^2\,b^4\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d+b^6\,c^2\right)}+\frac{C\,a^2-B\,a\,b+A\,b^2}{f\,\left(a\,d-b\,c\right)\,\left(a^2+b^2\right)\,\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}+\frac{d\,\ln\left(c+d\,\mathrm{tan}(e+f\,x)\right)\,\left(C\,c^2-B\,c\,d+A\,d^2\right)}{f\,{\left(a\,d-b\,c\right)}^2\,\left(c^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d*
tan(e + f*x))),x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(B - A*1i + C*1i))/(2*f*(a^2*c + a^2*d*1i - b^2*c -
 b^2*d*1i + a*b*c*2i - 2*a*b*d)) - (log(tan(e + f*x) + 1i)*(A*1i + B - C*1i
))/(2*f*(a^2*d*1i - a^2*c + b^2*c - b^2*d*1i + a*b*c*2i + 2*a*b*d)) - (log(
a + b*tan(e + f*x))*(b^4*(A*d - B*c) + a^2*b^2*(3*A*d + B*c - C*d) + C*a^4*
d - a*b^3*(2*A*c - 2*C*c) - 2*B*a^3*b*d))/(f*(a^6*d^2 + b^6*c^2 + 2*a^2*b^4
*c^2 + a^4*b^2*c^2 + a^2*b^4*d^2 + 2*a^4*b^2*d^2 - 2*a*b^5*c*d - 2*a^5*b*c*
d - 4*a^3*b^3*c*d)) + (A*b^2 + C*a^2 - B*a*b)/(f*(a*d - b*c)*(a^2 + b^2)*(a
 + b*tan(e + f*x))) + (d*log(c + d*tan(e + f*x))*(A*d^2 + C*c^2 - B*c*d))/(
f*(a*d - b*c)^2*(c^2 + d^2))
```

3.76 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$

Optimal. Leaf size=477

$$\frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)+3a^2b(Bc-(A-C)d)-b^3(Bc-(A-C)d))\,x}{(a^2+b^2)^3\,(c^2+d^2)}+\frac{(3ab^5Bc^2-3}{}$$

```
[Out] (a^3*(A*c+B*d-C*c)-3*a*b^2*(A*c+B*d-C*c)+3*a^2*b*(B*c-(A-C)*d)-b^3*(B*c-(A-
C)*d))*x/(a^2+b^2)^3/(c^2+d^2)+(3*a*b^5*B*c^2-3*a^5*b*B*d^2+a^6*C*d^2+3*a^4
*b^2*d*(2*A*d+B*c-C*d)+b^6*(c*(-B*d+C*c)-A*(c^2-d^2))-a^3*b^3*(8*c*(A-C)*d+
B*(c^2-d^2))-3*a^2*b^4*(c*(2*B*d+C*c)-A*(c^2+d^2)))*ln(a*cos(f*x+e)+b*sin(f
*x+e))/(a^2+b^2)^3/(-a*d+b*c)^3/f-d^2*(A*d^2-B*c*d+C*c^2)*ln(c*cos(f*x+e)+d
*sin(f*x+e))/(-a*d+b*c)^3/(c^2+d^2)/f+1/2*(-A*b^2+a*(B*b-C*a))/(a^2+b^2)/(-
a*d+b*c)/f/(a+b*tan(f*x+e))^2+(-2*a*b^3*c*(A-C)-2*a^3*b*B*d+a^4*C*d-b^4*(-A
*d+B*c)+a^2*b^2*(3*A*d+B*c-C*d))/(a^2+b^2)^2/(-a*d+b*c)^2/f/(a+b*tan(f*x+e)
)
```

Rubi [A]
time = 1.17, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3730, 3732, 3611}

$$-\frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2}+\frac{x(a^3(Ac+Bd-cC)+3a^2b(Bc-d(A-C))-3ab^2(Ac+Bd-cC)-b^3(Bc-d(A-C)))}{(a^2+b^2)^3(c^2+d^2)}-\frac{a^4(-C)d+2a^3bBd-a^2b^2(3Ad+Bc-Cd)+2ab^3c(A-C)+b^4(Bc-Ad)}{f(a^2+b^2)^2(bc-ad)^2(a+b\tan(e+fx))}+\frac{(a^6Cd^2-3a^5bBd^2+3a^4b^2d(2Ad+Bc-Cd)-a^3b^3(8cd(A-C)+B(c^2-d^2))-3a^2b^4(c(2Bd+cC)-A(c^2+d^2))+3ab^5Bc^2+b^6(c(cC-Bd)-A(c^2-d^2)))\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)^3(bc-ad)^3}-\frac{d^2(Ad^2-Bcd+c^2C)\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])),x]
```

```
[Out] ((a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c - c*C + B*d) + 3*a^2*b*(B*c - (A - C
)*d) - b^3*(B*c - (A - C)*d))*x)/((a^2 + b^2)^3*(c^2 + d^2)) + ((3*a*b^5*B*
c^2 - 3*a^5*b*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*d*(B*c + 2*A*d - C*d) + b^6*(c*
(c*C - B*d) - A*(c^2 - d^2)) - a^3*b^3*(8*c*(A - C)*d + B*(c^2 - d^2)) - 3*
a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + d^2)))*Log[a*Cos[e + f*x] + b*Sin[e + f
*x]])/((a^2 + b^2)^3*(b*c - a*d)^3*f) - (d^2*(c^2*C - B*c*d + A*d^2)*Log[c*
Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)*f) - (A*b^2 - a*
(b*B - a*C))/(2*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2) - (2*a*b^
3*c*(A - C) + 2*a^3*b*B*d - a^4*C*d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*
d - C*d))/((a^2 + b^2)^2*(b*c - a*d)^2*f*(a + b*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx = -\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2} - \frac{\int \frac{-2(abc(A-}{}}{}$$

$$= -\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2} - \frac{2ab^3c(A-}{}$$

$$= \frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)+3a^2b(Bc-(}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)}$$

$$= \frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)+3a^2b(Bc-(}{\left(a^2+b^2\right)^3\left(c^2+d^2\right)}$$

Mathematica [A]
time = 7.95, size = 898, normalized size = 1.88

$$-\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2} - \cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(
c + d*Tan[e + f*x])),x]
```

```
[Out] -1/2*(A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]
)^2) - (-((-((b*(b*c - a*d)^2*(3*a^2*A*b*c - A*b^3*c - a^3*B*c + 3*a*b^2*B*
c - 3*a^2*b*c*C + b^3*c*C + a^3*A*d - 3*a*A*b^2*d + 3*a^2*b*B*d - b^3*B*d -
 a^3*C*d + 3*a*b^2*C*d + (Sqrt[-b^2]*(a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c
- c*C + B*d) + 3*a^2*b*(B*c - (A - C)*d) - b^3*(B*c - (A - C)*d)))/b)*Log[S
qrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2))) + (2*b*(3*a*b^5*B*c
^2 - 3*a^5*b*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*d*(B*c + 2*A*d - C*d) + b^6*(c*(
c*C - B*d) - A*(c^2 - d^2)) - a^3*b^3*(8*c*(A - C)*d + B*(c^2 - d^2)) - 3*a
^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + d^2)))*Log[a + b*Tan[e + f*x]])/((a^2 +
b^2)*(b*c - a*d)) - (b*(b*c - a*d)^2*(3*a^2*A*b*c - A*b^3*c - a^3*B*c + 3*a
*b^2*B*c - 3*a^2*b*c*C + b^3*c*C + a^3*A*d - 3*a*A*b^2*d + 3*a^2*b*B*d - b^
3*B*d - a^3*C*d + 3*a*b^2*C*d - (Sqrt[-b^2]*(a^3*(A*c - c*C + B*d) - 3*a*b^
2*(A*c - c*C + B*d) + 3*a^2*b*(B*c - (A - C)*d) - b^3*(B*c - (A - C)*d)))/b
)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 +
 b^2)^2*d^2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(
c^2 + d^2)))/(b*(a^2 + b^2)*(b*c - a*d)*f)) - (-(a*(-2*a*(A*b^2 - a*(b*B -
a*C))*d + 2*b*(A*b - a*B - b*C)*(b*c - a*d))) - 2*b^2*(a*b*c*(A - C) - a^2*
A*d + b^2*(B*c - A*d)))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])))/(
2*(a^2 + b^2)*(b*c - a*d))
```

Maple [A]
time = 1.60, size = 647, normalized size = 1.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^3/(c^2+d^2)*(1/2*(-A*a^3*d-3*A*a^2*b*c+3*A*a*b^2*d+A*b^3*c
+B*a^3*c-3*B*a^2*b*d-3*B*a*b^2*c+B*b^3*d+C*a^3*d+3*C*a^2*b*c-3*C*a*b^2*d-C*
b^3*c)*ln(1+tan(f*x+e)^2)+(A*a^3*c-3*A*a^2*b*d-3*A*a*b^2*c+A*b^3*d+B*a^3*d+
3*B*a^2*b*c-3*B*a*b^2*d-B*b^3*c-C*a^3*c+3*C*a^2*b*d+3*C*a*b^2*c-C*b^3*d)*ar
ctan(tan(f*x+e)))+(3*A*a^2*b^2*d-2*A*a*b^3*c+A*b^4*d-2*B*a^3*b*d+B*a^2*b^2*
c-B*b^4*c+C*a^4*d-C*a^2*b^2*d+2*C*a*b^3*c)/(a*d-b*c)^2/(a^2+b^2)^2/(a+b*tan
(f*x+e))-(6*A*a^4*b^2*d^2-8*A*a^3*b^3*c*d+3*A*a^2*b^4*c^2+3*A*a^2*b^4*d^2-A
*b^6*c^2+A*b^6*d^2-3*B*a^5*b*d^2+3*B*a^4*b^2*c*d-B*a^3*b^3*c^2+B*a^3*b^3*d^
2-6*B*a^2*b^4*c*d+3*B*a*b^5*c^2-B*b^6*c*d+C*a^6*d^2-3*C*a^4*b^2*d^2+8*C*a^3
*b^3*c*d-3*C*a^2*b^4*c^2+C*b^6*c^2)/(a*d-b*c)^3/(a^2+b^2)^3*ln(a+b*tan(f*x+
e))+1/2*(A*b^2-B*a*b+C*a^2)/(a*d-b*c)/(a^2+b^2)/(a+b*tan(f*x+e))^2+d^2*(A*d
^2-B*c*d+C*c^2)/(c^2+d^2)/(a*d-b*c)^3*ln(c+d*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(481) = 962$.

time = 0.63, size = 1103, normalized size = 2.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c + (B*a^3 - 3*
(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d)*(f*x + e)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*c^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2) - 2*((B*a^3*
b^3 - 3*(A - C)*a^2*b^4 - 3*B*a*b^5 + (A - C)*b^6)*c^2 - (3*B*a^4*b^2 - 8*(
A - C)*a^3*b^3 - 6*B*a^2*b^4 - B*b^6)*c*d - (C*a^6 - 3*B*a^5*b + 3*(2*A - C
)*a^4*b^2 + B*a^3*b^3 + 3*A*a^2*b^4 + A*b^6)*d^2)*log(b*tan(f*x + e) + a)/(
(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*c^3 - 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^
3*b^6 + a*b^8)*c^2*d + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*c*d^2 -
(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*d^3) - 2*(C*c^2*d^2 - B*c*d^3 + A*d
^4)*log(d*tan(f*x + e) + c)/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c*d^4 - a^3*
d^5 + (3*a^2*b + b^3)*c^3*d^2 - (a^3 + 3*a*b^2)*c^2*d^3) + ((B*a^3 - 3*(A -
 C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c - ((A - C)*a^3 + 3*B*a^2*b - 3*(A -
C)*a*b^2 - B*b^3)*d)*log(tan(f*x + e)^2 + 1)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*c^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2) - ((C*a^4*b - 3*B*a^3
*b^2 + (5*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c - (3*C*a^5 - 5*B*a^4*b + (7
*A - C)*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4)*d - 2*((B*a^2*b^3 - 2*(A - C)*a*b^
4 - B*b^5)*c + (C*a^4*b - 2*B*a^3*b^2 + (3*A - C)*a^2*b^3 + A*b^5)*d)*tan(f
*x + e))/((a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*c^2 - 2*(a^7*b + 2*a^5*b^3 + a^3*
b^5)*c*d + (a^8 + 2*a^6*b^2 + a^4*b^4)*d^2 + ((a^4*b^4 + 2*a^2*b^6 + b^8)*c
^2 - 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*c*d + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*
d^2)*tan(f*x + e)^2 + 2*((a^5*b^3 + 2*a^3*b^5 + a*b^7)*c^2 - 2*(a^6*b^2 + 2
*a^4*b^4 + a^2*b^6)*c*d + (a^7*b + 2*a^5*b^3 + a^3*b^5)*d^2)*tan(f*x + e)))
/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3657 vs. $2(481) = 962$.
time = 12.07, size = 3657, normalized size = 7.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] -1/2*((3*C*a^4*b^4 - 5*B*a^3*b^5 + (7*A - 3*C)*a^2*b^6 + B*a*b^7 + A*b^8)*c
^4 - 4*(2*C*a^5*b^3 - 3*B*a^4*b^4 + (4*A - C)*a^3*b^5 + A*a*b^7)*c^3*d + (5
*C*a^6*b^2 - 7*B*a^5*b^3 + (9*A + 2*C)*a^4*b^4 - 6*B*a^3*b^5 + (10*A - 3*C)
```

*a^2*b^6 + B*a*b^7 + A*b^8)*c^2*d^2 - 4*(2*C*a^5*b^3 - 3*B*a^4*b^4 + (4*A - C)*a^3*b^5 + A*a*b^7)*c*d^3 + (5*C*a^6*b^2 - 7*B*a^5*b^3 + (9*A - C)*a^4*b^4 - B*a^3*b^5 + 3*A*a^2*b^6)*d^4 - 2*(((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*a^2*b^6)*c^4 - (3*(A - C)*a^6*b^2 + 8*B*a^5*b^3 - 6*(A - C)*a^4*b^4 - (A - C)*a^2*b^6)*c^3*d + 3*((A - C)*a^7*b + 2*B*a^6*b^2 + 2*B*a^4*b^4 - (A - C)*a^3*b^5)*c^2*d^2 - ((A - C)*a^8 + 6*(A - C)*a^6*b^2 + 8*B*a^5*b^3 - 3*(A - C)*a^4*b^4)*c*d^3 - (B*a^8 - 3*(A - C)*a^7*b - 3*B*a^6*b^2 + (A - C)*a^5*b^3)*d^4)*f*x - ((C*a^4*b^4 - 3*B*a^3*b^5 + 5*(A - C)*a^2*b^6 + 3*B*a*b^7 - A*b^8)*c^4 - 4*(C*a^5*b^3 - 2*B*a^4*b^4 + (3*A - 2*C)*a^3*b^5 + B*a^2*b^6)*c^3*d + (3*C*a^6*b^2 - 5*B*a^5*b^3 + (7*A - 2*C)*a^4*b^4 - 2*B*a^3*b^5 + (6*A - 5*C)*a^2*b^6 + 3*B*a*b^7 - A*b^8)*c^2*d^2 - 4*(C*a^5*b^3 - 2*B*a^4*b^4 + (3*A - 2*C)*a^3*b^5 + B*a^2*b^6)*c*d^3 + (3*C*a^6*b^2 - 5*B*a^5*b^3 + (7*A - 3*C)*a^4*b^4 + B*a^3*b^5 + A*a^2*b^6)*d^4 + 2*(((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8)*c^4 - (3*(A - C)*a^4*b^4 + 8*B*a^3*b^5 - 6*(A - C)*a^2*b^6 - (A - C)*b^8)*c^3*d + 3*((A - C)*a^5*b^3 + 2*B*a^4*b^4 + 2*B*a^2*b^6 - (A - C)*a*b^7)*c^2*d^2 - ((A - C)*a^6*b^2 + 6*(A - C)*a^4*b^4 + 8*B*a^3*b^5 - 3*(A - C)*a^2*b^6)*c*d^3 - (B*a^6*b^2 - 3*(A - C)*a^5*b^3 - 3*B*a^4*b^4 + (A - C)*a^3*b^5)*d^4)*f*x)*tan(f*x + e)^2 + ((B*a^5*b^3 - 3*(A - C)*a^4*b^4 - 3*B*a^3*b^5 + (A - C)*a^2*b^6)*c^4 - (3*B*a^6*b^2 - 8*(A - C)*a^5*b^3 - 6*B*a^4*b^4 - B*a^2*b^6)*c^3*d - (C*a^8 - 3*B*a^7*b + 3*(2*A - C)*a^6*b^2 + 3*(2*A - C)*a^4*b^4 + 3*B*a^3*b^5 + C*a^2*b^6)*c^2*d^2 - (3*B*a^6*b^2 - 8*(A - C)*a^5*b^3 - 6*B*a^4*b^4 - B*a^2*b^6)*c*d^3 - (C*a^8 - 3*B*a^7*b + 3*(2*A - C)*a^6*b^2 + B*a^5*b^3 + 3*A*a^4*b^4 + A*a^2*b^6)*d^4 + ((B*a^3*b^5 - 3*(A - C)*a^2*b^6 - 3*B*a*b^7 + (A - C)*b^8)*c^4 - (3*B*a^4*b^4 - 8*(A - C)*a^3*b^5 - 6*B*a^2*b^6 - B*b^8)*c^3*d - (C*a^6*b^2 - 3*B*a^5*b^3 + 3*(2*A - C)*a^4*b^4 + 3*(2*A - C)*a^2*b^6 + 3*B*a*b^7 + C*b^8)*c^2*d^2 - (3*B*a^4*b^4 - 8*(A - C)*a^3*b^5 - 6*B*a^2*b^6 - B*b^8)*c*d^3 - (C*a^6*b^2 - 3*B*a^5*b^3 + 3*(2*A - C)*a^4*b^4 + B*a^3*b^5 + 3*A*a^2*b^6 + A*b^8)*d^4)*tan(f*x + e)^2 + 2*((B*a^4*b^4 - 3*(A - C)*a^3*b^5 - 3*B*a^2*b^6 + (A - C)*a*b^7)*c^4 - (3*B*a^5*b^3 - 8*(A - C)*a^4*b^4 - 6*B*a^3*b^5 - B*a*b^7)*c^3*d - (C*a^7*b - 3*B*a^6*b^2 + 3*(2*A - C)*a^5*b^3 + 3*(2*A - C)*a^3*b^5 + 3*B*a^2*b^6 + C*a*b^7)*c^2*d^2 - (3*B*a^5*b^3 - 8*(A - C)*a^4*b^4 - 6*B*a^3*b^5 - B*a*b^7)*c*d^3 - (C*a^7*b - 3*B*a^6*b^2 + 3*(2*A - C)*a^5*b^3 + B*a^4*b^4 + 3*A*a^3*b^5 + A*a*b^7)*d^4)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) + ((C*a^8 + 3*C*a^6*b^2 + 3*C*a^4*b^4 + C*a^2*b^6)*c^2*d^2 - (B*a^8 + 3*B*a^6*b^2 + 3*B*a^4*b^4 + B*a^2*b^6)*c*d^3 + (A*a^8 + 3*A*a^6*b^2 + 3*A*a^4*b^4 + A*a^2*b^6)*d^4 + ((C*a^6*b^2 + 3*C*a^4*b^4 + 3*C*a^2*b^6 + C*b^8)*c^2*d^2 - (B*a^6*b^2 + 3*B*a^4*b^4 + 3*B*a^2*b^6 + B*b^8)*c*d^3 + (A*a^6*b^2 + 3*A*a^4*b^4 + 3*A*a^2*b^6 + A*b^8)*d^4)*tan(f*x + e)^2 + 2*((C*a^7*b + 3*C*a^5*b^3 + 3*C*a^3*b^5 + C*a*b^7)*c^2*d^2 - (B*a^7*b + 3*B*a^5*b^3 + 3*B*a^3*b^5 + B*a*b^7)*c*d^3 + (A*a^7*b + 3*A*a^5*b^3 + 3*A*a^3*b^5 + A*a*b^7)*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*((C*a^5*b^3 - 2*B*a^4*b^4 + 3*(A - C)*a^3*b^5 + 3*B*a^2*b^6 - (3*A - 2*C)*a*b^7 - B*b^8)*c^4 - (3*C*a^6*b^2 - 5*B*a^5*b^3 + (7*A - 6

```
*C)*a^4*b^4 + 6*B*a^3*b^5 - 3*(2*A - C)*a^2*b^6 - B*a*b^7 - A*b^8)*c^3*d +
(2*C*a^7*b - 3*B*a^6*b^2 + 2*(2*A - C)*a^5*b^3 + B*a^4*b^4 - 2*C*a^3*b^5 +
3*B*a^2*b^6 - 2*(2*A - C)*a*b^7 - B*b^8)*c^2*d^2 - (3*C*a^6*b^2 - 5*B*a^5*b
^3 + (7*A - 6*C)*a^4*b^4 + 6*B*a^3*b^5 - 3*(2*A - C)*a^2*b^6 - B*a*b^7 - A*
b^8)*c*d^3 + (2*C*a^7*b - 3*B*a^6*b^2 + (4*A - 3*C)*a^5*b^3 + 3*B*a^4*b^4 -
 (3*A - C)*a^3*b^5 - A*a*b^7)*d^4 + 2*(((A - C)*a^4*b^4 + 3*B*a^3*b^5 - 3*(
A - C)*a^2*b^6 - B*a*b^7)*c^4 - (3*(A - C)*a^5*b^3 + 8*B*a^4*b^4 - 6*(A - C
)*a^3*b^5 - (A - C)*a*b^7)*c^3*d + 3*((A - C)*a^6*b^2 + 2*B*a^5*b^3 + 2*B*a
^3*b^5 - (A - C)*a^2*b^6)*c^2*d^2 - ((A - C)*a^7*b + 6*(A - C)*a^5*b^3 + 8*
B*a^4*b^4 - 3*(A - C)*a^3*b^5)*c*d^3 - (B*a^7*b - 3*(A - C)*a^6*b^2 - 3*B*a
^5*b^3 + (A - C)*a^4*b^4)*d^4)*f*x)*tan(f*x + e))/(((a^6*b^5 + 3*a^4*b^7 +
3*a^2*b^9 + b^11)*c^5 - 3*(a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*c^4*d
+ (3*a^8*b^3 + 10*a^6*b^5 + 12*a^4*b^7 + 6*a^2*b^9 + b^11)*c^3*d^2 - (a^9*b
^2 + 6*a^7*b^4 + 12*a^5*b^6 + 10*a^3*b^8 + 3*a*b^10)*c^2*d^3 + 3*(a^8*b^3 +
 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*c*d^4 - (a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6
+ a^3*b^8)*d^5)*f*tan(f*x + e)^2 + 2*((a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*
b^10)*c^5 - 3*(a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*b^9)*c^4*d + (3*a^9*b^
2 + 10*a^7*b^4 + 12*a^5*b^6 + 6*a^3*b^8 + a*b^10)*c^3*d^2 - (a^10*b + 6*a^8
*b^3 + 12*a^6*b^5 + 10*a^4*b^7 + 3*a^2*b^9)*c^2...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**3/(c+d*tan(f*x
+e)),x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2127 vs. $2(481) = 962$.
time = 1.22, size = 2127, normalized size = 4.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c - C*a^3*c + 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c - B*b^3
*c + B*a^3*d - 3*A*a^2*b*d + 3*C*a^2*b*d - 3*B*a*b^2*d + A*b^3*d - C*b^3*d)
*(f*x + e)/(a^6*c^2 + 3*a^4*b^2*c^2 + 3*a^2*b^4*c^2 + b^6*c^2 + a^6*d^2 + 3
*a^4*b^2*d^2 + 3*a^2*b^4*d^2 + b^6*d^2) + (B*a^3*c - 3*A*a^2*b*c + 3*C*a^2*
b*c - 3*B*a*b^2*c + A*b^3*c - C*b^3*c - A*a^3*d + C*a^3*d - 3*B*a^2*b*d + 3
```

```
*A*a*b^2*d - 3*C*a*b^2*d + B*b^3*d)*log(tan(f*x + e)^2 + 1)/(a^6*c^2 + 3*a^
4*b^2*c^2 + 3*a^2*b^4*c^2 + b^6*c^2 + a^6*d^2 + 3*a^4*b^2*d^2 + 3*a^2*b^4*d
^2 + b^6*d^2) - 2*(B*a^3*b^4*c^2 - 3*A*a^2*b^5*c^2 + 3*C*a^2*b^5*c^2 - 3*B*
a*b^6*c^2 + A*b^7*c^2 - C*b^7*c^2 - 3*B*a^4*b^3*c*d + 8*A*a^3*b^4*c*d - 8*C
*a^3*b^4*c*d + 6*B*a^2*b^5*c*d + B*b^7*c*d - C*a^6*b*d^2 + 3*B*a^5*b^2*d^2
- 6*A*a^4*b^3*d^2 + 3*C*a^4*b^3*d^2 - B*a^3*b^4*d^2 - 3*A*a^2*b^5*d^2 - A*b
^7*d^2)*log(abs(b*tan(f*x + e) + a))/(a^6*b^4*c^3 + 3*a^4*b^6*c^3 + 3*a^2*b
^8*c^3 + b^10*c^3 - 3*a^7*b^3*c^2*d - 9*a^5*b^5*c^2*d - 9*a^3*b^7*c^2*d - 3
*a*b^9*c^2*d + 3*a^8*b^2*c*d^2 + 9*a^6*b^4*c*d^2 + 9*a^4*b^6*c*d^2 + 3*a^2*
b^8*c*d^2 - a^9*b*d^3 - 3*a^7*b^3*d^3 - 3*a^5*b^5*d^3 - a^3*b^7*d^3) - 2*(C
*c^2*d^3 - B*c*d^4 + A*d^5)*log(abs(d*tan(f*x + e) + c))/(b^3*c^5*d - 3*a*b
^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + b^3*c^3*d^3 - a^3*c^2*d^4 - 3*a*b^2*c^2*d^4
+ 3*a^2*b*c*d^5 - a^3*d^6) + (3*B*a^3*b^5*c^2*tan(f*x + e)^2 - 9*A*a^2*b^6*
c^2*tan(f*x + e)^2 + 9*C*a^2*b^6*c^2*tan(f*x + e)^2 - 9*B*a*b^7*c^2*tan(f*x
 + e)^2 + 3*A*b^8*c^2*tan(f*x + e)^2 - 3*C*b^8*c^2*tan(f*x + e)^2 - 9*B*a^4
*b^4*c*d*tan(f*x + e)^2 + 24*A*a^3*b^5*c*d*tan(f*x + e)^2 - 24*C*a^3*b^5*c*
d*tan(f*x + e)^2 + 18*B*a^2*b^6*c*d*tan(f*x + e)^2 + 3*B*b^8*c*d*tan(f*x +
e)^2 - 3*C*a^6*b^2*d^2*tan(f*x + e)^2 + 9*B*a^5*b^3*d^2*tan(f*x + e)^2 - 18
*A*a^4*b^4*d^2*tan(f*x + e)^2 + 9*C*a^4*b^4*d^2*tan(f*x + e)^2 - 3*B*a^3*b^
5*d^2*tan(f*x + e)^2 - 9*A*a^2*b^6*d^2*tan(f*x + e)^2 - 3*A*b^8*d^2*tan(f*x
 + e)^2 + 8*B*a^4*b^4*c^2*tan(f*x + e) - 22*A*a^3*b^5*c^2*tan(f*x + e) + 22
*C*a^3*b^5*c^2*tan(f*x + e) - 18*B*a^2*b^6*c^2*tan(f*x + e) + 2*A*a*b^7*c^2
*tan(f*x + e) - 2*C*a*b^7*c^2*tan(f*x + e) - 2*B*b^8*c^2*tan(f*x + e) + 2*C
*a^6*b^2*c*d*tan(f*x + e) - 24*B*a^5*b^3*c*d*tan(f*x + e) + 58*A*a^4*b^4*c*
d*tan(f*x + e) - 52*C*a^4*b^4*c*d*tan(f*x + e) + 32*B*a^3*b^5*c*d*tan(f*x +
 e) + 12*A*a^2*b^6*c*d*tan(f*x + e) - 6*C*a^2*b^6*c*d*tan(f*x + e) + 8*B*a*
b^7*c*d*tan(f*x + e) + 2*A*b^8*c*d*tan(f*x + e) - 8*C*a^7*b*d^2*tan(f*x + e
) + 22*B*a^6*b^2*d^2*tan(f*x + e) - 42*A*a^5*b^3*d^2*tan(f*x + e) + 18*C*a^
5*b^3*d^2*tan(f*x + e) - 2*B*a^4*b^4*d^2*tan(f*x + e) - 26*A*a^3*b^5*d^2*ta
n(f*x + e) + 2*C*a^3*b^5*d^2*tan(f*x + e) - 8*A*a*b^7*d^2*tan(f*x + e) - C*
a^6*b^2*c^2 + 6*B*a^5*b^3*c^2 - 14*A*a^4*b^4*c^2 + 11*C*a^4*b^4*c^2 - 7*B*a
^3*b^5*c^2 - 3*A*a^2*b^6*c^2 - B*a*b^7*c^2 - A*b^8*c^2 + 4*C*a^7*b*c*d - 17
*B*a^6*b^2*c*d + 36*A*a^5*b^3*c*d - 24*C*a^5*b^3*c*d + 10*B*a^4*b^4*c*d + 1
6*A*a^3*b^5*c*d - 4*C*a^3*b^5*c*d + 3*B*a^2*b^6*c*d + 4*A*a*b^7*c*d - 6*C*a
^8*d^2 + 14*B*a^7*b*d^2 - 25*A*a^6*b^2*d^2 + 7*C*a^6*b^2*d^2 + 3*B*a^5*b^3*
d^2 - 19*A*a^4*b^4*d^2 + C*a^4*b^4*d^2 + B*a^3*b^5*d^2 - 6*A*a^2*b^6*d^2)/(
(a^6*b^3*c^3 + 3*a^4*b^5*c^3 + 3*a^2*b^7*c^3 + b^9*c^3 - 3*a^7*b^2*c^2*d -
9*a^5*b^4*c^2*d - 9*a^3*b^6*c^2*d - 3*a*b^8*c^2*d + 3*a^8*b*c*d^2 + 9*a^6*b
^3*c*d^2 + 9*a^4*b^5*c*d^2 + 3*a^2*b^7*c*d^2 - a^9*d^3 - 3*a^7*b^2*d^3 - 3*
a^5*b^4*d^3 - a^3*b^6*d^3)*(b*tan(f*x + e) + a)^2))/f
```

Mupad [B]
time = 24.03, size = 2500, normalized size = 5.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^3*(c + d*
tan(e + f*x))),x)

[Out] -(((A*b^5*c - 3*C*a^5*d - 3*A*a*b^4*d + B*a*b^4*c + 5*B*a^4*b*d + C*a^4*b*c
 + 5*A*a^2*b^3*c - 7*A*a^3*b^2*d - 3*B*a^3*b^2*c + B*a^2*b^3*d - 3*C*a^2*b^
3*c + C*a^3*b^2*d)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 + 2*a^2*b^
2)) - (tan(e + f*x)*(A*b^5*d - B*b^5*c - 2*A*a*b^4*c + 2*C*a*b^4*c + C*a^4*
b*d + 3*A*a^2*b^3*d + B*a^2*b^3*c - 2*B*a^3*b^2*d - C*a^2*b^3*d))/((a^2*d^2
 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 + 2*a^2*b^2)))/(a^2 + b^2*tan(e + f*x)^2
 + 2*a*b*tan(e + f*x)) - symsum(log(- (A^3*b^8*c^2*d^4 - 4*A^3*a^2*b^6*d^6
- 7*A^3*a^4*b^4*d^6 - A^3*b^8*d^6 + A^2*C*b^8*d^6 - 3*A^3*a^2*b^6*c^2*d^4 -
 B^3*a^3*b^5*c^2*d^4 - C^3*a^2*b^6*c^2*d^4 - 2*C^3*a^3*b^5*c^3*d^3 + 7*C^3*
a^4*b^4*c^2*d^4 + A^2*B*a*b^7*d^6 + A^2*B*b^8*c*d^5 + A^3*a*b^7*c*d^5 + C^3
*a^7*b*c*d^5 - A*B^2*a^2*b^6*d^6 - 3*A*B^2*a^6*b^2*d^6 + 2*A^2*B*a^3*b^5*d^
6 + 9*A^2*B*a^5*b^3*d^6 - A*C^2*a^2*b^6*d^6 - 4*A*C^2*a^4*b^4*d^6 + A*C^2*a
^6*b^2*d^6 + 5*A^2*C*a^2*b^6*d^6 + 11*A^2*C*a^4*b^4*d^6 - A^2*C*a^6*b^2*d^6
 + A*C^2*b^8*c^2*d^4 - 2*A^2*C*b^8*c^2*d^4 - B*C^2*b^8*c^3*d^3 + B^2*C*b^8*
c^2*d^4 + 9*A^3*a^3*b^5*c*d^5 - B^3*a*b^7*c^2*d^4 + B^3*a^2*b^6*c*d^5 + B^3
*a^4*b^4*c*d^5 + 2*C^3*a*b^7*c^3*d^3 - 3*C^3*a^5*b^3*c*d^5 + A*B*C*a^7*b*d^
6 - 2*A*B*C*b^8*c*d^5 + 3*A*B^2*a^2*b^6*c^2*d^4 - A*B^2*a^4*b^4*c^2*d^4 + 3
*A^2*B*a^3*b^5*c^2*d^4 - A*C^2*a^2*b^6*c^2*d^4 + 4*A*C^2*a^3*b^5*c^3*d^3 -
14*A*C^2*a^4*b^4*c^2*d^4 + 5*A^2*C*a^2*b^6*c^2*d^4 - 2*A^2*C*a^3*b^5*c^3*d^
3 + 7*A^2*C*a^4*b^4*c^2*d^4 + 6*B*C^2*a^2*b^6*c^3*d^3 - 15*B*C^2*a^3*b^5*c^
2*d^4 - B*C^2*a^4*b^4*c^3*d^3 + 3*B*C^2*a^5*b^3*c^2*d^4 + 5*B^2*C*a^2*b^6*c
^2*d^4 + 2*B^2*C*a^3*b^5*c^3*d^3 - 4*B^2*C*a^4*b^4*c^2*d^4 + A*B*C*a^3*b^5*
d^6 - 6*A*B*C*a^5*b^3*d^6 + A*B*C*b^8*c^3*d^3 + 2*A*C^2*a*b^7*c*d^5 - A*C^2
*a^7*b*c*d^5 - 3*A^2*C*a*b^7*c*d^5 - 5*A*B^2*a^3*b^5*c*d^5 + 3*A*B^2*a^5*b^
3*c*d^5 - 5*A^2*B*a*b^7*c^2*d^4 + 7*A^2*B*a^2*b^6*c*d^5 - 10*A^2*B*a^4*b^4*
c*d^5 - 4*A*C^2*a*b^7*c^3*d^3 + 12*A*C^2*a^3*b^5*c*d^5 + 9*A*C^2*a^5*b^3*c*
d^5 + 2*A^2*C*a*b^7*c^3*d^3 - 21*A^2*C*a^3*b^5*c*d^5 - 6*A^2*C*a^5*b^3*c*d^
5 - 2*B*C^2*a*b^7*c^2*d^4 + B*C^2*a^2*b^6*c*d^5 + 5*B*C^2*a^4*b^4*c*d^5 - 4
*B*C^2*a^6*b^2*c*d^5 - 2*B^2*C*a*b^7*c^3*d^3 - B^2*C*a^3*b^5*c*d^5 + 3*B^2*
C*a^5*b^3*c*d^5 - 6*A*B*C*a^2*b^6*c^3*d^3 + 12*A*B*C*a^3*b^5*c^2*d^4 + A*B*
C*a^4*b^4*c^3*d^3 - 3*A*B*C*a^5*b^3*c^2*d^4 + 7*A*B*C*a*b^7*c^2*d^4 - 11*A*
B*C*a^2*b^6*c*d^5 + 2*A*B*C*a^4*b^4*c*d^5 + 3*A*B*C*a^6*b^2*c*d^5)/(a^12*d^
4 + b^12*c^4 + 4*a^2*b^10*c^4 + 6*a^4*b^8*c^4 + 4*a^6*b^6*c^4 + a^8*b^4*c^4
 + a^4*b^8*d^4 + 4*a^6*b^6*d^4 + 6*a^8*b^4*d^4 + 4*a^10*b^2*d^4 - 4*a^3*b^9
*c*d^3 - 16*a^3*b^9*c^3*d - 16*a^5*b^7*c*d^3 - 24*a^5*b^7*c^3*d - 24*a^7*b^
5*c*d^3 - 16*a^7*b^5*c^3*d - 16*a^9*b^3*c*d^3 - 4*a^9*b^3*c^3*d + 6*a^2*b^1
0*c^2*d^2 + 24*a^4*b^8*c^2*d^2 + 36*a^6*b^6*c^2*d^2 + 24*a^8*b^4*c^2*d^2 +
6*a^10*b^2*c^2*d^2 - 4*a*b^11*c^3*d - 4*a^11*b*c*d^3) - root(480*a^11*b^7*c
*d^9*f^4 + 480*a^7*b^11*c^9*d*f^4 + 360*a^13*b^5*c*d^9*f^4 + 360*a^9*b^9*c^
9*d*f^4 + 360*a^9*b^9*c*d^9*f^4 + 360*a^5*b^13*c^9*d*f^4 + 144*a^15*b^3*c*d
```

```
^9*f^4 + 144*a^11*b^7*c^9*d*f^4 + 144*a^7*b^11*c*d^9*f^4 + 144*a^3*b^15*c^9
*d*f^4 + 48*a^17*b*c^3*d^7*f^4 + 48*a*b^17*c^7*d^3*f^4 + 24*a^17*b*c^5*d^5*
f^4 + 24*a^13*b^5*c^9*d*f^4 + 24*a^5*b^13*c*d^9*f^4 + 24*a*b^17*c^5*d^5*f^4
 + 24*a^17*b*c*d^9*f^4 + 24*a*b^17*c^9*d*f^4 + 3920*a^9*b^9*c^5*d^5*f^4 - 3
360*a^10*b^8*c^4*d^6*f^4 - 3360*a^8*b^10*c^6*d^4*f^4 + 3024*a^11*b^7*c^5*d^
5*f^4 - 3024*a^10*b^8*c^6*d^4*f^4 - 3024*a^8*b^10*c^4*d^6*f^4 + 3024*a^7*b^
11*c^5*d^5*f^4 + 2320*a^9*b^9*c^7*d^3*f^4 + 2320*a^9*b^9*c^3*d^7*f^4 - 2240
*a^12*b^6*c^4*d^6*f^4 - 2240*a^6*b^12*c^6*d^4*f^4 + 2160*a^11*b^7*c^3*d^7*f
^4 + 2160*a^7*b^11*c^7*d^3*f^4 - 1624*a^12*b^6*c^6*d^4*f^4 - 1624*a^6*b^12*
c^4*d^6*f^4 + 1488*a^11*b^7*c^7*d^3*f^4 + 1488*a^7*b^11*c^3*d^7*f^4 + 1344*
a^13*b^5*c^5*d^5*f^4 + 1344*a^5*b^13*c^5*d^5*f^4 - 1320*a^10*b^8*c^2*d^8*f^
4 - 1320*a^8*b^10*c^8*d^2*f^4 + 1200*a^13*b^5*c^3*d^7*f^4 + 1200*a^5*b^13*c
^7*d^3*f^4 - 1060*a^12*b^6*c^2*d^8*f^4 - 1060*a^6*b^12*c^8*d^2*f^4 - 948*a^
10*b^8*c^8*d^2*f^4 - 948*a^8*b^10*c^2*d^8*f^4 - 840*a^14*b^4*c^4*d^6*f^4 -
840*a^4*b^14*c^6*d^4*f^4 + 528*a^13*b^5*c^7*d^3*f^4 + 528*a^5*b^13*c^3*d^7*
f^4 - 480*a^14*b^4*c^6*d^4*f^4 - 480*a^14*b^4*c^2*d^8*f^4 - 480*a^4*b^14*c^
8*d^2*f^4 - 480*a^4*b^14*c^4*d^6*f^4 + 368*a^15*b^3*c^3*d^7*f^4 - 368*a^12*
b^6*c^8*d^2*f^4 - 368*a^6*b^12*c^2*d^8*f^4 + 368*a^3*b^15*c^7*d^3*f^4 + 304
*a^15*b^3*c^5*d^5*f^4 + 304*a^3*b^15*c^5*d^5*f^4 - 144*a^16*b^2*c^4*d^6*f^4
 - 144*a^2*b^16*c^6*d^4*f^4 - 108*a^16*b^2*c^2*d^8*f^4 - 108*a^2*b^16*c^8*d
^2*f^4 + 80*a^15*b^3*c^7*d^3*f^4 + 80*a^3*b^15*c^3*d^7*f^4 - 60*a^16*b^2*c^
6*d^4*f^4 - 60*a^14*b^4*c^8*d^2*f^4 - 60*a^4*b^14*c^2*d^8*f^4 - 60*a^2*b^16
*c^4*d^6*f^4 - 8*b^18*c^8*d^2*f^4 - 4*b^18*c^6*d^4*f^4 - 8*a^18*c^2*d^8*f^4
 - 4*a^18*c^4*d^6*f^4 - 80*a^12*b^6*d^10*f^4 - 60*a^14*b^4*d^10*f^4 - 60*a^
10*b^8*d^10*f^4 - 24*a^16*b^2*d^10*f^4 - 24*a^8*b^10*d^10*f^4 - 4*a^6*b^12*
d^10*f^4 - 80*a^6*b^12*c^10*f^4 - 60*a^8*b^10*c^10*f^4 - 60*a^4*b^14*c^10*f
^4 - 24*a^10*b^8*c^10*f^4 - 24*a^2*b^16*c^10*f^...
```

3.77 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=579

$$-\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-3a^2b(2c(A-C)d-B(}{(c^2+d^2)^2}$$

```
[Out] -(a^3*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-3*a*b^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2
-d^2))-3*a^2*b*(2*c*(A-C)*d-B*(c^2-d^2))+b^3*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(
c^2+d^2)^2+(3*a^2*b*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-b^3*(c^2*C-2*B*c*d-C*
d^2-A*(c^2-d^2))+a^3*(2*c*(A-C)*d-B*(c^2-d^2))-3*a*b^2*(2*c*(A-C)*d-B*(c^2-
d^2)))*ln(cos(f*x+e))/(c^2+d^2)^2/f+(-a*d+b*c)^2*(b*(3*c^4*C-2*B*c^3*d+c^2*
(A+5*C)*d^2-4*B*c*d^3+3*A*d^4)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*ln(c+d*tan(
f*x+e))/d^4/(c^2+d^2)^2/f+b^2*(a*d*(3*c^2*C-B*c*d+(A+2*C)*d^2)-b*(3*c^3*C-2
*B*c^2*d+c*(A+2*C)*d^2-B*d^3))*tan(f*x+e)/d^3/(c^2+d^2)/f+1/2*b*(3*c^2*C-2*
B*c*d+(2*A+C)*d^2)*(a+b*tan(f*x+e))^2/d^2/(c^2+d^2)/f-(A*d^2-B*c*d+C*c^2)*(
a+b*tan(f*x+e))^3/d/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 1.40, antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3726, 3728, 3718, 3707, 3698, 31, 3556}

$$\frac{\log(\cos(e+fx))\left(a^3(2cd(A-C)-B(c^2-d^2))+3a^2b(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)-3ab^2(2cd(A-C)-B(c^2-d^2))-b^3(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)\right)}{f\left(c^2+d^2\right)^2}-\frac{x\left(a^3(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)-3a^2b(2cd(A-C)-B(c^2-d^2))-3ab^2(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)+b^3(2cd(A-C)-B(c^2-d^2))\right)}{\left(c^2+d^2\right)^2}+\frac{b^2\tan(e+fx)\left(ad(d^2(A+2C)-Bcd+3c^2C)-b(cd^2(A+2C)-2Bc^2d-Bd^3+3c^3C)\right)}{d^3f\left(c^2+d^2\right)}-\frac{(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}+\frac{b(d^2(2A+C)-2Bcd+3c^2C)(a+b\tan(e+fx))^2}{2d^2f\left(c^2+d^2\right)}+\frac{(bc-ad)^2\left(ad^2(2cd(A-C)-B(c^2-d^2))+b(c^2d^2(A+5C)+3Ad^4-2Bc^3d-4Bcd^3+3c^4C)\right)\log(c+d\tan(e+fx))}{d^4f\left(c^2+d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^2,x]
```

```
[Out] -(((a^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C - 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) - 3*a^2*b*(2*c*(A - C)*d - B*(c^2 - d^2)) + b^3*
(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/(c^2 + d^2)^2) + ((3*a^2*b*(c^2*C - 2*B
*c*d - C*d^2 - A*(c^2 - d^2)) - b^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2
)) + a^3*(2*c*(A - C)*d - B*(c^2 - d^2)) - 3*a*b^2*(2*c*(A - C)*d - B*(c^2
- d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)^2*f) + ((b*c - a*d)^2*(b*(3*c^4*C
- 2*B*c^3*d + c^2*(A + 5*C)*d^2 - 4*B*c*d^3 + 3*A*d^4) + a*d^2*(2*c*(A - C)
*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)^2*f) + (b^2*
(a*d*(3*c^2*C - B*c*d + (A + 2*C)*d^2) - b*(3*c^3*C - 2*B*c^2*d + c*(A + 2*
C)*d^2 - B*d^3))*Tan[e + f*x])/(d^3*(c^2 + d^2)*f) + (b*(3*c^2*C - 2*B*c*d
+ (2*A + C)*d^2)*(a + b*Tan[e + f*x])^2)/(2*d^2*(c^2 + d^2)*f) - ((c^2*C -
B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
```

```
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = -\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$

$$= \frac{b(3c^2C-2Bcd+(2A+C)d^2)\,(a+b\,t}{2d^2\,(c^2+d^2)\,f}$$

$$= \frac{b^2(ad(3c^2C-Bcd+(A+2C)d^2)-b}{}$$

$$= -\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2)}{}$$

$$= -\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2)}{}$$

$$= -\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.74, size = 2463, normalized size = 4.25

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((a^3*A*c^2 - 3*a*A*b^2*c^2 - 3*a^2*b*B*c^2 + b^3*B*c^2 - a^3*c^2*C + 3*a*b
^2*c^2*C + 6*a^2*A*b*c*d - 2*A*b^3*c*d + 2*a^3*B*c*d - 6*a*b^2*B*c*d - 6*a^
2*b*c*C*d + 2*b^3*c*C*d - a^3*A*d^2 + 3*a*A*b^2*d^2 + 3*a^2*b*B*d^2 - b^3*B
*d^2 + a^3*C*d^2 - 3*a*b^2*C*d^2)*(e + f*x)*Cos[e + f*x]*(c*Cos[e + f*x] +
d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)/((c - I*d)^2*(c + I*d)^2*f*(a*Cos
[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) + (((3*I)*b^3*c^11*C*
d^3 - (2*I)*b^3*B*c^10*d^4 - (6*I)*a*b^2*c^10*C*d^4 + 3*b^3*c^10*C*d^4 + I*
A*b^3*c^9*d^5 + (3*I)*a*b^2*B*c^9*d^5 - 2*b^3*B*c^9*d^5 + (3*I)*a^2*b*c^9*C
*d^5 - 6*a*b^2*c^9*C*d^5 + (8*I)*b^3*c^9*C*d^5 + A*b^3*c^8*d^6 + 3*a*b^2*B*
c^8*d^6 - (6*I)*b^3*B*c^8*d^6 + 3*a^2*b*c^8*C*d^6 - (18*I)*a*b^2*c^8*C*d^6
+ 8*b^3*c^8*C*d^6 - (3*I)*a^2*A*b*c^7*d^7 + (4*I)*A*b^3*c^7*d^7 - I*a^3*B*c
^7*d^7 + (12*I)*a*b^2*B*c^7*d^7 - 6*b^3*B*c^7*d^7 + (12*I)*a^2*b*c^7*C*d^7
- 18*a*b^2*c^7*C*d^7 + (5*I)*b^3*c^7*C*d^7 + (2*I)*a^3*A*c^6*d^8 - 3*a^2*A*
b*c^6*d^8 - (6*I)*a*A*b^2*c^6*d^8 + 4*A*b^3*c^6*d^8 - a^3*B*c^6*d^8 - (6*I)
*a^2*b*B*c^6*d^8 + 12*a*b^2*B*c^6*d^8 - (4*I)*b^3*B*c^6*d^8 - (2*I)*a^3*c^6
*C*d^8 + 12*a^2*b*c^6*C*d^8 - (12*I)*a*b^2*c^6*C*d^8 + 5*b^3*c^6*C*d^8 + 2*
a^3*A*c^5*d^9 - 6*a*A*b^2*c^5*d^9 + (3*I)*A*b^3*c^5*d^9 - 6*a^2*b*B*c^5*d^9
 + (9*I)*a*b^2*B*c^5*d^9 - 4*b^3*B*c^5*d^9 - 2*a^3*c^5*C*d^9 + (9*I)*a^2*b*
c^5*C*d^9 - 12*a*b^2*c^5*C*d^9 + (2*I)*a^3*A*c^4*d^10 - (6*I)*a*A*b^2*c^4*d
^10 + 3*A*b^3*c^4*d^10 - (6*I)*a^2*b*B*c^4*d^10 + 9*a*b^2*B*c^4*d^10 - (2*I
)*a^3*c^4*C*d^10 + 9*a^2*b*c^4*C*d^10 + 2*a^3*A*c^3*d^11 + (3*I)*a^2*A*b*c^
3*d^11 - 6*a*A*b^2*c^3*d^11 + I*a^3*B*c^3*d^11 - 6*a^2*b*B*c^3*d^11 - 2*a^3
*c^3*C*d^11 + 3*a^2*A*b*c^2*d^12 + a^3*B*c^2*d^12)*(e + f*x)*Cos[e + f*x]*(
c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)/(c^2*(c - I*d)^4
*(c + I*d)^3*d^7*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])
^2) - (I*(3*b^3*c^6*C - 2*b^3*B*c^5*d - 6*a*b^2*c^5*C*d + A*b^3*c^4*d^2 + 3
*a*b^2*B*c^4*d^2 + 3*a^2*b*c^4*C*d^2 + 5*b^3*c^4*C*d^2 - 4*b^3*B*c^3*d^3 -
12*a*b^2*c^3*C*d^3 - 3*a^2*A*b*c^2*d^4 + 3*A*b^3*c^2*d^4 - a^3*B*c^2*d^4 +
9*a*b^2*B*c^2*d^4 + 9*a^2*b*c^2*C*d^4 + 2*a^3*A*c*d^5 - 6*a*A*b^2*c*d^5 - 6
*a^2*b*B*c*d^5 - 2*a^3*c*C*d^5 + 3*a^2*A*b*d^6 + a^3*B*d^6)*ArcTan[Tan[e +
f*x]]*Cos[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])
^3)/(d^4*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e +
 f*x])^2) + ((-3*b^3*c^2*C + 2*b^3*B*c*d + 6*a*b^2*c*C*d - A*b^3*d^2 - 3*a*
b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3*C*d^2)*Cos[e + f*x]*Log[Cos[e + f*x]]*(c*Co
s[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)/(d^4*f*(a*Cos[e + f*
x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) + ((3*b^3*c^6*C - 2*b^3*B*c^
5*d - 6*a*b^2*c^5*C*d + A*b^3*c^4*d^2 + 3*a*b^2*B*c^4*d^2 + 3*a^2*b*c^4*C*d
^2 + 5*b^3*c^4*C*d^2 - 4*b^3*B*c^3*d^3 - 12*a*b^2*c^3*C*d^3 - 3*a^2*A*b*c^2
*d^4 + 3*A*b^3*c^2*d^4 - a^3*B*c^2*d^4 + 9*a*b^2*B*c^2*d^4 + 9*a^2*b*c^2*C*
d^4 + 2*a^3*A*c*d^5 - 6*a*A*b^2*c*d^5 - 6*a^2*b*B*c*d^5 - 2*a^3*c*C*d^5 + 3
*a^2*A*b*d^6 + a^3*B*d^6)*Cos[e + f*x]*Log[(c*Cos[e + f*x] + d*Sin[e + f*x]
)^2]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^3)/(2*d^4*(c^
2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) +
(b^3*C*Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x]
)^3)/(2*d^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) +
```

```
((c*Cos[e + f*x] + d*Sin[e + f*x])^2*(-2*b^3*c*C*Sin[e + f*x] + b^3*B*d*Si
n[e + f*x] + 3*a*b^2*C*d*Sin[e + f*x])*(a + b*Tan[e + f*x])^3)/(d^3*f*(a*Co
s[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2) + (Cos[e + f*x]*(c*C
os[e + f*x] + d*Sin[e + f*x])*(-(b^3*c^5*C*Sin[e + f*x]) + b^3*B*c^4*d*Sin[
e + f*x] + 3*a*b^2*c^4*C*d*Sin[e + f*x] - A*b^3*c^3*d^2*Sin[e + f*x] - 3*a*
b^2*B*c^3*d^2*Sin[e + f*x] - 3*a^2*b*c^3*C*d^2*Sin[e + f*x] + 3*a*A*b^2*c^2
*d^3*Sin[e + f*x] + 3*a^2*b*B*c^2*d^3*Sin[e + f*x] + a^3*c^2*C*d^3*Sin[e +
f*x] - 3*a^2*A*b*c*d^4*Sin[e + f*x] - a^3*B*c*d^4*Sin[e + f*x] + a^3*A*d^5*
Sin[e + f*x])*(a + b*Tan[e + f*x])^3)/(c*(c - I*d)*(c + I*d)*d^3*f*(a*Cos[e
 + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^2)
```

Maple [A]
time = 0.46, size = 829, normalized size = 1.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2/d^3*(1/2*C*d*tan(f*x+e)^2*b+b*tan(f*x+e)*B*d+3*tan(f*x+e)*C*a*d-2*
C*b*c*tan(f*x+e))-1/d^4*(A*a^3*d^5-3*A*a^2*b*c*d^4+3*A*a*b^2*c^2*d^3-A*b^3*
c^3*d^2-B*a^3*c*d^4+3*B*a^2*b*c^2*d^3-3*B*a*b^2*c^3*d^2+B*b^3*c^4*d+C*a^3*c
^2*d^3-3*C*a^2*b*c^3*d^2+3*C*a*b^2*c^4*d-C*b^3*c^5)/(c^2+d^2)/(c+d*tan(f*x+
e))+1/d^4*(2*A*a^3*c*d^5-3*A*a^2*b*c^2*d^4+3*A*a^2*b*d^6-6*A*a*b^2*c*d^5+A*
b^3*c^4*d^2+3*A*b^3*c^2*d^4-B*a^3*c^2*d^4+B*a^3*d^6-6*B*a^2*b*c*d^5+3*B*a*b
^2*c^4*d^2+9*B*a*b^2*c^2*d^4-2*B*b^3*c^5*d-4*B*b^3*c^3*d^3-2*C*a^3*c*d^5+3*
C*a^2*b*c^4*d^2+9*C*a^2*b*c^2*d^4-6*C*a*b^2*c^5*d-12*C*a*b^2*c^3*d^3+3*C*b^
3*c^6+5*C*b^3*c^4*d^2)/(c^2+d^2)^2*ln(c+d*tan(f*x+e))+1/(c^2+d^2)^2*(1/2*(-
2*A*a^3*c*d+3*A*a^2*b*c^2-3*A*a^2*b*d^2+6*A*a*b^2*c*d-A*b^3*c^2+A*b^3*d^2+B
*a^3*c^2-B*a^3*d^2+6*B*a^2*b*c*d-3*B*a*b^2*c^2+3*B*a*b^2*d^2-2*B*b^3*c*d+2*
C*a^3*c*d-3*C*a^2*b*c^2+3*C*a^2*b*d^2-6*C*a*b^2*c*d+C*b^3*c^2-C*b^3*d^2)*ln
(1+tan(f*x+e)^2)+(A*a^3*c^2-A*a^3*d^2+6*A*a^2*b*c*d-3*A*a*b^2*c^2+3*A*a*b^2
*d^2-2*A*b^3*c*d+2*B*a^3*c*d-3*B*a^2*b*c^2+3*B*a^2*b*d^2-6*B*a*b^2*c*d+B*b^
3*c^2-B*b^3*d^2-C*a^3*c^2+C*a^3*d^2-6*C*a^2*b*c*d+3*C*a*b^2*c^2-3*C*a*b^2*d
^2+2*C*b^3*c*d)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.54, size = 690, normalized size = 1.19

$$\frac{\frac{2\left(\left((A-C)a^3-3Ba^2b-3(A-C)ab^2+Bb^3\right)c^2+2\left(Ba^3+3(A-C)a^2b-3Bab^2-(A-C)b^3\right)cd-\left((A-C)a^3-3Ba^2b-3(A-C)ab^2+Bb^3\right)d^2\right)(fx+e)}{c^4+2c^2d^2+d^4}+\frac{2\left(3Cb^3c^6-2\left(3Cab^2+Bb^3\right)c^5d+\left(3Ca^2b+3Bab^2+(A+5C)b^3\right)c^4d^2-4\left(3Cab^2+Bb^3\right)c^3d^3-\left(Ba^3+3(A-3C)a^2b-9Bab^2-3Ab^3\right)c^2d^4+2\left((A-C)a^3-3Ba^2b-3Aab^2\right)cd^5+\left(Ba^3+3Aa^2b\right)d^6\right)\log\left(d\tan(fx+e)+c\right)}{c^4d^4+2c^2d^6+d^8}+\frac{\left(\left(Ba^3+3(A-C)a^2b-3Bab^2-(A-C)b^3\right)c^2-2\left((A-C)a^3-3Ba^2b-3(A-C)ab^2+Bb^3\right)cd-\left(Ba^3+3(A-C)a^2b-3Bab^2-(A-C)b^3\right)d^2\right)\log\left(\tan(fx+e)^2+1\right)}{c^4+2c^2d^2+d^4}+\frac{2\left(Cb^3c^5-Aa^3d^5-\left(3Cab^2+Bb^3\right)c^4d+\left(3Ca^2b+3Bab^2+Ab^3\right)c^3d^2-\left(Ca^3+3Ba^2b+3Aab^2\right)c^2d^3+\left(Ba^3+3Aa^2b\right)cd^4\right)}{c^3d^4+cd^6+\left(c^2d^5+d^7\right)\tan(fx+e)}+\frac{Cb^3d\tan(fx+e)^2-2\left(2Cb^3c-\left(3Cab^2+Bb^3\right)d\right)\tan(fx+e)}{d^3}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^2 + 2*(B*a^3
+ 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2*b
```

```
 - 3*(A - C)*a*b^2 + B*b^3)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + 2*(3*C
*b^3*c^6 - 2*(3*C*a*b^2 + B*b^3)*c^5*d + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)
*b^3)*c^4*d^2 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^3 - (B*a^3 + 3*(A - 3*C)*a^2*b
- 9*B*a*b^2 - 3*A*b^3)*c^2*d^4 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c*
d^5 + (B*a^3 + 3*A*a^2*b)*d^6)*log(d*tan(f*x + e) + c)/(c^4*d^4 + 2*c^2*d^6
 + d^8) + ((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2 - 2*((A
- C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d - (B*a^3 + 3*(A - C)*a^
2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^
2 + d^4) + 2*(C*b^3*c^5 - A*a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*
b + 3*B*a*b^2 + A*b^3)*c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 +
(B*a^3 + 3*A*a^2*b)*c*d^4)/(c^3*d^4 + c*d^6 + (c^2*d^5 + d^7)*tan(f*x + e))
 + (C*b^3*d*tan(f*x + e)^2 - 2*(2*C*b^3*c - (3*C*a*b^2 + B*b^3)*d)*tan(f*x
+ e))/d^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1487 vs. $2(583) = 1166$.
time = 12.85, size = 1487, normalized size = 2.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*C*b^3*c^5*d^2 - 2*A*a^3*d^7 - 2*(3*C*a*b^2 + B*b^3)*c^4*d^3 + 2*(3*C
*a^2*b + 3*B*a*b^2 + (A + C)*b^3)*c^3*d^4 - 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^
2)*c^2*d^5 + (2*B*a^3 + 6*A*a^2*b + C*b^3)*c*d^6 + (C*b^3*c^4*d^3 + 2*C*b^3
*c^2*d^5 + C*b^3*d^7)*tan(f*x + e)^3 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A -
 C)*a*b^2 + B*b^3)*c^3*d^4 + 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A -
C)*b^3)*c^2*d^5 - ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^6
)*f*x - (3*C*b^3*c^5*d^2 + 6*C*b^3*c^3*d^4 + 3*C*b^3*c*d^6 - 2*(3*C*a*b^2 +
 B*b^3)*c^4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^2*d^5 - 2*(3*C*a*b^2 + B*b^3)*d^7
)*tan(f*x + e)^2 + (3*C*b^3*c^7 - 2*(3*C*a*b^2 + B*b^3)*c^6*d + (3*C*a^2*b
+ 3*B*a*b^2 + (A + 5*C)*b^3)*c^5*d^2 - 4*(3*C*a*b^2 + B*b^3)*c^4*d^3 - (B*a
^3 + 3*(A - 3*C)*a^2*b - 9*B*a*b^2 - 3*A*b^3)*c^3*d^4 + 2*((A - C)*a^3 - 3*
B*a^2*b - 3*A*a*b^2)*c^2*d^5 + (B*a^3 + 3*A*a^2*b)*c*d^6 + (3*C*b^3*c^6*d -
 2*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)*b^3)*c^
4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^4 - (B*a^3 + 3*(A - 3*C)*a^2*b - 9*B*a*
b^2 - 3*A*b^3)*c^2*d^5 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c*d^6 + (B
*a^3 + 3*A*a^2*b)*d^7)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*
x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (3*C*b^3*c^7 - 2*(3*C*a*b^2 + B*b^3)*
c^6*d + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)*b^3)*c^5*d^2 - 4*(3*C*a*b^2 + B*
b^3)*c^4*d^3 + (6*C*a^2*b + 6*B*a*b^2 + (2*A + C)*b^3)*c^3*d^4 - 2*(3*C*a*b
^2 + B*b^3)*c^2*d^5 + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c*d^6 + (3*C*b^
3*c^6*d - 2*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C
```

```
)*b^3)*c^4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^4 + (6*C*a^2*b + 6*B*a*b^2 + (
2*A + C)*b^3)*c^2*d^5 - 2*(3*C*a*b^2 + B*b^3)*c*d^6 + (3*C*a^2*b + 3*B*a*b^
2 + (A - C)*b^3)*d^7)*tan(f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - (6*C*b^3*
c^6*d - C*b^3*d^7 - 4*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (6*C*a^2*b + 6*B*a*b^2
+ (2*A + 7*C)*b^3)*c^4*d^3 - 2*(C*a^3 + 3*B*a^2*b + 3*(A + 2*C)*a*b^2 + 2*B
*b^3)*c^3*d^4 + 2*(B*a^3 + 3*A*a^2*b + C*b^3)*c^2*d^5 - 2*(A*a^3 + 3*C*a*b^
2 + B*b^3)*c*d^6 - 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c
^2*d^5 + 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d^6 - ((A
- C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^7)*f*x)*tan(f*x + e))/((c
^4*d^5 + 2*c^2*d^7 + d^9)*f*tan(f*x + e) + (c^5*d^4 + 2*c^3*d^6 + c*d^8)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 33.48, size = 24300, normalized size = 41.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2,
Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), ((A*a**3*x + 3*A*a**2*b*log(tan(e + f*x)**
2 + 1)/(2*f) - 3*A*a*b**2*x + 3*A*a*b**2*tan(e + f*x)/f - A*b**3*log(tan(e
+ f*x)**2 + 1)/(2*f) + A*b**3*tan(e + f*x)**2/(2*f) + B*a**3*log(tan(e + f*
x)**2 + 1)/(2*f) - 3*B*a**2*b*x + 3*B*a**2*b*tan(e + f*x)/f - 3*B*a*b**2*lo
g(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*b**2*tan(e + f*x)**2/(2*f) + B*b**3*x
+ B*b**3*tan(e + f*x)**3/(3*f) - B*b**3*tan(e + f*x)/f - C*a**3*x + C*a**3*
tan(e + f*x)/f - 3*C*a**2*b*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a**2*b*tan
(e + f*x)**2/(2*f) + 3*C*a*b**2*x + C*a*b**2*tan(e + f*x)**3/f - 3*C*a*b**2
*tan(e + f*x)/f + C*b**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**3*tan(e + f*
x)**4/(4*f) - C*b**3*tan(e + f*x)**2/(2*f))/c**2, Eq(d, 0)), (-A*a**3*f*x*t
an(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2
*f) + 2*I*A*a**3*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*ta
n(e + f*x) - 4*d**2*f) + A*a**3*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*
tan(e + f*x) - 4*d**2*f) - A*a**3*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 -
8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a**3/(4*d**2*f*tan(e + f*x)**2
- 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 3*I*A*a**2*b*f*x*tan(e + f*x)**2/(4
*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*A*a**2*b*
f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d*
*2*f) - 3*I*A*a**2*b*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x
) - 4*d**2*f) + 3*I*A*a**2*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d
**2*f*tan(e + f*x) - 4*d**2*f) + 3*A*a*b**2*f*x*tan(e + f*x)**2/(4*d**2*f*t
an(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 6*I*A*a*b**2*f*x*tan
(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) -
 3*A*a*b**2*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**
```

```
2*f) - 9*A*a*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e
 + f*x) - 4*d**2*f) + 6*I*A*a*b**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*t
an(e + f*x) - 4*d**2*f) + 3*I*A*b**3*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e +
f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*A*b**3*f*x*tan(e + f*x)/(
4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*I*A*b**3
*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*A*
b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8
*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 4*I*A*b**3*log(tan(e + f*x)**2 + 1)*ta
n(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
- 2*A*b**3*log(tan(e + f*x)**2 + 1)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*
tan(e + f*x) - 4*d**2*f) - 5*I*A*b**3*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**
2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 4*A*b**3/(4*d**2*f*tan(e + f*x)**
2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*a**3*f*x*tan(e + f*x)**2/(4*d
**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*B*a**3*f*x*
tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f
) - I*B*a**3*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d*
*2*f) + I*B*a**3*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e
+ f*x) - 4*d**2*f) + 3*B*a**2*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)*
*2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 6*I*B*a**2*b*f*x*tan(e + f*x)/(4
*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*B*a**2*b*
f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 9*B*a
**2*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*
d**2*f) + 6*I*B*a**2*b/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x)
- 4*d**2*f) + 9*I*B*a*b**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 -
8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 18*B*a*b**2*f*x*tan(e + f*x)/(4*d**2*
f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 9*I*B*a*b**2*f*x/
(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6*B*a*b**
2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*
d**2*f*tan(e + f*x) - 4*d**2*f) - 12*I*B*a*b**2*log(tan(e + f*x)**2 + 1)*ta
n(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
- 6*B*a*b**2*log(tan(e + f*x)**2 + 1)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*
f*tan(e + f*x) - 4*d**2*f) - 15*I*B*a*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f
*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 12*B*a*b**2/(4*d**2*f*tan(e
+ f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 9*B*b**3*f*x*tan(e + f*x)
**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 18*I*
B*b**3*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x)
 - 4*d**2*f) + 9*B*b**3*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
f*x) - 4*d**2*f) + 4*I*B*b**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(4*d
**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 8*B*b**3*log(
tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*ta
n(e + f*x) - 4*d**2*f) - 4*I*B*b**3*log(tan(e + f*x)**2 + 1)/(4*d**2*f*tan(
e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(583) = 1166$.

time = 1.23, size = 1355, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c^2 - C*a^3*c^2 - 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*c
^2 + B*b^3*c^2 + 2*B*a^3*c*d + 6*A*a^2*b*c*d - 6*C*a^2*b*c*d - 6*B*a*b^2*c*
d - 2*A*b^3*c*d + 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 + 3*B*a^2*b*d^2 + 3*A
*a*b^2*d^2 - 3*C*a*b^2*d^2 - B*b^3*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) +
 (B*a^3*c^2 + 3*A*a^2*b*c^2 - 3*C*a^2*b*c^2 - 3*B*a*b^2*c^2 - A*b^3*c^2 + C
*b^3*c^2 - 2*A*a^3*c*d + 2*C*a^3*c*d + 6*B*a^2*b*c*d + 6*A*a*b^2*c*d - 6*C*
a*b^2*c*d - 2*B*b^3*c*d - B*a^3*d^2 - 3*A*a^2*b*d^2 + 3*C*a^2*b*d^2 + 3*B*a
*b^2*d^2 + A*b^3*d^2 - C*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2
+ d^4) + 2*(3*C*b^3*c^6 - 6*C*a*b^2*c^5*d - 2*B*b^3*c^5*d + 3*C*a^2*b*c^4*d
^2 + 3*B*a*b^2*c^4*d^2 + A*b^3*c^4*d^2 + 5*C*b^3*c^4*d^2 - 12*C*a*b^2*c^3*d
^3 - 4*B*b^3*c^3*d^3 - B*a^3*c^2*d^4 - 3*A*a^2*b*c^2*d^4 + 9*C*a^2*b*c^2*d^
4 + 9*B*a*b^2*c^2*d^4 + 3*A*b^3*c^2*d^4 + 2*A*a^3*c*d^5 - 2*C*a^3*c*d^5 - 6
*B*a^2*b*c*d^5 - 6*A*a*b^2*c*d^5 + B*a^3*d^6 + 3*A*a^2*b*d^6)*log(abs(d*tan
(f*x + e) + c))/(c^4*d^4 + 2*c^2*d^6 + d^8) - 2*(3*C*b^3*c^6*d*tan(f*x + e)
 - 6*C*a*b^2*c^5*d^2*tan(f*x + e) - 2*B*b^3*c^5*d^2*tan(f*x + e) + 3*C*a^2*
b*c^4*d^3*tan(f*x + e) + 3*B*a*b^2*c^4*d^3*tan(f*x + e) + A*b^3*c^4*d^3*tan
(f*x + e) + 5*C*b^3*c^4*d^3*tan(f*x + e) - 12*C*a*b^2*c^3*d^4*tan(f*x + e)
- 4*B*b^3*c^3*d^4*tan(f*x + e) - B*a^3*c^2*d^5*tan(f*x + e) - 3*A*a^2*b*c^2
*d^5*tan(f*x + e) + 9*C*a^2*b*c^2*d^5*tan(f*x + e) + 9*B*a*b^2*c^2*d^5*tan(
f*x + e) + 3*A*b^3*c^2*d^5*tan(f*x + e) + 2*A*a^3*c*d^6*tan(f*x + e) - 2*C*
a^3*c*d^6*tan(f*x + e) - 6*B*a^2*b*c*d^6*tan(f*x + e) - 6*A*a*b^2*c*d^6*tan
(f*x + e) + B*a^3*d^7*tan(f*x + e) + 3*A*a^2*b*d^7*tan(f*x + e) + 2*C*b^3*c
^7 - 3*C*a*b^2*c^6*d - B*b^3*c^6*d + 4*C*b^3*c^5*d^2 + C*a^3*c^4*d^3 + 3*B*
a^2*b*c^4*d^3 + 3*A*a*b^2*c^4*d^3 - 9*C*a*b^2*c^4*d^3 - 3*B*b^3*c^4*d^3 - 2
*B*a^3*c^3*d^4 - 6*A*a^2*b*c^3*d^4 + 6*C*a^2*b*c^3*d^4 + 6*B*a*b^2*c^3*d^4
+ 2*A*b^3*c^3*d^4 + 3*A*a^3*c^2*d^5 - C*a^3*c^2*d^5 - 3*B*a^2*b*c^2*d^5 - 3
*A*a*b^2*c^2*d^5 + A*a^3*d^7)/((c^4*d^4 + 2*c^2*d^6 + d^8)*(d*tan(f*x + e)
+ c)) + (C*b^3*d^2*tan(f*x + e)^2 - 4*C*b^3*c*d*tan(f*x + e) + 6*C*a*b^2*d^
2*tan(f*x + e) + 2*B*b^3*d^2*tan(f*x + e))/d^4)/f
```

Mupad [B]
time = 16.68, size = 701, normalized size = 1.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^2,x)
```

```
[Out] (tan(e + f*x)*((B*b^3 + 3*C*a*b^2)/d^2 - (2*C*b^3*c)/d^3))/f - (log(tan(e +
 f*x) + 1i)*(A*a^3*1i - A*b^3 + B*a^3 + B*b^3*1i - C*a^3*1i + C*b^3 - A*a*b
^2*3i + 3*A*a^2*b - 3*B*a*b^2 - B*a^2*b*3i + C*a*b^2*3i - 3*C*a^2*b))/(2*f*
(c*d*2i - c^2 + d^2)) + (log(c + d*tan(e + f*x))*(d^4*(3*A*b^3*c^2 - B*a^3*
c^2 - 3*A*a^2*b*c^2 + 9*B*a*b^2*c^2 + 9*C*a^2*b*c^2) - d^5*(2*C*a^3*c - 2*A
*a^3*c + 6*A*a*b^2*c + 6*B*a^2*b*c) - d^3*(4*B*b^3*c^3 + 12*C*a*b^2*c^3) +
d^6*(B*a^3 + 3*A*a^2*b) - d*(2*B*b^3*c^5 + 6*C*a*b^2*c^5) + d^2*(A*b^3*c^4
+ 5*C*b^3*c^4 + 3*B*a*b^2*c^4 + 3*C*a^2*b*c^4) + 3*C*b^3*c^6))/(f*(d^8 + 2*
c^2*d^6 + c^4*d^4)) - (log(tan(e + f*x) - 1i)*(A*a^3 - A*b^3*1i + B*a^3*1i
+ B*b^3 - C*a^3 + C*b^3*1i - 3*A*a*b^2 + A*a^2*b*3i - B*a*b^2*3i - 3*B*a^2*
b + 3*C*a*b^2 - C*a^2*b*3i))/(2*f*(2*c*d - c^2*1i + d^2*1i)) - (A*a^3*d^5 -
 C*b^3*c^5 - B*a^3*c*d^4 + B*b^3*c^4*d - A*b^3*c^3*d^2 + C*a^3*c^2*d^3 + 3*
A*a*b^2*c^2*d^3 - 3*B*a*b^2*c^3*d^2 + 3*B*a^2*b*c^2*d^3 - 3*C*a^2*b*c^3*d^2
 - 3*A*a^2*b*c*d^4 + 3*C*a*b^2*c^4*d)/(d*f*(c*d^3 + d^4*tan(e + f*x))*(c^2
+ d^2)) + (C*b^3*tan(e + f*x)^2)/(2*d^2*f)
```

3.78 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=417

$$-\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-2ab(2c(A-C)d-B(c^2-}{(c^2+d^2)^2}$$

```
[Out] -(a^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-b^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2
))-2*a*b*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(c^2+d^2)^2+(2*a*b*(c^2*C-2*B*c*d-C*d
^2-A*(c^2-d^2))+a^2*(2*c*(A-C)*d-B*(c^2-d^2))-b^2*(2*c*(A-C)*d-B*(c^2-d^2))
)*ln(cos(f*x+e))/(c^2+d^2)^2/f-(-a*d+b*c)*(b*(2*A*d^4-B*c^3*d-3*B*c*d^3+2*C
*c^4+4*C*c^2*d^2)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*ln(c+d*tan(f*x+e))/d^3/(
c^2+d^2)^2/f+b^2*(2*c^2*C-B*c*d+(A+C)*d^2)*tan(f*x+e)/d^2/(c^2+d^2)/f-(A*d^
2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.73, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3726, 3718, 3707, 3698, 31, 3556}

$$\frac{\log(\cos(e+fx))\left(a^2(2cd(A-C)-B(c^2-d^2))+2ab(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)-b^2(2cd(A-C)-B(c^2-d^2))\right)}{f\left(c^2+d^2\right)^2}-\frac{x\left(a^2(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)-2ab(2cd(A-C)-B(c^2-d^2))-b^2(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)\right)}{\left(c^2+d^2\right)^2}-\frac{(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}-\frac{(bc-ad)\left(ad^2(2cd(A-C)-B(c^2-d^2))+b(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2)\right)\log(c+d\tan(e+fx))}{d^3f\left(c^2+d^2\right)^2}+\frac{b^2\tan(e+fx)\left(d^2(A+C)-Bcd+2c^2C\right)}{d^2f\left(c^2+d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^2,x]
```

```
[Out] -(((a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) - 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/(c^2 + d
^2)^2) + ((2*a*b*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + a^2*(2*c*(A -
C)*d - B*(c^2 - d^2)) - b^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[Cos[e + f*
x]])/((c^2 + d^2)^2*f) - ((b*c - a*d)*(b*(2*c^4*C - B*c^3*d + 4*c^2*C*d^2 -
 3*B*c*d^3 + 2*A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c + d*Ta
n[e + f*x]])/(d^3*(c^2 + d^2)^2*f) + (b^2*(2*c^2*C - B*c*d + (A + C)*d^2)*T
an[e + f*x])/(d^2*(c^2 + d^2)*f) - ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e +
f*x])^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = -\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$

$$= \frac{b^2(2c^2C-Bcd+(A+C)d^2)\tan(e+}{d^2\,(c^2+d^2)\,f}$$

$$= -\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2}{}$$

$$= -\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2}{}$$

$$= -\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.35, size = 2636, normalized size = 6.32

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^2,x]
```

```
[Out] (((-2*I)*b^2*c^10*C*d^2 + I*b^2*B*c^9*d^3 + (2*I)*a*b*c^9*C*d^3 - 2*b^2*c^9
*C*d^3 + b^2*B*c^8*d^4 + 2*a*b*c^8*C*d^4 - (6*I)*b^2*c^8*C*d^4 - (2*I)*a*A*
b*c^7*d^5 - I*a^2*B*c^7*d^5 + (4*I)*b^2*B*c^7*d^5 + (8*I)*a*b*c^7*C*d^5 - 6
*b^2*c^7*C*d^5 + (2*I)*a^2*A*c^6*d^6 - 2*a*A*b*c^6*d^6 - (2*I)*A*b^2*c^6*d^
6 - a^2*B*c^6*d^6 - (4*I)*a*b*B*c^6*d^6 + 4*b^2*B*c^6*d^6 - (2*I)*a^2*c^6*C
*d^6 + 8*a*b*c^6*C*d^6 - (4*I)*b^2*c^6*C*d^6 + 2*a^2*A*c^5*d^7 - 2*A*b^2*c^
5*d^7 - 4*a*b*B*c^5*d^7 + (3*I)*b^2*B*c^5*d^7 - 2*a^2*c^5*C*d^7 + (6*I)*a*b
*c^5*C*d^7 - 4*b^2*c^5*C*d^7 + (2*I)*a^2*A*c^4*d^8 - (2*I)*A*b^2*c^4*d^8 -
(4*I)*a*b*B*c^4*d^8 + 3*b^2*B*c^4*d^8 - (2*I)*a^2*c^4*C*d^8 + 6*a*b*c^4*C*d
^8 + 2*a^2*A*c^3*d^9 + (2*I)*a*A*b*c^3*d^9 - 2*A*b^2*c^3*d^9 + I*a^2*B*c^3*
d^9 - 4*a*b*B*c^3*d^9 - 2*a^2*c^3*C*d^9 + 2*a*A*b*c^2*d^10 + a^2*B*c^2*d^10
)*(e + f*x)*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^2)/(c^
2*(c - I*d)^4*(c + I*d)^3*d^5*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*
Tan[e + f*x])^2) - (I*(-2*b^2*c^5*C + b^2*B*c^4*d + 2*a*b*c^4*C*d - 4*b^2*c
^3*C*d^2 - 2*a*A*b*c^2*d^3 - a^2*B*c^2*d^3 + 3*b^2*B*c^2*d^3 + 6*a*b*c^2*C*
d^3 + 2*a^2*A*c*d^4 - 2*A*b^2*c*d^4 - 4*a*b*B*c*d^4 - 2*a^2*c*C*d^4 + 2*a*A
*b*d^5 + a^2*B*d^5)*ArcTan[Tan[e + f*x]]*(c*Cos[e + f*x] + d*Sin[e + f*x])^
2*(a + b*Tan[e + f*x])^2)/(d^3*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e +
f*x])^2*(c + d*Tan[e + f*x])^2) + ((2*b^2*c*C - b^2*B*d - 2*a*b*C*d)*Log[Co
```

```
s[e + f*x]]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*Tan[e + f*x])^2)/(d^
3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^2) + ((-2*b^2*
c^5*C + b^2*B*c^4*d + 2*a*b*c^4*C*d - 4*b^2*c^3*C*d^2 - 2*a*A*b*c^2*d^3 - a
^2*B*c^2*d^3 + 3*b^2*B*c^2*d^3 + 6*a*b*c^2*C*d^3 + 2*a^2*A*c*d^4 - 2*A*b^2*
c*d^4 - 4*a*b*B*c*d^4 - 2*a^2*c*C*d^4 + 2*a*A*b*d^5 + a^2*B*d^5)*Log[(c*Cos
[e + f*x] + d*Sin[e + f*x])^2]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(a + b*T
an[e + f*x])^2)/(2*d^3*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*
(c + d*Tan[e + f*x])^2) + (Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])*(
b^2*c^5*C*d + 2*b^2*c^3*C*d^3 + b^2*c*C*d^5 + a^2*A*c^4*d^2*(e + f*x) - A*b
^2*c^4*d^2*(e + f*x) - 2*a*b*B*c^4*d^2*(e + f*x) - a^2*c^4*C*d^2*(e + f*x)
+ b^2*c^4*C*d^2*(e + f*x) + 4*a*A*b*c^3*d^3*(e + f*x) + 2*a^2*B*c^3*d^3*(e
+ f*x) - 2*b^2*B*c^3*d^3*(e + f*x) - 4*a*b*c^3*C*d^3*(e + f*x) - a^2*A*c^2*
d^4*(e + f*x) + A*b^2*c^2*d^4*(e + f*x) + 2*a*b*B*c^2*d^4*(e + f*x) + a^2*c
^2*C*d^4*(e + f*x) - b^2*c^2*C*d^4*(e + f*x) - b^2*c^5*C*d*Cos[2*(e + f*x)]
 - 2*b^2*c^3*C*d^3*Cos[2*(e + f*x)] - b^2*c*C*d^5*Cos[2*(e + f*x)] + a^2*A*
c^4*d^2*(e + f*x)*Cos[2*(e + f*x)] - A*b^2*c^4*d^2*(e + f*x)*Cos[2*(e + f*x
)] - 2*a*b*B*c^4*d^2*(e + f*x)*Cos[2*(e + f*x)] - a^2*c^4*C*d^2*(e + f*x)*C
os[2*(e + f*x)] + b^2*c^4*C*d^2*(e + f*x)*Cos[2*(e + f*x)] + 4*a*A*b*c^3*d^
3*(e + f*x)*Cos[2*(e + f*x)] + 2*a^2*B*c^3*d^3*(e + f*x)*Cos[2*(e + f*x)] -
 2*b^2*B*c^3*d^3*(e + f*x)*Cos[2*(e + f*x)] - 4*a*b*c^3*C*d^3*(e + f*x)*Cos
[2*(e + f*x)] - a^2*A*c^2*d^4*(e + f*x)*Cos[2*(e + f*x)] + A*b^2*c^2*d^4*(e
 + f*x)*Cos[2*(e + f*x)] + 2*a*b*B*c^2*d^4*(e + f*x)*Cos[2*(e + f*x)] + a^2
*c^2*C*d^4*(e + f*x)*Cos[2*(e + f*x)] - b^2*c^2*C*d^4*(e + f*x)*Cos[2*(e +
f*x)] + 2*b^2*c^6*C*Sin[2*(e + f*x)] - b^2*B*c^5*d*Sin[2*(e + f*x)] - 2*a*b
*c^5*C*d*Sin[2*(e + f*x)] + A*b^2*c^4*d^2*Sin[2*(e + f*x)] + 2*a*b*B*c^4*d^
2*Sin[2*(e + f*x)] + a^2*c^4*C*d^2*Sin[2*(e + f*x)] + 3*b^2*c^4*C*d^2*Sin[2
*(e + f*x)] - 2*a*A*b*c^3*d^3*Sin[2*(e + f*x)] - a^2*B*c^3*d^3*Sin[2*(e + f
*x)] - b^2*B*c^3*d^3*Sin[2*(e + f*x)] - 2*a*b*c^3*C*d^3*Sin[2*(e + f*x)] +
a^2*A*c^2*d^4*Sin[2*(e + f*x)] + A*b^2*c^2*d^4*Sin[2*(e + f*x)] + 2*a*b*B*c
^2*d^4*Sin[2*(e + f*x)] + a^2*c^2*C*d^4*Sin[2*(e + f*x)] + b^2*c^2*C*d^4*Si
n[2*(e + f*x)] - 2*a*A*b*c*d^5*Sin[2*(e + f*x)] - a^2*B*c*d^5*Sin[2*(e + f*
x)] + a^2*A*d^6*Sin[2*(e + f*x)] + a^2*A*c^3*d^3*(e + f*x)*Sin[2*(e + f*x)]
 - A*b^2*c^3*d^3*(e + f*x)*Sin[2*(e + f*x)] - 2*a*b*B*c^3*d^3*(e + f*x)*Sin
[2*(e + f*x)] - a^2*c^3*C*d^3*(e + f*x)*Sin[2*(e + f*x)] + b^2*c^3*C*d^3*(e
 + f*x)*Sin[2*(e + f*x)] + 4*a*A*b*c^2*d^4*(e + f*x)*Sin[2*(e + f*x)] + 2*a
^2*B*c^2*d^4*(e + f*x)*Sin[2*(e + f*x)] - 2*b^2*B*c^2*d^4*(e + f*x)*Sin[2*(
e + f*x)] - 4*a*b*c^2*C*d^4*(e + f*x)*Sin[2*(e + f*x)] - a^2*A*c*d^5*(e + f
*x)*Sin[2*(e + f*x)] + A*b^2*c*d^5*(e + f*x)*Sin[2*(e + f*x)] + 2*a*b*B*c*d
^5*(e + f*x)*Sin[2*(e + f*x)] + a^2*c*C*d^5*(e + f*x)*Sin[2*(e + f*x)] - b^
2*c*C*d^5*(e + f*x)*Sin[2*(e + f*x)])*(a + b*Tan[e + f*x])^2)/(2*c*(c - I*d
)^2*(c + I*d)^2*d^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*
x])^2)
```

Maple [A]
time = 0.35, size = 552, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{b^2C\tan(fx+e)}{d^2}-\frac{A\,a^2d^4-2Aabc\,d^3+A\,b^2c^2d^2-B\,a^2c\,d^3+2Bab\,c^2d^2-B\,b^2c^3d+C\,a^2c^2d^2-2Cab\,c^3d+C\,b^2c^4}{d^3\left(c^2+d^2\right)(c+d\tan(fx+e))}+\frac{\left(2A\,a^2c\,d^4-2Aab\,c^2\right.}{}}{}$
default	$\frac{\frac{b^2C\tan(fx+e)}{d^2}-\frac{A\,a^2d^4-2Aabc\,d^3+A\,b^2c^2d^2-B\,a^2c\,d^3+2Bab\,c^2d^2-B\,b^2c^3d+C\,a^2c^2d^2-2Cab\,c^3d+C\,b^2c^4}{d^3\left(c^2+d^2\right)(c+d\tan(fx+e))}+\frac{\left(2A\,a^2c\,d^4-2Aab\,c^2\right.}{}}{}$
norman	$\frac{\frac{c\left(A\,a^2c^2-A\,a^2d^2+4Aabcd-A\,b^2c^2+A\,b^2d^2+2B\,a^2cd-2Bab\,c^2+2Bab\,d^2-2B\,b^2cd-C\,a^2c^2+a^2C\,d^2-4Cabcd+C\,b^2c^2-C\,b^2d^2\right)x}{c^4+2c^2d^2+d^4}+}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b^2*C/d^2*tan(f*x+e)-1/d^3*(A*a^2*d^4-2*A*a*b*c*d^3+A*b^2*c^2*d^2-B*a^
2*c*d^3+2*B*a*b*c^2*d^2-B*b^2*c^3*d+C*a^2*c^2*d^2-2*C*a*b*c^3*d+C*b^2*c^4)/
(c^2+d^2)/(c+d*tan(f*x+e))+1/d^3*(2*A*a^2*c*d^4-2*A*a*b*c^2*d^3+2*A*a*b*d^5
-2*A*b^2*c*d^4-B*a^2*c^2*d^3+B*a^2*d^5-4*B*a*b*c*d^4+B*b^2*c^4*d+3*B*b^2*c^
2*d^3-2*C*a^2*c*d^4+2*C*a*b*c^4*d+6*C*a*b*c^2*d^3-2*C*b^2*c^5-4*C*b^2*c^3*d
^2)/(c^2+d^2)^2*ln(c+d*tan(f*x+e))+1/(c^2+d^2)^2*(1/2*(-2*A*a^2*c*d+2*A*a*b
*c^2-2*A*a*b*d^2+2*A*b^2*c*d+B*a^2*c^2-B*a^2*d^2+4*B*a*b*c*d-B*b^2*c^2+B*b^
2*d^2+2*C*a^2*c*d-2*C*a*b*c^2+2*C*a*b*d^2-2*C*b^2*c*d)*ln(1+tan(f*x+e)^2)+(
A*a^2*c^2-A*a^2*d^2+4*A*a*b*c*d-A*b^2*c^2+A*b^2*d^2+2*B*a^2*c*d-2*B*a*b*c^2
+2*B*a*b*d^2-2*B*b^2*c*d-C*a^2*c^2+C*a^2*d^2-4*C*a*b*c*d+C*b^2*c^2-C*b^2*d^
2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.56, size = 498, normalized size = 1.19

$$\frac{\frac{2\,Cb^2\tan(fx+e)}{d^2}+\frac{2\,(((A-C)a^2-2\,Bab-(A-C)b^2)c^2+2\,(Ba^2+2\,(A-C)ab-Bb^2)cd-((A-C)a^2-2\,Bab-(A-C)b^2)d^2)(fx+e)}{c^4+2\,c^2d^2+d^4}-\frac{2\,(2\,Cb^2c^5+4\,Cb^2c^3d^2-(2\,Cab+Bb^2)c^4d+(Ba^2+2\,(A-3\,C)ab-3\,Bb^2)c^2d^3-2\,((A-C)a^2-2\,Bab-Ab^2)cd^4-(Ba^2+2\,Aab)d^5)\log(d\tan(fx+e)+c)}{c^4d^3+2\,c^2d^5+d^7}+\frac{((Ba^2+2\,(A-C)ab-Bb^2)c^2-2\,((A-C)a^2-2\,Bab-(A-C)b^2)cd-(Ba^2+2\,(A-C)ab-Bb^2)d^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}-\frac{2\,(Cb^2c^4+Aa^2d^4-(2\,Cab+Bb^2)c^3d+(Ca^2+2\,Bab+Ab^2)c^2d^2-(Ba^2+2\,Aab)cd^3)}{c^3d^3+cd^5+(c^2d^4+d^6)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*b^2*tan(f*x + e)/d^2 + 2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^
2 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B*a*b - (A - C
)*b^2)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) - 2*(2*C*b^2*c^5 + 4*C*b^2*c^
3*d^2 - (2*C*a*b + B*b^2)*c^4*d + (B*a^2 + 2*(A - 3*C)*a*b - 3*B*b^2)*c^2*d
^3 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*log(d
*tan(f*x + e) + c)/(c^4*d^3 + 2*c^2*d^5 + d^7) + ((B*a^2 + 2*(A - C)*a*b -
B*b^2)*c^2 - 2*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d - (B*a^2 + 2*(A -
```

```
C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) - 2*(C
*b^2*c^4 + A*a^2*d^4 - (2*C*a*b + B*b^2)*c^3*d + (C*a^2 + 2*B*a*b + A*b^2)*
c^2*d^2 - (B*a^2 + 2*A*a*b)*c*d^3)/(c^3*d^3 + c*d^5 + (c^2*d^4 + d^6)*tan(f
*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(422) = 844$.
time = 4.40, size = 948, normalized size = 2.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*C*b^2*c^4*d^2 + 2*A*a^2*d^6 - 2*(2*C*a*b + B*b^2)*c^3*d^3 + 2*(C*a^
2 + 2*B*a*b + A*b^2)*c^2*d^4 - 2*(B*a^2 + 2*A*a*b)*c*d^5 - 2*(((A - C)*a^2
- 2*B*a*b - (A - C)*b^2)*c^3*d^3 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^
4 - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^5)*f*x - 2*(C*b^2*c^4*d^2 + 2
*C*b^2*c^2*d^4 + C*b^2*d^6)*tan(f*x + e)^2 + (2*C*b^2*c^6 + 4*C*b^2*c^4*d^2
 - (2*C*a*b + B*b^2)*c^5*d + (B*a^2 + 2*(A - 3*C)*a*b - 3*B*b^2)*c^3*d^3 -
2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b)*c*d^5 + (2*C*
b^2*c^5*d + 4*C*b^2*c^3*d^3 - (2*C*a*b + B*b^2)*c^4*d^2 + (B*a^2 + 2*(A - 3
*C)*a*b - 3*B*b^2)*c^2*d^4 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^5 - (B*a
^2 + 2*A*a*b)*d^6)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x +
e) + c^2)/(tan(f*x + e)^2 + 1)) - (2*C*b^2*c^6 + 4*C*b^2*c^4*d^2 + 2*C*b^2*
c^2*d^4 - (2*C*a*b + B*b^2)*c^5*d - 2*(2*C*a*b + B*b^2)*c^3*d^3 - (2*C*a*b
+ B*b^2)*c*d^5 + (2*C*b^2*c^5*d + 4*C*b^2*c^3*d^3 + 2*C*b^2*c*d^5 - (2*C*a*
b + B*b^2)*c^4*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^4 - (2*C*a*b + B*b^2)*d^6)*t
an(f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - 2*(2*C*b^2*c^5*d - (2*C*a*b + B*
b^2)*c^4*d^2 + (C*a^2 + 2*B*a*b + (A + 2*C)*b^2)*c^3*d^3 - (B*a^2 + 2*A*a*b
)*c^2*d^4 + (A*a^2 + C*b^2)*c*d^5 + (((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*
c^2*d^4 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^5 - ((A - C)*a^2 - 2*B*a*b
- (A - C)*b^2)*d^6)*f*x)*tan(f*x + e))/((c^4*d^4 + 2*c^2*d^6 + d^8)*f*tan(f
*x + e) + (c^5*d^3 + 2*c^3*d^5 + c*d^7)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.74, size = 16225, normalized size = 38.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2,
Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), (-A*a**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan
(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a**2*f*x*tan(e +
 f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*a
**2*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A
*a**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*
d**2*f) + 2*I*A*a**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) -
4*d**2*f) + 2*I*A*a*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d
**2*f*tan(e + f*x) - 4*d**2*f) + 4*A*a*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e +
 f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*A*a*b*f*x/(4*d**2*f*ta
n(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a*b*tan(e + f*x
)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*b**2*
f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4
*d**2*f) - 2*I*A*b**2*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2
*f*tan(e + f*x) - 4*d**2*f) - A*b**2*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d*
*2*f*tan(e + f*x) - 4*d**2*f) - 3*A*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x
)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*b**2/(4*d**2*f*tan(e + f
*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*a**2*f*x*tan(e + f*x)**2
/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*B*a**2
*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d
**2*f) - I*B*a**2*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) -
 4*d**2*f) + I*B*a**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*t
an(e + f*x) - 4*d**2*f) + 2*B*a*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x
)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 4*I*B*a*b*f*x*tan(e + f*x)/(4*
d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*B*a*b*f*x/
(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 6*B*a*b*t
an(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
 + 4*I*B*a*b/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f
) + 3*I*B*b**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*t
an(e + f*x) - 4*d**2*f) + 6*B*b**2*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)*
*2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*I*B*b**2*f*x/(4*d**2*f*tan(e +
 f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*B*b**2*log(tan(e + f*x)*
*2 + 1)*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x)
 - 4*d**2*f) - 4*I*B*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f*t
an(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*B*b**2*log(tan(e +
 f*x)**2 + 1)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*
f) - 5*I*B*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
 f*x) - 4*d**2*f) - 4*B*b**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
 f*x) - 4*d**2*f) + C*a**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 -
8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*C*a**2*f*x*tan(e + f*x)/(4*d**2*f
*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - C*a**2*f*x/(4*d**2
*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*C*a**2*tan(e +
 f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I
*C*a**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 6
*I*C*a*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
```

```
 f*x) - 4*d**2*f) + 12*C*a*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8
*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 6*I*C*a*b*f*x/(4*d**2*f*tan(e + f*x)**
2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 4*C*a*b*log(tan(e + f*x)**2 + 1)*
tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**
2*f) - 8*I*C*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f*tan(e + f*
x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 4*C*a*b*log(tan(e + f*x)**2 +
 1)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 10*I*
C*a*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*
d**2*f) - 8*C*a*b/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d
**2*f) - 9*C*b**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*
f*tan(e + f*x) - 4*d**2*f) + 18*I*C*b**2*f*x*tan(e + f*x)/(4*d**2*f*tan(e +
 f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 9*C*b**2*f*x/(4*d**2*f*tan
(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 4*I*C*b**2*log(tan(e +
 f*x)**2 + 1)*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e
+ f*x) - 4*d**2*f) + 8*C*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2
*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 4*I*C*b**2*log(t
an(e + f*x)**2 + 1)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4
*d**2*f) + 4*C*b**2*tan(e + f*x)**3/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*
tan(e + f*x) - 4*d**2*f) + 19*C*b**2*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2
 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 14*I*C*b**2/(4*d**2*f*tan(e + f*x)
**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f), Eq(c, -I*d)), (-A*a**2*f*x*tan(e
 + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(422) = 844.
time = 0.94, size = 912, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*b^2*tan(f*x + e)/d^2 + 2*(A*a^2*c^2 - C*a^2*c^2 - 2*B*a*b*c^2 - A*
b^2*c^2 + C*b^2*c^2 + 2*B*a^2*c*d + 4*A*a*b*c*d - 4*C*a*b*c*d - 2*B*b^2*c*d
 - A*a^2*d^2 + C*a^2*d^2 + 2*B*a*b*d^2 + A*b^2*d^2 - C*b^2*d^2)*(f*x + e)/(
c^4 + 2*c^2*d^2 + d^4) + (B*a^2*c^2 + 2*A*a*b*c^2 - 2*C*a*b*c^2 - B*b^2*c^2
 - 2*A*a^2*c*d + 2*C*a^2*c*d + 4*B*a*b*c*d + 2*A*b^2*c*d - 2*C*b^2*c*d - B*
a^2*d^2 - 2*A*a*b*d^2 + 2*C*a*b*d^2 + B*b^2*d^2)*log(tan(f*x + e)^2 + 1)/(c
^4 + 2*c^2*d^2 + d^4) - 2*(2*C*b^2*c^5 - 2*C*a*b*c^4*d - B*b^2*c^4*d + 4*C*
b^2*c^3*d^2 + B*a^2*c^2*d^3 + 2*A*a*b*c^2*d^3 - 6*C*a*b*c^2*d^3 - 3*B*b^2*c
^2*d^3 - 2*A*a^2*c*d^4 + 2*C*a^2*c*d^4 + 4*B*a*b*c*d^4 + 2*A*b^2*c*d^4 - B*
a^2*d^5 - 2*A*a*b*d^5)*log(abs(d*tan(f*x + e) + c))/(c^4*d^3 + 2*c^2*d^5 +
d^7) + 2*(2*C*b^2*c^5*d*tan(f*x + e) - 2*C*a*b*c^4*d^2*tan(f*x + e) - B*b^2
*c^4*d^2*tan(f*x + e) + 4*C*b^2*c^3*d^3*tan(f*x + e) + B*a^2*c^2*d^4*tan(f*
```

```
x + e) + 2*A*a*b*c^2*d^4*tan(f*x + e) - 6*C*a*b*c^2*d^4*tan(f*x + e) - 3*B*
b^2*c^2*d^4*tan(f*x + e) - 2*A*a^2*c*d^5*tan(f*x + e) + 2*C*a^2*c*d^5*tan(f
*x + e) + 4*B*a*b*c*d^5*tan(f*x + e) + 2*A*b^2*c*d^5*tan(f*x + e) - B*a^2*d
^6*tan(f*x + e) - 2*A*a*b*d^6*tan(f*x + e) + C*b^2*c^6 - C*a^2*c^4*d^2 - 2*
B*a*b*c^4*d^2 - A*b^2*c^4*d^2 + 3*C*b^2*c^4*d^2 + 2*B*a^2*c^3*d^3 + 4*A*a*b
*c^3*d^3 - 4*C*a*b*c^3*d^3 - 2*B*b^2*c^3*d^3 - 3*A*a^2*c^2*d^4 + C*a^2*c^2*
d^4 + 2*B*a*b*c^2*d^4 + A*b^2*c^2*d^4 - A*a^2*d^6)/((c^4*d^3 + 2*c^2*d^5 +
d^7)*(d*tan(f*x + e) + c)))/f
```

Mupad [B]
time = 35.26, size = 2500, normalized size = 6.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^2,x)
```

```
[Out] (log((2*C^2*b^4*c^5 - 2*C^2*a^2*b^2*c^5 + 4*C^2*b^4*c^3*d^2 - A*B*a^4*d^5 -
 2*A*C*b^4*c^5 + B*C*a^4*d^5 + 2*A^2*a*b^3*d^5 - 2*A^2*a^3*b*d^5 - A^2*a^4*
c*d^4 + 2*B^2*a^3*b*d^5 - A^2*b^4*c*d^4 + B^2*a^4*c*d^4 + B^2*b^4*c*d^4 - C
^2*a^4*c*d^4 + C^2*b^4*c*d^4 - 4*C^2*a^2*b^2*c^3*d^2 + 5*A*B*a^2*b^2*d^5 +
2*A*C*a^2*b^2*c^5 + A*B*a^4*c^2*d^3 + 3*A*B*b^4*c^2*d^3 - B*C*a^2*b^2*d^5 -
 4*A*C*b^4*c^3*d^2 - B*C*a^4*c^2*d^3 - 3*B*C*b^4*c^2*d^3 + 2*B^2*a*b^3*c^4*
d - 2*C^2*a*b^3*c^4*d + 2*C^2*a^3*b*c^4*d - 2*A^2*a*b^3*c^2*d^3 + 6*A^2*a^2
*b^2*c*d^4 + 2*A^2*a^3*b*c^2*d^3 + 6*B^2*a*b^3*c^2*d^3 - 6*B^2*a^2*b^2*c*d^
4 - 2*B^2*a^3*b*c^2*d^3 - 6*C^2*a*b^3*c^2*d^3 + 4*C^2*a^2*b^2*c*d^4 + 6*C^2
*a^3*b*c^2*d^3 - 2*A*C*a*b^3*d^5 + 2*A*C*a^3*b*d^5 - 4*B*C*a*b^3*c^5 + A*B*
b^4*c^4*d + 2*A*C*a^4*c*d^4 - B*C*b^4*c^4*d - 8*A*B*a*b^3*c*d^4 + 8*A*B*a^3
*b*c*d^4 + 2*A*C*a*b^3*c^4*d - 2*A*C*a^3*b*c^4*d + 4*B*C*a*b^3*c*d^4 - 8*B*
C*a^3*b*c*d^4 - A*B*a^2*b^2*c^4*d + 8*A*C*a*b^3*c^2*d^3 - 10*A*C*a^2*b^2*c*
d^4 - 8*A*C*a^3*b*c^2*d^3 - 8*B*C*a*b^3*c^3*d^2 + 5*B*C*a^2*b^2*c^4*d - 8*A
*B*a^2*b^2*c^2*d^3 + 4*A*C*a^2*b^2*c^3*d^2 + 16*B*C*a^2*b^2*c^2*d^3)/(d^2*(
c^2 + d^2)^2) + ((a*1i - b)^2*((A*b^2*d^2 - A*a^2*d^2 + C*a^2*d^2 - 8*C*b^2
*c^2 - C*b^2*d^2 + 2*B*a*b*d^2 + 4*B*b^2*c*d + 8*C*a*b*c*d)/d - (tan(e + f*
x)*(3*B*a^2*d^5 - 5*B*b^2*d^5 - 4*C*b^2*c^5 + 6*A*a*b*d^5 - 10*C*a*b*d^5 +
4*A*a^2*c*d^4 - 4*A*b^2*c*d^4 + 2*B*b^2*c^4*d - 4*C*a^2*c*d^4 + 8*C*b^2*c*d
^4 - B*a^2*c^2*d^3 + B*b^2*c^2*d^3 - 8*B*a*b*c*d^4 + 4*C*a*b*c^4*d - 2*A*a*
b*c^2*d^3 + 2*C*a*b*c^2*d^3))/(d^2*(c^2 + d^2)) + (d*(a*1i - b)^2*(4*c*d -
c^2*tan(e + f*x) + 3*d^2*tan(e + f*x))*(A + B*1i - C)*1i)/(c*1i - d)^2)*(A
+ B*1i - C)*1i)/(2*(c*1i - d)^2) + (tan(e + f*x)*(A^2*a^4*d^5 + A^2*b^4*d^5
 + B^2*b^4*d^5 + C^2*a^4*d^5 + C^2*b^4*d^5 - 2*A^2*a^2*b^2*d^5 + 3*B^2*a^2*
b^2*d^5 + B^2*a^4*c^2*d^3 + 2*C^2*a^2*b^2*d^5 + 3*B^2*b^4*c^2*d^3 - 2*A*C*a
^4*d^5 - 2*A*C*b^4*d^5 - 2*B*C*b^4*c^5 - 4*C^2*a*b^3*c^5 + B^2*b^4*c^4*d +
4*A^2*a^2*b^2*c^2*d^3 - 4*B^2*a^2*b^2*c^2*d^3 + 12*C^2*a^2*b^2*c^2*d^3 + 2*
```

B*C*a^2*b^2*c^5 - 4*B*C*b^4*c^3*d^2 + 4*A^2*a*b^3*c*d^4 - 4*A^2*a^3*b*c*d^4 - 4*B^2*a*b^3*c*d^4 + 4*B^2*a^3*b*c*d^4 - 4*C^2*a^3*b*c*d^4 - B^2*a^2*b^2*c^4*d - 8*C^2*a*b^3*c^3*d^2 + 4*C^2*a^2*b^2*c^4*d + 2*A*B*a*b^3*d^5 - 4*A*B*a^3*b*d^5 + 4*A*C*a*b^3*c^5 - 2*A*B*a^4*c*d^4 - 2*A*B*b^4*c*d^4 + 2*B*C*a^3*b*d^5 + 2*B*C*a^4*c*d^4 - 2*A*B*a*b^3*c^4*d - 4*A*C*a*b^3*c*d^4 + 8*A*C*a^3*b*c*d^4 + 4*B*C*a*b^3*c^4*d - 2*B*C*a^3*b*c^4*d - 8*A*B*a*b^3*c^2*d^3 + 12*A*B*a^2*b^2*c*d^4 + 4*A*B*a^3*b*c^2*d^3 + 8*A*C*a*b^3*c^3*d^2 - 4*A*C*a^2*b^2*c^4*d + 12*B*C*a*b^3*c^2*d^3 - 10*B*C*a^2*b^2*c*d^4 - 8*B*C*a^3*b*c^2*d^3 - 16*A*C*a^2*b^2*c^2*d^3 + 4*B*C*a^2*b^2*c^3*d^2))/(d^2*(c^2 + d^2)^2))*(A*b^2 - A*a^2 - B*a^2*1i + B*b^2*1i + C*a^2 - C*b^2 - A*a*b*2i + 2*B*a*b + C*a*b*2i))/(2*f*(2*c*d - c^2*1i + d^2*1i)) + (log((2*C^2*b^4*c^5 - 2*C^2*a^2*b^2*c^5 + 4*C^2*b^4*c^3*d^2 - A*B*a^4*d^5 - 2*A*C*b^4*c^5 + B*C*a^4*d^5 + 2*A^2*a*b^3*d^5 - 2*A^2*a^3*b*d^5 - A^2*a^4*c*d^4 + 2*B^2*a^3*b*d^5 - A^2*b^4*c*d^4 + B^2*a^4*c*d^4 + B^2*b^4*c*d^4 - C^2*a^4*c*d^4 + C^2*b^4*c*d^4 - 4*C^2*a^2*b^2*c^3*d^2 + 5*A*B*a^2*b^2*d^5 + 2*A*C*a^2*b^2*c^5 + A*B*a^4*c^2*d^3 + 3*A*B*b^4*c^2*d^3 - B*C*a^2*b^2*d^5 - 4*A*C*b^4*c^3*d^2 - B*C*a^4*c^2*d^3 - 3*B*C*b^4*c^2*d^3 + 2*B^2*a*b^3*c^4*d - 2*C^2*a*b^3*c^4*d + 2*C^2*a^3*b*c^4*d - 2*A^2*a*b^3*c^2*d^3 + 6*A^2*a^2*b^2*c*d^4 + 2*A^2*a^3*b*c^2*d^3 + 6*B^2*a*b^3*c^2*d^3 - 6*B^2*a^2*b^2*c*d^4 - 2*B^2*a^3*b*c^2*d^3 - 6*C^2*a*b^3*c^2*d^3 + 4*C^2*a^2*b^2*c*d^4 + 6*C^2*a^3*b*c^2*d^3 - 2*A*C*a*b^3*d^5 + 2*A*C*a^3*b*d^5 - 4*B*C*a*b^3*c^5 + A*B*b^4*c^4*d + 2*A*C*a^4*c*d^4 - B*C*b^4*c^4*d - 8*A*B*a*b^3*c*d^4 + 8*A*B*a^3*b*c*d^4 + 2*A*C*a*b^3*c^4*d - 2*A*C*a^3*b*c^4*d + 4*B*C*a*b^3*c*d^4 - 8*B*C*a^3*b*c*d^4 - A*B*a^2*b^2*c^4*d + 8*A*C*a*b^3*c^2*d^3 - 10*A*C*a^2*b^2*c*d^4 - 8*A*C*a^3*b*c^2*d^3 - 8*B*C*a*b^3*c^3*d^2 + 5*B*C*a^2*b^2*c^4*d - 8*A*B*a^2*b^2*c^2*d^3 + 4*A*C*a^2*b^2*c^3*d^2 + 16*B*C*a^2*b^2*c^2*d^3)/(d^2*(c^2 + d^2)^2) + (tan(e + f*x)*(A^2*a^4*d^5 + A^2*b^4*d^5 + B^2*b^4*d^5 + C^2*a^4*d^5 + C^2*b^4*d^5 - 2*A^2*a^2*b^2*d^5 + 3*B^2*a^2*b^2*d^5 + B^2*a^4*c^2*d^3 + 2*C^2*a^2*b^2*d^5 + 3*B^2*b^4*c^2*d^3 - 2*A*C*a^4*d^5 - 2*A*C*b^4*d^5 - 2*B*C*b^4*c^5 - 4*C^2*a*b^3*c^5 + B^2*b^4*c^4*d + 4*A^2*a^2*b^2*c^2*d^3 - 4*B^2*a^2*b^2*c^2*d^3 + 12*C^2*a^2*b^2*c^2*d^3 + 2*B*C*a^2*b^2*c^5 - 4*B*C*b^4*c^3*d^2 + 4*A^2*a*b^3*c*d^4 - 4*A^2*a^3*b*c*d^4 - 4*B^2*a*b^3*c*d^4 + 4*B^2*a^3*b*c*d^4 - 4*C^2*a^3*b*c*d^4 - B^2*a^2*b^2*c^4*d - 8*C^2*a*b^3*c^3*d^2 + 4*C^2*a^2*b^2*c^4*d + 2*A*B*a*b^3*d^5 - 4*A*B*a^3*b*d^5 + 4*A*C*a*b^3*c^5 - 2*A*B*a^4*c*d^4 - 2*A*B*b^4*c*d^4 + 2*B*C*a^3*b*d^5 + 2*B*C*a^4*c*d^4 - 2*A*B*a*b^3*c^4*d - 4*A*C*a*b^3*c*d^4 + 8*A*C*a^3*b*c*d^4 + 4*B*C*a*b^3*c^4*d - 2*B*C*a^3*b*c^4*d - 8*A*B*a*b^3*c^2*d^3 + 12*A*B*a^2*b^2*c*d^4 + 4*A*B*a^3*b*c^2*d^3 + 8*A*C*a*b^3*c^3*d^2 - 4*A*C*a^2*b^2*c^4*d + 12*B*C*a*b^3*c^2*d^3 - 10*B*C*a^2*b^2*c*d^4 - 8*B*C*a^3*b*c^2*d^3 - 16*A*C*a^2*b^2*c^2*d^3 + 4*B*C*a^2*b^2*c^3*d^2))/(d^2*(c^2 + d^2)^2) + ((a*1i + b)^2*(...

3.79 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=292

$$-\frac{(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b(2c(A-C)d-B(c^2-d^2)))\,x}{(c^2+d^2)^2}-\frac{(a(Bc^2+2cCd-Bd^2)-b(c^2C$$

```
[Out] -(a*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-b*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(c^2+d
^2)^2-(a*(B*c^2-B*d^2+2*C*c*d)-b*(-2*B*c*d+C*c^2-C*d^2)-A*(2*a*c*d-b*(c^2-d
^2)))*ln(cos(f*x+e))/(c^2+d^2)^2/f+(b*(c^4*C-c^2*(A-3*C)*d^2-2*B*c*d^3+A*d^
4)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*ln(c+d*tan(f*x+e))/d^2/(c^2+d^2)^2/f+(-
a*d+b*c)*(A*d^2-B*c*d+C*c^2)/d^2/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.36, antiderivative size = 288, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3716, 3707, 3698, 31, 3556}

$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))}+\frac{\log(\cos(e+fx))\left(2aAcd-aB(c^2-d^2)-2acCd-Ab(c^2-d^2)+b(-2Bcd+c^2C-Cd^2)\right)}{f\left(c^2+d^2\right)^2}-\frac{x\left(a(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)-b(2cd(A-C)-B(c^2-d^2))\right)}{\left(c^2+d^2\right)^2}+\frac{\left(ad^2(2cd(A-C)-B(c^2-d^2))+b(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C)\right)\log(c+d\tan(e+fx))}{d^2f\left(c^2+d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x])^2,x]
```

```
[Out] -(((a*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b*(2*c*(A - C)*d - B*(c^2
 - d^2)))*x)/(c^2 + d^2)^2) + ((2*a*A*c*d - 2*a*c*C*d - A*b*(c^2 - d^2) - a
*B*(c^2 - d^2) + b*(c^2*C - 2*B*c*d - C*d^2))*Log[Cos[e + f*x]])/((c^2 + d^
2)^2*f) + ((b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*
(A - C)*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2*f)
+ ((b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(d^2*(c^2 + d^2)*f*(c + d*Tan[e + f
*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
```

```
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx = \frac{(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int ad\cdots}{\cdots}$$

$$= -\frac{\left(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))-\cdots\right.}{(c^2+d^2\cdots}$$

$$= -\frac{\left(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))-\cdots\right.}{(c^2+d^2\cdots}$$

$$= -\frac{\left(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))-\cdots\right.}{(c^2+d^2\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.46, size = 606, normalized size = 2.08

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^2,x]
```

```
[Out] (c^2*(2*(c + I*d)^2*(a*(A - I*B - C)*d^2 + b*(I*c^2*C + 2*c*C*d + ((-I)*A -
 B)*d^2))*(e + f*x) - 2*b*C*(c^2 + d^2)^2*Log[Cos[e + f*x]] + (b*(c^4*C - c
^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d + B*(-c^2 + d^
2)))*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]) + d*(2*(c + I*d)*(b*c*(I*c^3
*C*(I + e + f*x) + d^3*((-I)*B*(e + f*x) + A*(I + e + f*x)) - I*c*d^2*(-2*C
*(e + f*x) + A*(-I + e + f*x) - I*B*(I + e + f*x)) + c^2*d*(B + C*(I + e +
f*x))) + a*d*(c^3*C - I*A*d^3 + c*d^2*(A*(1 + I*e + I*f*x) - I*C*(e + f*x)
+ B*(I + e + f*x)) - c^2*d*(B*(1 + I*e + I*f*x) - A*(e + f*x) + C*(I + e +
f*x)))) - 2*b*c*C*(c^2 + d^2)^2*Log[Cos[e + f*x]] + c*(b*(c^4*C - c^2*(A -
3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d + B*(-c^2 + d^2)))*Log
[(c*Cos[e + f*x] + d*Sin[e + f*x])^2])*Tan[e + f*x] - (2*I)*c*(b*(c^4*C - c
^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d + B*(-c^2 + d^
2)))*ArcTan[Tan[e + f*x]]*(c + d*Tan[e + f*x]))/(2*c*d^2*(c^2 + d^2)^2*f*(c
 + d*Tan[e + f*x]))
```

Maple [A]

time = 0.36, size = 321, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{\left(-2Aacd+Ab\,c^{2}-Ab\,d^{2}+Ba\,c^{2}-Ba\,d^{2}+2Bbcd+2Cacd-Cb\,c^{2}+Cb\,d^{2}\right)\ln\left(1+\tan^{2}(fx+e)\right)}{2}+\left(Aa\,c^{2}-Aa\,d^{2}+2Abcd+2Bacd-Bb\,c^{2}+\right.}{\left(c^{2}+d^{2}\right)^{2}}$
default	$\frac{\frac{\left(-2Aacd+Ab\,c^{2}-Ab\,d^{2}+Ba\,c^{2}-Ba\,d^{2}+2Bbcd+2Cacd-Cb\,c^{2}+Cb\,d^{2}\right)\ln\left(1+\tan^{2}(fx+e)\right)}{2}+\left(Aa\,c^{2}-Aa\,d^{2}+2Abcd+2Bacd-Bb\,c^{2}+\right.}{\left(c^{2}+d^{2}\right)^{2}}$
norman	$\frac{\frac{c\left(Aa\,c^{2}-Aa\,d^{2}+2Abcd+2Bacd-Bb\,c^{2}+Bb\,d^{2}-Ca\,c^{2}+Ca\,d^{2}-2Cbcd\right)x}{c^{4}+2c^{2}d^{2}+d^{4}}+\frac{d\left(Aa\,c^{2}-Aa\,d^{2}+2Abcd+2Bacd-Bb\,c^{2}+Bb\,d^{2}-Ca\,c^{2}+C\right.}{c^{4}+2c^{2}d^{2}+d^{4}}}{c+d\tan(fx+e)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(c^2+d^2)^2*(1/2*(-2*A*a*c*d+A*b*c^2-A*b*d^2+B*a*c^2-B*a*d^2+2*B*b*c
*d+2*C*a*c*d-C*b*c^2+C*b*d^2)*ln(1+tan(f*x+e)^2)+(A*a*c^2-A*a*d^2+2*A*b*c*d
+2*B*a*c*d-B*b*c^2+B*b*d^2-C*a*c^2+C*a*d^2-2*C*b*c*d)*arctan(tan(f*x+e)))-(
A*a*d^3-A*b*c*d^2-B*a*c*d^2+B*b*c^2*d+C*a*c^2*d-C*b*c^3)/d^2/(c^2+d^2)/(c+d
*tan(f*x+e))+(2*A*a*c*d^3-A*b*c^2*d^2+A*b*d^4-B*a*c^2*d^2+B*a*d^4-2*B*b*c*d
^3-2*C*a*c*d^3+C*b*c^4+3*C*b*c^2*d^2)/(c^2+d^2)^2/d^2*ln(c+d*tan(f*x+e)))
```

Maxima [A]

time = 0.53, size = 323, normalized size = 1.11

$$\frac{\frac{2\,(((A-C)a-Bb)c^2+2\,(Ba+(A-C)b)cd-((A-C)a-Bb)d^2)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{2\,(Cbc^4-(Ba+(A-3\,C)b)c^2d^2+2\,((A-C)a-Bb)cd^3+(Ba+Ab)d^4)\log(d\tan(fx+e)+c)}{c^4d^2+2\,c^2d^4+d^6}+\frac{((Ba+(A-C)b)c^2-2\,((A-C)a-Bb)cd-(Ba+(A-C)b)d^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}+\frac{2\,(Cbc^3-Aad^3-(Ca+Bb)c^2d+(Ba+Ab)cd^2)}{c^3d^2+cd^4+(c^2d^3+d^5)\tan(fx+e)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a - B*b)*c^2 + 2*(B*a + (A - C)*b)*c*d - ((A - C)*a - B*b)
*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + 2*(C*b*c^4 - (B*a + (A - 3*C)*b)*
c^2*d^2 + 2*((A - C)*a - B*b)*c*d^3 + (B*a + A*b)*d^4)*log(d*tan(f*x + e) +
 c)/(c^4*d^2 + 2*c^2*d^4 + d^6) + ((B*a + (A - C)*b)*c^2 - 2*((A - C)*a - B
*b)*c*d - (B*a + (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 +
 d^4) + 2*(C*b*c^3 - A*a*d^3 - (C*a + B*b)*c^2*d + (B*a + A*b)*c*d^2)/(c^3*
d^2 + c*d^4 + (c^2*d^3 + d^5)*tan(f*x + e)))/f
```

Fricas [A]
time = 3.10, size = 513, normalized size = 1.76

$$\frac{2\,Cbc^3d^2-2\,Aad^5-2\,(Ca+Bb)c^2d^3+2\,(Ba+Ab)cd^4+2\,(((A-C)a-Bb)c^3d^2+2\,(Ba+(A-C)b)c^2d^3-((A-C)a-Bb)cd^4)fx+(Cbc^5-(Ba+(A-3\,C)b)c^3d^2+2\,((A-C)a-Bb)c^2d^3+(Ba+Ab)cd^4+(Cbc^4d-(Ba+(A-3\,C)b)c^2d^3+2\,((A-C)a-Bb)cd^4+(Ba+Ab)d^5)\tan(fx+e))\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-(Cbc^5+2\,Cbc^3d^2+Cbcd^4+(Cbc^4d+2\,Cbc^2d^3+Cbd^5)\tan(fx+e))\log\left(\frac{1}{\tan(fx+e)^2+1}\right)-2\,(Cbc^4d-Aacd^4-(Ca+Bb)c^3d^2+(Ba+Ab)c^2d^3-(((A-C)a-Bb)c^2d^3+2\,(Ba+(A-C)b)cd^4-((A-C)a-Bb)d^5)fx)\tan(fx+e)}{2\,((c^4d^3+2\,c^2d^5+d^7)f\tan(fx+e)+(c^5d^2+2\,c^3d^4+cd^6)f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*b*c^3*d^2 - 2*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 2*(B*a + A*b)*c*d^
4 + 2*(((A - C)*a - B*b)*c^3*d^2 + 2*(B*a + (A - C)*b)*c^2*d^3 - ((A - C)*a
 - B*b)*c*d^4)*f*x + (C*b*c^5 - (B*a + (A - 3*C)*b)*c^3*d^2 + 2*((A - C)*a
- B*b)*c^2*d^3 + (B*a + A*b)*c*d^4 + (C*b*c^4*d - (B*a + (A - 3*C)*b)*c^2*d
^3 + 2*((A - C)*a - B*b)*c*d^4 + (B*a + A*b)*d^5)*tan(f*x + e))*log((d^2*ta
n(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (C*b*c^5 +
 2*C*b*c^3*d^2 + C*b*c*d^4 + (C*b*c^4*d + 2*C*b*c^2*d^3 + C*b*d^5)*tan(f*x
+ e))*log(1/(tan(f*x + e)^2 + 1)) - 2*(C*b*c^4*d - A*a*c*d^4 - (C*a + B*b)*
c^3*d^2 + (B*a + A*b)*c^2*d^3 - (((A - C)*a - B*b)*c^2*d^3 + 2*(B*a + (A -
C)*b)*c*d^4 - ((A - C)*a - B*b)*d^5)*f*x)*tan(f*x + e))/((c^4*d^3 + 2*c^2*d
^5 + d^7)*f*tan(f*x + e) + (c^5*d^2 + 2*c^3*d^4 + c*d^6)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.69, size = 9721, normalized size = 33.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**2,x)
```

```
[Out] Piecewise((zoo*x*(a + b*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2, Eq(
c, 0) & Eq(d, 0) & Eq(f, 0)), (-A*a*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f
*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a*f*x*tan(e + f*x)/(4*
d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*a*f*x/(4*d
**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A*a*tan(e + f
*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A
*a/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*A*b*
f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4
*d**2*f) + 2*A*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*ta
n(e + f*x) - 4*d**2*f) - I*A*b*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*t
an(e + f*x) - 4*d**2*f) + I*A*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*
I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*a*f*x*tan(e + f*x)**2/(4*d**2*f*tan
(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*B*a*f*x*tan(e + f*x)
/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - I*B*a*f*
x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*a*t
an(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
 + B*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f
*x) - 4*d**2*f) - 2*I*B*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*
d**2*f*tan(e + f*x) - 4*d**2*f) - B*b*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d
**2*f*tan(e + f*x) - 4*d**2*f) - 3*B*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)*
*2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*B*b/(4*d**2*f*tan(e + f*x)**
2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + C*a*f*x*tan(e + f*x)**2/(4*d**2*f
*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*C*a*f*x*tan(e
 + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - C*
a*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*C
*a*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**
2*f) + 2*I*C*a/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2
*f) + 3*I*C*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*ta
n(e + f*x) - 4*d**2*f) + 6*C*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 -
 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*I*C*b*f*x/(4*d**2*f*tan(e + f*x)**
2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*C*b*log(tan(e + f*x)**2 + 1)*ta
n(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*
f) - 4*I*C*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**
2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*C*b*log(tan(e + f*x)**2 + 1)/(4
*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 5*I*C*b*tan
(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) -
 4*C*b/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f), Eq(
c, -I*d)), (-A*a*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f
*tan(e + f*x) - 4*d**2*f) - 2*I*A*a*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)
**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*a*f*x/(4*d**2*f*tan(e + f*x)*
*2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A*a*tan(e + f*x)/(4*d**2*f*tan(e
 + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*A*a/(4*d**2*f*tan(e
+ f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - I*A*b*f*x*tan(e + f*x)**2
/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*A*b*f*
x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2
```

```
*f) + I*A*b*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**
2*f) - I*A*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*
x) - 4*d**2*f) - I*B*a*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*
d**2*f*tan(e + f*x) - 4*d**2*f) + 2*B*a*f*x*tan(e + f*x)/(4*d**2*f*tan(e +
f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*a*f*x/(4*d**2*f*tan(e +
 f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - I*B*a*tan(e + f*x)/(4*d**2
*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + B*b*f*x*tan(e +
f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2
*I*B*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x)
 - 4*d**2*f) - B*b*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x)
- 4*d**2*f) - 3*B*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan
(e + f*x) - 4*d**2*f) - 2*I*B*b/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(
e + f*x) - 4*d**2*f) + C*a*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 +
8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*C*a*f*x*tan(e + f*x)/(4*d**2*f*ta
n(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - C*a*f*x/(4*d**2*f*tan
(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*C*a*tan(e + f*x)/(4*
d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*C*a/(4*d
**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*I*C*b*f*x*t
an(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2
*f) + 6*C*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e +
 f*x) - 4*d**2*f) + 3*I*C*b*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(
e + f*x) - 4*d**2*f) + 2*C*b*log(tan(e + f*x)**...
```

Giac [A]
time = 0.77, size = 528, normalized size = 1.81

$$\frac{\frac{2\,(Aac^2-Cac^2-Bbc^2+2\,Bacd+2\,Abcd-2\,Cbcd-Aad^2+Cad^2+Bbd^2)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{(Bac^2+Abc^2-Cbc^2-2\,Aacd+2\,Cacd+2\,Bbcd-Bad^2-Abd^2+Cbd^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}+\frac{2\,(Cbc^4-Bac^2d^2-Abc^2d^2+3\,Cbc^2d^2+2\,Aacd^3-2\,Cacd^3-2\,Bbcd^3+Bad^4+Abd^4)\log(|d\tan(fx+e)+c|)}{c^4d^2+2\,c^2d^4+d^6}-\frac{2\,(Cbc^4\tan(fx+e)-Bac^2d^2\tan(fx+e)-Abc^2d^2\tan(fx+e)+3\,Cbc^2d^2\tan(fx+e)+2\,Aacd^3\tan(fx+e)-2\,Cacd^3\tan(fx+e)-2\,Bbcd^3\tan(fx+e)+Bad^4\tan(fx+e)+Abd^4\tan(fx+e)+Cac^4+Bbc^4-2\,Bac^3d-2\,Abc^3d+2\,Cbc^3d+3\,Aac^2d^2-Cac^2d^2-Bbc^2d^2+Aad^4)}{(c^4d+2\,c^2d^3+d^5)(d\tan(fx+e)+c)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^2 - C*a*c^2 - B*b*c^2 + 2*B*a*c*d + 2*A*b*c*d - 2*C*b*c*d - A
*a*d^2 + C*a*d^2 + B*b*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + (B*a*c^2 +
A*b*c^2 - C*b*c^2 - 2*A*a*c*d + 2*C*a*c*d + 2*B*b*c*d - B*a*d^2 - A*b*d^2 +
 C*b*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) + 2*(C*b*c^4 - B*
a*c^2*d^2 - A*b*c^2*d^2 + 3*C*b*c^2*d^2 + 2*A*a*c*d^3 - 2*C*a*c*d^3 - 2*B*b
*c*d^3 + B*a*d^4 + A*b*d^4)*log(abs(d*tan(f*x + e) + c))/(c^4*d^2 + 2*c^2*d
^4 + d^6) - 2*(C*b*c^4*tan(f*x + e) - B*a*c^2*d^2*tan(f*x + e) - A*b*c^2*d^
2*tan(f*x + e) + 3*C*b*c^2*d^2*tan(f*x + e) + 2*A*a*c*d^3*tan(f*x + e) - 2*
C*a*c*d^3*tan(f*x + e) - 2*B*b*c*d^3*tan(f*x + e) + B*a*d^4*tan(f*x + e) +
A*b*d^4*tan(f*x + e) + C*a*c^4 + B*b*c^4 - 2*B*a*c^3*d - 2*A*b*c^3*d + 2*C*
b*c^3*d + 3*A*a*c^2*d^2 - C*a*c^2*d^2 - B*b*c^2*d^2 + A*a*d^4)/((c^4*d + 2*
c^2*d^3 + d^5)*(d*tan(f*x + e) + c)))/f
```

Mupad [B]

time = 22.01, size = 1875, normalized size = 6.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*t
an(e + f*x))^2,x)
[Out] (log(c + d*tan(e + f*x))*(d^4*(A*b + B*a) - d^3*(2*B*b*c - 2*A*a*c + 2*C*a*
c) - d^2*(A*b*c^2 + B*a*c^2 - 3*C*b*c^2) + C*b*c^4))/(f*(d^6 + 2*c^2*d^4 +
c^4*d^2)) - (log((A*B*b^2*d^4 - A*B*a^2*d^4 + B*C*a^2*d^4 + B*C*b^2*c^4 - A
^2*a*b*d^4 + B^2*a*b*d^4 + C^2*a*b*c^4 - A^2*a^2*c*d^3 + A^2*b^2*c*d^3 + B^
2*a^2*c*d^3 - B^2*b^2*c*d^3 - C^2*a^2*c*d^3 + C^2*b^2*c*d^3 + A*B*a^2*c^2*d
^2 - A*B*b^2*c^2*d^2 - B*C*a^2*c^2*d^2 + 3*B*C*b^2*c^2*d^2 + A^2*a*b*c^2*d^
2 - B^2*a*b*c^2*d^2 + 3*C^2*a*b*c^2*d^2 - A*C*a*b*c^4 + A*C*a*b*d^4 + 2*A*C
*a^2*c*d^3 - 2*A*C*b^2*c*d^3 - 4*A*C*a*b*c^2*d^2 + 4*A*B*a*b*c*d^3 - 4*B*C*
a*b*c*d^3)/(d*(c^2 + d^2)^2) + (tan(e + f*x)*(A^2*a^2*d^4 + B^2*b^2*d^4 + C
^2*a^2*d^4 + C^2*b^2*c^4 + C^2*b^2*d^4 + A^2*b^2*c^2*d^2 + B^2*a^2*c^2*d^2
+ 3*C^2*b^2*c^2*d^2 - 2*A*C*a^2*d^4 - A*C*b^2*c^4 - A*C*b^2*d^4 - 4*A*C*b^2
*c^2*d^2 - 2*A*B*a*b*d^4 - B*C*a*b*c^4 + B*C*a*b*d^4 - 2*A*B*a^2*c*d^3 + 2*
A*B*b^2*c*d^3 + 2*B*C*a^2*c*d^3 - 2*B*C*b^2*c*d^3 - 2*A^2*a*b*c*d^3 + 2*B^2
*a*b*c*d^3 - 2*C^2*a*b*c*d^3 + 2*A*B*a*b*c^2*d^2 - 4*B*C*a*b*c^2*d^2 + 4*A*
C*a*b*c*d^3))/(d*(c^2 + d^2)^2) + ((a*1i + b)*(B*1i - A + C)*(A*a*d - B*b*d
 - C*a*d - 4*C*b*c + (tan(e + f*x)*(3*A*b*d^4 + 3*B*a*d^4 + 2*C*b*c^4 - 5*C
*b*d^4 + 4*A*a*c*d^3 - 4*B*b*c*d^3 - 4*C*a*c*d^3 - A*b*c^2*d^2 - B*a*c^2*d^
2 + C*b*c^2*d^2))/(d*(c^2 + d^2)) + (d*(a*1i + b)*(4*c*d - c^2*tan(e + f*x)
 + 3*d^2*tan(e + f*x))*(B*1i - A + C))/(c*1i + d)^2))/(2*(c*1i + d)^2))*(A*
a*1i + A*b + B*a - B*b*1i - C*a*1i - C*b))/(2*f*(c*d*2i - c^2 + d^2)) - (lo
g((A*B*b^2*d^4 - A*B*a^2*d^4 + B*C*a^2*d^4 + B*C*b^2*c^4 - A^2*a*b*d^4 + B^
2*a*b*d^4 + C^2*a*b*c^4 - A^2*a^2*c*d^3 + A^2*b^2*c*d^3 + B^2*a^2*c*d^3 - B
^2*b^2*c*d^3 - C^2*a^2*c*d^3 + C^2*b^2*c*d^3 + A*B*a^2*c^2*d^2 - A*B*b^2*c^
2*d^2 - B*C*a^2*c^2*d^2 + 3*B*C*b^2*c^2*d^2 + A^2*a*b*c^2*d^2 - B^2*a*b*c^2
*d^2 + 3*C^2*a*b*c^2*d^2 - A*C*a*b*c^4 + A*C*a*b*d^4 + 2*A*C*a^2*c*d^3 - 2*
A*C*b^2*c*d^3 - 4*A*C*a*b*c^2*d^2 + 4*A*B*a*b*c*d^3 - 4*B*C*a*b*c*d^3)/(d*(
c^2 + d^2)^2) + (tan(e + f*x)*(A^2*a^2*d^4 + B^2*b^2*d^4 + C^2*a^2*d^4 + C^
2*b^2*c^4 + C^2*b^2*d^4 + A^2*b^2*c^2*d^2 + B^2*a^2*c^2*d^2 + 3*C^2*b^2*c^2
*d^2 - 2*A*C*a^2*d^4 - A*C*b^2*c^4 - A*C*b^2*d^4 - 4*A*C*b^2*c^2*d^2 - 2*A*
B*a*b*d^4 - B*C*a*b*c^4 + B*C*a*b*d^4 - 2*A*B*a^2*c*d^3 + 2*A*B*b^2*c*d^3 +
 2*B*C*a^2*c*d^3 - 2*B*C*b^2*c*d^3 - 2*A^2*a*b*c*d^3 + 2*B^2*a*b*c*d^3 - 2*
C^2*a*b*c*d^3 + 2*A*B*a*b*c^2*d^2 - 4*B*C*a*b*c^2*d^2 + 4*A*C*a*b*c*d^3))/(
d*(c^2 + d^2)^2) + ((a + b*1i)*(A + B*1i - C)*(A*a*d - B*b*d - C*a*d - 4*C*
b*c + (tan(e + f*x)*(3*A*b*d^4 + 3*B*a*d^4 + 2*C*b*c^4 - 5*C*b*d^4 + 4*A*a*
c*d^3 - 4*B*b*c*d^3 - 4*C*a*c*d^3 - A*b*c^2*d^2 - B*a*c^2*d^2 + C*b*c^2*d^2
))/(d*(c^2 + d^2)) + (d*(a + b*1i)*(4*c*d - c^2*tan(e + f*x) + 3*d^2*tan(e
+ f*x))*(A + B*1i - C)*1i)/(c*1i - d)^2)*1i)/(2*(c*1i - d)^2))*(A*a + A*b*1
```

```
i + B*a*1i - B*b - C*a - C*b*1i))/(2*f*(2*c*d - c^2*1i + d^2*1i)) - (A*a*d^
3 - C*b*c^3 - A*b*c*d^2 - B*a*c*d^2 + B*b*c^2*d + C*a*c^2*d)/(d^2*f*(c^2 +
d^2)*(c + d*tan(e + f*x)))
```

3.80 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=140

$$-\frac{(c^2C-2Bcd-Cd^2-A(c^2-d^2))\,x}{(c^2+d^2)^2}+\frac{(2c(A-C)d-B(c^2-d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{(c^2+d^2)^2\,f}-\frac{}{d\,(}$$

[Out] -(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))*x/(c^2+d^2)^2+(2*c*(A-C)*d-B*(c^2-d^2))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)^2/f+(-A*d^2+B*c*d-C*c^2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3709, 3612, 3611}

$$-\frac{Ad^2-Bcd+c^2C}{df\,(c^2+d^2)\,(c+d\tan(e+fx))}+\frac{(2cd(A-C)-B(c^2-d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^2}-\frac{x(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)}{(c^2+d^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^2,x]

[Out] -(((c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2))*x)/(c^2 + d^2)^2) + ((2*c*(A - C)*d - B*(c^2 - d^2))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)^2*f) - (c^2*C - B*c*d + A*d^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
```

```
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx = -\frac{c^2C-Bcd+Ad^2}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))} + \frac{\int \frac{Ac-cC+Bd+(Bc-(A-C)d)\,\text{ta}}{c+d\tan(e+fx)}}{c^2+d^2}$$

$$= -\frac{(c^2C-2Bcd-Cd^2-A(c^2-d^2))\,x}{(c^2+d^2)^2} - \frac{c^2C-Bcd+}{d\,(c^2+d^2)\,f(c+d\,\text{ta}}$$

$$= -\frac{(c^2C-2Bcd-Cd^2-A(c^2-d^2))\,x}{(c^2+d^2)^2} + \frac{(2c(A-C)d-B(c^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.78, size = 207, normalized size = 1.48

$$\frac{\frac{B((-ic-d)\log(i-\tan(e+fx))+i(c+id)\log(i+\tan(e+fx))+2d\log(c+d\tan(e+fx)))}{c^2+d^2}-\frac{2C}{c+d\tan(e+fx)}+(Bc+(-A+C)d)\left(\frac{i\log(i-\tan(e+fx))}{(c+id)^2}-\frac{i\log(i+\tan(e+fx))}{(c-id)^2}+\frac{2d\left(-2c\log(c+d\tan(e+fx))+\frac{c^2+d^2}{c+d\tan(e+fx)}\right)}{(c^2+d^2)^2}\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((B*(((-I)*c - d)*Log[I - Tan[e + f*x]] + I*(c + I*d)*Log[I + Tan[e + f*x]]
 + 2*d*Log[c + d*Tan[e + f*x]]))/(c^2 + d^2) - (2*C)/(c + d*Tan[e + f*x]) +
 (B*c + (-A + C)*d)*((I*Log[I - Tan[e + f*x]])/(c + I*d)^2 - (I*Log[I + Tan
[e + f*x]])/(c - I*d)^2 + (2*d*(-2*c*Log[c + d*Tan[e + f*x]] + (c^2 + d^2)/
(c + d*Tan[e + f*x])))/(c^2 + d^2)^2))/(2*d*f)
```

Maple [A]
time = 0.21, size = 173, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{A\,d^2-Bcd+c^2C}{(c^2+d^2)\,d(c+d\tan(fx+e))}+\frac{(2Acd-B\,c^2+B\,d^2-2cCd)\ln(c+d\tan(fx+e))}{(c^2+d^2)^2}+\frac{\frac{(-2Acd+B\,c^2-B\,d^2+2cCd)\ln(1+\tan^2(fx+e))}{2}+(A}{(c^2+d^2}}{f}$
default	$\frac{-\frac{A\,d^2-Bcd+c^2C}{(c^2+d^2)\,d(c+d\tan(fx+e))}+\frac{(2Acd-B\,c^2+B\,d^2-2cCd)\ln(c+d\tan(fx+e))}{(c^2+d^2)^2}+\frac{\frac{(-2Acd+B\,c^2-B\,d^2+2cCd)\ln(1+\tan^2(fx+e))}{2}+(A}{(c^2+d^2}}{f}$
norman	$\frac{\frac{c(A\,c^2-A\,d^2+2Bcd-c^2C+C\,d^2)x}{c^4+2c^2d^2+d^4}+\frac{d(A\,c^2-A\,d^2+2Bcd-c^2C+C\,d^2)x\tan(fx+e)}{c^4+2c^2d^2+d^4}-\frac{A\,d^2-Bcd+c^2C}{(c^2+d^2)\,df}}{c+d\tan(fx+e)}+\frac{(2Acd-B\,c^2+B\,d^2-2c}{f(c^4+2c}$

risch	$-\frac{4iAcdx}{c^4+2c^2d^2+d^4}-\frac{xA}{2icd-c^2+d^2}+\frac{xC}{2icd-c^2+d^2}+\frac{2iA\,d^2}{(id+c)f(-id+c)^2\left(-ie^{2i(fx+e)}d+id+e^{2i(fx+e)}c+c\right)}+\frac{2i}{f(c^4+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x,method=_RETURNVERB
OSE)
```

```
[Out] 1/f*(-(A*d^2-B*c*d+C*c^2)/(c^2+d^2)/d/(c+d*tan(f*x+e))+(2*A*c*d-B*c^2+B*d^2
-2*C*c*d)/(c^2+d^2)^2*ln(c+d*tan(f*x+e))+1/(c^2+d^2)^2*(1/2*(-2*A*c*d+B*c^2
-B*d^2+2*C*c*d)*ln(1+tan(f*x+e)^2)+(A*c^2-A*d^2+2*B*c*d-C*c^2+C*d^2)*arctan
(tan(f*x+e))))
```

Maxima [A]
time = 0.51, size = 209, normalized size = 1.49

$$\frac{\frac{2\left((A-C)c^2+2\,Bcd-(A-C)d^2\right)(fx+e)}{c^4+2\,c^2d^2+d^4}-\frac{2\left(Bc^2-2\,(A-C)cd-Bd^2\right)\log\left(d\tan\left(fx+e\right)+c\right)}{c^4+2\,c^2d^2+d^4}+\frac{\left(Bc^2-2\,(A-C)cd-Bd^2\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^4+2\,c^2d^2+d^4}-\frac{2\left(Cc^2-Bcd+Ad^2\right)}{c^3d+cd^3+\left(c^2d^2+d^4\right)\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] 1/2*(2*((A - C)*c^2 + 2*B*c*d - (A - C)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d
^4) - 2*(B*c^2 - 2*(A - C)*c*d - B*d^2)*log(d*tan(f*x + e) + c)/(c^4 + 2*c^
2*d^2 + d^4) + (B*c^2 - 2*(A - C)*c*d - B*d^2)*log(tan(f*x + e)^2 + 1)/(c^4
 + 2*c^2*d^2 + d^4) - 2*(C*c^2 - B*c*d + A*d^2)/(c^3*d + c*d^3 + (c^2*d^2 +
 d^4)*tan(f*x + e)))/f
```

Fricas [A]
time = 3.29, size = 262, normalized size = 1.87

$$-\frac{2\,Cc^2d-2\,Bcd^2+2\,Ad^3-2\,\left((A-C)c^3+2\,Bc^2d-(A-C)cd^2\right)fx+\left(Bc^3-2\,(A-C)c^2d-Bcd^2+\left(Bc^2d-2\,(A-C)cd^2-Bd^3\right)\tan\left(fx+e\right)\right)\log\left(\frac{d^2\tan\left(fx+e\right)^2+2\,cd\tan\left(fx+e\right)+c^2}{\tan\left(fx+e\right)^2+1}\right)-2\,\left(Cc^3-Bc^2d+Acd^2+\left((A-C)c^2d+2\,Bcd^2-(A-C)d^3\right)fx\right)\tan\left(fx+e\right)}{2\,\left(\left(c^4d+2\,c^2d^3+d^5\right)f\tan\left(fx+e\right)+\left(c^5+2\,c^3d^2+cd^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] -1/2*(2*C*c^2*d - 2*B*c*d^2 + 2*A*d^3 - 2*((A - C)*c^3 + 2*B*c^2*d - (A - C
)*c*d^2)*f*x + (B*c^3 - 2*(A - C)*c^2*d - B*c*d^2 + (B*c^2*d - 2*(A - C)*c*
d^2 - B*d^3)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c
^2)/(tan(f*x + e)^2 + 1)) - 2*(C*c^3 - B*c^2*d + A*c*d^2 + ((A - C)*c^2*d +
 2*B*c*d^2 - (A - C)*d^3)*f*x)*tan(f*x + e))/((c^4*d + 2*c^2*d^3 + d^5)*f*t
an(f*x + e) + (c^5 + 2*c^3*d^2 + c*d^4)*f)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.99, size = 4396, normalized size = 31.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**2,x)
```

```
[Out] Piecewise((zoo*x*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2, Eq(c, 0) & Eq(d, 0
) & Eq(f, 0)), (-A*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2
*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)
**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*f*x/(4*d**2*f*tan(e + f*x)**2
 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A*tan(e + f*x)/(4*d**2*f*tan(e + f
*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A/(4*d**2*f*tan(e + f*x)
**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*f*x*tan(e + f*x)**2/(4*d**2
*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*B*f*x*tan(e +
f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - I*B*
f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*t
an(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
 + C*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x
) - 4*d**2*f) - 2*I*C*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2
*f*tan(e + f*x) - 4*d**2*f) - C*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*
tan(e + f*x) - 4*d**2*f) - 3*C*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I
*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*C/(4*d**2*f*tan(e + f*x)**2 - 8*I*d*
*2*f*tan(e + f*x) - 4*d**2*f), Eq(c, -I*d)), (-A*f*x*tan(e + f*x)**2/(4*d**
2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*A*f*x*tan(e
 + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A
*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A*ta
n(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
- 2*I*A/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - I
*B*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x)
- 4*d**2*f) + 2*B*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*t
an(e + f*x) - 4*d**2*f) + I*B*f*x/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*ta
n(e + f*x) - 4*d**2*f) - I*B*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d
**2*f*tan(e + f*x) - 4*d**2*f) + C*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*
x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*C*f*x*tan(e + f*x)/(4*d**
2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - C*f*x/(4*d**2*f
*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*C*tan(e + f*x)/(
4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*C/(4*d
**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) - 4*d**2*f), Eq(c, I*d)), (
(A*x + B*log(tan(e + f*x)**2 + 1)/(2*f) - C*x + C*tan(e + f*x)/f)/c**2, Eq(
d, 0)), (x*(A + B*tan(e) + C*tan(e)**2)/(c + d*tan(e))**2, Eq(f, 0)), (2*A*
c**3*d*f*x/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**
2*d**4*f*tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) + 2*A*c**2*d**2
*f*x*tan(e + f*x)/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f
+ 4*c**2*d**4*f*tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) + 4*A*c*
*2*d**2*log(c/d + tan(e + f*x))/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) +
4*c**3*d**3*f + 4*c**2*d**4*f*tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e +
f*x)) - 2*A*c**2*d**2*log(tan(e + f*x)**2 + 1)/(2*c**5*d*f + 2*c**4*d**2*f*
```

```
tan(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*f*tan(e + f*x) + 2*c*d**5*f + 2*
d**6*f*tan(e + f*x)) - 2*A*c**2*d**2/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*
x) + 4*c**3*d**3*f + 4*c**2*d**4*f*tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan
(e + f*x)) - 2*A*c*d**3*f*x/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c*
*3*d**3*f + 4*c**2*d**4*f*tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)
) + 4*A*c*d**3*log(c/d + tan(e + f*x))*tan(e + f*x)/(2*c**5*d*f + 2*c**4*d*
*2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*f*tan(e + f*x) + 2*c*d**5*f
 + 2*d**6*f*tan(e + f*x)) - 2*A*c*d**3*log(tan(e + f*x)**2 + 1)*tan(e + f*x
)/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*f*
tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) - 2*A*d**4*f*x*tan(e + f
*x)/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*
f*tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) - 2*A*d**4/(2*c**5*d*f
 + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*f*tan(e + f*x)
+ 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) - 2*B*c**3*d*log(c/d + tan(e + f*x))/
(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*f*ta
n(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) + B*c**3*d*log(tan(e + f*x
)**2 + 1)/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**2
*d**4*f*tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) + 2*B*c**3*d/(2*
c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*f*tan(e
 + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) + 4*B*c**2*d**2*f*x/(2*c**5*d
*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*f*tan(e + f*x
) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) - 2*B*c**2*d**2*log(c/d + tan(e + f
*x))*tan(e + f*x)/(2*c**5*d*f + 2*c**4*d**2*f*tan(e + f*x) + 4*c**3*d**3*f
+ 4*c**2*d**4*f*tan(e + f*x) + 2*c*d**5*f + 2*d**6*f*tan(e + f*x)) + B*c**2
*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c**5*d*f + 2*c**4*d**2*f*tan
(e + f*x) + 4*c**3*d**3*f + 4*c**2*d**4*f*tan(e...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(143) = 286$.

time = 0.71, size = 299, normalized size = 2.14

$$\frac{\frac{2\,(Ac^2-Cc^2+2\,Bcd-Ad^2+Cd^2)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{(Bc^2-2\,Acd+2\,Ccd-Bd^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}-\frac{2\,(Bc^2d-2\,Acd^2+2\,Ccd^2-Bd^3)\log(|d\tan(fx+e)+c|)}{c^4d+2\,c^2d^3+d^5}+\frac{2\,(Bc^2d^2\tan(fx+e)-2\,Acd^3\tan(fx+e)+2\,Ccd^3\tan(fx+e)-Bd^4\tan(fx+e)-Cc^4+2\,Bc^3d-3\,Ac^2d^2+Cc^2d^2-Ad^4)}{(c^4d+2\,c^2d^3+d^5)(d\tan(fx+e)+c)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] 1/2*(2*(A*c^2 - C*c^2 + 2*B*c*d - A*d^2 + C*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2
 + d^4) + (B*c^2 - 2*A*c*d + 2*C*c*d - B*d^2)*log(tan(f*x + e)^2 + 1)/(c^4
+ 2*c^2*d^2 + d^4) - 2*(B*c^2*d - 2*A*c*d^2 + 2*C*c*d^2 - B*d^3)*log(abs(d*
tan(f*x + e) + c))/(c^4*d + 2*c^2*d^3 + d^5) + 2*(B*c^2*d^2*tan(f*x + e) -
2*A*c*d^3*tan(f*x + e) + 2*C*c*d^3*tan(f*x + e) - B*d^4*tan(f*x + e) - C*c^
4 + 2*B*c^3*d - 3*A*c^2*d^2 + C*c^2*d^2 - A*d^4)/((c^4*d + 2*c^2*d^3 + d^5)
*(d*tan(f*x + e) + c)))/f
```

Mupad [B]
time = 11.35, size = 184, normalized size = 1.31

$$\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(-B\,c^2+(2\,A-2\,C)\,c\,d+B\,d^2\right)}{f\,\left(c^4+2\,c^2\,d^2+d^4\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A-C+B\,1\mathrm{i}\right)}{2\,f\,\left(-c^2\,1\mathrm{i}+2\,c\,d+d^2\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,1\mathrm{i}+B-C\,1\mathrm{i}\right)}{2\,f\,\left(-c^2+c\,d\,2\mathrm{i}+d^2\right)}-\frac{C\,c^2-B\,c\,d+A\,d^2}{d\,f\,\left(c^2+d^2\right)\,\left(c+d\tan\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^2,x)
```

```
[Out] (log(c + d*tan(e + f*x))*(B*d^2 - B*c^2 + c*d*(2*A - 2*C)))/(f*(c^4 + d^4 +
 2*c^2*d^2)) - (log(tan(e + f*x) - 1i)*(A + B*1i - C))/(2*f*(2*c*d - c^2*1i
 + d^2*1i)) - (log(tan(e + f*x) + 1i)*(A*1i + B - C*1i))/(2*f*(c*d*2i - c^2
 + d^2)) - (A*d^2 + C*c^2 - B*c*d)/(d*f*(c^2 + d^2)*(c + d*tan(e + f*x)))
```

3.81 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=293

$$-\frac{(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))+b(2c(A-C)d-B(c^2-d^2)))\,x}{(a^2+b^2)\,(c^2+d^2)^2}+\frac{b(Ab^2-a(bB-aC))\log(a\cos(e}{(a^2+b^2)\,(bc-}$$

```
[Out] -(a*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))+b*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(a^2+b
^2)/(c^2+d^2)^2+b*(A*b^2-a*(B*b-C*a))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^
2)/(-a*d+b*c)^2/f-(b*(c^4*C-2*B*c^3*d+c^2*(3*A-C)*d^2+A*d^4)-a*d^2*(2*c*(A-
C)*d-B*(c^2-d^2)))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^2/(c^2+d^2)^2/f
+(A*d^2-B*c*d+C*c^2)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

Rubi [A]

time = 0.54, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3730, 3732, 3611}

$$-\frac{x(a(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)+b(2cd(A-C)-B(c^2-d^2)))}{(a^2+b^2)\,(c^2+d^2)^2}+\frac{b(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{f\,(a^2+b^2)\,(bc-ad)^2}+\frac{Ad^2-Bcd+c^2C}{f\,(c^2+d^2)\,(bc-ad)(c+d\tan(e+fx))}-\frac{(b(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C)-ad^2(2cd(A-C)-B(c^2-d^2)))\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^2\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])^2),x]
```

```
[Out] -(((a*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d - B*(c^2
 - d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)^2)) + (b*(A*b^2 - a*(b*B - a*C))*Log[
a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^2*f) - ((b*(c^4*
C - 2*B*c^3*d + c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2
- d^2)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^2*(c^2 + d^2)^2
*f) + (c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x
]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx = \frac{c^2C-Bcd+Ad^2}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))} + \frac{\int \frac{-aAcd+ad(cC-Bc}{}}{}$$

$$= -\frac{(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))+b(2c(A-C)d-B}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2}$$

$$= -\frac{(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))+b(2c(A-C)d-B}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2}$$

Mathematica [A]
time = 4.92, size = 561, normalized size = 1.91

$$\frac{\frac{(bc-ad)\left(Abc^2-aBc^2-bc^2C+2aAcd+2bBcd-2acCd-Abd^2+aBd^2+bCd^2+\frac{\sqrt{-b^2}\left(a\left(-c^2C+2Bcd+Cd^2+A\left(c^2-d^2\right)\right)+b\left(2c(-A+C)d+B\left(c^2-d^2\right)\right)\right)}{b}\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)}{2(a^2+b^2)(c^2+d^2)} - \frac{b(Ab^2+a(-bB+aC))\left(c^2+d^2\right)\log(a+b\tan(e+fx))}{(a^2+b^2)(bc-ad)} + \frac{(bc-ad)\left(Abc^2-aBc^2-bc^2C+2aAcd+2bBcd-2acCd-Abd^2+aBd^2+bCd^2+\frac{\sqrt{-b^2}\left(a\left(c^2C-2Bcd-Cd^2+A\left(-c^2+d^2\right)\right)+b\left(2c(A-C)d+B\left(-c^2+d^2\right)\right)\right)}{b}\right)\log\left(\sqrt{-b^2}+b\tan(e+fx)\right)}{2(a^2+b^2)(c^2+d^2)} + \frac{\left(b\left(c^4C-2Bc^3d+c^2(3A-C)d^2+Ad^4\right)+ad^2\left(2c(-A+C)d+B\left(c^2-d^2\right)\right)\right)\log(c+d\tan(e+fx))}{(bc-ad)(c^2+d^2)} - \frac{Ad^2}{c+d\tan(e+fx)} + \frac{c(-cC+Bd)}{c+d\tan(e+fx)}}{(-bc+ad)\left(c^2+d^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])^2),x]
```

```
[Out] (((b*c - a*d)*(A*b*c^2 - a*B*c^2 - b*c^2*C + 2*a*A*c*d + 2*b*B*c*d - 2*a*c*
C*d - A*b*d^2 + a*B*d^2 + b*C*d^2 + (Sqrt[-b^2]*(a*(-(c^2*C) + 2*B*c*d + C*
d^2 + A*(c^2 - d^2)) + b*(2*c*(-A + C)*d + B*(c^2 - d^2))))/b)*Log[Sqrt[-b^
2] - b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b*(A*b^2 + a*(-(b*B) +
 a*C))*(c^2 + d^2)*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)) + ((b
```

```
*c - a*d)*(A*b*c^2 - a*B*c^2 - b*c^2*C + 2*a*A*c*d + 2*b*B*c*d - 2*a*c*C*d
- A*b*d^2 + a*B*d^2 + b*C*d^2 + (Sqrt[-b^2]*(a*(c^2*C - 2*B*c*d - C*d^2 + A
*(-c^2 + d^2)) + b*(2*c*(A - C)*d + B*(-c^2 + d^2))))/b)*Log[Sqrt[-b^2] + b
*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) + ((b*(c^4*C - 2*B*c^3*d + c^2*
(3*A - C)*d^2 + A*d^4) + a*d^2*(2*c*(-A + C)*d + B*(c^2 - d^2)))*Log[c + d*
Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)) - (A*d^2)/(c + d*Tan[e + f*x]) + (
c*(-(c*C) + B*d))/(c + d*Tan[e + f*x]))/((-(b*c) + a*d)*(c^2 + d^2)*f)
```

Maple [A]
time = 0.76, size = 365, normalized size = 1.25

method	result
derivativedivides	$\frac{\frac{\left(-2Aacd-Abc^2+Abd^2+Bac^2-Bad^2-2Bbcd+2Cacd+Cbc^2-Cbd^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(Aac^2-Aad^2-2Abcd+2Bacd+Bbc^2-\right.}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2}$
default	$\frac{\frac{\left(-2Aacd-Abc^2+Abd^2+Bac^2-Bad^2-2Bbcd+2Cacd+Cbc^2-Cbd^2\right)\ln\left(1+\tan^2(fx+e)\right)}{2}+\left(Aac^2-Aad^2-2Abcd+2Bacd+Bbc^2-\right.}{\left(a^2+b^2\right)\left(c^2+d^2\right)^2}$
norman	$\frac{\frac{\left(Aac^2-Aad^2-2Abcd+2Bacd+Bbc^2-Bbd^2-Cac^2+Cad^2+2Cbcd\right)cx}{\left(a^2+b^2\right)\left(c^4+2c^2d^2+d^4\right)}+\frac{d\left(Aac^2-Aad^2-2Abcd+2Bacd+Bbc^2-Bbd^2-Cac^2+C\right.}{\left(a^2+b^2\right)\left(c^4+2c^2d^2+d^4\right)}}{c+d\tan(fx+e)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)/(c^2+d^2)^2*(1/2*(-2*A*a*c*d-A*b*c^2+A*b*d^2+B*a*c^2-B*a*d
^2-2*B*b*c*d+2*C*a*c*d+C*b*c^2-C*b*d^2)*ln(1+tan(f*x+e)^2)+(A*a*c^2-A*a*d^2
-2*A*b*c*d+2*B*a*c*d+B*b*c^2-B*b*d^2-C*a*c^2+C*a*d^2+2*C*b*c*d)*arctan(tan(
f*x+e)))+(A*b^2-B*a*b+C*a^2)*b/(a*d-b*c)^2/(a^2+b^2)*ln(a+b*tan(f*x+e))+(2*
A*a*c*d^3-3*A*b*c^2*d^2-A*b*d^4-B*a*c^2*d^2+B*a*d^4+2*B*b*c^3*d-2*C*a*c*d^3
-C*b*c^4+C*b*c^2*d^2)/(c^2+d^2)^2/(a*d-b*c)^2*ln(c+d*tan(f*x+e))-(A*d^2-B*c
*d+C*c^2)/(c^2+d^2)/(a*d-b*c)/(c+d*tan(f*x+e)))
```

Maxima [A]
time = 0.53, size = 518, normalized size = 1.77

$$\frac{\frac{2\,\left(\left(\left(A-C\right)a+Bb\right)c^2+2\,\left(Ba-\left(A-C\right)b\right)cd-\left(\left(A-C\right)a+Bb\right)d^2\right)\left(fx+e\right)}{\left(a^2+b^2\right)c^4+2\,\left(a^2+b^2\right)c^2d^2+\left(a^2+b^2\right)d^4}+\frac{2\,\left(Ca^2b-Bab^2+Ab^3\right)\log\left(b\tan\left(fx+e\right)+a\right)}{\left(a^2b^2+b^4\right)c^2-2\,\left(a^3b+ab^3\right)cd+\left(a^4+a^2b^2\right)d^2}-\frac{2\,\left(Cbc^4-2\,Bbc^3d-2\,\left(A-C\right)acd^3+\left(Ba+\left(3\,A-C\right)b\right)c^2d^2-\left(Ba-Ab\right)d^4\right)\log\left(d\tan\left(fx+e\right)+c\right)}{b^2c^6-2\,abc^5d-4\,abc^3d^3-2\,abcd^5+a^2d^6+\left(a^2+2\,b^2\right)c^4d^2+\left(2\,a^2+b^2\right)c^2d^4}+\frac{\left(\left(Ba-\left(A-C\right)b\right)c^2-2\,\left(\left(A-C\right)a+Bb\right)cd-\left(Ba-\left(A-C\right)b\right)d^2\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{\left(a^2+b^2\right)c^4+2\,\left(a^2+b^2\right)c^2d^2+\left(a^2+b^2\right)d^4}+\frac{2\,\left(Cc^2-Bcd+Ad^2\right)}{bc^4-ac^3d+bc^2d^2-acd^3+\left(bc^3d-ac^2d^2+bcd^3-ad^4\right)\tan\left(fx+e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a + B*b)*c^2 + 2*(B*a - (A - C)*b)*c*d - ((A - C)*a + B*b)
*d^2)*(f*x + e)/((a^2 + b^2)*c^4 + 2*(a^2 + b^2)*c^2*d^2 + (a^2 + b^2)*d^4)
```

```
 + 2*(C*a^2*b - B*a*b^2 + A*b^3)*log(b*tan(f*x + e) + a)/((a^2*b^2 + b^4)*c
^2 - 2*(a^3*b + a*b^3)*c*d + (a^4 + a^2*b^2)*d^2) - 2*(C*b*c^4 - 2*B*b*c^3*
d - 2*(A - C)*a*c*d^3 + (B*a + (3*A - C)*b)*c^2*d^2 - (B*a - A*b)*d^4)*log(
d*tan(f*x + e) + c)/(b^2*c^6 - 2*a*b*c^5*d - 4*a*b*c^3*d^3 - 2*a*b*c*d^5 +
a^2*d^6 + (a^2 + 2*b^2)*c^4*d^2 + (2*a^2 + b^2)*c^2*d^4) + ((B*a - (A - C)*
b)*c^2 - 2*((A - C)*a + B*b)*c*d - (B*a - (A - C)*b)*d^2)*log(tan(f*x + e)^
2 + 1)/((a^2 + b^2)*c^4 + 2*(a^2 + b^2)*c^2*d^2 + (a^2 + b^2)*d^4) + 2*(C*c
^2 - B*c*d + A*d^2)/(b*c^4 - a*c^3*d + b*c^2*d^2 - a*c*d^3 + (b*c^3*d - a*c
^2*d^2 + b*c*d^3 - a*d^4)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1285 vs. $2(296) = 592$.

time = 10.13, size = 1285, normalized size = 4.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(C*a^2*b + C*b^3)*c^3*d^2 - 2*(C*a^3 + B*a^2*b + C*a*b^2 + B*b^3)*c^
2*d^3 + 2*(B*a^3 + A*a^2*b + B*a*b^2 + A*b^3)*c*d^4 - 2*(A*a^3 + A*a*b^2)*d
^5 + 2*(((A - C)*a*b^2 + B*b^3)*c^5 - 2*((A - C)*a^2*b + (A - C)*b^3)*c^4*d
 + ((A - C)*a^3 - 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c^3*d^2 + 2*(B*a^3 +
 B*a*b^2)*c^2*d^3 - ((A - C)*a^3 + B*a^2*b)*c*d^4)*f*x + ((C*a^2*b - B*a*b^
2 + A*b^3)*c^5 + 2*(C*a^2*b - B*a*b^2 + A*b^3)*c^3*d^2 + (C*a^2*b - B*a*b^2
 + A*b^3)*c*d^4 + ((C*a^2*b - B*a*b^2 + A*b^3)*c^4*d + 2*(C*a^2*b - B*a*b^2
 + A*b^3)*c^2*d^3 + (C*a^2*b - B*a*b^2 + A*b^3)*d^5)*tan(f*x + e))*log((b^2
*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - ((C*a^2
*b + C*b^3)*c^5 - 2*(B*a^2*b + B*b^3)*c^4*d + (B*a^3 + (3*A - C)*a^2*b + B*
a*b^2 + (3*A - C)*b^3)*c^3*d^2 - 2*((A - C)*a^3 + (A - C)*a*b^2)*c^2*d^3 -
(B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)*c*d^4 + ((C*a^2*b + C*b^3)*c^4*d - 2*(B
*a^2*b + B*b^3)*c^3*d^2 + (B*a^3 + (3*A - C)*a^2*b + B*a*b^2 + (3*A - C)*b^
3)*c^2*d^3 - 2*((A - C)*a^3 + (A - C)*a*b^2)*c*d^4 - (B*a^3 - A*a^2*b + B*a
*b^2 - A*b^3)*d^5)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x +
e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*((C*a^2*b + C*b^3)*c^4*d - (C*a^3 + B*a
^2*b + C*a*b^2 + B*b^3)*c^3*d^2 + (B*a^3 + A*a^2*b + B*a*b^2 + A*b^3)*c^2*d
^3 - (A*a^3 + A*a*b^2)*c*d^4 - (((A - C)*a*b^2 + B*b^3)*c^4*d - 2*((A - C)*
a^2*b + (A - C)*b^3)*c^3*d^2 + ((A - C)*a^3 - 3*B*a^2*b + 3*(A - C)*a*b^2 -
 B*b^3)*c^2*d^3 + 2*(B*a^3 + B*a*b^2)*c*d^4 - ((A - C)*a^3 + B*a^2*b)*d^5)*
f*x)*tan(f*x + e))/(((a^2*b^2 + b^4)*c^6*d - 2*(a^3*b + a*b^3)*c^5*d^2 + (a
^4 + 3*a^2*b^2 + 2*b^4)*c^4*d^3 - 4*(a^3*b + a*b^3)*c^3*d^4 + (2*a^4 + 3*a^
2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b + a*b^3)*c*d^6 + (a^4 + a^2*b^2)*d^7)*f*tan
(f*x + e) + ((a^2*b^2 + b^4)*c^7 - 2*(a^3*b + a*b^3)*c^6*d + (a^4 + 3*a^2*b
^2 + 2*b^4)*c^5*d^2 - 4*(a^3*b + a*b^3)*c^4*d^3 + (2*a^4 + 3*a^2*b^2 + b^4)
*c^3*d^4 - 2*(a^3*b + a*b^3)*c^2*d^5 + (a^4 + a^2*b^2)*c*d^6)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(296) = 592.
time = 0.94, size = 846, normalized size = 2.89

$$\frac{\frac{2\,(Aac^2-Cac^2+Bbc^2+2\,Bacd-2\,Abcd+2\,Cbcd-Aad^2+Cad^2-Bbd^2)(fx+e)}{a^2c^4+b^2c^4+2\,a^2c^2d^2+2\,b^2c^2d^2+a^2d^4+b^2d^4}+\frac{(Bac^2-Abc^2+Cbc^2-2\,Aacd+2\,Cacd-2\,Bbcd-Bad^2+Abd^2-Cbd^2)\log\left(\tan(fx+e)^2+1\right)}{a^2c^4+b^2c^4+2\,a^2c^2d^2+2\,b^2c^2d^2+a^2d^4+b^2d^4}+\frac{2\,(Ca^2b^2-Bab^3+Ab^4)\log(|b\tan(fx+e)+a|)}{a^2b^3c^2+b^5c^2-2\,a^3b^2cd-2\,ab^4cd+a^4bd^2+a^2b^3d^2}-\frac{2\,(Cbc^4d-2\,Bbc^3d^2+Bac^2d^3+3\,Abc^2d^3-Cbc^2d^3-2\,Aacd^4+2\,Cacd^4-Bad^5+Abd^5)\log(|d\tan(fx+e)+c|)}{b^2c^6d-2\,abc^5d^2+a^2c^4d^3+2\,b^2c^4d^3-4\,abc^3d^4+2\,a^2c^2d^5+b^2c^2d^5-2\,abcd^6+a^2d^7}+\frac{2\,(Cbc^4d\tan(fx+e)-2\,Bbc^3d^2\tan(fx+e)+Bac^2d^3\tan(fx+e)+3\,Abc^2d^3\tan(fx+e)-Cbc^2d^3\tan(fx+e)-2\,Aacd^4\tan(fx+e)+2\,Cacd^4\tan(fx+e)-Bad^5\tan(fx+e)+Abd^5\tan(fx+e)+2\,Cbc^5-Cac^4d-3\,Bbc^4d+2\,Bac^3d^2+4\,Abc^3d^2-3\,Aac^2d^3+Cac^2d^3-Bbc^2d^3+2\,Abcd^4-Aad^5)}{(b^2c^6-2\,abc^5d+a^2c^4d^2+2\,b^2c^4d^2-4\,abc^3d^3+2\,a^2c^2d^4+b^2c^2d^4-2\,abcd^5+a^2d^6)(d\tan(fx+e)+c)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^2 - C*a*c^2 + B*b*c^2 + 2*B*a*c*d - 2*A*b*c*d + 2*C*b*c*d - A
*a*d^2 + C*a*d^2 - B*b*d^2)*(f*x + e)/(a^2*c^4 + b^2*c^4 + 2*a^2*c^2*d^2 +
2*b^2*c^2*d^2 + a^2*d^4 + b^2*d^4) + (B*a*c^2 - A*b*c^2 + C*b*c^2 - 2*A*a*c
*d + 2*C*a*c*d - 2*B*b*c*d - B*a*d^2 + A*b*d^2 - C*b*d^2)*log(tan(f*x + e)^
2 + 1)/(a^2*c^4 + b^2*c^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + a^2*d^4 + b^2*d
^4) + 2*(C*a^2*b^2 - B*a*b^3 + A*b^4)*log(abs(b*tan(f*x + e) + a))/(a^2*b^3
*c^2 + b^5*c^2 - 2*a^3*b^2*c*d - 2*a*b^4*c*d + a^4*b*d^2 + a^2*b^3*d^2) - 2
*(C*b*c^4*d - 2*B*b*c^3*d^2 + B*a*c^2*d^3 + 3*A*b*c^2*d^3 - C*b*c^2*d^3 - 2
*A*a*c*d^4 + 2*C*a*c*d^4 - B*a*d^5 + A*b*d^5)*log(abs(d*tan(f*x + e) + c))/
(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3 + 2*b^2*c^4*d^3 - 4*a*b*c^3*d^4 +
2*a^2*c^2*d^5 + b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 2*(C*b*c^4*d*tan(f*x
 + e) - 2*B*b*c^3*d^2*tan(f*x + e) + B*a*c^2*d^3*tan(f*x + e) + 3*A*b*c^2*d
^3*tan(f*x + e) - C*b*c^2*d^3*tan(f*x + e) - 2*A*a*c*d^4*tan(f*x + e) + 2*C
*a*c*d^4*tan(f*x + e) - B*a*d^5*tan(f*x + e) + A*b*d^5*tan(f*x + e) + 2*C*b
*c^5 - C*a*c^4*d - 3*B*b*c^4*d + 2*B*a*c^3*d^2 + 4*A*b*c^3*d^2 - 3*A*a*c^2*
d^3 + C*a*c^2*d^3 - B*b*c^2*d^3 + 2*A*b*c*d^4 - A*a*d^5)/((b^2*c^6 - 2*a*b*
c^5*d + a^2*c^4*d^2 + 2*b^2*c^4*d^2 - 4*a*b*c^3*d^3 + 2*a^2*c^2*d^4 + b^2*c
^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*(d*tan(f*x + e) + c)))/f
```

Mupad [**B**]
time = 85.86, size = 430, normalized size = 1.47

$$\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B-A\,1\mathrm{i}+C\,1\mathrm{i}\right)}{2\,f\,\left(a\,c^2-a\,d^2-2\,b\,c\,d+b\,c^2\,1\mathrm{i}-b\,d^2\,1\mathrm{i}+a\,c\,d\,2\mathrm{i}\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,1\mathrm{i}+B-C\,1\mathrm{i}\right)}{2\,f\,\left(a\,d^2-a\,c^2+2\,b\,c\,d+b\,c^2\,1\mathrm{i}-b\,d^2\,1\mathrm{i}+a\,c\,d\,2\mathrm{i}\right)}+\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(C\,a^2\,b-B\,a\,b^2+A\,b^3\right)}{f\,\left(a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2+a^2\,b^2\,d^2-2\,a\,b^3\,c\,d+b^4\,c^2\right)}-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(C\,b\,c^4-2\,B\,b\,c^3\,d+\left(3\,A\,b+B\,a-C\,b\right)\,c^2\,d^2+\left(2\,C\,a-2\,A\,a\right)\,c\,d^3+\left(A\,b-B\,a\right)\,d^4\right)}{f\,\left(a^2\,c^4\,d^2+2\,a^2\,c^2\,d^4+a^2\,d^6-2\,a\,b\,c^5\,d-4\,a\,b\,c^3\,d^3-2\,a\,b\,c\,d^5+b^2\,c^6+2\,b^2\,c^4\,d^2+b^2\,c^2\,d^4\right)}-\frac{C\,c^2-B\,c\,d+A\,d^2}{f\,\left(a\,d-b\,c\right)\,\left(c^2+d^2\right)\,\left(c+d\tan\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*ta
n(e + f*x))^2),x)
```

```
[Out] (log(tan(e + f*x) - 1i)*(B - A*1i + C*1i))/(2*f*(a*c^2 - a*d^2 + b*c^2*1i -
 b*d^2*1i + a*c*d*2i - 2*b*c*d)) - (log(tan(e + f*x) + 1i)*(A*1i + B - C*1i
))/(2*f*(a*d^2 - a*c^2 + b*c^2*1i - b*d^2*1i + a*c*d*2i + 2*b*c*d)) + (log(
a + b*tan(e + f*x))*(A*b^3 - B*a*b^2 + C*a^2*b))/(f*(a^4*d^2 + b^4*c^2 + a^
2*b^2*c^2 + a^2*b^2*d^2 - 2*a*b^3*c*d - 2*a^3*b*c*d)) - (log(c + d*tan(e +
f*x))*(d^4*(A*b - B*a) + c^2*d^2*(3*A*b + B*a - C*b) + C*b*c^4 - c*d^3*(2*A
*a - 2*C*a) - 2*B*b*c^3*d))/(f*(a^2*d^6 + b^2*c^6 + 2*a^2*c^2*d^4 + a^2*c^4
*d^2 + b^2*c^2*d^4 + 2*b^2*c^4*d^2 - 2*a*b*c*d^5 - 2*a*b*c^5*d - 4*a*b*c^3*
d^3)) - (A*d^2 + C*c^2 - B*c*d)/(f*(a*d - b*c)*(c^2 + d^2)*(c + d*tan(e + f
*x)))
```

3.82 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=509

$$-\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))+2ab(2c(A-C)d-B(c^2-}{(a^2+b^2)^2\,(c^2+d^2)^2}$$

```
[Out] -(a^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-b^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2
))+2*a*b*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(a^2+b^2)^2/(c^2+d^2)^2+b*(3*a^3*b*B*
d-2*a^4*C*d+b^4*(-2*A*d+B*c)-a^2*b^2*(4*A*d+B*c)+a*b^3*(2*A*c+B*d-2*C*c))*l
n(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^2/(-a*d+b*c)^3/f+d*(b*(4*A*c^2*d^2+2
*A*d^4-3*B*c^3*d-B*c*d^3+2*C*c^4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*ln(c*cos
(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^3/(c^2+d^2)^2/f-d*(b^2*c*(-B*d+C*c)-a*b*B*
(c^2+d^2)+a^2*(-B*c*d+2*C*c^2+C*d^2)+A*(a^2*d^2+b^2*(c^2+2*d^2)))/(a^2+b^2)
/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))+(-A*b^2+a*(B*b-C*a))/(a^2+b^2)/(
-a*d+b*c)/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
```

Rubi [A]
time = 1.40, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3730, 3732, 3611}

$$-\frac{d(a^2Ad^2+a^2(-Bcd+2c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+2d^2)+b^2c(cC-Bd))}{f(a^2+b^2)(c^2+d^2)(bc-ad)^2(c+d\tan(e+fx))}-\frac{x(a^2(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)+2ab(2cd(A-C)-B(c^2-d^2))-b^2(-A(c^2-d^2)-2Bcd+c^2C-Cd^2))}{(a^2+b^2)^2(c^2+d^2)^2}-\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}+\frac{b(-2a^4Cd+3a^3bBd-a^2b^2(4Ad+Bc)+ab^3(2Ac+Bd-2cC)+b^4(Bc-2Ad))\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)^2(bc-ad)^3}+\frac{d(b(4Ac^2d^2+2Ad^4-3Bc^3d-Bcd^3+2c^4C)-ad^2(2cd(A-C)-B(c^2-d^2)))\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^2),x]
```

```
[Out] -(((a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d -
C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/((a^2 +
b^2)^2*(c^2 + d^2)^2)) + (b*(3*a^3*b*B*d - 2*a^4*C*d + b^4*(B*c - 2*A*d) -
a^2*b^2*(B*c + 4*A*d) + a*b^3*(2*A*c - 2*c*C + B*d))*Log[a*Cos[e + f*x] + b
*Sin[e + f*x]])/((a^2 + b^2)^2*(b*c - a*d)^3*f) + (d*(b*(2*c^4*C - 3*B*c^3*
d + 4*A*c^2*d^2 - B*c*d^3 + 2*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)
))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)^2*f) -
(d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - a*b*B*(c^2 + d^2) + A*b^2*(c^2 + 2*d^2)
 + a^2*(2*c^2*C - B*c*d + C*d^2)))/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f
*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx = -\frac{Ab^2-a(bB-aC)}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx}$$

$$= -\frac{d(a^2Ad^2+b^2c(cC-Bd)-abB(c^2+d^2)+Ab^2(c^2+2d^2}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d\,\mathrm{ta}}$$

$$= -\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b^2(c^2C-2Bcd}{(a^2+b^2)^2}$$

$$= -\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b^2(c^2C-2Bcd}{(a^2+b^2)^2}$$

Mathematica [A]
time = 7.84, size = 984, normalized size = 1.93

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^2),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(
c + d*Tan[e + f*x]))) - (-(((b*(b*c - a*d)^2*(2*a*A*b*c^2 - a^2*B*c^2 + b^2
*B*c^2 - 2*a*b*c^2*C + 2*a^2*A*c*d - 2*A*b^2*c*d + 4*a*b*B*c*d - 2*a^2*c*C*
d + 2*b^2*c*C*d - 2*a*A*b*d^2 + a^2*B*d^2 - b^2*B*d^2 + 2*a*b*C*d^2 - (Sqrt
[-b^2]*(a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log
[Sqrt[-b^2] - b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b^2*(c^2 + d^
2)*(3*a^3*b*B*d - 2*a^4*C*d + b^4*(B*c - 2*A*d) - a^2*b^2*(B*c + 4*A*d) + a
*b^3*(2*A*c - 2*c*C + B*d))*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*
d)) + (b*(b*c - a*d)^2*(2*a*A*b*c^2 - a^2*B*c^2 + b^2*B*c^2 - 2*a*b*c^2*C +
 2*a^2*A*c*d - 2*A*b^2*c*d + 4*a*b*B*c*d - 2*a^2*c*C*d + 2*b^2*c*C*d - 2*a*
A*b*d^2 + a^2*B*d^2 - b^2*B*d^2 + 2*a*b*C*d^2 + (Sqrt[-b^2]*(a^2*(c^2*C - 2
*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d
^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log[Sqrt[-b^2] + b*Tan[e
+ f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b*(a^2 + b^2)*d*(b*(2*c^4*C - 3*B*c
^3*d + 4*A*c^2*d^2 - B*c*d^3 + 2*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d
^2)))*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)
*(c^2 + d^2)*f)) - (-(c*(-2*c*(A*b^2 - a*(b*B - a*C))*d + (A*b - a*B - b*C)
*d*(b*c - a*d))) + d^2*(2*A*b^2*d - a*A*(b*c - a*d) - (b*B - a*C)*(b*c + a*
d)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/((a^2 + b^2)*(b*c
 - a*d))
```

Maple [A]
time = 1.60, size = 577, normalized size = 1.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(a^2+b^2)^2/(c^2+d^2)^2*(1/2*(-2*A*a^2*c*d-2*A*a*b*c^2+2*A*a*b*d^2+2
*A*b^2*c*d+B*a^2*c^2-B*a^2*d^2-4*B*a*b*c*d-B*b^2*c^2+B*b^2*d^2+2*C*a^2*c*d+
2*C*a*b*c^2-2*C*a*b*d^2-2*C*b^2*c*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c^2-A*a^2*d^
2-4*A*a*b*c*d-A*b^2*c^2+A*b^2*d^2+2*B*a^2*c*d+2*B*a*b*c^2-2*B*a*b*d^2-2*B*b
^2*c*d-C*a^2*c^2+C*a^2*d^2+4*C*a*b*c*d+C*b^2*c^2-C*b^2*d^2)*arctan(tan(f*x+
e)))+d*(2*A*a*c*d^3-4*A*b*c^2*d^2-2*A*b*d^4-B*a*c^2*d^2+B*a*d^4+3*B*b*c^3*d
+B*b*c*d^3-2*C*a*c*d^3-2*C*b*c^4)/(a*d-b*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e)
)-(A*d^2-B*c*d+C*c^2)*d/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))-b*(A*b^2-B*a
*b+C*a^2)/(a^2+b^2)/(a*d-b*c)^2/(a+b*tan(f*x+e))+b*(4*A*a^2*b^2*d-2*A*a*b^3
*c+2*A*b^4*d-3*B*a^3*b*d+B*a^2*b^2*c-B*a*b^3*d-B*b^4*c+2*C*a^4*d+2*C*a*b^3*
c)/(a^2+b^2)^2/(a*d-b*c)^3*ln(a+b*tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. 2(519) = 1038.

time = 0.62, size = 1192, normalized size = 2.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^2 + 2*(B*a^2 - 2*(A - C)*a*
b - B*b^2)*c*d - ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d^2)*(f*x + e)/((a^4
 + 2*a^2*b^2 + b^4)*c^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^2 + (a^4 + 2*a^2*
b^2 + b^4)*d^4) - 2*((B*a^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c + (2*C*a^4*b -
 3*B*a^3*b^2 + 4*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*d)*log(b*tan(f*x + e) + a)/
((a^4*b^3 + 2*a^2*b^5 + b^7)*c^3 - 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^2*d +
3*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c*d^2 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^3) +
2*(2*C*b*c^4*d - 3*B*b*c^3*d^2 + (B*a + 4*A*b)*c^2*d^3 - (2*(A - C)*a + B*b
)*c*d^4 - (B*a - 2*A*b)*d^5)*log(d*tan(f*x + e) + c)/(b^3*c^7 - 3*a*b^2*c^6
*d + 3*a^2*b*c*d^6 - a^3*d^7 + (3*a^2*b + 2*b^3)*c^5*d^2 - (a^3 + 6*a*b^2)*
c^4*d^3 + (6*a^2*b + b^3)*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5) + ((B*a^2 -
2*(A - C)*a*b - B*b^2)*c^2 - 2*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d -
(B*a^2 - 2*(A - C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^2*
b^2 + b^4)*c^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^2 + (a^4 + 2*a^2*b^2 + b^4
)*d^4) - 2*((C*a^2*b - B*a*b^2 + A*b^3)*c^3 + (C*a^3 + C*a*b^2)*c^2*d - (B*
a^3 - C*a^2*b + 2*B*a*b^2 - A*b^3)*c*d^2 + (A*a^3 + A*a*b^2)*d^3 + ((2*C*a^
2*b - B*a*b^2 + (A + C)*b^3)*c^2*d - (B*a^2*b + B*b^3)*c*d^2 + ((A + C)*a^2
*b - B*a*b^2 + 2*A*b^3)*d^3)*tan(f*x + e))/((a^3*b^2 + a*b^4)*c^5 - 2*(a^4*
b + a^2*b^3)*c^4*d + (a^5 + 2*a^3*b^2 + a*b^4)*c^3*d^2 - 2*(a^4*b + a^2*b^3
)*c^2*d^3 + (a^5 + a^3*b^2)*c*d^4 + ((a^2*b^3 + b^5)*c^4*d - 2*(a^3*b^2 + a
*b^4)*c^3*d^2 + (a^4*b + 2*a^2*b^3 + b^5)*c^2*d^3 - 2*(a^3*b^2 + a*b^4)*c*d
^4 + (a^4*b + a^2*b^3)*d^5)*tan(f*x + e)^2 + ((a^2*b^3 + b^5)*c^5 - (a^3*b^
2 + a*b^4)*c^4*d - (a^4*b - b^5)*c^3*d^2 + (a^5 - a*b^4)*c^2*d^3 - (a^4*b +
 a^2*b^3)*c*d^4 + (a^5 + a^3*b^2)*d^5)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4188 vs. 2(519) = 1038.
time = 13.10, size = 4188, normalized size = 8.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*a^2*b^4 - B*a*b^5 + A*b^6)*c^6 - 2*(C*a^3*b^3 - B*a^2*b^4 + A*a*
b^5)*c^5*d + 4*(C*a^2*b^4 - B*a*b^5 + A*b^6)*c^4*d^2 + 2*(C*a^5*b + 2*B*a^2
```

```
*b^4 - (2*A - C)*a*b^5)*c^3*d^3 - 2*(C*a^6 + B*a^5*b + 2*C*a^4*b^2 + 2*B*a^
3*b^3 + 2*B*a*b^5 - A*b^6)*c^2*d^4 + 2*(B*a^6 + A*a^5*b + 2*B*a^4*b^2 + (2*
A - C)*a^3*b^3 + 2*B*a^2*b^4)*c*d^5 - 2*(A*a^6 + 2*A*a^4*b^2 + A*a^2*b^4)*d
^6 - 2*(((A - C)*a^3*b^3 + 2*B*a^2*b^4 - (A - C)*a*b^5)*c^6 - (3*(A - C)*a^
4*b^2 + 4*B*a^3*b^3 + (A - C)*a^2*b^4 + 2*B*a*b^5)*c^5*d + (3*(A - C)*a^5*b
 + 8*(A - C)*a^3*b^3 + 4*B*a^2*b^4 + (A - C)*a*b^5)*c^4*d^2 - ((A - C)*a^6
- 4*B*a^5*b + 8*(A - C)*a^4*b^2 + 3*(A - C)*a^2*b^4)*c^3*d^3 - (2*B*a^6 - (
A - C)*a^5*b + 4*B*a^4*b^2 - 3*(A - C)*a^3*b^3)*c^2*d^4 + ((A - C)*a^6 + 2*
B*a^5*b - (A - C)*a^4*b^2)*c*d^5)*f*x - 2*((C*a^3*b^3 - B*a^2*b^4 + A*a*b^5
)*c^5*d + (B*a^3*b^3 - (A - 2*C)*a^2*b^4 + C*b^6)*c^4*d^2 - (C*a^5*b + B*a^
4*b^2 + 4*B*a^2*b^4 - (2*A - C)*a*b^5 + B*b^6)*c^3*d^3 + (B*a^5*b + (A - 2*
C)*a^4*b^2 + 4*B*a^3*b^3 + B*a*b^5 + A*b^6)*c^2*d^4 - (A*a^5*b + (2*A - C)*
a^3*b^3 + B*a^2*b^4)*c*d^5 - (C*a^4*b^2 - B*a^3*b^3 + A*a^2*b^4)*d^6 + (((A
 - C)*a^2*b^4 + 2*B*a*b^5 - (A - C)*b^6)*c^5*d - (3*(A - C)*a^3*b^3 + 4*B*a
^2*b^4 + (A - C)*a*b^5 + 2*B*b^6)*c^4*d^2 + (3*(A - C)*a^4*b^2 + 8*(A - C)*
a^2*b^4 + 4*B*a*b^5 + (A - C)*b^6)*c^3*d^3 - ((A - C)*a^5*b - 4*B*a^4*b^2 +
 8*(A - C)*a^3*b^3 + 3*(A - C)*a*b^5)*c^2*d^4 - (2*B*a^5*b - (A - C)*a^4*b^
2 + 4*B*a^3*b^3 - 3*(A - C)*a^2*b^4)*c*d^5 + ((A - C)*a^5*b + 2*B*a^4*b^2 -
 (A - C)*a^3*b^3)*d^6)*f*x)*tan(f*x + e)^2 + ((B*a^3*b^3 - 2*(A - C)*a^2*b^
4 - B*a*b^5)*c^6 + (2*C*a^5*b - 3*B*a^4*b^2 + 4*A*a^3*b^3 - B*a^2*b^4 + 2*A
*a*b^5)*c^5*d + 2*(B*a^3*b^3 - 2*(A - C)*a^2*b^4 - B*a*b^5)*c^4*d^2 + 2*(2*
C*a^5*b - 3*B*a^4*b^2 + 4*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*c^3*d^3 + (B*a
^3*b^3 - 2*(A - C)*a^2*b^4 - B*a*b^5)*c^2*d^4 + (2*C*a^5*b - 3*B*a^4*b^2 +
4*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*c*d^5 + ((B*a^2*b^4 - 2*(A - C)*a*b^5
- B*b^6)*c^5*d + (2*C*a^4*b^2 - 3*B*a^3*b^3 + 4*A*a^2*b^4 - B*a*b^5 + 2*A*b
^6)*c^4*d^2 + 2*(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^3*d^3 + 2*(2*C*a^4*
b^2 - 3*B*a^3*b^3 + 4*A*a^2*b^4 - B*a*b^5 + 2*A*b^6)*c^2*d^4 + (B*a^2*b^4 -
 2*(A - C)*a*b^5 - B*b^6)*c*d^5 + (2*C*a^4*b^2 - 3*B*a^3*b^3 + 4*A*a^2*b^4
- B*a*b^5 + 2*A*b^6)*d^6)*tan(f*x + e)^2 + ((B*a^2*b^4 - 2*(A - C)*a*b^5 -
B*b^6)*c^6 + 2*(C*a^4*b^2 - B*a^3*b^3 + (A + C)*a^2*b^4 - B*a*b^5 + A*b^6)*
c^5*d + (2*C*a^5*b - 3*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*(A - 2*C)*a*
b^5 - 2*B*b^6)*c^4*d^2 + 4*(C*a^4*b^2 - B*a^3*b^3 + (A + C)*a^2*b^4 - B*a*b
^5 + A*b^6)*c^3*d^3 + (4*C*a^5*b - 6*B*a^4*b^2 + 8*A*a^3*b^3 - B*a^2*b^4 +
2*(A + C)*a*b^5 - B*b^6)*c^2*d^4 + 2*(C*a^4*b^2 - B*a^3*b^3 + (A + C)*a^2*b
^4 - B*a*b^5 + A*b^6)*c*d^5 + (2*C*a^5*b - 3*B*a^4*b^2 + 4*A*a^3*b^3 - B*a^
2*b^4 + 2*A*a*b^5)*d^6)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f
*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (2*(C*a^5*b + 2*C*a^3*b^3 + C*a*b^5)
*c^5*d - 3*(B*a^5*b + 2*B*a^3*b^3 + B*a*b^5)*c^4*d^2 + (B*a^6 + 4*A*a^5*b +
 2*B*a^4*b^2 + 8*A*a^3*b^3 + B*a^2*b^4 + 4*A*a*b^5)*c^3*d^3 - (2*(A - C)*a^
6 + B*a^5*b + 4*(A - C)*a^4*b^2 + 2*B*a^3*b^3 + 2*(A - C)*a^2*b^4 + B*a*b^5
)*c^2*d^4 - (B*a^6 - 2*A*a^5*b + 2*B*a^4*b^2 - 4*A*a^3*b^3 + B*a^2*b^4 - 2*
A*a*b^5)*c*d^5 + (2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*c^4*d^2 - 3*(B*a^4*b^
2 + 2*B*a^2*b^4 + B*b^6)*c^3*d^3 + (B*a^5*b + 4*A*a^4*b^2 + 2*B*a^3*b^3 + 8
*A*a^2*b^4 + B*a*b^5 + 4*A*b^6)*c^2*d^4 - (2*(A - C)*a^5*b + B*a^4*b^2 + 4*
(A - C)*a^3*b^3 + 2*B*a^2*b^4 + 2*(A - C)*a*b^5 + B*b^6)*c*d^5 - (B*a^5*b -
```

```
 2*A*a^4*b^2 + 2*B*a^3*b^3 - 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*d^6)*tan(f*x
+ e)^2 + (2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*c^5*d + (2*C*a^5*b - 3*B*a^4*
b^2 + 4*C*a^3*b^3 - 6*B*a^2*b^4 + 2*C*a*b^5 - 3*B*b^6)*c^4*d^2 - 2*(B*a^5*b
 - 2*A*a^4*b^2 + 2*B*a^3*b^3 - 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*c^3*d^3 + (
B*a^6 + 2*(A + C)*a^5*b + B*a^4*b^2 + 4*(A + C)*a^3*b^3 - B*a^2*b^4 + 2*(A
+ C)*a*b^5 - B*b^6)*c^2*d^4 - 2*((A - C)*a^6 + B*a^5*b + (A - 2*C)*a^4*b^2
+ 2*B*a^3*b^3 - (A + C)*a^2*b^4 + B*a*b^5 - A*b^6)*c*d^5 - (B*a^6 - 2*A*a^5
*b + 2*B*a^4*b^2 - 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*d^6)*tan(f*x + e))*
log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) -
 2*((C*a^3*b^3 - B*a^2*b^4 + A*a*b^5)*c^6 - (C*a^4*b^2 - B*a^3*b^3 + (A + C
)*a^2*b^4 - B*a*b^5 + A*b^6)*c^5*d + (C*a^5*b + 5*C*a^3*b^3 - 3*B*a^2*b^4 +
 (3*A + C)*a*b^5)*c^4*d^2 - (C*a^6 + B*a^5*b + 5*C*a^4*b^2 + (2*A + 5*C)*a^
2*b^4 - B*a*b^5 + (2*A + C)*b^6)*c^3*d^3 + (B*a^6 + (A + C)*a^5*b + 3*B*a^4
*b^2 + (2*A + 5*C)*a^3*b^3 + (4*A + C)*a*b^5 + B*b^6)*c^2*d^4 - (A*a^6 + B*
a^5*b + (3*A + C)*a^4*b^2 + B*a^3*b^3 + (4*A + C)*a^2*b^4 + 2*A*b^6)*c*d^5
+ (A*a^5*b + (2*A + C)*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5)*d^6 + (((A - C)*a^2
*b^4 + 2*B*a*b^5 - (A - C)*b^6)*c^6 - 2*((A - C)*a^3*b^3 + B*a^2*b^4 + (A -
 C)*a*b^5 + B*b^6)*c^5*d - (4*B*a^3*b^3 - 7*(A - C)*a^2*b^4 - 2*B*a*b^5 - (
A - C)*b^6)*c^4*d^2 + 2*((A - C)*a^5*b + 2*B*a^4*b^2 + 2*B*a^2*b^4 - (A - C
)*a*b^5)*c^3*d^3 - ((A - C)*a^6 - 2*B*a^5*b + 7...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2893 vs. $2(519) = 1038$.
time = 1.23, size = 2893, normalized size = 5.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c^2 - C*a^2*c^2 + 2*B*a*b*c^2 - A*b^2*c^2 + C*b^2*c^2 + 2*B*a
^2*c*d - 4*A*a*b*c*d + 4*C*a*b*c*d - 2*B*b^2*c*d - A*a^2*d^2 + C*a^2*d^2 -
2*B*a*b*d^2 + A*b^2*d^2 - C*b^2*d^2)*(f*x + e)/(a^4*c^4 + 2*a^2*b^2*c^4 + b
^4*c^4 + 2*a^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2 + 2*b^4*c^2*d^2 + a^4*d^4 + 2*a^
```

2*b^2*d^4 + b^4*d^4) + (B*a^2*c^2 - 2*A*a*b*c^2 + 2*C*a*b*c^2 - B*b^2*c^2 - 2*A*a^2*c*d + 2*C*a^2*c*d - 4*B*a*b*c*d + 2*A*b^2*c*d - 2*C*b^2*c*d - B*a^2*d^2 + 2*A*a*b*d^2 - 2*C*a*b*d^2 + B*b^2*d^2)*log(tan(f*x + e)^2 + 1)/(a^4*c^4 + 2*a^2*b^2*c^4 + b^4*c^4 + 2*a^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2 + 2*b^4*c^2*d^2 + a^4*d^4 + 2*a^2*b^2*d^4 + b^4*d^4) - 2*(B*a^2*b^4*c - 2*A*a*b^5*c + 2*C*a*b^5*c - B*b^6*c + 2*C*a^4*b^2*d - 3*B*a^3*b^3*d + 4*A*a^2*b^4*d - B*a*b^5*d + 2*A*b^6*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^4*c^3 + 2*a^2*b^6*c^3 + b^8*c^3 - 3*a^5*b^3*c^2*d - 6*a^3*b^5*c^2*d - 3*a*b^7*c^2*d + 3*a^6*b^2*c*d^2 + 6*a^4*b^4*c*d^2 + 3*a^2*b^6*c*d^2 - a^7*b*d^3 - 2*a^5*b^3*d^3 - a^3*b^5*d^3) + 2*(2*C*b*c^4*d^2 - 3*B*b*c^3*d^3 + B*a*c^2*d^4 + 4*A*b*c^2*d^4 - 2*A*a*c*d^5 + 2*C*a*c*d^5 - B*b*c*d^5 - B*a*d^6 + 2*A*b*d^6)*log(abs(d*tan(f*x + e) + c))/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 + 2*b^3*c^5*d^3 - a^3*c^4*d^4 - 6*a*b^2*c^4*d^4 + 6*a^2*b*c^3*d^5 + b^3*c^3*d^5 - 2*a^3*c^2*d^6 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8) + (B*a^2*b^3*c^4*d*tan(f*x + e)^2 - 2*A*a*b^4*c^4*d*tan(f*x + e)^2 + 2*C*a*b^4*c^4*d*tan(f*x + e)^2 - B*b^5*c^4*d*tan(f*x + e)^2 - 2*B*a^3*b^2*c^3*d^2*tan(f*x + e)^2 + 2*A*a^2*b^3*c^3*d^2*tan(f*x + e)^2 - 2*C*a^2*b^3*c^3*d^2*tan(f*x + e)^2 - 2*B*a*b^4*c^3*d^2*tan(f*x + e)^2 + 2*A*b^5*c^3*d^2*tan(f*x + e)^2 - 2*C*b^5*c^3*d^2*tan(f*x + e)^2 + B*a^4*b*c^2*d^3*tan(f*x + e)^2 + 2*A*a^3*b^2*c^2*d^3*tan(f*x + e)^2 - 2*C*a^3*b^2*c^2*d^3*tan(f*x + e)^2 + 6*B*a^2*b^3*c^2*d^3*tan(f*x + e)^2 - 2*A*a*b^4*c^2*d^3*tan(f*x + e)^2 + 2*C*a*b^4*c^2*d^3*tan(f*x + e)^2 + B*b^5*c^2*d^3*tan(f*x + e)^2 - 2*A*a^4*b*c*d^4*tan(f*x + e)^2 + 2*C*a^4*b*c*d^4*tan(f*x + e)^2 - 2*B*a^3*b^2*c*d^4*tan(f*x + e)^2 - 2*A*a^2*b^3*c*d^4*tan(f*x + e)^2 + 2*C*a^2*b^3*c*d^4*tan(f*x + e)^2 - 2*B*a*b^4*c*d^4*tan(f*x + e)^2 - B*a^4*b*d^5*tan(f*x + e)^2 + 2*A*a^3*b^2*d^5*tan(f*x + e)^2 - 2*C*a^3*b^2*d^5*tan(f*x + e)^2 + B*a^2*b^3*d^5*tan(f*x + e)^2 + B*a^2*b^3*c^5*tan(f*x + e) - 2*A*a*b^4*c^5*tan(f*x + e) + 2*C*a*b^4*c^5*tan(f*x + e) - B*b^5*c^5*tan(f*x + e) - 4*C*a^4*b*c^4*d*tan(f*x + e) + B*a^3*b^2*c^4*d*tan(f*x + e) - 2*A*a^2*b^3*c^4*d*tan(f*x + e) - 6*C*a^2*b^3*c^4*d*tan(f*x + e) - B*a*b^4*c^4*d*tan(f*x + e) - 4*C*b^5*c^4*d*tan(f*x + e) + B*a^4*b*c^3*d^2*tan(f*x + e) + 4*A*a^3*b^2*c^3*d^2*tan(f*x + e) - 4*C*a^3*b^2*c^3*d^2*tan(f*x + e) + 8*B*a^2*b^3*c^3*d^2*tan(f*x + e) + 3*B*b^5*c^3*d^2*tan(f*x + e) + B*a^5*c^2*d^3*tan(f*x + e) - 2*A*a^4*b*c^2*d^3*tan(f*x + e) - 6*C*a^4*b*c^2*d^3*tan(f*x + e) + 8*B*a^3*b^2*c^2*d^3*tan(f*x + e) - 12*A*a^2*b^3*c^2*d^3*tan(f*x + e) - 4*C*a^2*b^3*c^2*d^3*tan(f*x + e) + 3*B*a*b^4*c^2*d^3*tan(f*x + e) - 6*A*b^5*c^2*d^3*tan(f*x + e) - 2*C*b^5*c^2*d^3*tan(f*x + e) - 2*A*a^5*c*d^4*tan(f*x + e) + 2*C*a^5*c*d^4*tan(f*x + e) - B*a^4*b*c*d^4*tan(f*x + e) + 3*B*a^2*b^3*c*d^4*tan(f*x + e) + 2*B*b^5*c*d^4*tan(f*x + e) - B*a^5*d^5*tan(f*x + e) - 4*C*a^4*b*d^5*tan(f*x + e) + 3*B*a^3*b^2*d^5*tan(f*x + e) - 6*A*a^2*b^3*d^5*tan(f*x + e) - 2*C*a^2*b^3*d^5*tan(f*x + e) + 2*B*a*b^4*d^5*tan(f*x + e) - 4*A*b^5*d^5*tan(f*x + e) - 2*C*a^4*b*c^5 + 3*B*a^3*b^2*c^5 - 4*A*a^2*b^3*c^5 + B*a*b^4*c^5 - 2*A*b^5*c^5 - 2*C*a^5*c^4*d - 2*B*a^4*b*c^4*d + 2*A*a^3*b^2*c^4*d - 6*C*a^3*b^2*c^4*d - 2*B*a^2*b^3*c^4*d + 2*A*a*b^4*c^4*d - 4*C*a*b^4*c^4*d + 3*B*a^5*c^3*d^2 + 2*A*a^4*b*c^3*d^2 - 6*C*a^4*b*c^3*d^2 + 14*B*a^3*b^2*c^3*d^2 - 6*A*a^2*b^3*c^3*d^

```
2 - 2*C*a^2*b^3*c^3*d^2 + 7*B*a*b^4*c^3*d^2 - 4*A*b^5*c^3*d^2 - 4*A*a^5*c^2
*d^3 - 2*B*a^4*b*c^2*d^3 - 6*A*a^3*b^2*c^2*d^3 - 2*C*a^3*b^2*c^2*d^3 - 2*B*
a^2*b^3*c^2*d^3 - 2*A*a*b^4*c^2*d^3 - 2*C*a*b^4*c^2*d^3 + B*a^5*c*d^4 + 2*A
*a^4*b*c*d^4 - 4*C*a^4*b*c*d^4 + 7*B*a^3*b^2*c*d^4 - 2*A*a^2*b^3*c*d^4 - 2*
C*a^2*b^3*c*d^4 + 4*B*a*b^4*c*d^4 - 2*A*b^5*c*d^4 - 2*A*a^5*d^5 - 4*A*a^3*b
^2*d^5 - 2*A*a*b^4*d^5)/((a^4*b^2*c^6 + 2*a^2*b^4*c^6 + b^6*c^6 - 2*a^5*b*c
^5*d - 4*a^3*b^3*c^5*d - 2*a*b^5*c^5*d + a^6*c^4*d^2 + 4*a^4*b^2*c^4*d^2 +
5*a^2*b^4*c^4*d^2 + 2*b^6*c^4*d^2 - 4*a^5*b*c^3*d^3 - 8*a^3*b^3*c^3*d^3 - 4
*a*b^5*c^3*d^3 + 2*a^6*c^2*d^4 + 5*a^4*b^2*c^2*d^4 + 4*a^2*b^4*c^2*d^4 + b^
6*c^2*d^4 - 2*a^5*b*c*d^5 - 4*a^3*b^3*c*d^5 - 2*a*b^5*c*d^5 + a^6*d^6 + 2*a
^4*b^2*d^6 + a^2*b^4*d^6)*(b*d*tan(f*x + e)^2 + b*c*tan(f*x + e) + a*d*tan(
f*x + e) + a*c)))/f
```

Mupad [B]
time = 31.51, size = 2500, normalized size = 4.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d*
tan(e + f*x))^2),x)
```

```
[Out] (symsum(log((tan(e + f*x)*(4*A^3*a^3*b^4*d^7 + B^3*a^2*b^5*d^7 + 4*A^3*b^7*
c^3*d^4 + 2*C^3*a^5*b^2*d^7 + B^3*b^7*c^2*d^5 + 2*C^3*b^7*c^5*d^2 + 4*A^2*B
*b^7*d^7 - 4*B^3*a^2*b^5*c^2*d^5 - 3*B^3*a^2*b^5*c^4*d^3 + 10*B^3*a^3*b^4*c
^3*d^4 - 3*B^3*a^4*b^3*c^2*d^5 - 4*A*B^2*a*b^6*d^7 - 4*A*B^2*b^7*c*d^6 + 2*
B^3*a*b^6*c*d^6 - 6*A*B^2*a^3*b^4*d^7 + 8*A^2*B*a^2*b^5*d^7 - 3*A^2*B*a^4*b
^3*d^7 + 4*A*C^2*a^3*b^4*d^7 - 4*A*C^2*a^5*b^2*d^7 - 8*A^2*C*a^3*b^4*d^7 +
2*A^2*C*a^5*b^2*d^7 - 6*A*B^2*b^7*c^3*d^4 - 3*B*C^2*a^4*b^3*d^7 + 8*A^2*B*b
^7*c^2*d^5 - 3*A^2*B*b^7*c^4*d^3 + 4*A*C^2*b^7*c^3*d^4 - 4*A*C^2*b^7*c^5*d^
2 - 8*A^2*C*b^7*c^3*d^4 + 2*A^2*C*b^7*c^5*d^2 - 3*B*C^2*b^7*c^4*d^3 - 4*A^3
*a*b^6*c^2*d^5 - 4*A^3*a^2*b^5*c*d^6 + 6*B^3*a*b^6*c^3*d^4 + 6*B^3*a^3*b^4*
c*d^6 - 2*C^3*a*b^6*c^4*d^3 - 2*C^3*a^4*b^3*c*d^6 - 10*A*B^2*a^2*b^5*c^3*d^
4 - 10*A*B^2*a^3*b^4*c^2*d^5 + 18*A^2*B*a^2*b^5*c^2*d^5 + 2*B*C^2*a^2*b^5*c
^2*d^5 + 4*B*C^2*a^4*b^3*c^4*d^3 + 2*B^2*C*a^2*b^5*c^3*d^4 + 2*B^2*C*a^2*b^
5*c^5*d^2 + 2*B^2*C*a^3*b^4*c^2*d^5 - 6*B^2*C*a^3*b^4*c^4*d^3 - 6*B^2*C*a^4
*b^3*c^3*d^4 + 2*B^2*C*a^5*b^2*c^2*d^5 + 10*A*B*C*a^4*b^3*d^7 + 10*A*B*C*b^
7*c^4*d^3 - 8*A^2*B*a*b^6*c*d^6 - 2*A*B^2*a*b^6*c^2*d^5 + 6*A*B^2*a*b^6*c^4
*d^3 - 2*A*B^2*a^2*b^5*c*d^6 + 6*A*B^2*a^4*b^3*c*d^6 - 4*A^2*B*a*b^6*c^3*d^
4 - 4*A^2*B*a^3*b^4*c*d^6 - 4*A*C^2*a*b^6*c^2*d^5 + 4*A*C^2*a*b^6*c^4*d^3 -
 4*A*C^2*a^2*b^5*c*d^6 + 4*A*C^2*a^4*b^3*c*d^6 + 8*A^2*C*a*b^6*c^2*d^5 - 2*
A^2*C*a*b^6*c^4*d^3 + 8*A^2*C*a^2*b^5*c*d^6 - 2*A^2*C*a^4*b^3*c*d^6 + 4*B*C
^2*a*b^6*c^3*d^4 + 4*B*C^2*a*b^6*c^5*d^2 + 4*B*C^2*a^3*b^4*c*d^6 + 4*B*C^2*
a^5*b^2*c*d^6 - 4*B^2*C*a*b^6*c^2*d^5 - 10*B^2*C*a*b^6*c^4*d^3 - 4*B^2*C*a^
2*b^5*c*d^6 - 10*B^2*C*a^4*b^3*c*d^6 - 4*A*B*C*a^2*b^5*c^2*d^5 + 8*A*B*C*a^
```

2*b^5*c^4*d^3 + 8*A*B*C*a^4*b^3*c^2*d^5 + 8*A*B*C*a*b^6*c*d^6 - 4*A*B*C*a*b
^6*c^5*d^2 - 4*A*B*C*a^5*b^2*c*d^6))/(a^8*d^8 + b^8*c^8 + 2*a^2*b^6*c^8 + a
^4*b^4*c^8 + a^4*b^4*d^8 + 2*a^6*b^2*d^8 + 2*a^8*c^2*d^6 + a^8*c^4*d^4 + b^
8*c^4*d^4 + 2*b^8*c^6*d^2 - 4*a*b^7*c^3*d^5 - 8*a*b^7*c^5*d^3 - 4*a^3*b^5*c
*d^7 - 8*a^3*b^5*c^7*d - 8*a^5*b^3*c*d^7 - 4*a^5*b^3*c^7*d - 8*a^7*b*c^3*d^
5 - 4*a^7*b*c^5*d^3 + 6*a^2*b^6*c^2*d^6 + 14*a^2*b^6*c^4*d^4 + 10*a^2*b^6*c
^6*d^2 - 16*a^3*b^5*c^3*d^5 - 20*a^3*b^5*c^5*d^3 + 14*a^4*b^4*c^2*d^6 + 26*
a^4*b^4*c^4*d^4 + 14*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^3*d^5 - 16*a^5*b^3*c^5*
d^3 + 10*a^6*b^2*c^2*d^6 + 14*a^6*b^2*c^4*d^4 + 6*a^6*b^2*c^6*d^2 - 4*a*b^7
*c^7*d - 4*a^7*b*c*d^7) - (4*A^2*C*b^7*d^7 - 6*A^3*a^2*b^5*d^7 - B^3*a^3*b^
4*d^7 - 6*A^3*b^7*c^2*d^5 - B^3*b^7*c^3*d^4 - 4*A^3*b^7*d^7 - 8*A^3*a^2*b^5
*c^2*d^5 - 3*B^3*a^2*b^5*c^3*d^4 - 3*B^3*a^3*b^4*c^2*d^5 + 2*C^3*a^2*b^5*c^
4*d^3 + 2*C^3*a^4*b^3*c^2*d^5 + 4*C^3*a^4*b^3*c^4*d^3 + 4*A^2*B*a*b^6*d^7 +
 4*A^2*B*b^7*c*d^6 + 4*A^3*a*b^6*c*d^6 + A*B^2*a^2*b^5*d^7 - 3*A*B^2*a^4*b^
3*d^7 + 9*A^2*B*a^3*b^4*d^7 + 2*A*C^2*a^2*b^5*d^7 + 4*A*C^2*a^4*b^3*d^7 + 4
*A^2*C*a^2*b^5*d^7 - 4*A^2*C*a^4*b^3*d^7 + A*B^2*b^7*c^2*d^5 - 3*A*B^2*b^7*
c^4*d^3 - B*C^2*a^3*b^4*d^7 - 2*B*C^2*a^5*b^2*d^7 + 9*A^2*B*b^7*c^3*d^4 + B
^2*C*a^2*b^5*d^7 + 3*B^2*C*a^4*b^3*d^7 + 2*A*C^2*b^7*c^2*d^5 + 4*A*C^2*b^7*
c^4*d^3 + 4*A^2*C*b^7*c^2*d^5 - 4*A^2*C*b^7*c^4*d^3 - B*C^2*b^7*c^3*d^4 - 2
*B*C^2*b^7*c^5*d^2 + B^2*C*b^7*c^2*d^5 + 3*B^2*C*b^7*c^4*d^3 + 2*A^3*a*b^6*
c^3*d^4 + 2*A^3*a^3*b^4*c*d^6 + B^3*a*b^6*c^2*d^5 + 3*B^3*a*b^6*c^4*d^3 + B
^3*a^2*b^5*c*d^6 + 3*B^3*a^4*b^3*c*d^6 + 2*C^3*a*b^6*c^3*d^4 + 2*C^3*a*b^6*
c^5*d^2 + 2*C^3*a^3*b^4*c*d^6 + 2*C^3*a^5*b^2*c*d^6 - 4*A*B*C*a*b^6*d^7 - 4
*A*B*C*b^7*c*d^6 + 14*A*B^2*a^2*b^5*c^2*d^5 + 3*A*B^2*a^2*b^5*c^4*d^3 - 10*
A*B^2*a^3*b^4*c^3*d^4 + 3*A*B^2*a^4*b^3*c^2*d^5 + 7*A^2*B*a^2*b^5*c^3*d^4 +
 7*A^2*B*a^3*b^4*c^2*d^5 + 8*A*C^2*a^2*b^5*c^2*d^5 + 4*A*C^2*a^2*b^5*c^4*d^
3 + 4*A*C^2*a^4*b^3*c^2*d^5 - 4*A*C^2*a^4*b^3*c^4*d^3 - 6*A^2*C*a^2*b^5*c^4
*d^3 - 6*A^2*C*a^4*b^3*c^2*d^5 - B*C^2*a^2*b^5*c^3*d^4 + 2*B*C^2*a^2*b^5*c^
5*d^2 - B*C^2*a^3*b^4*c^2*d^5 - 6*B*C^2*a^3*b^4*c^4*d^3 - 6*B*C^2*a^4*b^3*c
^3*d^4 + 2*B*C^2*a^5*b^2*c^2*d^5 - 6*B^2*C*a^2*b^5*c^2*d^5 - B^2*C*a^2*b^5*
c^4*d^3 + 10*B^2*C*a^3*b^4*c^3*d^4 - B^2*C*a^4*b^3*c^2*d^5 - 8*A*B*C*a^3*b^
4*d^7 + 2*A*B*C*a^5*b^2*d^7 - 8*A*B*C*b^7*c^3*d^4 + 2*A*B*C*b^7*c^5*d^2 - 6
*A*B^2*a*b^6*c*d^6 + 4*A*C^2*a*b^6*c*d^6 - 8*A^2*C*a*b^6*c*d^6 + 2*B^2*C*a*
b^6*c*d^6 - 8*A*B^2*a*b^6*c^3*d^4 - 8*A*B^2*a^3*b^4*c*d^6 - A^2*B*a*b^6*c^2
*d^5 - 3*A^2*B*a*b^6*c^4*d^3 - A^2*B*a^2*b^5*c*d^6 - 3*A^2*B*a^4*b^3*c*d^6
 - 2*A*C^2*a*b^6*c^3*d^4 - 4*A*C^2*a*b^6*c^5*d^2 - 2*A*C^2*a^3*b^4*c*d^6 - 4
*A*C^2*a^5*b^2*c*d^6 - 2*A^2*C*a*b^6*c^3*d^4 + 2*A^2*C*a*b^6*c^5*d^2 - 2*A^
2*C*a^3*b^4*c*d^6 + 2*A^2*C*a^5*b^2*c*d^6 - 3*B*C^2*a*b^6*c^2*d^5 - 5*B*C^2
*a*b^6*c^4*d^3 - 3*B*C^2*a^2*b^5*c*d^6 - 5*B*C^2*a^4*b^3*c*d^6 + 4*B^2*C*a*
b^6*c^3*d^4 - 2*B^2*C*a*b^6*c^5*d^2 + 4*B^2*C*a^3*b^4*c*d^6 - 2*B^2*C*a^5*b
^2*c*d^6 - 6*A*B*C*a^2*b^5*c^3*d^4 - 2*A*B*C*a^2*b^5*c^5*d^2 - 6*A*B*C*a^3*
b^4*c^2*d^5 + 6*A*B*C*a^3*b^4*c^4*d^3 + 6*A*B*C*a^4*b^3*c^3*d^4 - 2*A*B*C*a
^5*b^2*c^2*d^5 + 4*A*B*C*a*b^6*c^2*d^5 + 8*A*B*C*a*b^6*c^4*d^3 + 4*A*B*C*a^
2*b^5*c*d^6 + 8*A*B*C*a^4*b^3*c*d^6)/(a^8*d^8 +...

3.83 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=841

$$-\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))+3a^2b(2c(A-C)d-B(c^2}{(a^2+b^2)^3\,(c^2+d^2)^2}$$

```
[Out] -(a^3*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-3*a*b^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2
-d^2))+3*a^2*b*(2*c*(A-C)*d-B*(c^2-d^2))-b^3*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(
a^2+b^2)^3/(c^2+d^2)^2-b*(6*a^5*b*B*d^2-3*a^6*C*d^2-a^4*b^2*d*(4*B*c+(10*A-
C)*d)-b^6*(c*(-2*B*d+C*c)-A*(c^2-3*d^2))+a*b^5*(2*c*(A-C)*d-B*(3*c^2-d^2))+
3*a^2*b^4*(c*(2*B*d+C*c)-A*(c^2+3*d^2))+a^3*b^3*(10*c*(A-C)*d+B*(c^2+3*d^2)
))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^3/(-a*d+b*c)^4/f-d^2*(b*(3*c^4*C
-4*B*c^3*d+c^2*(5*A+C)*d^2-2*B*c*d^3+3*A*d^4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2
)))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^4/(c^2+d^2)^2/f-d*(3*a^3*b*B*d
*(c^2+d^2)+a*b^3*(2*A*c+B*d-2*C*c)*(c^2+d^2)-a^4*d*(3*c^2*C-B*c*d+(A+2*C)*d
^2)-a^2*b^2*(4*A*c^2*d+6*A*d^3+B*c^3-B*c*d^2+2*C*c^2*d)-b^4*(d*(2*A*c^2+3*A
*d^2+C*c^2)-B*(c^3+2*c*d^2)))/(a^2+b^2)^2/(-a*d+b*c)^3/(c^2+d^2)/f/(c+d*tan
(f*x+e))+1/2*(-A*b^2+a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^2
/(c+d*tan(f*x+e))+1/2*(-5*a^3*b*B*d+3*a^4*C*d-b^4*(-3*A*d+2*B*c)-a*b^3*(4*A
*c+B*d-4*C*c)+a^2*b^2*(2*B*c+(7*A-C)*d))/(a^2+b^2)^2/(-a*d+b*c)^2/f/(a+b*ta
n(f*x+e))/(c+d*tan(f*x+e))
```

Rubi [A]
time = 2.72, antiderivative size = 841, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3730, 3732, 3611}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])^2),x]
```

```
[Out] -(((a^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C - 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) + 3*a^2*b*(2*c*(A - C)*d - B*(c^2 - d^2)) - b^3*
(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/((a^2 + b^2)^3*(c^2 + d^2)^2)) - (b*(6*
a^5*b*B*d^2 - 3*a^6*C*d^2 - a^4*b^2*d*(4*B*c + (10*A - C)*d) - b^6*(c*(c*C
- 2*B*d) - A*(c^2 - 3*d^2)) + a*b^5*(2*c*(A - C)*d - B*(3*c^2 - d^2)) + 3*a
^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + 3*d^2)) + a^3*b^3*(10*c*(A - C)*d + B*(c
^2 + 3*d^2)))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)^3*(b*c - a
*d)^4*f) - (d^2*(b*(3*c^4*C - 4*B*c^3*d + c^2*(5*A + C)*d^2 - 2*B*c*d^3 + 3
*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c*Cos[e + f*x] + d*Sin
[e + f*x]])/((b*c - a*d)^4*(c^2 + d^2)^2*f) - (d*(3*a^3*b*B*d*(c^2 + d^2) +
```

```
 a*b^3*(2*A*c - 2*c*C + B*d)*(c^2 + d^2) - a^4*d*(3*c^2*C - B*c*d + (A + 2*
C)*d^2) - a^2*b^2*(B*c^3 + 4*A*c^2*d + 2*c^2*C*d - B*c*d^2 + 6*A*d^3) - b^4
*(d*(2*A*c^2 + c^2*C + 3*A*d^2) - B*(c^3 + 2*c*d^2))))/((a^2 + b^2)^2*(b*c
- a*d)^3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])) - (A*b^2 - a*(b*B - a*C))/(2*(
a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])) - (5*
a^3*b*B*d - 3*a^4*C*d + b^4*(2*B*c - 3*A*d) + a*b^3*(4*A*c - 4*c*C + B*d) -
 a^2*b^2*(2*B*c + (7*A - C)*d))/(2*(a^2 + b^2)^2*(b*c - a*d)^2*f*(a + b*Tan
[e + f*x])*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx = -\frac{Ab^2-a(bB-aC)}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+$$

$$= -\frac{Ab^2-a(bB-aC)}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+$$

$$= -\frac{d(3a^3bBd(c^2+d^2)+ab^3(2Ac-2cC+Bd)(c^2+d^2)-a}{}$$

$$= -\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C-2Bc}{}$$

$$= -\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C-2Bc}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1758 vs. $2(841) = 1682$.
time = 7.70, size = 1758, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(
c + d*Tan[e + f*x])^2),x]
```

```
[Out] -1/2*(A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]
)^2*(c + d*Tan[e + f*x])) - (-((-(a*(-3*a*(A*b^2 - a*(b*B - a*C))*d + 2*b*(
A*b - a*B - b*C)*(b*c - a*d))) + b^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B
- a*C)*(2*b*c + a*d)))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c +
 d*Tan[e + f*x]))) - (-((-(((b*c - a*d)^3*(-(b^2*(-3*a^2*A*b*c^2 + A*b^3*c^
2 + a^3*B*c^2 - 3*a*b^2*B*c^2 + 3*a^2*b*c^2*C - b^3*c^2*C - 2*a^3*A*c*d + 6
*a*A*b^2*c*d - 6*a^2*b*B*c*d + 2*b^3*B*c*d + 2*a^3*c*C*d - 6*a*b^2*c*C*d +
3*a^2*A*b*d^2 - A*b^3*d^2 - a^3*B*d^2 + 3*a*b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3
*C*d^2)) + Sqrt[-b^2]*(a^3*A*b*c^2 - 3*a*A*b^3*c^2 + 3*a^2*b^2*B*c^2 - b^4*
B*c^2 - a^3*b*c^2*C + 3*a*b^3*c^2*C - 6*a^2*A*b^2*c*d + 2*A*b^4*c*d + 2*a^3
*b*B*c*d - 6*a*b^3*B*c*d + 6*a^2*b^2*c*C*d - 2*b^4*c*C*d - a^3*A*b*d^2 + 3*
a*A*b^3*d^2 - 3*a^2*b^2*B*d^2 + b^4*B*d^2 + a^3*b*C*d^2 - 3*a*b^3*C*d^2))*L
og[Sqrt[-b^2] - b*Tan[e + f*x]])/(b*(a^2 + b^2)*(c^2 + d^2))) - (2*b^2*(c^2
 + d^2)*(6*a^5*b*B*d^2 - 3*a^6*C*d^2 - a^4*b^2*d*(4*B*c + (10*A - C)*d) - b
^6*(c*(c*C - 2*B*d) - A*(c^2 - 3*d^2)) + a*b^5*(2*c*(A - C)*d - B*(3*c^2 -
d^2)) + 3*a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + 3*d^2)) + a^3*b^3*(10*c*(A -
C)*d + B*(c^2 + 3*d^2)))*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d))
 + ((b*c - a*d)^3*(b^2*(-3*a^2*A*b*c^2 + A*b^3*c^2 + a^3*B*c^2 - 3*a*b^2*B*
```

```
c^2 + 3*a^2*b*c^2*C - b^3*c^2*C - 2*a^3*A*c*d + 6*a*A*b^2*c*d - 6*a^2*b*B*c
*d + 2*b^3*B*c*d + 2*a^3*c*C*d - 6*a*b^2*c*C*d + 3*a^2*A*b*d^2 - A*b^3*d^2
- a^3*B*d^2 + 3*a*b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3*C*d^2) + Sqrt[-b^2]*(a^3*
A*b*c^2 - 3*a*A*b^3*c^2 + 3*a^2*b^2*B*c^2 - b^4*B*c^2 - a^3*b*c^2*C + 3*a*b
^3*c^2*C - 6*a^2*A*b^2*c*d + 2*A*b^4*c*d + 2*a^3*b*B*c*d - 6*a*b^3*B*c*d +
6*a^2*b^2*c*C*d - 2*b^4*c*C*d - a^3*A*b*d^2 + 3*a*A*b^3*d^2 - 3*a^2*b^2*B*d
^2 + b^4*B*d^2 + a^3*b*C*d^2 - 3*a*b^3*C*d^2))*Log[Sqrt[-b^2] + b*Tan[e + f
*x]])/(b*(a^2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 + b^2)^2*d^2*(b*(3*c^4*C - 4*
B*c^3*d + c^2*(5*A + C)*d^2 - 2*B*c*d^3 + 3*A*d^4) - a*d^2*(2*c*(A - C)*d -
 B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(
b*c) + a*d)*(c^2 + d^2)*f)) - (d^2*((-(b*c) - a*d)*(-3*a*(A*b^2 - a*(b*B -
a*C))*d + 2*b*(A*b - a*B - b*C)*(b*c - a*d)) + (2*b^2*d - a*(b*c - a*d))*(3
*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d))) - c*(d*(b*c - a*
d)*(-3*b*(A*b^2 - a*(b*B - a*C))*d - 2*a*(A*b - a*B - b*C)*(b*c - a*d) + b*
(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d))) - 2*c*d*(-(a*(
-3*a*(A*b^2 - a*(b*B - a*C))*d + 2*b*(A*b - a*B - b*C)*(b*c - a*d))) + b^2*
(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))))/((-(b*c) + a
*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/((a^2 + b^2)*(b*c - a*d)))/(2*(a^2
 + b^2)*(b*c - a*d))
```

Maple [A]
time = 3.59, size = 951, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{d^{2}\left(2 A a c d^{3}-5 A b c^{2} d^{2}-3 A b d^{4}-B a c^{2} d^{2}+B a d^{4}+4 B b c^{3} d+2 B b c d^{3}-2 C a c d^{3}-3 C b c^{4}-C b c^{2} d^{2}\right) \ln (c+d \tan (f x+e))}{(a d-b c)^{4}\left(c^{2}+d^{2}\right)^{2}}-\frac{\left(A d^{2}\right.}{(a d-b c)^{3}\left(c^{2}\right.}}{}$
default	$\frac{\frac{d^{2}\left(2 A a c d^{3}-5 A b c^{2} d^{2}-3 A b d^{4}-B a c^{2} d^{2}+B a d^{4}+4 B b c^{3} d+2 B b c d^{3}-2 C a c d^{3}-3 C b c^{4}-C b c^{2} d^{2}\right) \ln (c+d \tan (f x+e))}{(a d-b c)^{4}\left(c^{2}+d^{2}\right)^{2}}-\frac{\left(A d^{2}\right.}{(a d-b c)^{3}\left(c^{2}\right.}}{}$
norman	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(d^2*(2*A*a*c*d^3-5*A*b*c^2*d^2-3*A*b*d^4-B*a*c^2*d^2+B*a*d^4+4*B*b*c^3
*d+2*B*b*c*d^3-2*C*a*c*d^3-3*C*b*c^4-C*b*c^2*d^2)/(a*d-b*c)^4/(c^2+d^2)^2*l
n(c+d*tan(f*x+e))-(A*d^2-B*c*d+C*c^2)*d^2/(a*d-b*c)^3/(c^2+d^2)/(c+d*tan(f*
x+e))+1/(a^2+b^2)^3/(c^2+d^2)^2*(1/2*(-2*A*a^3*c*d-3*A*a^2*b*c^2+3*A*a^2*b*
d^2+6*A*a*b^2*c*d+A*b^3*c^2-A*b^3*d^2+B*a^3*c^2-B*a^3*d^2-6*B*a^2*b*c*d-3*B
*a*b^2*c^2+3*B*a*b^2*d^2+2*B*b^3*c*d+2*C*a^3*c*d+3*C*a^2*b*c^2-3*C*a^2*b*d^
2-6*C*a*b^2*c*d-C*b^3*c^2+C*b^3*d^2)*ln(1+tan(f*x+e)^2)+(A*a^3*c^2-A*a^3*d^
2-6*A*a^2*b*c*d-3*A*a*b^2*c^2+3*A*a*b^2*d^2+2*A*b^3*c*d+2*B*a^3*c*d+3*B*a^2
```

```
*b*c^2-3*B*a^2*b*d^2-6*B*a*b^2*c*d-B*b^3*c^2+B*b^3*d^2-C*a^3*c^2+C*a^3*d^2+
6*C*a^2*b*c*d+3*C*a*b^2*c^2-3*C*a*b^2*d^2-2*C*b^3*c*d)*arctan(tan(f*x+e))-
1/2*b*(A*b^2-B*a*b+C*a^2)/(a*d-b*c)^2/(a^2+b^2)/(a+b*tan(f*x+e))^2-b*(4*A*a
^2*b^2*d-2*A*a*b^3*c+2*A*b^4*d-3*B*a^3*b*d+B*a^2*b^2*c-B*a*b^3*d-B*b^4*c+2*
C*a^4*d+2*C*a*b^3*c)/(a*d-b*c)^3/(a^2+b^2)^2/(a+b*tan(f*x+e))+b*(10*A*a^4*b
^2*d^2-10*A*a^3*b^3*c*d+3*A*a^2*b^4*c^2+9*A*a^2*b^4*d^2-2*A*a*b^5*c*d-A*b^6
*c^2+3*A*b^6*d^2-6*B*a^5*b*d^2+4*B*a^4*b^2*c*d-B*a^3*b^3*c^2-3*B*a^3*b^3*d^
2-6*B*a^2*b^4*c*d+3*B*a*b^5*c^2-B*a*b^5*d^2-2*B*b^6*c*d+3*C*a^6*d^2-C*a^4*b
^2*d^2+10*C*a^3*b^3*c*d-3*C*a^2*b^4*c^2+2*C*a*b^5*c*d+C*b^6*c^2)/(a*d-b*c)^
4/(a^2+b^2)^3*ln(a+b*tan(f*x+e)))
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(844) = 1688$.
time = 0.72, size = 2528, normalized size = 3.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c^2 + 2*(B*a^3
- 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c*d - ((A - C)*a^3 + 3*B*a^2*b
 - 3*(A - C)*a*b^2 - B*b^3)*d^2)*(f*x + e)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
b^6)*c^4 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d^2 + (a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*d^4) - 2*((B*a^3*b^4 - 3*(A - C)*a^2*b^5 - 3*B*a*b^6 +
(A - C)*b^7)*c^2 - 2*(2*B*a^4*b^3 - 5*(A - C)*a^3*b^4 - 3*B*a^2*b^5 - (A -
C)*a*b^6 - B*b^7)*c*d - (3*C*a^6*b - 6*B*a^5*b^2 + (10*A - C)*a^4*b^3 - 3*B
*a^3*b^4 + 9*A*a^2*b^5 - B*a*b^6 + 3*A*b^7)*d^2)*log(b*tan(f*x + e) + a)/((
a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*c^4 - 4*(a^7*b^3 + 3*a^5*b^5 + 3*a^
3*b^7 + a*b^9)*c^3*d + 6*(a^8*b^2 + 3*a^6*b^4 + 3*a^4*b^6 + a^2*b^8)*c^2*d^
2 - 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*c*d^3 + (a^10 + 3*a^8*b^2 +
 3*a^6*b^4 + a^4*b^6)*d^4) - 2*(3*C*b*c^4*d^2 - 4*B*b*c^3*d^3 + (B*a + (5*A
 + C)*b)*c^2*d^4 - 2*((A - C)*a + B*b)*c*d^5 - (B*a - 3*A*b)*d^6)*log(d*tan
(f*x + e) + c)/(b^4*c^8 - 4*a*b^3*c^7*d - 4*a^3*b*c*d^7 + a^4*d^8 + 2*(3*a^
2*b^2 + b^4)*c^6*d^2 - 4*(a^3*b + 2*a*b^3)*c^5*d^3 + (a^4 + 12*a^2*b^2 + b^
4)*c^4*d^4 - 4*(2*a^3*b + a*b^3)*c^3*d^5 + 2*(a^4 + 3*a^2*b^2)*c^2*d^6) + (
(B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c^2 - 2*((A - C)*a^3 +
3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c*d - (B*a^3 - 3*(A - C)*a^2*b - 3*B*a
*b^2 + (A - C)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*c^4 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d^2 + (a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*d^4) - ((C*a^4*b^2 - 3*B*a^3*b^3 + (5*A - 3*C)*a^
2*b^4 + B*a*b^5 + A*b^6)*c^4 - (5*C*a^5*b - 7*B*a^4*b^2 + (9*A + C)*a^3*b^3
 - 3*B*a^2*b^4 + 5*A*a*b^5)*c^3*d - (2*C*a^6 + 3*C*a^4*b^2 + 3*B*a^3*b^3 -
5*(A - C)*a^2*b^4 - B*a*b^5 - A*b^6)*c^2*d^2 + (2*B*a^6 - 5*C*a^5*b + 11*B*
```

```
a^4*b^2 - (9*A + C)*a^3*b^3 + 5*B*a^2*b^4 - 5*A*a*b^5)*c*d^3 - 2*(A*a^6 + 2
*A*a^4*b^2 + A*a^2*b^4)*d^4 - 2*((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^3*
d + (3*C*a^4*b^2 - 3*B*a^3*b^3 + 2*(2*A + C)*a^2*b^4 - B*a*b^5 + (2*A + C)*
b^6)*c^2*d^2 - (B*a^4*b^2 + B*a^2*b^4 + 2*(A - C)*a*b^5 + 2*B*b^6)*c*d^3 +
((A + 2*C)*a^4*b^2 - 3*B*a^3*b^3 + 6*A*a^2*b^4 - B*a*b^5 + 3*A*b^6)*d^4)*ta
n(f*x + e)^2 - (2*(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^4 + 3*(C*a^4*b^2
- B*a^3*b^3 + (A + C)*a^2*b^4 - B*a*b^5 + A*b^6)*c^3*d + (9*C*a^5*b - 7*B*a
^4*b^2 + 9*(A + C)*a^3*b^3 - B*a^2*b^4 + (A + 8*C)*a*b^5 - 2*B*b^6)*c^2*d^2
 - (4*B*a^5*b - 3*C*a^4*b^2 + 11*B*a^3*b^3 - 3*(A + C)*a^2*b^4 + 7*B*a*b^5
- 3*A*b^6)*c*d^3 + ((4*A + 5*C)*a^5*b - 7*B*a^4*b^2 + (17*A + C)*a^3*b^3 -
3*B*a^2*b^4 + 9*A*a*b^5)*d^4)*tan(f*x + e))/((a^6*b^3 + 2*a^4*b^5 + a^2*b^7
)*c^6 - 3*(a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*c^5*d + (3*a^8*b + 7*a^6*b^3 + 5*
a^4*b^5 + a^2*b^7)*c^4*d^2 - (a^9 + 5*a^7*b^2 + 7*a^5*b^4 + 3*a^3*b^6)*c^3*
d^3 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*c^2*d^4 - (a^9 + 2*a^7*b^2 + a^5*b^4)
*c*d^5 + ((a^4*b^5 + 2*a^2*b^7 + b^9)*c^5*d - 3*(a^5*b^4 + 2*a^3*b^6 + a*b^
8)*c^4*d^2 + (3*a^6*b^3 + 7*a^4*b^5 + 5*a^2*b^7 + b^9)*c^3*d^3 - (a^7*b^2 +
 5*a^5*b^4 + 7*a^3*b^6 + 3*a*b^8)*c^2*d^4 + 3*(a^6*b^3 + 2*a^4*b^5 + a^2*b^
7)*c*d^5 - (a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*d^6)*tan(f*x + e)^3 + ((a^4*b^5
+ 2*a^2*b^7 + b^9)*c^6 - (a^5*b^4 + 2*a^3*b^6 + a*b^8)*c^5*d - (3*a^6*b^3 +
 5*a^4*b^5 + a^2*b^7 - b^9)*c^4*d^2 + (5*a^7*b^2 + 9*a^5*b^4 + 3*a^3*b^6 -
a*b^8)*c^3*d^3 - (2*a^8*b + 7*a^6*b^3 + 8*a^4*b^5 + 3*a^2*b^7)*c^2*d^4 + 5*
(a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*c*d^5 - 2*(a^8*b + 2*a^6*b^3 + a^4*b^5)*d^6
)*tan(f*x + e)^2 + (2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*c^6 - 5*(a^6*b^3 + 2*a^
4*b^5 + a^2*b^7)*c^5*d + (3*a^7*b^2 + 8*a^5*b^4 + 7*a^3*b^6 + 2*a*b^8)*c^4*
d^2 + (a^8*b - 3*a^6*b^3 - 9*a^4*b^5 - 5*a^2*b^7)*c^3*d^3 - (a^9 - a^7*b^2
- 5*a^5*b^4 - 3*a^3*b^6)*c^2*d^4 + (a^8*b + 2*a^6*b^3 + a^4*b^5)*c*d^5 - (a
^9 + 2*a^7*b^2 + a^5*b^4)*d^6)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9612 vs. $2(844) = 1688$.
time = 20.93, size = 9612, normalized size = 11.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((3*C*a^4*b^5 - 5*B*a^3*b^6 + (7*A - 3*C)*a^2*b^7 + B*a*b^8 + A*b^9)*c
^7 - 2*(5*C*a^5*b^4 - 7*B*a^4*b^5 + (9*A - C)*a^3*b^6 - B*a^2*b^7 + 3*A*a*b
^8)*c^6*d + (7*C*a^6*b^3 - 9*B*a^5*b^4 + (11*A + 7*C)*a^4*b^5 - 13*B*a^3*b^
6 + (19*A - 6*C)*a^2*b^7 + 2*B*a*b^8 + 2*A*b^9)*c^5*d^2 - 4*(5*C*a^5*b^4 -
7*B*a^4*b^5 + (9*A - C)*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*c^4*d^3 - (2*C*a^8
*b - 8*C*a^6*b^3 + 18*B*a^5*b^4 - (22*A - C)*a^4*b^5 + 11*B*a^3*b^6 - (17*A
 - 5*C)*a^2*b^7 - B*a*b^8 - A*b^9)*c^3*d^4 + 2*(C*a^9 + B*a^8*b + 3*C*a^7*b
```

^2 + 3*B*a^6*b^3 - 2*C*a^5*b^4 + 10*B*a^4*b^5 - (9*A - 2*C)*a^3*b^6 + 2*B*a
^2*b^7 - 3*A*a*b^8)*c^2*d^5 - (2*B*a^9 + 2*A*a^8*b + 6*B*a^7*b^2 + (6*A - 7
*C)*a^6*b^3 + 15*B*a^5*b^4 - (5*A + C)*a^4*b^5 + 5*B*a^3*b^6 - 3*A*a^2*b^7)
*c*d^6 + 2*(A*a^9 + 3*A*a^7*b^2 + 3*A*a^5*b^4 + A*a^3*b^6)*d^7 - ((C*a^4*b^
5 - 3*B*a^3*b^6 + 5*(A - C)*a^2*b^7 + 3*B*a*b^8 - A*b^9)*c^6*d - 2*(3*C*a^5
*b^4 - 5*B*a^4*b^5 + (7*A - 3*C)*a^3*b^6 + B*a^2*b^7 + A*a*b^8)*c^5*d^2 + (
3*C*a^6*b^3 - 7*B*a^5*b^4 + (9*A - 5*C)*a^4*b^5 - 7*B*a^3*b^6 + (13*A - 16*
C)*a^2*b^7 + 6*B*a*b^8 - 2*(A + C)*b^9)*c^4*d^3 + 2*(C*a^7*b^2 + B*a^6*b^3
- 3*C*a^5*b^4 + 13*B*a^4*b^5 - (14*A - 9*C)*a^3*b^6 + B*a^2*b^7 - (2*A - C)
*a*b^8 + B*b^9)*c^3*d^4 - (2*B*a^7*b^2 + 2*(A - 5*C)*a^6*b^3 + 20*B*a^5*b^4
- (12*A - C)*a^4*b^5 + 11*B*a^3*b^6 - 5*(A - C)*a^2*b^7 - B*a*b^8 + 3*A*b^
9)*c^2*d^5 + 2*(A*a^7*b^2 + 3*(A - C)*a^5*b^4 + 5*B*a^4*b^5 - (4*A - 3*C)*a
^3*b^6 - B*a^2*b^7)*c*d^6 + (5*C*a^6*b^3 - 7*B*a^5*b^4 + (9*A - C)*a^4*b^5
- B*a^3*b^6 + 3*A*a^2*b^7)*d^7 + 2*(((A - C)*a^3*b^6 + 3*B*a^2*b^7 - 3*(A -
C)*a*b^8 - B*b^9)*c^6*d - 2*(2*(A - C)*a^4*b^5 + 5*B*a^3*b^6 - 3*(A - C)*a
^2*b^7 + B*a*b^8 - (A - C)*b^9)*c^5*d^2 + (6*(A - C)*a^5*b^4 + 10*B*a^4*b^5
+ 5*(A - C)*a^3*b^6 + 15*B*a^2*b^7 - 5*(A - C)*a*b^8 + B*b^9)*c^4*d^3 - 4*
((A - C)*a^6*b^3 + 5*(A - C)*a^4*b^5 + 5*B*a^3*b^6 + B*a*b^8)*c^3*d^4 + ((A
- C)*a^7*b^2 - 5*B*a^6*b^3 + 15*(A - C)*a^5*b^4 + 5*B*a^4*b^5 + 10*(A - C)
*a^3*b^6 + 6*B*a^2*b^7)*c^2*d^5 + 2*(B*a^7*b^2 - (A - C)*a^6*b^3 + 3*B*a^5*
b^4 - 5*(A - C)*a^4*b^5 - 2*B*a^3*b^6)*c*d^6 - ((A - C)*a^7*b^2 + 3*B*a^6*b
^3 - 3*(A - C)*a^5*b^4 - B*a^4*b^5)*d^7)*f*x)*tan(f*x + e)^3 - 2*(((A - C)*
a^5*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*b^6 - B*a^2*b^7)*c^7 - 2*(2*(A - C)*a
^6*b^3 + 5*B*a^5*b^4 - 3*(A - C)*a^4*b^5 + B*a^3*b^6 - (A - C)*a^2*b^7)*c^6
d + (6(A - C)*a^7*b^2 + 10*B*a^6*b^3 + 5*(A - C)*a^5*b^4 + 15*B*a^4*b^5 -
5*(A - C)*a^3*b^6 + B*a^2*b^7)*c^5*d^2 - 4*((A - C)*a^8*b + 5*(A - C)*a^6*
b^3 + 5*B*a^5*b^4 + B*a^3*b^6)*c^4*d^3 + ((A - C)*a^9 - 5*B*a^8*b + 15*(A -
C)*a^7*b^2 + 5*B*a^6*b^3 + 10*(A - C)*a^5*b^4 + 6*B*a^4*b^5)*c^3*d^4 + 2*(
B*a^9 - (A - C)*a^8*b + 3*B*a^7*b^2 - 5*(A - C)*a^6*b^3 - 2*B*a^5*b^4)*c^2*
d^5 - ((A - C)*a^9 + 3*B*a^8*b - 3*(A - C)*a^7*b^2 - B*a^6*b^3)*c*d^6)*f*x
- ((C*a^4*b^5 - 3*B*a^3*b^6 + 5*(A - C)*a^2*b^7 + 3*B*a*b^8 - A*b^9)*c^7 -
2*(2*C*a^5*b^4 - 3*B*a^4*b^5 + 4*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(2*A - C)*a*b^
8 + B*b^9)*c^6*d - (3*C*a^6*b^3 - 5*B*a^5*b^4 + (7*A - 13*C)*a^4*b^5 + 19*B
*a^3*b^6 - (25*A - 14*C)*a^2*b^7 - 6*B*a*b^8 - 2*A*b^9)*c^5*d^2 + 2*(C*a^7*
b^2 - 4*B*a^6*b^3 + (5*A - 13*C)*a^5*b^4 + 9*B*a^4*b^5 - (11*A + 6*C)*a^3*b
^6 + 5*B*a^2*b^7 - 2*(5*A - C)*a*b^8 - 2*B*b^9)*c^4*d^3 + (4*C*a^8*b + 4*B*
a^7*b^2 + 8*C*a^6*b^3 + 22*B*a^5*b^4 - (14*A - 41*C)*a^4*b^5 - 17*B*a^3*b^6
+ (35*A - 3*C)*a^2*b^7 + 7*B*a*b^8 + (7*A + 2*C)*b^9)*c^3*d^4 - 2*(2*B*a^8
*b + (2*A - 5*C)*a^7*b^2 + 15*B*a^6*b^3 - (4*A - 11*C)*a^5*b^4 + (16*A + 3*
C)*a^3*b^6 + B*a^2*b^7 + (10*A - C)*a*b^8 + 2*B*b^9)*c^2*d^5 + (4*A*a^8*b +
2*B*a^7*b^2 + (14*A - 3*C)*a^6*b^3 + 11*B*a^5*b^4 + 11*(A + C)*a^4*b^5 - 7
*B*a^3*b^6 + (25*A - 4*C)*a^2*b^7 + 2*B*a*b^8 + 6*A*b^9)*c*d^6 - 2*((A - 3*
C)*a^7*b^2 + 4*B*a^6*b^3 - (2*A - 3*C)*a^5*b^4 - 3*B*a^4*b^5 + 6*A*a^3*b^6
- B*a^2*b^7 + 3*A*a*b^8)*d^7 + 2*(((A - C)*a^3*b^6 + 3*B*a^2*b^7 - 3*(A - C
)*a*b^8 - B*b^9)*c^7 - 2*((A - C)*a^4*b^5 + 2*B*a^3*b^6 + 2*B*a*b^8 - (A -

```
C)*b^9)*c^6*d - (2*(A - C)*a^5*b^4 + 10*B*a^4*b^5 - 17*(A - C)*a^3*b^6 - 11
*B*a^2*b^7 + (A - C)*a*b^8 - B*b^9)*c^5*d^2 + 2*(4*(A - C)*a^6*b^3 + 10*B*a
^5*b^4 - 5*(A - C)*a^4*b^5 + 5*B*a^3*b^6 - 5*(A - C)*a^2*b^7 - B*a*b^8)*c^4
*d^3 - (7*(A - C)*a^7*b^2 + 5*B*a^6*b^3 + 25*(A - C)*a^5*b^4 + 35*B*a^4*b^5
 - 10*(A - C)*a^3*b^6 + 2*B*a^2*b^7)*c^3*d^4 + 2*((A - C)*a^8*b - 4*B*a^7*b
^2 + 14*(A - C)*a^6*b^3 + 8*B*a^5*b^4 + 5*(A - C)*a^4*b^5 + 4*B*a^3*b^6)*c^
2*d^5 + (4*B*a^8*b - 5*(A - C)*a^7*b^2 + 9*B*a^6*b^3 - 17*(A - C)*a^5*b^4 -
 7*B*a^4*b^5)*c*d^6 - 2*((A - C)*a^8*b + 3*B*a^7*b^2 - 3*(A - C)*a^6*b^3 -
B*a^5*b^4)*d^7)*f*x)*tan(f*x + e)^2 + ((B*a^5*b^4 - 3*(A - C)*a^4*b^5 - 3*B
*a^3*b^6 + (A - C)*a^2*b^7)*c^7 - 2*(2*B*a^6*b^3 - 5*(A - C)*a^5*b^4 - 3*B*
a^4*b^5 - (A - C)*a^3*b^6 - B*a^2*b^7)*c^6*d - (3*C*a^8*b - 6*B*a^7*b^2 + (
10*A - C)*a^6*b^3 - 5*B*a^5*b^4 + 3*(5*A - 2*C)*a^4*b^5 + 5*B*a^3*b^6 + (A
+ 2*C)*a^2*b^7)*c^5*d^2 - 4*(2*B*a^6*b^3 - 5*(A - C)*a^5*b^4 - 3*B*a^4*b^5
- (A - C)*a^3*b^6 - B*a^2*b^7)*c^4*d^3 - (6*C*a^8*b - 12*B*a^7*b^2 + 2*(10*
A - C)*a^6*b^3 - 7*B*a^5*b^4 + 3*(7*A - C)*a^4*...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**3/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3176 vs. $2(844) = 1688$.
time = 1.23, size = 3176, normalized size = 3.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^3*c^2 - C*a^3*c^2 + 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*c
^2 - B*b^3*c^2 + 2*B*a^3*c*d - 6*A*a^2*b*c*d + 6*C*a^2*b*c*d - 6*B*a*b^2*c*
d + 2*A*b^3*c*d - 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 - 3*B*a^2*b*d^2 + 3*A
*a*b^2*d^2 - 3*C*a*b^2*d^2 + B*b^3*d^2)*(f*x + e)/(a^6*c^4 + 3*a^4*b^2*c^4
+ 3*a^2*b^4*c^4 + b^6*c^4 + 2*a^6*c^2*d^2 + 6*a^4*b^2*c^2*d^2 + 6*a^2*b^4*c
^2*d^2 + 2*b^6*c^2*d^2 + a^6*d^4 + 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 + b^6*d^4)
 + (B*a^3*c^2 - 3*A*a^2*b*c^2 + 3*C*a^2*b*c^2 - 3*B*a*b^2*c^2 + A*b^3*c^2 -
 C*b^3*c^2 - 2*A*a^3*c*d + 2*C*a^3*c*d - 6*B*a^2*b*c*d + 6*A*a*b^2*c*d - 6*
C*a*b^2*c*d + 2*B*b^3*c*d - B*a^3*d^2 + 3*A*a^2*b*d^2 - 3*C*a^2*b*d^2 + 3*B
```

*a*b^2*d^2 - A*b^3*d^2 + C*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(a^6*c^4 + 3*a^
4*b^2*c^4 + 3*a^2*b^4*c^4 + b^6*c^4 + 2*a^6*c^2*d^2 + 6*a^4*b^2*c^2*d^2 + 6
*a^2*b^4*c^2*d^2 + 2*b^6*c^2*d^2 + a^6*d^4 + 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4
+ b^6*d^4) - 2*(B*a^3*b^5*c^2 - 3*A*a^2*b^6*c^2 + 3*C*a^2*b^6*c^2 - 3*B*a*b
^7*c^2 + A*b^8*c^2 - C*b^8*c^2 - 4*B*a^4*b^4*c*d + 10*A*a^3*b^5*c*d - 10*C*
a^3*b^5*c*d + 6*B*a^2*b^6*c*d + 2*A*a*b^7*c*d - 2*C*a*b^7*c*d + 2*B*b^8*c*d
 - 3*C*a^6*b^2*d^2 + 6*B*a^5*b^3*d^2 - 10*A*a^4*b^4*d^2 + C*a^4*b^4*d^2 + 3
*B*a^3*b^5*d^2 - 9*A*a^2*b^6*d^2 + B*a*b^7*d^2 - 3*A*b^8*d^2)*log(abs(b*tan
(f*x + e) + a))/(a^6*b^5*c^4 + 3*a^4*b^7*c^4 + 3*a^2*b^9*c^4 + b^11*c^4 - 4
*a^7*b^4*c^3*d - 12*a^5*b^6*c^3*d - 12*a^3*b^8*c^3*d - 4*a*b^10*c^3*d + 6*a
^8*b^3*c^2*d^2 + 18*a^6*b^5*c^2*d^2 + 18*a^4*b^7*c^2*d^2 + 6*a^2*b^9*c^2*d^
2 - 4*a^9*b^2*c*d^3 - 12*a^7*b^4*c*d^3 - 12*a^5*b^6*c*d^3 - 4*a^3*b^8*c*d^3
 + a^10*b*d^4 + 3*a^8*b^3*d^4 + 3*a^6*b^5*d^4 + a^4*b^7*d^4) - 2*(3*C*b*c^4
*d^3 - 4*B*b*c^3*d^4 + B*a*c^2*d^5 + 5*A*b*c^2*d^5 + C*b*c^2*d^5 - 2*A*a*c*
d^6 + 2*C*a*c*d^6 - 2*B*b*c*d^6 - B*a*d^7 + 3*A*b*d^7)*log(abs(d*tan(f*x +
e) + c))/(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 + 2*b^4*c^6*d^3 -
 4*a^3*b*c^5*d^4 - 8*a*b^3*c^5*d^4 + a^4*c^4*d^5 + 12*a^2*b^2*c^4*d^5 + b^4
*c^4*d^5 - 8*a^3*b*c^3*d^6 - 4*a*b^3*c^3*d^6 + 2*a^4*c^2*d^7 + 6*a^2*b^2*c^
2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9) + 2*(3*C*b*c^4*d^3*tan(f*x + e) - 4*B*b*c^
3*d^4*tan(f*x + e) + B*a*c^2*d^5*tan(f*x + e) + 5*A*b*c^2*d^5*tan(f*x + e)
+ C*b*c^2*d^5*tan(f*x + e) - 2*A*a*c*d^6*tan(f*x + e) + 2*C*a*c*d^6*tan(f*x
 + e) - 2*B*b*c*d^6*tan(f*x + e) - B*a*d^7*tan(f*x + e) + 3*A*b*d^7*tan(f*x
 + e) + 4*C*b*c^5*d^2 - C*a*c^4*d^3 - 5*B*b*c^4*d^3 + 2*B*a*c^3*d^4 + 6*A*b
*c^3*d^4 + 2*C*b*c^3*d^4 - 3*A*a*c^2*d^5 + C*a*c^2*d^5 - 3*B*b*c^2*d^5 + 4*
A*b*c*d^6 - A*a*d^7)/((b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 + 2*b^4*
c^6*d^2 - 4*a^3*b*c^5*d^3 - 8*a*b^3*c^5*d^3 + a^4*c^4*d^4 + 12*a^2*b^2*c^4*
d^4 + b^4*c^4*d^4 - 8*a^3*b*c^3*d^5 - 4*a*b^3*c^3*d^5 + 2*a^4*c^2*d^6 + 6*a
^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*(d*tan(f*x + e) + c)) + (3*B*a^3*
b^6*c^2*tan(f*x + e)^2 - 9*A*a^2*b^7*c^2*tan(f*x + e)^2 + 9*C*a^2*b^7*c^2*t
an(f*x + e)^2 - 9*B*a*b^8*c^2*tan(f*x + e)^2 + 3*A*b^9*c^2*tan(f*x + e)^2 -
 3*C*b^9*c^2*tan(f*x + e)^2 - 12*B*a^4*b^5*c*d*tan(f*x + e)^2 + 30*A*a^3*b^
6*c*d*tan(f*x + e)^2 - 30*C*a^3*b^6*c*d*tan(f*x + e)^2 + 18*B*a^2*b^7*c*d*t
an(f*x + e)^2 + 6*A*a*b^8*c*d*tan(f*x + e)^2 - 6*C*a*b^8*c*d*tan(f*x + e)^2
 + 6*B*b^9*c*d*tan(f*x + e)^2 - 9*C*a^6*b^3*d^2*tan(f*x + e)^2 + 18*B*a^5*b
^4*d^2*tan(f*x + e)^2 - 30*A*a^4*b^5*d^2*tan(f*x + e)^2 + 3*C*a^4*b^5*d^2*t
an(f*x + e)^2 + 9*B*a^3*b^6*d^2*tan(f*x + e)^2 - 27*A*a^2*b^7*d^2*tan(f*x +
 e)^2 + 3*B*a*b^8*d^2*tan(f*x + e)^2 - 9*A*b^9*d^2*tan(f*x + e)^2 + 8*B*a^4
*b^5*c^2*tan(f*x + e) - 22*A*a^3*b^6*c^2*tan(f*x + e) + 22*C*a^3*b^6*c^2*ta
n(f*x + e) - 18*B*a^2*b^7*c^2*tan(f*x + e) + 2*A*a*b^8*c^2*tan(f*x + e) - 2
*C*a*b^8*c^2*tan(f*x + e) - 2*B*b^9*c^2*tan(f*x + e) + 4*C*a^6*b^3*c*d*tan(
f*x + e) - 32*B*a^5*b^4*c*d*tan(f*x + e) + 72*A*a^4*b^5*c*d*tan(f*x + e) -
60*C*a^4*b^5*c*d*tan(f*x + e) + 28*B*a^3*b^6*c*d*tan(f*x + e) + 28*A*a^2*b^
7*c*d*tan(f*x + e) - 16*C*a^2*b^7*c*d*tan(f*x + e) + 12*B*a*b^8*c*d*tan(f*x
 + e) + 4*A*b^9*c*d*tan(f*x + e) - 22*C*a^7*b^2*d^2*tan(f*x + e) + 42*B*a^6
*b^3*d^2*tan(f*x + e) - 68*A*a^5*b^4*d^2*tan(f*x + e) + 2*C*a^5*b^4*d^2*tan

```
(f*x + e) + 26*B*a^4*b^5*d^2*tan(f*x + e) - 66*A*a^3*b^6*d^2*tan(f*x + e) +
 8*B*a^2*b^7*d^2*tan(f*x + e) - 22*A*a*b^8*d^2*tan(f*x + e) - C*a^6*b^3*c^2
 + 6*B*a^5*b^4*c^2 - 14*A*a^4*b^5*c^2 + 11*C*a^4*b^5*c^2 - 7*B*a^3*b^6*c^2
- 3*A*a^2*b^7*c^2 - B*a*b^8*c^2 - A*b^9*c^2 + 6*C*a^7*b^2*c*d - 22*B*a^6*b^
3*c*d + 44*A*a^5*b^4*c*d - 26*C*a^5*b^4*c*d + 6*B*a^4*b^5*c*d + 26*A*a^3*b^
6*c*d - 8*C*a^3*b^6*c*d + 4*B*a^2*b^7*c*d + 6*A*a*b^8*c*d - 14*C*a^8*b*d^2
+ 25*B*a^7*b^2*d^2 - 39*A*a^6*b^3*d^2 - 3*C*a^6*b^3*d^2 + 19*B*a^5*b^4*d^2
- 41*A*a^4*b^5*d^2 - C*a^4*b^5*d^2 + 6*B*a^3*b^6*d^2 - 14*A*a^2*b^7*d^2)/((
a^6*b^4*c^4 + 3*a^4*b^6*c^4 + 3*a^2*b^8*c^4 + b^10*c^4 - 4*a^7*b^3*c^3*d -
12*a^5*b^5*c^3*d - 12*a^3*b^7*c^3*d - 4*a*b^9*c^3*d + 6*a^8*b^2*c^2*d^2 + 1
8*a^6*b^4*c^2*d^2 + 18*a^4*b^6*c^2*d^2 + 6*a^2*b^8*c^2*d^2 - 4*a^9*b*c*d^3
- 12*a^7*b^3*c*d^3 - 12*a^5*b^5*c*d^3 - 4*a^3*b^7*c*d^3 + a^10*d^4 + 3*a^8*
b^2*d^4 + 3*a^6*b^4*d^4 + a^4*b^6*d^4)*(b*tan(f...
```

Mupad [B]
time = 58.47, size = 2500, normalized size = 2.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^3*(c + d*
tan(e + f*x))^2),x)
```

```
[Out] (symsum(log((24*A^3*a^3*b^7*d^9 + 27*A^3*a^5*b^5*d^9 + B^3*a^2*b^8*d^9 + 4*
B^3*a^4*b^6*d^9 + 7*B^3*a^6*b^4*d^9 + 3*A^3*b^10*c^3*d^6 - A^3*b^10*c^5*d^4
 + 4*B^3*b^10*c^2*d^7 + 6*B^3*b^10*c^4*d^5 + C^3*b^10*c^5*d^4 + 9*A^2*B*b^1
0*d^9 + 9*A^3*a*b^9*d^9 + 16*A^3*a^2*b^8*c^3*d^6 + 3*A^3*a^2*b^8*c^5*d^4 +
26*A^3*a^3*b^7*c^2*d^7 - 6*A^3*a^3*b^7*c^4*d^5 - 11*A^3*a^4*b^6*c^3*d^6 + 3
1*A^3*a^5*b^5*c^2*d^7 + 5*B^3*a^2*b^8*c^2*d^7 + 6*B^3*a^2*b^8*c^4*d^5 + 28*
B^3*a^3*b^7*c^3*d^6 + 7*B^3*a^3*b^7*c^5*d^4 - 14*B^3*a^4*b^6*c^2*d^7 - 20*B
^3*a^4*b^6*c^4*d^5 + 19*B^3*a^5*b^5*c^3*d^6 + 9*B^3*a^6*b^4*c^2*d^7 - 7*C^3
*a^2*b^8*c^3*d^6 - 3*C^3*a^2*b^8*c^5*d^4 + C^3*a^3*b^7*c^2*d^7 + 15*C^3*a^3
*b^7*c^4*d^5 + 6*C^3*a^3*b^7*c^6*d^3 - 28*C^3*a^4*b^6*c^3*d^6 - 24*C^3*a^4*
b^6*c^5*d^4 - 4*C^3*a^5*b^5*c^2*d^7 + 3*C^3*a^6*b^4*c^3*d^6 - 9*C^3*a^7*b^3
*c^2*d^7 - 9*C^3*a^7*b^3*c^4*d^5 - 6*A*B^2*a*b^9*d^9 - 9*A^2*C*a*b^9*d^9 -
12*A*B^2*b^10*c*d^8 + 4*B^3*a*b^9*c*d^8 - 20*A*B^2*a^3*b^7*d^9 - 28*A*B^2*a
^5*b^5*d^9 + 6*A*B^2*a^7*b^3*d^9 + 21*A^2*B*a^2*b^8*d^9 + 13*A^2*B*a^4*b^6*
d^9 - 27*A^2*B*a^6*b^4*d^9 - 3*A*C^2*a^3*b^7*d^9 - 9*A*C^2*a^7*b^3*d^9 - 21
*A^2*C*a^3*b^7*d^9 - 27*A^2*C*a^5*b^5*d^9 + 9*A^2*C*a^7*b^3*d^9 - 17*A*B^2*
b^10*c^3*d^6 + 3*A*B^2*b^10*c^5*d^4 + B*C^2*a^4*b^6*d^9 + 3*B*C^2*a^8*b^2*d
^9 + 12*A^2*B*b^10*c^2*d^7 - 7*A^2*B*b^10*c^4*d^5 - B^2*C*a^3*b^7*d^9 - 2*B
^2*C*a^5*b^5*d^9 - 9*B^2*C*a^7*b^3*d^9 + 3*A*C^2*b^10*c^3*d^6 - 3*A*C^2*b^1
0*c^5*d^4 - 6*A^2*C*b^10*c^3*d^6 + 3*A^2*C*b^10*c^5*d^4 - B*C^2*b^10*c^4*d^
5 + 3*B*C^2*b^10*c^6*d^3 - 4*B^2*C*b^10*c^3*d^6 - 9*B^2*C*b^10*c^5*d^4 + 3*
A^3*a*b^9*c^2*d^7 - 10*A^3*a*b^9*c^4*d^5 - 3*A^3*a^2*b^8*c*d^8 - 31*A^3*a^4
```

*b^6*c*d^8 - 8*A^3*a^6*b^4*c*d^8 + B^3*a*b^9*c^3*d^6 - 5*B^3*a*b^9*c^5*d^4
+ 11*B^3*a^3*b^7*c*d^8 + 5*B^3*a^5*b^5*c*d^8 - 6*B^3*a^7*b^3*c*d^8 - 2*C^3*
a*b^9*c^4*d^5 - 6*C^3*a*b^9*c^6*d^3 - 2*C^3*a^4*b^6*c*d^8 - C^3*a^6*b^4*c*d
^8 - 3*C^3*a^8*b^2*c*d^8 - 60*A*B^2*a^2*b^8*c^3*d^6 - 21*A*B^2*a^2*b^8*c^5*
d^4 - 4*A*B^2*a^3*b^7*c^2*d^7 + 44*A*B^2*a^3*b^7*c^4*d^5 + 25*A*B^2*a^4*b^6
*c^3*d^6 + 4*A*B^2*a^4*b^6*c^5*d^4 - 77*A*B^2*a^5*b^5*c^2*d^7 - 17*A*B^2*a^
5*b^5*c^4*d^5 + 28*A*B^2*a^6*b^4*c^3*d^6 - 6*A*B^2*a^7*b^3*c^2*d^7 + 71*A^2
*B*a^2*b^8*c^2*d^7 + 16*A^2*B*a^2*b^8*c^4*d^5 - 116*A^2*B*a^3*b^7*c^3*d^6 -
 9*A^2*B*a^3*b^7*c^5*d^4 + 86*A^2*B*a^4*b^6*c^2*d^7 + 35*A^2*B*a^4*b^6*c^4*
d^5 - 37*A^2*B*a^5*b^5*c^3*d^6 - 13*A^2*B*a^6*b^4*c^2*d^7 + 30*A*C^2*a^2*b^
8*c^3*d^6 + 9*A*C^2*a^2*b^8*c^5*d^4 - 30*A*C^2*a^3*b^7*c^2*d^7 - 63*A*C^2*a
^3*b^7*c^4*d^5 - 12*A*C^2*a^3*b^7*c^6*d^3 + 45*A*C^2*a^4*b^6*c^3*d^6 + 48*A
*C^2*a^4*b^6*c^5*d^4 - 15*A*C^2*a^5*b^5*c^2*d^7 - 27*A*C^2*a^5*b^5*c^4*d^5
- 6*A*C^2*a^6*b^4*c^3*d^6 + 9*A*C^2*a^7*b^3*c^4*d^5 - 39*A^2*C*a^2*b^8*c^3*
d^6 - 9*A^2*C*a^2*b^8*c^5*d^4 + 3*A^2*C*a^3*b^7*c^2*d^7 + 54*A^2*C*a^3*b^7*
c^4*d^5 + 6*A^2*C*a^3*b^7*c^6*d^3 - 6*A^2*C*a^4*b^6*c^3*d^6 - 24*A^2*C*a^4*
b^6*c^5*d^4 - 12*A^2*C*a^5*b^5*c^2*d^7 + 27*A^2*C*a^5*b^5*c^4*d^5 + 3*A^2*C
*a^6*b^4*c^3*d^6 + 9*A^2*C*a^7*b^3*c^2*d^7 + 11*B*C^2*a^2*b^8*c^2*d^7 - 17*
B*C^2*a^2*b^8*c^4*d^5 - 18*B*C^2*a^2*b^8*c^6*d^3 + 16*B*C^2*a^3*b^7*c^3*d^6
 + 39*B*C^2*a^3*b^7*c^5*d^4 + 47*B*C^2*a^4*b^6*c^2*d^7 + 47*B*C^2*a^4*b^6*c
^4*d^5 + 3*B*C^2*a^4*b^6*c^6*d^3 - 25*B*C^2*a^5*b^5*c^3*d^6 - 12*B*C^2*a^5*
b^5*c^5*d^4 + 17*B*C^2*a^6*b^4*c^2*d^7 + 27*B*C^2*a^6*b^4*c^4*d^5 + 12*B*C^
2*a^7*b^3*c^3*d^6 - 3*B*C^2*a^8*b^2*c^2*d^7 + 9*B^2*C*a^2*b^8*c^3*d^6 + 9*B
^2*C*a^2*b^8*c^5*d^4 - 35*B^2*C*a^3*b^7*c^2*d^7 - 68*B^2*C*a^3*b^7*c^4*d^5
- 6*B^2*C*a^3*b^7*c^6*d^3 - 16*B^2*C*a^4*b^6*c^3*d^6 + 14*B^2*C*a^4*b^6*c^5
*d^4 + 26*B^2*C*a^5*b^5*c^2*d^7 - 4*B^2*C*a^5*b^5*c^4*d^5 - 37*B^2*C*a^6*b^
4*c^3*d^6 + 3*B^2*C*a^7*b^3*c^2*d^7 + 6*A*B*C*a^2*b^8*d^9 + 13*A*B*C*a^4*b^
6*d^9 + 36*A*B*C*a^6*b^4*d^9 - 3*A*B*C*a^8*b^2*d^9 + 6*A*B*C*b^10*c^2*d^7 +
 17*A*B*C*b^10*c^4*d^5 - 3*A*B*C*b^10*c^6*d^3 - 24*A^2*B*a*b^9*c*d^8 + 11*A
*B^2*a*b^9*c^2*d^7 + 25*A*B^2*a*b^9*c^4*d^5 - 19*A*B^2*a^2*b^8*c*d^8 + 37*A
*B^2*a^4*b^6*c*d^8 + 32*A*B^2*a^6*b^4*c*d^8 - 23*A^2*B*a*b^9*c^3*d^6 + 11*A
^2*B*a*b^9*c^5*d^4 - 81*A^2*B*a^3*b^7*c*d^8 - 15*A^2*B*a^5*b^5*c*d^8 + 6*A^
2*B*a^7*b^3*c*d^8 - 15*A*C^2*a*b^9*c^2*d^7 - 15*A*C^2*a*b^9*c^4*d^5 + 12*A*
C^2*a*b^9*c^6*d^3 - 3*A*C^2*a^2*b^8*c*d^8 - 27*A*C^2*a^4*b^6*c*d^8 - 6*A*C^
2*a^6*b^4*c*d^8 + 6*A*C^2*a^8*b^2*c*d^8 + 12*A^2*C*a*b^9*c^2*d^7 + 27*A^2*C
*a*b^9*c^4*d^5 - 6*A^2*C*a*b^9*c^6*d^3 + 6*A^2*C*a^2*b^8*c*d^8 + 60*A^2*C*a
^4*b^6*c*d^8 + 15*A^2*C*a^6*b^4*c*d^8 - 3*A^2*C*a^8*b^2*c*d^8 + 13*B*C^2*a*
b^9*c^3*d^6 + 23*B*C^2*a*b^9*c^5*d^4 + 3*B*C^2*a^3*b^7*c*d^8 + 9*B*C^2*a^5*
b^5*c*d^8 + 18*B*C^2*a^7*b^3*c*d^8 - 14*B^2*C*a*b^9*c^2*d^7 - 16*B^2*C*a*b^
9*c^4*d^5 + 6*B^2*C*a*b^9*c^6*d^3 - 8*B^2*C*a^2*b^8*c*d^8 - 28*B^2*C*a^4*b^
6*c*d^8 - 29*B^2*C*a^6*b^4*c*d^8 + 3*B^2*C*a^8*b^2*c*d^8 - 28*A*B*C*a^2*b^8
*c^2*d^7 + 28*A*B*C*a^2*b^8*c^4*d^5 + 18*A*B*C*a^2*b^8*c^6*d^3 + 100*A*B*C*
a^3*b^7*c^3*d^6 - 30*A*B*C*a^3*b^7*c^5*d^4 - 79*A*B*C*a^4*b^6*c^2*d^7 - 55*
A*B*C*a^4*b^6*c^4*d^5 - 3*A*B*C*a^4*b^6*c^6*d^3 + 62*A*B*C*a^5*b^5*c^3*d^6
+ 12*A*B*C*a^5*b^5*c^5*d^4 + 14*A*B*C*a^6*b^4*c...

3.84 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=804

$$-\frac{(3ab^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)+a^3(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))\ \cdots}{(c^2+d^2)^3}$$

```
[Out] -(3*a*b^2*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)+a^3*(c^3*C-3*B*
c^2*d-3*c*C*d^2+B*d^3-A*(c^3-3*c*d^2))-3*a^2*b*((A-C)*d*(3*c^2-d^2)-B*(c^3-
3*c*d^2))+b^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2)))*x/(c^2+d^2)^3-(3*a^2*b
*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)-b^3*(A*c^3-3*A*c*d^2+3*B
*c^2*d-B*d^3-C*c^3+3*C*c*d^2)-a^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2))+3*a
*b^2*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2)))*ln(cos(f*x+e))/(c^2+d^2)^3/f-(-
a*d+b*c)*(b^2*(3*c^6*C-B*c^5*d+9*c^4*C*d^2-3*B*c^3*d^3-c^2*(A-10*C)*d^4-6*B
*c*d^5+3*A*d^6)+a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2))+a*b*d^2*(8*c*
(A-C)*d^3-B*(c^4+6*c^2*d^2-3*d^4)))*ln(c+d*tan(f*x+e))/d^4/(c^2+d^2)^3/f+b^
2*(b*(3*c^4*C-B*c^3*d+6*c^2*C*d^2-3*B*c*d^3+(2*A+C)*d^4)+a*d^2*(2*c*(A-C)*d
-B*(c^2-d^2)))*tan(f*x+e)/d^3/(c^2+d^2)^2/f-1/2*(A*d^2-B*c*d+C*c^2)*(a+b*ta
n(f*x+e))^3/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^2-1/2*(b*(3*c^4*C-B*c^3*d-c^2*(A
-7*C)*d^2-5*B*c*d^3+3*A*d^4)+2*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*
x+e))^2/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]

time = 1.79, antiderivative size = 804, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3726, 3718, 3707, 3698, 31, 3556}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^3,x]
```

```
[Out] -(((3*a*b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a
^3*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - 3*a^2*b*((
A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(3*c^2 - d^2)
- B*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3) - ((3*a^2*b*(A*c^3 - c^3*C + 3*B*c^
2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - b^3*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A
*c*d^2 + 3*c*C*d^2 - B*d^3) - a^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d
^2)) + 3*a*b^2*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[Cos[e + f
*x]])/((c^2 + d^2)^3*f) - ((b*c - a*d)*(b^2*(3*c^6*C - B*c^5*d + 9*c^4*C*d^
2 - 3*B*c^3*d^3 - c^2*(A - 10*C)*d^4 - 6*B*c*d^5 + 3*A*d^6) + a^2*d^3*((A -
 C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + a*b*d^2*(8*c*(A - C)*d^3 - B*(c^
4 + 6*c^2*d^2 - 3*d^4)))*Log[c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)^3*f) + (
```

```
b^2*(b*(3*c^4*C - B*c^3*d + 6*c^2*C*d^2 - 3*B*c*d^3 + (2*A + C)*d^4) + a*d^
2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Tan[e + f*x])/(d^3*(c^2 + d^2)^2*f) - ((
c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(2*d*(c^2 + d^2)*f*(c + d*Ta
n[e + f*x])^2) - ((b*(3*c^4*C - B*c^3*d - c^2*(A - 7*C)*d^2 - 5*B*c*d^3 + 3
*A*d^4) + 2*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^2)/
(2*d^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx = -\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$

$$= -\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$

$$= \frac{b^2(b(3c^4C-Bc^3d+6c^2Cd^2-3Bcd^3}{}$$

$$= -\frac{(3ab^2(Ac^3-c^3C+3Bc^2d-3Acd^2}{}$$

$$= -\frac{(3ab^2(Ac^3-c^3C+3Bc^2d-3Acd^2}{}$$

$$= -\frac{(3ab^2(Ac^3-c^3C+3Bc^2d-3Acd^2}{}$$

Mathematica [A]
time = 13.71, size = 1445, normalized size = 1.80

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((3*a*b^2*(-(A*c^3) + c^3*C - 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 + B*d^3) +
a^3*(-(c^3*C) + 3*B*c^2*d + 3*c*C*d^2 - B*d^3 + A*(c^3 - 3*c*d^2)) - 3*a^2*
b*((A - C)*d*(-3*c^2 + d^2) + B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(-3*c^2 +
 d^2) + B*(c^3 - 3*c*d^2)))*(e + f*x)*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(
a + b*Tan[e + f*x])^3)/((c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3
```

```
*(c + d*Tan[e + f*x])^3) - (b^2*(-3*b*c*C + b*B*d + 3*a*C*d)*Log[1 - Tan[(e
 + f*x)/2]^2]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^3)/(
d^4*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c + d*Tan[e + f*x])^3) + ((-3*a^
2*b*(-(A*c^3) + c^3*C - 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 + B*d^3) + b^3*(-
(A*c^3) + c^3*C - 3*B*c^2*d + 3*A*c*d^2 - 3*c*C*d^2 + B*d^3) + a^3*((A - C)
*d*(-3*c^2 + d^2) + B*(c^3 - 3*c*d^2)) - 3*a*b^2*((A - C)*d*(-3*c^2 + d^2)
+ B*(c^3 - 3*c*d^2)))*Log[1 + Tan[(e + f*x)/2]^2]*(c*Cos[e + f*x] + d*Sin[e
 + f*x])^3*(a + b*Tan[e + f*x])^3)/((c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin
[e + f*x])^3*(c + d*Tan[e + f*x])^3) + ((-(b*c) + a*d)*(b^2*(3*c^6*C - B*c^
5*d + 9*c^4*C*d^2 - 3*B*c^3*d^3 - c^2*(A - 10*C)*d^4 - 6*B*c*d^5 + 3*A*d^6)
 + a^2*d^3*(-((A - C)*d*(-3*c^2 + d^2)) - B*(c^3 - 3*c*d^2)) - a*b*d^2*(8*c
*(-A + C)*d^3 + B*(c^4 + 6*c^2*d^2 - 3*d^4)))*Log[-2*d*Tan[(e + f*x)/2] + c
*(-1 + Tan[(e + f*x)/2]^2)]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[
e + f*x])^3)/(d^4*(c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(c +
d*Tan[e + f*x])^3) - (2*b^3*C*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*Tan[(e +
f*x)/2]*(a + b*Tan[e + f*x])^3)/(d^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*
(-1 + Tan[(e + f*x)/2]^2)*(c + d*Tan[e + f*x])^3) + (2*(b*c - a*d)^3*(c^2*C
 - B*c*d + A*d^2)*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(c + 2*d*Tan[(e + f*x
)/2])*(a + b*Tan[e + f*x])^3)/(c^3*d^2*(c^2 + d^2)*f*(a*Cos[e + f*x] + b*Si
n[e + f*x])^3*(c + 2*d*Tan[(e + f*x)/2] - c*Tan[(e + f*x)/2]^2)^2*(c + d*Ta
n[e + f*x])^3) - (2*(b*c - a*d)^2*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a*d*
(c^2*(A + C)*d^3 + A*d^5 + c^5*C*Tan[(e + f*x)/2] + c*d^4*(-B + A*Tan[(e +
f*x)/2]) + c^4*d*(C - 2*B*Tan[(e + f*x)/2]) - c^3*d^2*(B - 3*A*Tan[(e + f*x
)/2] + C*Tan[(e + f*x)/2])) + b*c*(-(A*d^5) + 2*c^5*C*Tan[(e + f*x)/2] + c*
d^4*(B + 2*A*Tan[(e + f*x)/2]) - c^4*d*(C + B*Tan[(e + f*x)/2]) - c^2*d^3*(
A + C + 3*B*Tan[(e + f*x)/2]) + c^3*d^2*(B + 4*C*Tan[(e + f*x)/2])))*(a + b
*Tan[e + f*x])^3)/(c^3*d^3*(c^2 + d^2)^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x]
)^3*(-2*d*Tan[(e + f*x)/2] + c*(-1 + Tan[(e + f*x)/2]^2))*(c + d*Tan[e + f*
x])^3)
```

Maple [A]
time = 1.01, size = 1271, normalized size = 1.58 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(C*b^3/d^3*tan(f*x+e)-1/2/d^4*(A*a^3*d^5-3*A*a^2*b*c*d^4+3*A*a*b^2*c^2*
d^3-A*b^3*c^3*d^2-B*a^3*c*d^4+3*B*a^2*b*c^2*d^3-3*B*a*b^2*c^3*d^2+B*b^3*c^4
*d+C*a^3*c^2*d^3-3*C*a^2*b*c^3*d^2+3*C*a*b^2*c^4*d-C*b^3*c^5)/(c^2+d^2)/(c+
d*tan(f*x+e))^2-1/d^4*(2*A*a^3*c*d^5-3*A*a^2*b*c^2*d^4+3*A*a^2*b*d^6-6*A*a*
b^2*c*d^5+A*b^3*c^4*d^2+3*A*b^3*c^2*d^4-B*a^3*c^2*d^4+B*a^3*d^6-6*B*a^2*b*c
*d^5+3*B*a*b^2*c^4*d^2+9*B*a*b^2*c^2*d^4-2*B*b^3*c^5*d-4*B*b^3*c^3*d^3-2*C*
a^3*c*d^5+3*C*a^2*b*c^4*d^2+9*C*a^2*b*c^2*d^4-6*C*a*b^2*c^5*d-12*C*a*b^2*c^
3*d^3+3*C*b^3*c^6+5*C*b^3*c^4*d^2)/(c^2+d^2)^2/(c+d*tan(f*x+e))+1/d^4*(3*A*
a^3*c^2*d^5-A*a^3*d^7-3*A*a^2*b*c^3*d^4+9*A*a^2*b*c*d^6-9*A*a*b^2*c^2*d^5+3
```

```
*A*a*b^2*d^7+A*b^3*c^3*d^4-3*A*b^3*c*d^6-B*a^3*c^3*d^4+3*B*a^3*c*d^6-9*B*a^
2*b*c^2*d^5+3*B*a^2*b*d^7+3*B*a*b^2*c^3*d^4-9*B*a*b^2*c*d^6+B*b^3*c^6*d+3*B
*b^3*c^4*d^3+6*B*b^3*c^2*d^5-3*C*a^3*c^2*d^5+C*a^3*d^7+3*C*a^2*b*c^3*d^4-9*
C*a^2*b*c*d^6+3*C*a*b^2*c^6*d+9*C*a*b^2*c^4*d^3+18*C*a*b^2*c^2*d^5-3*C*b^3*
c^7-9*C*b^3*c^5*d^2-10*C*b^3*c^3*d^4)/(c^2+d^2)^3*ln(c+d*tan(f*x+e))+1/(c^2
+d^2)^3*(1/2*(-3*A*a^3*c^2*d+A*a^3*d^3+3*A*a^2*b*c^3-9*A*a^2*b*c*d^2+9*A*a*
b^2*c^2*d-3*A*a*b^2*d^3-A*b^3*c^3+3*A*b^3*c*d^2+B*a^3*c^3-3*B*a^3*c*d^2+9*B
*a^2*b*c^2*d-3*B*a^2*b*d^3-3*B*a*b^2*c^3+9*B*a*b^2*c*d^2-3*B*b^3*c^2*d+B*b^
3*d^3+3*C*a^3*c^2*d-C*a^3*d^3-3*C*a^2*b*c^3+9*C*a^2*b*c*d^2-9*C*a*b^2*c^2*d
+3*C*a*b^2*d^3+C*b^3*c^3-3*C*b^3*c*d^2)*ln(1+tan(f*x+e)^2)+(A*a^3*c^3-3*A*a
^3*c*d^2+9*A*a^2*b*c^2*d-3*A*a^2*b*d^3-3*A*a*b^2*c^3+9*A*a*b^2*c*d^2-3*A*b^
3*c^2*d+A*b^3*d^3+3*B*a^3*c^2*d-B*a^3*d^3-3*B*a^2*b*c^3+9*B*a^2*b*c*d^2-9*B
*a*b^2*c^2*d+3*B*a*b^2*d^3+B*b^3*c^3-3*B*b^3*c*d^2-C*a^3*c^3+3*C*a^3*c*d^2-
9*C*a^2*b*c^2*d+3*C*a^2*b*d^3+3*C*a*b^2*c^3-9*C*a*b^2*c*d^2+3*C*b^3*c^2*d-C
*b^3*d^3)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.60, size = 1117, normalized size = 1.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*C*b^3*tan(f*x + e)/d^3 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b
^2 + B*b^3)*c^3 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2
*d - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^2 - (B*a^3 +
 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2
 + 3*c^2*d^4 + d^6) - 2*(3*C*b^3*c^7 + 9*C*b^3*c^5*d^2 - (3*C*a*b^2 + B*b^3
)*c^6*d - 3*(3*C*a*b^2 + B*b^3)*c^4*d^3 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*
b^2 - (A - 10*C)*b^3)*c^3*d^4 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - 2*C)*a*
b^2 + 2*B*b^3)*c^2*d^5 - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A*b^3)*c*
d^6 + ((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*d^7)*log(d*tan(f*x + e) + c)/(c
^6*d^4 + 3*c^4*d^6 + 3*c^2*d^8 + d^10) + ((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*
b^2 - (A - C)*b^3)*c^3 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b
^3)*c^2*d - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d^2 + (
(A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^3)*log(tan(f*x + e)^2
+ 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (5*C*b^3*c^7 + A*a^3*d^7 - 3*(3*
C*a*b^2 + B*b^3)*c^6*d + (3*C*a^2*b + 3*B*a*b^2 + (A + 9*C)*b^3)*c^5*d^2 +
(C*a^3 + 3*B*a^2*b + 3*(A - 7*C)*a*b^2 - 7*B*b^3)*c^4*d^3 - (3*B*a^3 + 3*(3
*A - 5*C)*a^2*b - 15*B*a*b^2 - 5*A*b^3)*c^3*d^4 + ((5*A - 3*C)*a^3 - 9*B*a^
2*b - 9*A*a*b^2)*c^2*d^5 + (B*a^3 + 3*A*a^2*b)*c*d^6 + 2*(3*C*b^3*c^6*d - 2
*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)*b^3)*c^4*
d^3 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^4 - (B*a^3 + 3*(A - 3*C)*a^2*b - 9*B*a*b^
```

```
2 - 3*A*b^3)*c^2*d^5 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c*d^6 + (B*a
^3 + 3*A*a^2*b)*d^7)*tan(f*x + e))/(c^6*d^4 + 2*c^4*d^6 + c^2*d^8 + (c^4*d^
6 + 2*c^2*d^8 + d^10)*tan(f*x + e)^2 + 2*(c^5*d^5 + 2*c^3*d^7 + c*d^9)*tan(
f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2503 vs. $2(804) = 1608$.
time = 4.64, size = 2503, normalized size = 3.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="fricas")

[Out] -1/2*(3*C*b^3*c^7*d^2 + A*a^3*d^9 - (3*C*a*b^2 + B*b^3)*c^6*d^3 - (3*C*a^2*
b + 3*B*a*b^2 + (A - 9*C)*b^3)*c^5*d^4 + (3*C*a^3 + 9*B*a^2*b + 3*(3*A - 7*
C)*a*b^2 - 7*B*b^3)*c^4*d^5 - 5*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A*b^
3)*c^3*d^6 + ((7*A - 3*C)*a^3 - 9*B*a^2*b - 9*A*a*b^2)*c^2*d^7 + (B*a^3 + 3
*A*a^2*b)*c*d^8 - 2*(C*b^3*c^6*d^3 + 3*C*b^3*c^4*d^5 + 3*C*b^3*c^2*d^7 + C*
b^3*d^9)*tan(f*x + e)^3 - 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B
*b^3)*c^5*d^4 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^4*d
^5 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^3*d^6 - (B*a^3
 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2*d^7)*f*x - (9*C*b^3*c^7*d
^2 - A*a^3*d^9 - 3*(3*C*a*b^2 + B*b^3)*c^6*d^3 + (3*C*a^2*b + 3*B*a*b^2 + (
A + 23*C)*b^3)*c^5*d^4 + (C*a^3 + 3*B*a^2*b + 3*(A - 9*C)*a*b^2 - 9*B*b^3)*
c^4*d^5 - (3*B*a^3 + 3*(3*A - 7*C)*a^2*b - 21*B*a*b^2 - (7*A + 12*C)*b^3)*c
^3*d^6 + 5*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c^2*d^7 + (3*B*a^3 + 9*A*a
^2*b + 4*C*b^3)*c*d^8 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b
^3)*c^3*d^6 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2*d^7
 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^8 - (B*a^3 + 3
*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^9)*f*x)*tan(f*x + e)^2 + (3*C*b
^3*c^9 + 9*C*b^3*c^7*d^2 - (3*C*a*b^2 + B*b^3)*c^8*d - 3*(3*C*a*b^2 + B*b^3
)*c^6*d^3 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - 10*C)*b^3)*c^5*d^4
- 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - 2*C)*a*b^2 + 2*B*b^3)*c^4*d^5 - 3*(B*
a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A*b^3)*c^3*d^6 + ((A - C)*a^3 - 3*B*a^2
*b - 3*A*a*b^2)*c^2*d^7 + (3*C*b^3*c^7*d^2 + 9*C*b^3*c^5*d^4 - (3*C*a*b^2 +
 B*b^3)*c^6*d^3 - 3*(3*C*a*b^2 + B*b^3)*c^4*d^5 + (B*a^3 + 3*(A - C)*a^2*b
- 3*B*a*b^2 - (A - 10*C)*b^3)*c^3*d^6 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A -
 2*C)*a*b^2 + 2*B*b^3)*c^2*d^7 - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A
*b^3)*c*d^8 + ((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*d^9)*tan(f*x + e)^2 + 2
*(3*C*b^3*c^8*d + 9*C*b^3*c^6*d^3 - (3*C*a*b^2 + B*b^3)*c^7*d^2 - 3*(3*C*a*
b^2 + B*b^3)*c^5*d^4 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - 10*C)*b^
3)*c^4*d^5 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - 2*C)*a*b^2 + 2*B*b^3)*c^3*
d^6 - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A*b^3)*c^2*d^7 + ((A - C)*a^
```

```
3 - 3*B*a^2*b - 3*A*a*b^2)*c*d^8)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2
*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (3*C*b^3*c^9 + 9*C*b^3*c^7
*d^2 + 9*C*b^3*c^5*d^4 + 3*C*b^3*c^3*d^6 - (3*C*a*b^2 + B*b^3)*c^8*d - 3*(3
*C*a*b^2 + B*b^3)*c^6*d^3 - 3*(3*C*a*b^2 + B*b^3)*c^4*d^5 - (3*C*a*b^2 + B*
b^3)*c^2*d^7 + (3*C*b^3*c^7*d^2 + 9*C*b^3*c^5*d^4 + 9*C*b^3*c^3*d^6 + 3*C*b
^3*c*d^8 - (3*C*a*b^2 + B*b^3)*c^6*d^3 - 3*(3*C*a*b^2 + B*b^3)*c^4*d^5 - 3*
(3*C*a*b^2 + B*b^3)*c^2*d^7 - (3*C*a*b^2 + B*b^3)*d^9)*tan(f*x + e)^2 + 2*(
3*C*b^3*c^8*d + 9*C*b^3*c^6*d^3 + 9*C*b^3*c^4*d^5 + 3*C*b^3*c^2*d^7 - (3*C*
a*b^2 + B*b^3)*c^7*d^2 - 3*(3*C*a*b^2 + B*b^3)*c^5*d^4 - 3*(3*C*a*b^2 + B*b
^3)*c^3*d^6 - (3*C*a*b^2 + B*b^3)*c*d^8)*tan(f*x + e))*log(1/(tan(f*x + e)^
2 + 1)) - 2*(3*C*b^3*c^8*d + 6*C*b^3*c^6*d^3 - (3*C*a*b^2 + B*b^3)*c^7*d^2
+ (C*a^3 + 3*B*a^2*b + 3*(A - 3*C)*a*b^2 - 3*B*b^3)*c^5*d^4 - (2*B*a^3 + 3*
(2*A - 3*C)*a^2*b - 9*B*a*b^2 - (3*A - 2*C)*b^3)*c^4*d^5 + (3*(A - C)*a^3 -
 9*B*a^2*b - 3*(3*A - 4*C)*a*b^2 + 4*B*b^3)*c^3*d^6 + (3*B*a^3 + 9*(A - C)*
a^2*b - 9*B*a*b^2 - (3*A - C)*b^3)*c^2*d^7 - ((3*A - 2*C)*a^3 - 6*B*a^2*b -
 6*A*a*b^2)*c*d^8 - (B*a^3 + 3*A*a^2*b)*d^9 + 2*(((A - C)*a^3 - 3*B*a^2*b -
 3*(A - C)*a*b^2 + B*b^3)*c^4*d^5 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2
- (A - C)*b^3)*c^3*d^6 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b
^3)*c^2*d^7 - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d^8)*f*
x)*tan(f*x + e))/((c^6*d^6 + 3*c^4*d^8 + 3*c^2*d^10 + d^12)*f*tan(f*x + e)^
2 + 2*(c^7*d^5 + 3*c^5*d^7 + 3*c^3*d^9 + c*d^11)*f*tan(f*x + e) + (c^8*d^4
+ 3*c^6*d^6 + 3*c^4*d^8 + c^2*d^10)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2505 vs. 2(804) = 1608.
time = 1.49, size = 2505, normalized size = 3.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*C*b^3*tan(f*x + e)/d^3 + 2*(A*a^3*c^3 - C*a^3*c^3 - 3*B*a^2*b*c^3 -
3*A*a*b^2*c^3 + 3*C*a*b^2*c^3 + B*b^3*c^3 + 3*B*a^3*c^2*d + 9*A*a^2*b*c^2*d
 - 9*C*a^2*b*c^2*d - 9*B*a*b^2*c^2*d - 3*A*b^3*c^2*d + 3*C*b^3*c^2*d - 3*A*
a^3*c*d^2 + 3*C*a^3*c*d^2 + 9*B*a^2*b*c*d^2 + 9*A*a*b^2*c*d^2 - 9*C*a*b^2*c
*d^2 - 3*B*b^3*c*d^2 - B*a^3*d^3 - 3*A*a^2*b*d^3 + 3*C*a^2*b*d^3 + 3*B*a*b^
2*d^3 + A*b^3*d^3 - C*b^3*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6
) + (B*a^3*c^3 + 3*A*a^2*b*c^3 - 3*C*a^2*b*c^3 - 3*B*a*b^2*c^3 - A*b^3*c^3
+ C*b^3*c^3 - 3*A*a^3*c^2*d + 3*C*a^3*c^2*d + 9*B*a^2*b*c^2*d + 9*A*a*b^2*c
^2*d - 9*C*a*b^2*c^2*d - 3*B*b^3*c^2*d - 3*B*a^3*c*d^2 - 9*A*a^2*b*c*d^2 +
9*C*a^2*b*c*d^2 + 9*B*a*b^2*c*d^2 + 3*A*b^3*c*d^2 - 3*C*b^3*c*d^2 + A*a^3*d
^3 - C*a^3*d^3 - 3*B*a^2*b*d^3 - 3*A*a*b^2*d^3 + 3*C*a*b^2*d^3 + B*b^3*d^3)
*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(3*C*b^3*c
^7 - 3*C*a*b^2*c^6*d - B*b^3*c^6*d + 9*C*b^3*c^5*d^2 - 9*C*a*b^2*c^4*d^3 -
3*B*b^3*c^4*d^3 + B*a^3*c^3*d^4 + 3*A*a^2*b*c^3*d^4 - 3*C*a^2*b*c^3*d^4 - 3
*B*a*b^2*c^3*d^4 - A*b^3*c^3*d^4 + 10*C*b^3*c^3*d^4 - 3*A*a^3*c^2*d^5 + 3*C
*a^3*c^2*d^5 + 9*B*a^2*b*c^2*d^5 + 9*A*a*b^2*c^2*d^5 - 18*C*a*b^2*c^2*d^5 -
 6*B*b^3*c^2*d^5 - 3*B*a^3*c*d^6 - 9*A*a^2*b*c*d^6 + 9*C*a^2*b*c*d^6 + 9*B*
a*b^2*c*d^6 + 3*A*b^3*c*d^6 + A*a^3*d^7 - C*a^3*d^7 - 3*B*a^2*b*d^7 - 3*A*a
*b^2*d^7)*log(abs(d*tan(f*x + e) + c))/(c^6*d^4 + 3*c^4*d^6 + 3*c^2*d^8 + d
^10) + (9*C*b^3*c^7*d^2*tan(f*x + e)^2 - 9*C*a*b^2*c^6*d^3*tan(f*x + e)^2 -
 3*B*b^3*c^6*d^3*tan(f*x + e)^2 + 27*C*b^3*c^5*d^4*tan(f*x + e)^2 - 27*C*a*
b^2*c^4*d^5*tan(f*x + e)^2 - 9*B*b^3*c^4*d^5*tan(f*x + e)^2 + 3*B*a^3*c^3*d
^6*tan(f*x + e)^2 + 9*A*a^2*b*c^3*d^6*tan(f*x + e)^2 - 9*C*a^2*b*c^3*d^6*ta
n(f*x + e)^2 - 9*B*a*b^2*c^3*d^6*tan(f*x + e)^2 - 3*A*b^3*c^3*d^6*tan(f*x +
 e)^2 + 30*C*b^3*c^3*d^6*tan(f*x + e)^2 - 9*A*a^3*c^2*d^7*tan(f*x + e)^2 +
9*C*a^3*c^2*d^7*tan(f*x + e)^2 + 27*B*a^2*b*c^2*d^7*tan(f*x + e)^2 + 27*A*a
*b^2*c^2*d^7*tan(f*x + e)^2 - 54*C*a*b^2*c^2*d^7*tan(f*x + e)^2 - 18*B*b^3*
c^2*d^7*tan(f*x + e)^2 - 9*B*a^3*c*d^8*tan(f*x + e)^2 - 27*A*a^2*b*c*d^8*ta
n(f*x + e)^2 + 27*C*a^2*b*c*d^8*tan(f*x + e)^2 + 27*B*a*b^2*c*d^8*tan(f*x +
 e)^2 + 9*A*b^3*c*d^8*tan(f*x + e)^2 + 3*A*a^3*d^9*tan(f*x + e)^2 - 3*C*a^3
*d^9*tan(f*x + e)^2 - 9*B*a^2*b*d^9*tan(f*x + e)^2 - 9*A*a*b^2*d^9*tan(f*x
+ e)^2 + 12*C*b^3*c^8*d*tan(f*x + e) - 6*C*a*b^2*c^7*d^2*tan(f*x + e) - 2*B
*b^3*c^7*d^2*tan(f*x + e) - 6*C*a^2*b*c^6*d^3*tan(f*x + e) - 6*B*a*b^2*c^6*
d^3*tan(f*x + e) - 2*A*b^3*c^6*d^3*tan(f*x + e) + 38*C*b^3*c^6*d^3*tan(f*x
+ e) - 18*C*a*b^2*c^5*d^4*tan(f*x + e) - 6*B*b^3*c^5*d^4*tan(f*x + e) + 8*B
*a^3*c^4*d^5*tan(f*x + e) + 24*A*a^2*b*c^4*d^5*tan(f*x + e) - 42*C*a^2*b*c^
4*d^5*tan(f*x + e) - 42*B*a*b^2*c^4*d^5*tan(f*x + e) - 14*A*b^3*c^4*d^5*tan
(f*x + e) + 50*C*b^3*c^4*d^5*tan(f*x + e) - 22*A*a^3*c^3*d^6*tan(f*x + e) +
 22*C*a^3*c^3*d^6*tan(f*x + e) + 66*B*a^2*b*c^3*d^6*tan(f*x + e) + 66*A*a*b
^2*c^3*d^6*tan(f*x + e) - 84*C*a*b^2*c^3*d^6*tan(f*x + e) - 28*B*b^3*c^3*d^
6*tan(f*x + e) - 18*B*a^3*c^2*d^7*tan(f*x + e) - 54*A*a^2*b*c^2*d^7*tan(f*x
 + e) + 36*C*a^2*b*c^2*d^7*tan(f*x + e) + 36*B*a*b^2*c^2*d^7*tan(f*x + e) +
 12*A*b^3*c^2*d^7*tan(f*x + e) + 2*A*a^3*c*d^8*tan(f*x + e) - 2*C*a^3*c*d^8
*tan(f*x + e) - 6*B*a^2*b*c*d^8*tan(f*x + e) - 6*A*a*b^2*c*d^8*tan(f*x + e)
 - 2*B*a^3*d^9*tan(f*x + e) - 6*A*a^2*b*d^9*tan(f*x + e) + 4*C*b^3*c^9 - 3*
```

```
C*a^2*b*c^7*d^2 - 3*B*a*b^2*c^7*d^2 - A*b^3*c^7*d^2 + 13*C*b^3*c^7*d^2 - C*
a^3*c^6*d^3 - 3*B*a^2*b*c^6*d^3 - 3*A*a*b^2*c^6*d^3 + 3*C*a*b^2*c^6*d^3 + B
*b^3*c^6*d^3 + 6*B*a^3*c^5*d^4 + 18*A*a^2*b*c^5*d^4 - 27*C*a^2*b*c^5*d^4 -
27*B*a*b^2*c^5*d^4 - 9*A*b^3*c^5*d^4 + 21*C*b^3*c^5*d^4 - 14*A*a^3*c^4*d^5
+ 11*C*a^3*c^4*d^5 + 33*B*a^2*b*c^4*d^5 + 33*A*a*b^2*c^4*d^5 - 33*C*a*b^2*c
^4*d^5 - 11*B*b^3*c^4*d^5 - 7*B*a^3*c^3*d^6 - 21*A*a^2*b*c^3*d^6 + 12*C*a^2
*b*c^3*d^6 + 12*B*a*b^2*c^3*d^6 + 4*A*b^3*c^3*d^6 - 3*A*a^3*c^2*d^7 - B*a^3
*c*d^8 - 3*A*a^2*b*c*d^8 - A*a^3*d^9)/((c^6*d^4 + 3*c^4*d^6 + 3*c^2*d^8 + d
^10)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [B]
time = 20.60, size = 1172, normalized size = 1.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^3,x)
```

```
[Out] (log(tan(e + f*x) + 1i)*(A*a^3 + A*b^3*1i - B*a^3*1i + B*b^3 - C*a^3 - C*b^
3*1i - 3*A*a*b^2 - A*a^2*b*3i + B*a*b^2*3i - 3*B*a^2*b + 3*C*a*b^2 + C*a^2*
b*3i))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3)) - ((A*a^3*d^7 + 5*C*b^3*c^
7 + B*a^3*c*d^6 - 3*B*b^3*c^6*d + 5*A*a^3*c^2*d^5 + 5*A*b^3*c^3*d^4 + A*b^3
*c^5*d^2 - 3*B*a^3*c^3*d^4 - 7*B*b^3*c^4*d^3 - 3*C*a^3*c^2*d^5 + C*a^3*c^4*
d^3 + 9*C*b^3*c^5*d^2 - 9*A*a*b^2*c^2*d^5 + 3*A*a*b^2*c^4*d^3 - 9*A*a^2*b*c
^3*d^4 + 15*B*a*b^2*c^3*d^4 + 3*B*a*b^2*c^5*d^2 - 9*B*a^2*b*c^2*d^5 + 3*B*a
^2*b*c^4*d^3 - 21*C*a*b^2*c^4*d^3 + 15*C*a^2*b*c^3*d^4 + 3*C*a^2*b*c^5*d^2
+ 3*A*a^2*b*c*d^6 - 9*C*a*b^2*c^6*d)/(2*d*(c^4 + d^4 + 2*c^2*d^2)) + (tan(e
 + f*x)*(B*a^3*d^6 + 3*C*b^3*c^6 + 3*A*a^2*b*d^6 + 2*A*a^3*c*d^5 - 2*B*b^3*
c^5*d - 2*C*a^3*c*d^5 + 3*A*b^3*c^2*d^4 + A*b^3*c^4*d^2 - B*a^3*c^2*d^4 - 4
*B*b^3*c^3*d^3 + 5*C*b^3*c^4*d^2 - 3*A*a^2*b*c^2*d^4 + 9*B*a*b^2*c^2*d^4 +
3*B*a*b^2*c^4*d^2 - 12*C*a*b^2*c^3*d^3 + 9*C*a^2*b*c^2*d^4 + 3*C*a^2*b*c^4*
d^2 - 6*A*a*b^2*c*d^5 - 6*B*a^2*b*c*d^5 - 6*C*a*b^2*c^5*d))/(c^4 + d^4 + 2*
c^2*d^2))/(f*(c^2*d^3 + d^5*tan(e + f*x)^2 + 2*c*d^4*tan(e + f*x))) + (log(
c + d*tan(e + f*x))*(d^3*(3*B*b^3*c^4 + 9*C*a*b^2*c^4) - d^6*(3*A*b^3*c - 3
*B*a^3*c - 9*A*a^2*b*c + 9*B*a*b^2*c + 9*C*a^2*b*c) + d^5*(3*A*a^3*c^2 + 6*
B*b^3*c^2 - 3*C*a^3*c^2 - 9*A*a*b^2*c^2 - 9*B*a^2*b*c^2 + 18*C*a*b^2*c^2) +
 d^4*(A*b^3*c^3 - B*a^3*c^3 - 10*C*b^3*c^3 - 3*A*a^2*b*c^3 + 3*B*a*b^2*c^3
+ 3*C*a^2*b*c^3) + d^7*(C*a^3 - A*a^3 + 3*A*a*b^2 + 3*B*a^2*b) + d*(B*b^3*c
^6 + 3*C*a*b^2*c^6) - 3*C*b^3*c^7 - 9*C*b^3*c^5*d^2))/(f*(d^10 + 3*c^2*d^8
+ 3*c^4*d^6 + c^6*d^4)) + (log(tan(e + f*x) - 1i)*(A*a^3*1i + A*b^3 - B*a^3
 + B*b^3*1i - C*a^3*1i - C*b^3 - A*a*b^2*3i - 3*A*a^2*b + 3*B*a*b^2 - B*a^2
*b*3i + C*a*b^2*3i + 3*C*a^2*b))/(2*f*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i))
+ (C*b^3*tan(e + f*x))/(d^3*f)
```

3.85 $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=597

$$-\frac{(b^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)+a^2(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))-2a}{(c^2+d^2)^3}$$

```
[Out] -(b^2*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)+a^2*(c^3*C-3*B*c^2*
d-3*c*C*d^2+B*d^3-A*(c^3-3*c*d^2))-2*a*b*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^
2)))*x/(c^2+d^2)^3-(2*a*b*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)
-a^2*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2))+b^2*((A-C)*d*(3*c^2-d^2)-B*(c^3-
3*c*d^2)))*ln(cos(f*x+e))/(c^2+d^2)^3/f-(2*a*b*d^3*(A*c^3-3*A*c*d^2+3*B*c^2
*d-B*d^3-C*c^3+3*C*c*d^2)-b^2*(c^6*C+3*c^4*C*d^2+B*c^3*d^3-3*c^2*(A-2*C)*d^
4-3*B*c*d^5+A*d^6)-a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2)))*ln(c+d*ta
n(f*x+e))/d^3/(c^2+d^2)^3/f-1/2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^2/d/(c
^2+d^2)/f/(c+d*tan(f*x+e))^2+(-a*d+b*c)*(b*(c^4*C-c^2*(A-3*C)*d^2-2*B*c*d^3
+A*d^4)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/d^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 0.95, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3726, 3716, 3707, 3698, 31, 3556}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^3,x]
```

```
[Out] -(((b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a^2*(
c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)
*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3) - ((2*a*b*(A*c^3 -
 c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - a^2*((A - C)*d*(3*c^2
 - d^2) - B*(c^3 - 3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*
d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)^3*f) - ((2*a*b*d^3*(A*c^3 - c^3*C +
3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - b^2*(c^6*C + 3*c^4*C*d^2 + B*c
^3*d^3 - 3*c^2*(A - 2*C)*d^4 - 3*B*c*d^5 + A*d^6) - a^2*d^3*((A - C)*d*(3*c
^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)^3
*f) - ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^2)/(2*d*(c^2 + d^2)*f*(
c + d*Tan[e + f*x])^2) + ((b*c - a*d)*(b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c
*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/(d^3*(c^2 + d^2)^2*
f*(c + d*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx = -\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2}$$

$$= -\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2}$$

$$= \frac{(a^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cC}{}$$

$$= \frac{(a^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cC}{}$$

$$= \frac{(a^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cC}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.49, size = 2499, normalized size = 4.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^3,x]
```

```
[Out] ((-(b^2*c^4*C) + b^2*B*c^3*d + 2*a*b*c^3*C*d - A*b^2*c^2*d^2 - 2*a*b*B*c^2*
d^2 - a^2*c^2*C*d^2 + 2*a*A*b*c*d^3 + a^2*B*c*d^3 - a^2*A*d^4)*Sec[e + f*x]
*(c*Cos[e + f*x] + d*Sin[e + f*x])*(a + b*Tan[e + f*x])^2)/(2*(c - I*d)^2*(
c + I*d)^2*d*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)
+ ((a^2*A*c^3 - A*b^2*c^3 - 2*a*b*B*c^3 - a^2*c^3*C + b^2*c^3*C + 6*a*A*b*c
^2*d + 3*a^2*B*c^2*d - 3*b^2*B*c^2*d - 6*a*b*c^2*C*d - 3*a^2*A*c*d^2 + 3*A*
b^2*c*d^2 + 6*a*b*B*c*d^2 + 3*a^2*c*C*d^2 - 3*b^2*c*C*d^2 - 2*a*A*b*d^3 - a
^2*B*d^3 + b^2*B*d^3 + 2*a*b*C*d^3)*(e + f*x)*Sec[e + f*x]*(c*Cos[e + f*x]
+ d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)/((c - I*d)^3*(c + I*d)^3*f*(a*C
os[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3) + ((I*b^2*c^13*C*d^
2 + b^2*c^12*C*d^3 + (5*I)*b^2*c^11*C*d^4 - (2*I)*a*A*b*c^10*d^5 - I*a^2*B*
c^10*d^5 + I*b^2*B*c^10*d^5 + (2*I)*a*b*c^10*C*d^5 + 5*b^2*c^10*C*d^5 + (3*
I)*a^2*A*c^9*d^6 - 2*a*A*b*c^9*d^6 - (3*I)*A*b^2*c^9*d^6 - a^2*B*c^9*d^6 -
(6*I)*a*b*B*c^9*d^6 + b^2*B*c^9*d^6 - (3*I)*a^2*c^9*C*d^6 + 2*a*b*c^9*C*d^6
```

```
 + (13*I)*b^2*c^9*C*d^6 + 3*a^2*A*c^8*d^7 + (2*I)*a*A*b*c^8*d^7 - 3*A*b^2*c
^8*d^7 + I*a^2*B*c^8*d^7 - 6*a*b*B*c^8*d^7 - I*b^2*B*c^8*d^7 - 3*a^2*c^8*C*
d^7 - (2*I)*a*b*c^8*C*d^7 + 13*b^2*c^8*C*d^7 + (5*I)*a^2*A*c^7*d^8 + 2*a*A*
b*c^7*d^8 - (5*I)*A*b^2*c^7*d^8 + a^2*B*c^7*d^8 - (10*I)*a*b*B*c^7*d^8 - b^
2*B*c^7*d^8 - (5*I)*a^2*c^7*C*d^8 - 2*a*b*c^7*C*d^8 + (15*I)*b^2*c^7*C*d^8
+ 5*a^2*A*c^6*d^9 + (10*I)*a*A*b*c^6*d^9 - 5*A*b^2*c^6*d^9 + (5*I)*a^2*B*c^
6*d^9 - 10*a*b*B*c^6*d^9 - (5*I)*b^2*B*c^6*d^9 - 5*a^2*c^6*C*d^9 - (10*I)*a
*b*c^6*C*d^9 + 15*b^2*c^6*C*d^9 + I*a^2*A*c^5*d^10 + 10*a*A*b*c^5*d^10 - I*
A*b^2*c^5*d^10 + 5*a^2*B*c^5*d^10 - (2*I)*a*b*B*c^5*d^10 - 5*b^2*B*c^5*d^10
 - I*a^2*c^5*C*d^10 - 10*a*b*c^5*C*d^10 + (6*I)*b^2*c^5*C*d^10 + a^2*A*c^4*
d^11 + (6*I)*a*A*b*c^4*d^11 - A*b^2*c^4*d^11 + (3*I)*a^2*B*c^4*d^11 - 2*a*b
*B*c^4*d^11 - (3*I)*b^2*B*c^4*d^11 - a^2*c^4*C*d^11 - (6*I)*a*b*c^4*C*d^11
+ 6*b^2*c^4*C*d^11 - I*a^2*A*c^3*d^12 + 6*a*A*b*c^3*d^12 + I*A*b^2*c^3*d^12
 + 3*a^2*B*c^3*d^12 + (2*I)*a*b*B*c^3*d^12 - 3*b^2*B*c^3*d^12 + I*a^2*c^3*C
*d^12 - 6*a*b*c^3*C*d^12 - a^2*A*c^2*d^13 + A*b^2*c^2*d^13 + 2*a*b*B*c^2*d^
13 + a^2*c^2*C*d^13)*(e + f*x)*Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x
])^3*(a + b*Tan[e + f*x])^2)/(c^2*(c - I*d)^6*(c + I*d)^5*d^5*f*(a*Cos[e +
f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3) - (I*(b^2*c^6*C + 3*b^2*c^
4*C*d^2 - 2*a*A*b*c^3*d^3 - a^2*B*c^3*d^3 + b^2*B*c^3*d^3 + 2*a*b*c^3*C*d^3
 + 3*a^2*A*c^2*d^4 - 3*A*b^2*c^2*d^4 - 6*a*b*B*c^2*d^4 - 3*a^2*c^2*C*d^4 +
6*b^2*c^2*C*d^4 + 6*a*A*b*c*d^5 + 3*a^2*B*c*d^5 - 3*b^2*B*c*d^5 - 6*a*b*c*C
*d^5 - a^2*A*d^6 + A*b^2*d^6 + 2*a*b*B*d^6 + a^2*C*d^6)*ArcTan[Tan[e + f*x]
]*Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)/
(d^3*(c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x
])^3) - (b^2*C*Log[Cos[e + f*x]]*Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f
*x])^3*(a + b*Tan[e + f*x])^2)/(d^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(
c + d*Tan[e + f*x])^3) + ((b^2*c^6*C + 3*b^2*c^4*C*d^2 - 2*a*A*b*c^3*d^3 -
a^2*B*c^3*d^3 + b^2*B*c^3*d^3 + 2*a*b*c^3*C*d^3 + 3*a^2*A*c^2*d^4 - 3*A*b^2
*c^2*d^4 - 6*a*b*B*c^2*d^4 - 3*a^2*c^2*C*d^4 + 6*b^2*c^2*C*d^4 + 6*a*A*b*c*
d^5 + 3*a^2*B*c*d^5 - 3*b^2*B*c*d^5 - 6*a*b*c*C*d^5 - a^2*A*d^6 + A*b^2*d^6
 + 2*a*b*B*d^6 + a^2*C*d^6)*Log[(c*Cos[e + f*x] + d*Sin[e + f*x])^2]*Sec[e
+ f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^3*(a + b*Tan[e + f*x])^2)/(2*d^3*(
c^2 + d^2)^3*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e + f*x])^3)
+ (Sec[e + f*x]*(c*Cos[e + f*x] + d*Sin[e + f*x])^2*(-(b^2*c^5*C*Sin[e + f*
x]) + A*b^2*c^3*d^2*Sin[e + f*x] + 2*a*b*B*c^3*d^2*Sin[e + f*x] + a^2*c^3*C
*d^2*Sin[e + f*x] - 4*b^2*c^3*C*d^2*Sin[e + f*x] - 4*a*A*b*c^2*d^3*Sin[e +
f*x] - 2*a^2*B*c^2*d^3*Sin[e + f*x] + 3*b^2*B*c^2*d^3*Sin[e + f*x] + 6*a*b*
c^2*C*d^3*Sin[e + f*x] + 3*a^2*A*c*d^4*Sin[e + f*x] - 2*A*b^2*c*d^4*Sin[e +
 f*x] - 4*a*b*B*c*d^4*Sin[e + f*x] - 2*a^2*c*C*d^4*Sin[e + f*x] + 2*a*A*b*d
^5*Sin[e + f*x] + a^2*B*d^5*Sin[e + f*x])*(a + b*Tan[e + f*x])^2)/(c*(c - I
*d)^2*(c + I*d)^2*d^2*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(c + d*Tan[e +
f*x])^3)
```

Maple [A]
time = 0.59, size = 865, normalized size = 1.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*(A*a^2*d^4-2*A*a*b*c*d^3+A*b^2*c^2*d^2-B*a^2*c*d^3+2*B*a*b*c^2*d^
2-B*b^2*c^3*d+C*a^2*c^2*d^2-2*C*a*b*c^3*d+C*b^2*c^4)/d^3/(c^2+d^2)/(c+d*tan
(f*x+e))^2-(2*A*a^2*c*d^4-2*A*a*b*c^2*d^3+2*A*a*b*d^5-2*A*b^2*c*d^4-B*a^2*c
^2*d^3+B*a^2*d^5-4*B*a*b*c*d^4+B*b^2*c^4*d+3*B*b^2*c^2*d^3-2*C*a^2*c*d^4+2*
C*a*b*c^4*d+6*C*a*b*c^2*d^3-2*C*b^2*c^5-4*C*b^2*c^3*d^2)/d^3/(c^2+d^2)^2/(c
+d*tan(f*x+e))+(3*A*a^2*c^2*d^4-A*a^2*d^6-2*A*a*b*c^3*d^3+6*A*a*b*c*d^5-3*A
*b^2*c^2*d^4+A*b^2*d^6-B*a^2*c^3*d^3+3*B*a^2*c*d^5-6*B*a*b*c^2*d^4+2*B*a*b*
d^6+B*b^2*c^3*d^3-3*B*b^2*c*d^5-3*C*a^2*c^2*d^4+C*a^2*d^6+2*C*a*b*c^3*d^3-6
*C*a*b*c*d^5+C*b^2*c^6+3*C*b^2*c^4*d^2+6*C*b^2*c^2*d^4)/(c^2+d^2)^3/d^3*ln(
c+d*tan(f*x+e))+1/(c^2+d^2)^3*(1/2*(-3*A*a^2*c^2*d+A*a^2*d^3+2*A*a*b*c^3-6*
A*a*b*c*d^2+3*A*b^2*c^2*d-A*b^2*d^3+B*a^2*c^3-3*B*a^2*c*d^2+6*B*a*b*c^2*d-2
*B*a*b*d^3-B*b^2*c^3+3*B*b^2*c*d^2+3*C*a^2*c^2*d-C*a^2*d^3-2*C*a*b*c^3+6*C*
a*b*c*d^2-3*C*b^2*c^2*d+C*b^2*d^3)*ln(1+tan(f*x+e)^2)+(A*a^2*c^3-3*A*a^2*c*
d^2+6*A*a*b*c^2*d-2*A*a*b*d^3-A*b^2*c^3+3*A*b^2*c*d^2+3*B*a^2*c^2*d-B*a^2*d
^3-2*B*a*b*c^3+6*B*a*b*c*d^2-3*B*b^2*c^2*d+B*b^2*d^3-C*a^2*c^3+3*C*a^2*c*d^
2-6*C*a*b*c^2*d+2*C*a*b*d^3+C*b^2*c^3-3*C*b^2*c*d^2)*arctan(tan(f*x+e))))
```

Maxima [A]
time = 0.55, size = 833, normalized size = 1.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a*
b - B*b^2)*c^2*d - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 +
 2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6)
+ 2*(C*b^2*c^6 + 3*C*b^2*c^4*d^2 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^3
+ 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^2*d^4 + 3*(B*a^2 + 2*(A - C)*
a*b - B*b^2)*c*d^5 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*d^6)*log(d*tan(f*x + e
) + c)/(c^6*d^3 + 3*c^4*d^5 + 3*c^2*d^7 + d^9) + ((B*a^2 + 2*(A - C)*a*b -
B*b^2)*c^3 - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 + 2*(
A - C)*a*b - B*b^2)*c*d^2 + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*log(
tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (3*C*b^2*c^6 - A*
a^2*d^6 - (2*C*a*b + B*b^2)*c^5*d - (C*a^2 + 2*B*a*b + (A - 7*C)*b^2)*c^4*d
^2 + (3*B*a^2 + 2*(3*A - 5*C)*a*b - 5*B*b^2)*c^3*d^3 - ((5*A - 3*C)*a^2 - 6
*B*a*b - 3*A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b)*c*d^5 + 2*(2*C*b^2*c^5*d + 4*
C*b^2*c^3*d^3 - (2*C*a*b + B*b^2)*c^4*d^2 + (B*a^2 + 2*(A - 3*C)*a*b - 3*B*
b^2)*c^2*d^4 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^5 - (B*a^2 + 2*A*a*b)*
```

```
d^6)*tan(f*x + e))/(c^6*d^3 + 2*c^4*d^5 + c^2*d^7 + (c^4*d^5 + 2*c^2*d^7 +
d^9)*tan(f*x + e)^2 + 2*(c^5*d^4 + 2*c^3*d^6 + c*d^8)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1630 vs. $2(596) = 1192$.
time = 3.37, size = 1630, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*b^2*c^6*d^2 - A*a^2*d^8 + (2*C*a*b + B*b^2)*c^5*d^3 - (3*C*a^2 + 6*B
*a*b + (3*A - 7*C)*b^2)*c^4*d^4 + 5*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^5
 - ((7*A - 3*C)*a^2 - 6*B*a*b - 3*A*b^2)*c^2*d^6 - (B*a^2 + 2*A*a*b)*c*d^7
+ 2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^5*d^3 + 3*(B*a^2 + 2*(A - C)*a
*b - B*b^2)*c^4*d^4 - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^3*d^5 - (B*
a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^6)*f*x - (3*C*b^2*c^6*d^2 + A*a^2*d^8 -
(2*C*a*b + B*b^2)*c^5*d^3 - (C*a^2 + 2*B*a*b + (A - 9*C)*b^2)*c^4*d^4 + (3*
B*a^2 + 2*(3*A - 7*C)*a*b - 7*B*b^2)*c^3*d^5 - 5*((A - C)*a^2 - 2*B*a*b - A
*b^2)*c^2*d^6 - 3*(B*a^2 + 2*A*a*b)*c*d^7 - 2*(((A - C)*a^2 - 2*B*a*b - (A
- C)*b^2)*c^3*d^5 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^6 - 3*((A - C)*
a^2 - 2*B*a*b - (A - C)*b^2)*c*d^7 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^8)*f
*x)*tan(f*x + e)^2 + (C*b^2*c^8 + 3*C*b^2*c^6*d^2 - (B*a^2 + 2*(A - C)*a*b
- B*b^2)*c^5*d^3 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^4*d^4 + 3*(B
*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^5 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*c^2
*d^6 + (C*b^2*c^6*d^2 + 3*C*b^2*c^4*d^4 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c
^3*d^5 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^2*d^6 + 3*(B*a^2 + 2*(
A - C)*a*b - B*b^2)*c*d^7 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*d^8)*tan(f*x +
e)^2 + 2*(C*b^2*c^7*d + 3*C*b^2*c^5*d^3 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c
^4*d^4 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^3*d^5 + 3*(B*a^2 + 2*(
A - C)*a*b - B*b^2)*c^2*d^6 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^7)*tan(f*
x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2
 + 1)) - (C*b^2*c^8 + 3*C*b^2*c^6*d^2 + 3*C*b^2*c^4*d^4 + C*b^2*c^2*d^6 + (
C*b^2*c^6*d^2 + 3*C*b^2*c^4*d^4 + 3*C*b^2*c^2*d^6 + C*b^2*d^8)*tan(f*x + e)
^2 + 2*(C*b^2*c^7*d + 3*C*b^2*c^5*d^3 + 3*C*b^2*c^3*d^5 + C*b^2*c*d^7)*tan(
f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - 2*(C*b^2*c^7*d - (C*a^2 + 2*B*a*b +
 (A - 3*C)*b^2)*c^5*d^3 + (2*B*a^2 + 2*(2*A - 3*C)*a*b - 3*B*b^2)*c^4*d^4 -
 (3*(A - C)*a^2 - 6*B*a*b - (3*A - 4*C)*b^2)*c^3*d^5 - 3*(B*a^2 + 2*(A - C)
*a*b - B*b^2)*c^2*d^6 + ((3*A - 2*C)*a^2 - 4*B*a*b - 2*A*b^2)*c*d^7 + (B*a^
2 + 2*A*a*b)*d^8 - 2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^4*d^4 + 3*(B*
a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^5 - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b
^2)*c^2*d^6 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^7)*f*x)*tan(f*x + e))/((c
^6*d^5 + 3*c^4*d^7 + 3*c^2*d^9 + d^11)*f*tan(f*x + e)^2 + 2*(c^7*d^4 + 3*c^
```

```
5*d^6 + 3*c^3*d^8 + c*d^10)*f*tan(f*x + e) + (c^8*d^3 + 3*c^6*d^5 + 3*c^4*d
^7 + c^2*d^9)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1709 vs. 2(596) = 1192.
time = 1.14, size = 1709, normalized size = 2.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c^3 - C*a^2*c^3 - 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 + 3*B*a
^2*c^2*d + 6*A*a*b*c^2*d - 6*C*a*b*c^2*d - 3*B*b^2*c^2*d - 3*A*a^2*c*d^2 +
3*C*a^2*c*d^2 + 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 - B*a^2*d^3 -
 2*A*a*b*d^3 + 2*C*a*b*d^3 + B*b^2*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*
d^4 + d^6) + (B*a^2*c^3 + 2*A*a*b*c^3 - 2*C*a*b*c^3 - B*b^2*c^3 - 3*A*a^2*c
^2*d + 3*C*a^2*c^2*d + 6*B*a*b*c^2*d + 3*A*b^2*c^2*d - 3*C*b^2*c^2*d - 3*B*
a^2*c*d^2 - 6*A*a*b*c*d^2 + 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 + A*a^2*d^3 - C*a
^2*d^3 - 2*B*a*b*d^3 - A*b^2*d^3 + C*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(c^6
+ 3*c^4*d^2 + 3*c^2*d^4 + d^6) + 2*(C*b^2*c^6 + 3*C*b^2*c^4*d^2 - B*a^2*c^3
*d^3 - 2*A*a*b*c^3*d^3 + 2*C*a*b*c^3*d^3 + B*b^2*c^3*d^3 + 3*A*a^2*c^2*d^4
- 3*C*a^2*c^2*d^4 - 6*B*a*b*c^2*d^4 - 3*A*b^2*c^2*d^4 + 6*C*b^2*c^2*d^4 + 3
*B*a^2*c*d^5 + 6*A*a*b*c*d^5 - 6*C*a*b*c*d^5 - 3*B*b^2*c*d^5 - A*a^2*d^6 +
C*a^2*d^6 + 2*B*a*b*d^6 + A*b^2*d^6)*log(abs(d*tan(f*x + e) + c))/(c^6*d^3
+ 3*c^4*d^5 + 3*c^2*d^7 + d^9) - (3*C*b^2*c^6*d*tan(f*x + e)^2 + 9*C*b^2*c^
4*d^3*tan(f*x + e)^2 - 3*B*a^2*c^3*d^4*tan(f*x + e)^2 - 6*A*a*b*c^3*d^4*tan
(f*x + e)^2 + 6*C*a*b*c^3*d^4*tan(f*x + e)^2 + 3*B*b^2*c^3*d^4*tan(f*x + e)
^2 + 9*A*a^2*c^2*d^5*tan(f*x + e)^2 - 9*C*a^2*c^2*d^5*tan(f*x + e)^2 - 18*B
*a*b*c^2*d^5*tan(f*x + e)^2 - 9*A*b^2*c^2*d^5*tan(f*x + e)^2 + 18*C*b^2*c^2
*d^5*tan(f*x + e)^2 + 9*B*a^2*c*d^6*tan(f*x + e)^2 + 18*A*a*b*c*d^6*tan(f*x
 + e)^2 - 18*C*a*b*c*d^6*tan(f*x + e)^2 - 9*B*b^2*c*d^6*tan(f*x + e)^2 - 3*
A*a^2*d^7*tan(f*x + e)^2 + 3*C*a^2*d^7*tan(f*x + e)^2 + 6*B*a*b*d^7*tan(f*x
```

```
 + e)^2 + 3*A*b^2*d^7*tan(f*x + e)^2 + 2*C*b^2*c^7*tan(f*x + e) + 4*C*a*b*c
^6*d*tan(f*x + e) + 2*B*b^2*c^6*d*tan(f*x + e) + 6*C*b^2*c^5*d^2*tan(f*x +
e) - 8*B*a^2*c^4*d^3*tan(f*x + e) - 16*A*a*b*c^4*d^3*tan(f*x + e) + 28*C*a*
b*c^4*d^3*tan(f*x + e) + 14*B*b^2*c^4*d^3*tan(f*x + e) + 22*A*a^2*c^3*d^4*t
an(f*x + e) - 22*C*a^2*c^3*d^4*tan(f*x + e) - 44*B*a*b*c^3*d^4*tan(f*x + e)
 - 22*A*b^2*c^3*d^4*tan(f*x + e) + 28*C*b^2*c^3*d^4*tan(f*x + e) + 18*B*a^2
*c^2*d^5*tan(f*x + e) + 36*A*a*b*c^2*d^5*tan(f*x + e) - 24*C*a*b*c^2*d^5*ta
n(f*x + e) - 12*B*b^2*c^2*d^5*tan(f*x + e) - 2*A*a^2*c*d^6*tan(f*x + e) + 2
*C*a^2*c*d^6*tan(f*x + e) + 4*B*a*b*c*d^6*tan(f*x + e) + 2*A*b^2*c*d^6*tan(
f*x + e) + 2*B*a^2*d^7*tan(f*x + e) + 4*A*a*b*d^7*tan(f*x + e) + 2*C*a*b*c^
7 + B*b^2*c^7 + C*a^2*c^6*d + 2*B*a*b*c^6*d + A*b^2*c^6*d - C*b^2*c^6*d - 6
*B*a^2*c^5*d^2 - 12*A*a*b*c^5*d^2 + 18*C*a*b*c^5*d^2 + 9*B*b^2*c^5*d^2 + 14
*A*a^2*c^4*d^3 - 11*C*a^2*c^4*d^3 - 22*B*a*b*c^4*d^3 - 11*A*b^2*c^4*d^3 + 1
1*C*b^2*c^4*d^3 + 7*B*a^2*c^3*d^4 + 14*A*a*b*c^3*d^4 - 8*C*a*b*c^3*d^4 - 4*
B*b^2*c^3*d^4 + 3*A*a^2*c^2*d^5 + B*a^2*c*d^6 + 2*A*a*b*c*d^6 + A*a^2*d^7)/
((c^6*d^2 + 3*c^4*d^4 + 3*c^2*d^6 + d^8)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [B]
time = 30.69, size = 807, normalized size = 1.35

$$-\frac{\frac{Aa^2d^6-3Cb^2c^6+Ba^2cd^5+Bb^2c^5d+5Aa^2c^2d^4-3Ab^2c^2d^4+Ab^2c^4d^2-3Ba^2c^3d^3+5Bb^2c^3d^3-3Ca^2c^2d^4+Ca^2c^4d^2-7Cb^2c^4d^2+2Aabcd^5+2Cabc^5d-6Aabc^3d^3-6Babc^2d^4+2Babc^4d^2+10Cabc^3d^3}{2d^3(c^4+2c^2d^2+d^4)}+\frac{\tan(e+fx)(Ba^2d^5-2Cb^2c^5+2Aabd^5+2Aa^2cd^4-2Ab^2cd^4+Bb^2c^4d-2Ca^2cd^4-Ba^2c^2d^3+3Bb^2c^2d^3-4Cb^2c^3d^2-4Babcd^4+2Cabc^4d-2Aabc^2d^3+6Cabc^2d^3)}{d^2(c^4+2c^2d^2+d^4)}}{f\left(c^2+2cd\tan(e+fx)+d^2\tan(e+fx)^2\right)}-\frac{\ln(c+d\tan(e+fx))\left(\frac{c^2(d^4(3Ab^2-3Aa^2+3Ca^2-6Cb^2+6Bab)+3Cb^2d^4)-d^6(Ab^2-Aa^2+Ca^2+2Bab)+Cb^2d^6-cd^5(3Ba^2-3Bb^2+6Aab-6Cab)+c^3d^3(Ba^2-Bb^2+2Aab-2Cab)}{c^6d^3+3c^4d^5+3c^2d^7+d^9}-\frac{Cb^2}{d^3}\right)}{f}-\frac{\ln(\tan(e+fx)-\mathrm{i})(Ba^2-Bb^2+2Aab-2Cab+Ab^2\mathrm{i}-Aa^2\mathrm{i}+Ca^2\mathrm{i}-Cb^2\mathrm{i}+Bab\,2\mathrm{i})}{2f(-c^3-c^2d\,3\mathrm{i}+3cd^2+d^3\mathrm{i})}-\frac{\ln(\tan(e+fx)+\mathrm{i})(Ab^2-Aa^2+Ca^2-Cb^2+2Bab+Ba^2\mathrm{i}-Bb^2\mathrm{i}+Aab\,2\mathrm{i}-Cab\,2\mathrm{i})}{2f(-c^3\mathrm{i}-3c^2d+cd^2\,3\mathrm{i}+d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^3,x)
```

```
[Out] - ((A*a^2*d^6 - 3*C*b^2*c^6 + B*a^2*c*d^5 + B*b^2*c^5*d + 5*A*a^2*c^2*d^4 -
 3*A*b^2*c^2*d^4 + A*b^2*c^4*d^2 - 3*B*a^2*c^3*d^3 + 5*B*b^2*c^3*d^3 - 3*C*
a^2*c^2*d^4 + C*a^2*c^4*d^2 - 7*C*b^2*c^4*d^2 + 2*A*a*b*c*d^5 + 2*C*a*b*c^5
*d - 6*A*a*b*c^3*d^3 - 6*B*a*b*c^2*d^4 + 2*B*a*b*c^4*d^2 + 10*C*a*b*c^3*d^3
)/(2*d^3*(c^4 + d^4 + 2*c^2*d^2)) + (tan(e + f*x)*(B*a^2*d^5 - 2*C*b^2*c^5
+ 2*A*a*b*d^5 + 2*A*a^2*c*d^4 - 2*A*b^2*c*d^4 + B*b^2*c^4*d - 2*C*a^2*c*d^4
 - B*a^2*c^2*d^3 + 3*B*b^2*c^2*d^3 - 4*C*b^2*c^3*d^2 - 4*B*a*b*c*d^4 + 2*C*
a*b*c^4*d - 2*A*a*b*c^2*d^3 + 6*C*a*b*c^2*d^3))/(d^2*(c^4 + d^4 + 2*c^2*d^2
)))/(f*(c^2 + d^2*tan(e + f*x)^2 + 2*c*d*tan(e + f*x))) - (log(c + d*tan(e
+ f*x))*((c^2*(d^4*(3*A*b^2 - 3*A*a^2 + 3*C*a^2 - 6*C*b^2 + 6*B*a*b) + 3*C*
b^2*d^4) - d^6*(A*b^2 - A*a^2 + C*a^2 + 2*B*a*b) + C*b^2*d^6 - c*d^5*(3*B*a
^2 - 3*B*b^2 + 6*A*a*b - 6*C*a*b) + c^3*d^3*(B*a^2 - B*b^2 + 2*A*a*b - 2*C*
a*b))/(d^9 + 3*c^2*d^7 + 3*c^4*d^5 + c^6*d^3) - (C*b^2)/d^3))/f - (log(tan(
e + f*x) - 1i)*(A*b^2*1i - A*a^2*1i + B*a^2 - B*b^2 + C*a^2*1i - C*b^2*1i +
 2*A*a*b + B*a*b*2i - 2*C*a*b))/(2*f*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)) -
 (log(tan(e + f*x) + 1i)*(A*b^2 - A*a^2 + B*a^2*1i - B*b^2*1i + C*a^2 - C*b
^2 + A*a*b*2i + 2*B*a*b - C*a*b*2i))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1i + d^
3))
```

3.86 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=352

$$-\frac{(a(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))-b((A-C)d(3c^2-d^2)-B(c^3-3cd^2)))\,x}{(c^2+d^2)^3}+\frac{(b(c^3C-3\ldots}{}$$

```
[Out] -(a*(c^3*C-3*B*c^2*d-3*c*C*d^2+B*d^3-A*(c^3-3*c*d^2))-b*((A-C)*d*(3*c^2-d^2
)-B*(c^3-3*c*d^2)))*x/(c^2+d^2)^3+(b*(-3*B*c^2*d+B*d^3+C*c^3-3*C*c*d^2)-a*(
B*c^3-3*B*c*d^2+3*C*c^2*d-C*d^3)+A*(a*d*(3*c^2-d^2)-b*(c^3-3*c*d^2)))*ln(c*
cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)^3/f+1/2*(-a*d+b*c)*(A*d^2-B*c*d+C*c^2)/d
^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(-b*(c^4*C-c^2*(A-3*C)*d^2-2*B*c*d^3+A*d^
4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]

time = 0.51, antiderivative size = 349, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {3716, 3709, 3612, 3611}

$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{2d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^2}-\frac{ad^2(2cd(A-C)-B(c^2-d^2))+b(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C)}{d^2f\left(c^2+d^2\right)^2(c+d\tan(e+fx))}+\frac{(aAd(3c^2-d^2)-a(Bc^3-3Bcd^2+3c^2Cd-Cd^3)-Ab(c^3-3cd^2)+b(-3Bc^2d+Bd^3+c^3C-3cCd^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f\left(c^2+d^2\right)^3}+\frac{x(-a(-A(c^3-3cd^2)-3Bc^2d+Bd^3+c^3C-3cCd^2)+bd(A-C)(3c^2-d^2)-bB(c^3-3cd^2))}{\left(c^2+d^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x])^3,x]
```

```
[Out] ((b*(A - C)*d*(3*c^2 - d^2) - b*B*(c^3 - 3*c*d^2) - a*(c^3*C - 3*B*c^2*d -
3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)))*x)/(c^2 + d^2)^3 + ((a*A*d*(3*c^2 -
 d^2) - A*b*(c^3 - 3*c*d^2) + b*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3) - a
*(B*c^3 + 3*c^2*C*d - 3*B*c*d^2 - C*d^3))*Log[c*Cos[e + f*x] + d*Sin[e + f*
x]])/((c^2 + d^2)^3*f) + ((b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(2*d^2*(c^2
+ d^2)*f*(c + d*Tan[e + f*x])^2) - (b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c*d^
3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))/(d^2*(c^2 + d^2)^2*f*(c
 + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx = \frac{(bc-ad)\,(c^2C-Bcd+Ad^2)}{2d^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} +$$

$$= \frac{(bc-ad)\,(c^2C-Bcd+Ad^2)}{2d^2\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} -$$

$$= \frac{(b(A-C)d(3c^2-d^2)-bB(c^3-3cd^2)}{}$$

$$= \frac{(b(A-C)d(3c^2-d^2)-bB(c^3-3cd^2)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.62, size = 331, normalized size = 0.94

$$\frac{\frac{aCd-b(cC+Bd)}{(c+d\tan(e+fx))^2}-\frac{2Cd(a+b\tan(e+fx))}{(c+d\tan(e+fx))^2}+2(Ab+aB-bC)d\left(-\frac{i\log(i-\tan(e+fx))}{2(c+id)^2}+\frac{i\log(i+\tan(e+fx))}{2(c-id)^2}+\frac{d\left(2c\log(c+d\tan(e+fx))-\frac{c^2+d^2}{c+d\tan(e+fx)}\right)}{(c^2+d^2)^2}\right)-d(Abc+aBc-bcC-aAd+bBd+aCd)\left(\frac{\log(i-\tan(e+fx))}{(-ic+d)^3}+\frac{\log(i+\tan(e+fx))}{(ic+d)^3}+\frac{d\left((6c^2-2d^2)\log(c+d\tan(e+fx))-\frac{(c^2+d^2)(5c^2+d^2+4cd\tan(e+fx))}{(c+d\tan(e+fx))^2}\right)}{(c^2+d^2)^3}\right)}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^3,x]
```

```
[Out] ((a*C*d - b*(c*C + B*d))/(c + d*Tan[e + f*x])^2 - (2*C*d*(a + b*Tan[e + f*x
]))/(c + d*Tan[e + f*x])^2 + 2*(A*b + a*B - b*C)*d*(((-1/2*I)*Log[I - Tan[e
 + f*x]])/(c + I*d)^2 + ((I/2)*Log[I + Tan[e + f*x]])/(c - I*d)^2 + (d*(2*c
*Log[c + d*Tan[e + f*x]] - (c^2 + d^2)/(c + d*Tan[e + f*x])))/(c^2 + d^2)^2
) - d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*(Log[I - Tan[e + f*x]
]/((-I)*c + d)^3 + Log[I + Tan[e + f*x]]/(I*c + d)^3 + (d*((6*c^2 - 2*d^2)*
Log[c + d*Tan[e + f*x]] - ((c^2 + d^2)*(5*c^2 + d^2 + 4*c*d*Tan[e + f*x]))/
(c + d*Tan[e + f*x])^2))/(c^2 + d^2)^3))/(2*d^2*f)
```

Maple [A]

time = 0.32, size = 493, normalized size = 1.40

method	result
derivativedivides	$\frac{\frac{\left(-3Aa\,c^{2}d+Aa\,d^{3}+Ab\,c^{3}-3Abc\,d^{2}+Ba\,c^{3}-3Bac\,d^{2}+3Bb\,c^{2}d-Bb\,d^{3}+3Ca\,c^{2}d-Ca\,d^{3}-Cb\,c^{3}+3Cbc\,d^{2}\right)\ln\left(1+\tan^{2}(fx+e)\right)}{2}+\left(Aa\,c^{3}\right.}{\left(c^{2}+d^{2}\right)^{3}}$
default	$\frac{\frac{\left(-3Aa\,c^{2}d+Aa\,d^{3}+Ab\,c^{3}-3Abc\,d^{2}+Ba\,c^{3}-3Bac\,d^{2}+3Bb\,c^{2}d-Bb\,d^{3}+3Ca\,c^{2}d-Ca\,d^{3}-Cb\,c^{3}+3Cbc\,d^{2}\right)\ln\left(1+\tan^{2}(fx+e)\right)}{2}+\left(Aa\,c^{3}\right.}{\left(c^{2}+d^{2}\right)^{3}}$
norman	$\frac{\left(Aa\,c^{3}-3Aac\,d^{2}+3Ab\,c^{2}d-Ab\,d^{3}+3Ba\,c^{2}d-Ba\,d^{3}-Bb\,c^{3}+3Bbc\,d^{2}-Ca\,c^{3}+3Cac\,d^{2}-3Cb\,c^{2}d+Cb\,d^{3}\right)c^{2}x}{\left(c^{4}+2c^{2}d^{2}+d^{4}\right)\left(c^{2}+d^{2}\right)}+\frac{d^{2}\left(Aa\,c^{3}-3Aac\,d^{2}+3Ab\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/(c^2+d^2)^3*(1/2*(-3*A*a*c^2*d+A*a*d^3+A*b*c^3-3*A*b*c*d^2+B*a*c^3-3
*B*a*c*d^2+3*B*b*c^2*d-B*b*d^3+3*C*a*c^2*d-C*a*d^3-C*b*c^3+3*C*b*c*d^2)*ln(
1+tan(f*x+e)^2)+(A*a*c^3-3*A*a*c*d^2+3*A*b*c^2*d-A*b*d^3+3*B*a*c^2*d-B*a*d^
3-B*b*c^3+3*B*b*c*d^2-C*a*c^3+3*C*a*c*d^2-3*C*b*c^2*d+C*b*d^3)*arctan(tan(f
*x+e)))+(3*A*a*c^2*d-A*a*d^3-A*b*c^3+3*A*b*c*d^2-B*a*c^3+3*B*a*c*d^2-3*B*b*
c^2*d+B*b*d^3-3*C*a*c^2*d+C*a*d^3+C*b*c^3-3*C*b*c*d^2)/(c^2+d^2)^3*ln(c+d*t
an(f*x+e))-1/2*(A*a*d^3-A*b*c*d^2-B*a*c*d^2+B*b*c^2*d+C*a*c^2*d-C*b*c^3)/d^
2/(c^2+d^2)/(c+d*tan(f*x+e))^2-(2*A*a*c*d^3-A*b*c^2*d^2+A*b*d^4-B*a*c^2*d^2
+B*a*d^4-2*B*b*c*d^3-2*C*a*c*d^3+C*b*c^4+3*C*b*c^2*d^2)/(c^2+d^2)^2/d^2/(c+
d*tan(f*x+e)))
```

Maxima [A]

time = 0.55, size = 549, normalized size = 1.56

$$\frac{\frac{2\left(\left((A-C)a-Bb\right)c^{3}+3\left(Ba+(A-C)b\right)c^{2}d-3\left((A-C)a-Bb\right)cd^{2}-\left(Ba+(A-C)b\right)d^{3}\right)(fx+e)}{c^{6}+3c^{4}d^{2}+3c^{2}d^{4}+d^{6}}-\frac{2\left(\left(Ba+(A-C)b\right)c^{3}-3\left((A-C)a-Bb\right)c^{2}d-3\left(Ba+(A-C)b\right)cd^{2}+\left((A-C)a-Bb\right)d^{3}\right)\log\left(d\tan(fx+e)+c\right)}{c^{6}+3c^{4}d^{2}+3c^{2}d^{4}+d^{6}}+\frac{\left(\left(Ba+(A-C)b\right)c^{3}-3\left((A-C)a-Bb\right)c^{2}d-3\left(Ba+(A-C)b\right)cd^{2}+\left((A-C)a-Bb\right)d^{3}\right)\log\left(\tan(fx+e)^{2}+1\right)}{c^{6}+3c^{4}d^{2}+3c^{2}d^{4}+d^{6}}-\frac{Cbc^{5}+Aad^{5}+(Ca+Bb)c^{4}d-\left(3Ba+(3A-5C)b\right)c^{3}d^{2}+\left((5A-3C)a-3Bb\right)c^{2}d^{3}+(Ba+Ab)cd^{4}+2\left(Cbc^{4}d-\left(Ba+(A-3C)b\right)c^{2}d^{3}+2\left((A-C)a-Bb\right)cd^{4}+(Ba+Ab)d^{5}\right)\tan(fx+e)}{c^{6}d^{2}+2c^{4}d^{4}+c^{2}d^{6}+\left(c^{4}d^{4}+2c^{2}d^{6}+d^{8}\right)\tan(fx+e)^{2}+2\left(c^{5}d^{3}+2c^{3}d^{5}+cd^{7}\right)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a - B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d - 3*((A - C)*a -
B*b)*c*d^2 - (B*a + (A - C)*b)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4
+ d^6) - 2*((B*a + (A - C)*b)*c^3 - 3*((A - C)*a - B*b)*c^2*d - 3*(B*a + (A
 - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^
4*d^2 + 3*c^2*d^4 + d^6) + ((B*a + (A - C)*b)*c^3 - 3*((A - C)*a - B*b)*c^2
*d - 3*(B*a + (A - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*log(tan(f*x + e)^2
+ 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (C*b*c^5 + A*a*d^5 + (C*a + B*b)
*c^4*d - (3*B*a + (3*A - 5*C)*b)*c^3*d^2 + ((5*A - 3*C)*a - 3*B*b)*c^2*d^3
+ (B*a + A*b)*c*d^4 + 2*(C*b*c^4*d - (B*a + (A - 3*C)*b)*c^2*d^3 + 2*((A -
C)*a - B*b)*c*d^4 + (B*a + A*b)*d^5)*tan(f*x + e))/(c^6*d^2 + 2*c^4*d^4 + c
^2*d^6 + (c^4*d^4 + 2*c^2*d^6 + d^8)*tan(f*x + e)^2 + 2*(c^5*d^3 + 2*c^3*d^
5 + c*d^7)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(354) = 708$.
time = 4.61, size = 906, normalized size = 2.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*b*c^5 - A*a*d^5 - 3*(C*a + B*b)*c^4*d + 5*(B*a + (A - C)*b)*c^3*d^2
- ((7*A - 3*C)*a - 3*B*b)*c^2*d^3 - (B*a + A*b)*c*d^4 + 2*(((A - C)*a - B*b
)*c^5 + 3*(B*a + (A - C)*b)*c^4*d - 3*((A - C)*a - B*b)*c^3*d^2 - (B*a + (A
 - C)*b)*c^2*d^3)*f*x + (C*b*c^5 - A*a*d^5 + (C*a + B*b)*c^4*d - (3*B*a + (
3*A - 7*C)*b)*c^3*d^2 + 5*((A - C)*a - B*b)*c^2*d^3 + 3*(B*a + A*b)*c*d^4 +
 2*(((A - C)*a - B*b)*c^3*d^2 + 3*(B*a + (A - C)*b)*c^2*d^3 - 3*((A - C)*a
- B*b)*c*d^4 - (B*a + (A - C)*b)*d^5)*f*x)*tan(f*x + e)^2 - ((B*a + (A - C)
*b)*c^5 - 3*((A - C)*a - B*b)*c^4*d - 3*(B*a + (A - C)*b)*c^3*d^2 + ((A - C
)*a - B*b)*c^2*d^3 + ((B*a + (A - C)*b)*c^3*d^2 - 3*((A - C)*a - B*b)*c^2*d
^3 - 3*(B*a + (A - C)*b)*c*d^4 + ((A - C)*a - B*b)*d^5)*tan(f*x + e)^2 + 2*
((B*a + (A - C)*b)*c^4*d - 3*((A - C)*a - B*b)*c^3*d^2 - 3*(B*a + (A - C)*b
)*c^2*d^3 + ((A - C)*a - B*b)*c*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2
+ 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) + 2*((C*a + B*b)*c^5 - (2
*B*a + (2*A - 3*C)*b)*c^4*d + 3*((A - C)*a - B*b)*c^3*d^2 + 3*(B*a + (A - C
)*b)*c^2*d^3 - ((3*A - 2*C)*a - 2*B*b)*c*d^4 - (B*a + A*b)*d^5 + 2*(((A - C
)*a - B*b)*c^4*d + 3*(B*a + (A - C)*b)*c^3*d^2 - 3*((A - C)*a - B*b)*c^2*d^
3 - (B*a + (A - C)*b)*c*d^4)*f*x)*tan(f*x + e))/((c^6*d^2 + 3*c^4*d^4 + 3*c
^2*d^6 + d^8)*f*tan(f*x + e)^2 + 2*(c^7*d + 3*c^5*d^3 + 3*c^3*d^5 + c*d^7)*
f*tan(f*x + e) + (c^8 + 3*c^6*d^2 + 3*c^4*d^4 + c^2*d^6)*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. 2(354) = 708.
time = 0.95, size = 1037, normalized size = 2.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^3 - C*a*c^3 - B*b*c^3 + 3*B*a*c^2*d + 3*A*b*c^2*d - 3*C*b*c^2
*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 + 3*B*b*c*d^2 - B*a*d^3 - A*b*d^3 + C*b*d^3)
*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (B*a*c^3 + A*b*c^3 - C*b*c
^3 - 3*A*a*c^2*d + 3*C*a*c^2*d + 3*B*b*c^2*d - 3*B*a*c*d^2 - 3*A*b*c*d^2 +
3*C*b*c*d^2 + A*a*d^3 - C*a*d^3 - B*b*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3
*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(B*a*c^3*d + A*b*c^3*d - C*b*c^3*d - 3*A*a*
c^2*d^2 + 3*C*a*c^2*d^2 + 3*B*b*c^2*d^2 - 3*B*a*c*d^3 - 3*A*b*c*d^3 + 3*C*b
*c*d^3 + A*a*d^4 - C*a*d^4 - B*b*d^4)*log(abs(d*tan(f*x + e) + c))/(c^6*d +
 3*c^4*d^3 + 3*c^2*d^5 + d^7) + (3*B*a*c^3*d^4*tan(f*x + e)^2 + 3*A*b*c^3*d
^4*tan(f*x + e)^2 - 3*C*b*c^3*d^4*tan(f*x + e)^2 - 9*A*a*c^2*d^5*tan(f*x +
e)^2 + 9*C*a*c^2*d^5*tan(f*x + e)^2 + 9*B*b*c^2*d^5*tan(f*x + e)^2 - 9*B*a*
c*d^6*tan(f*x + e)^2 - 9*A*b*c*d^6*tan(f*x + e)^2 + 9*C*b*c*d^6*tan(f*x + e
)^2 + 3*A*a*d^7*tan(f*x + e)^2 - 3*C*a*d^7*tan(f*x + e)^2 - 3*B*b*d^7*tan(f
*x + e)^2 - 2*C*b*c^6*d*tan(f*x + e) + 8*B*a*c^4*d^3*tan(f*x + e) + 8*A*b*c
^4*d^3*tan(f*x + e) - 14*C*b*c^4*d^3*tan(f*x + e) - 22*A*a*c^3*d^4*tan(f*x
+ e) + 22*C*a*c^3*d^4*tan(f*x + e) + 22*B*b*c^3*d^4*tan(f*x + e) - 18*B*a*c
^2*d^5*tan(f*x + e) - 18*A*b*c^2*d^5*tan(f*x + e) + 12*C*b*c^2*d^5*tan(f*x
+ e) + 2*A*a*c*d^6*tan(f*x + e) - 2*C*a*c*d^6*tan(f*x + e) - 2*B*b*c*d^6*ta
n(f*x + e) - 2*B*a*d^7*tan(f*x + e) - 2*A*b*d^7*tan(f*x + e) - C*b*c^7 - C*
a*c^6*d - B*b*c^6*d + 6*B*a*c^5*d^2 + 6*A*b*c^5*d^2 - 9*C*b*c^5*d^2 - 14*A*
a*c^4*d^3 + 11*C*a*c^4*d^3 + 11*B*b*c^4*d^3 - 7*B*a*c^3*d^4 - 7*A*b*c^3*d^4
 + 4*C*b*c^3*d^4 - 3*A*a*c^2*d^5 - B*a*c*d^6 - A*b*c*d^6 - A*a*d^7)/((c^6*d
^2 + 3*c^4*d^4 + 3*c^2*d^6 + d^8)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [B]
time = 16.53, size = 502, normalized size = 1.43

$$-\frac{\frac{A a d^5+C b c^5+A b c d^4+B a c d^4+B b c^4 d+C a c^4 d+5 A a c^2 d^3-3 A b c^3 d^2-3 B a c^3 d^2-3 B b c^2 d^3-3 C a c^2 d^3+5 C b c^3 d^2}{2 d^2 \left(c^4+2 c^2 d^2+d^4\right)}+\frac{\tan(e+f x) \left(A b d^4+B a d^4+C b c^4+2 A a c d^3-2 B b c d^3-2 C a c d^3-A b c^2 d^2-B a c^2 d^2+3 C b c^2 d^2\right)}{d \left(c^4+2 c^2 d^2+d^4\right)}}{f \left(c^2+2 c d \tan\left(e+f x\right)+d^2 \tan\left(e+f x\right)^2\right)}-\frac{\ln\left(\tan(e+f x)+1\mathrm{i}\right) \left(B b+A b 1\mathrm{i}+B a 1\mathrm{i}-A a+C a-C b 1\mathrm{i}\right)}{2 f \left(-c^3 1\mathrm{i}-3 c^2 d+c d^2 3\mathrm{i}+d^3\right)}-\frac{\ln\left(\tan(e+f x)-\mathrm{i}\right) \left(A b+B a-C b-A a 1\mathrm{i}+B b 1\mathrm{i}+C a 1\mathrm{i}\right)}{2 f \left(-c^3-c^2 d 3\mathrm{i}+3 c d^2+d^3 1\mathrm{i}\right)}-\frac{\ln\left(c+d \tan(e+f x)\right) \left(\left(A b+B a-C b\right) c^3+\left(3 B b-3 A a+3 C a\right) c^2 d+\left(3 C b-3 B a-3 A b\right) c d^2+\left(A a-B b-C a\right) d^3\right)}{f \left(c^6+3 c^4 d^2+3 c^2 d^4+d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*t
an(e + f*x))^3,x)

[Out] - ((A*a*d^5 + C*b*c^5 + A*b*c*d^4 + B*a*c*d^4 + B*b*c^4*d + C*a*c^4*d + 5*A
*a*c^2*d^3 - 3*A*b*c^3*d^2 - 3*B*a*c^3*d^2 - 3*B*b*c^2*d^3 - 3*C*a*c^2*d^3
+ 5*C*b*c^3*d^2)/(2*d^2*(c^4 + d^4 + 2*c^2*d^2)) + (tan(e + f*x)*(A*b*d^4 +
 B*a*d^4 + C*b*c^4 + 2*A*a*c*d^3 - 2*B*b*c*d^3 - 2*C*a*c*d^3 - A*b*c^2*d^2
- B*a*c^2*d^2 + 3*C*b*c^2*d^2))/(d*(c^4 + d^4 + 2*c^2*d^2)))/(f*(c^2 + d^2*
tan(e + f*x)^2 + 2*c*d*tan(e + f*x))) - (log(tan(e + f*x) + 1i)*(A*b*1i - A
*a + B*a*1i + B*b + C*a - C*b*1i))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3)
) - (log(tan(e + f*x) - 1i)*(A*b - A*a*1i + B*a + B*b*1i + C*a*1i - C*b))/(
2*f*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)) - (log(c + d*tan(e + f*x))*(c^3*(A
*b + B*a - C*b) - d^3*(B*b - A*a + C*a) + c^2*d*(3*B*b - 3*A*a + 3*C*a) - c
*d^2*(3*A*b + 3*B*a - 3*C*b)))/(f*(c^6 + d^6 + 3*c^2*d^4 + 3*c^4*d^2))
```

3.87 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=209

$$-\frac{(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))\,x}{(c^2+d^2)^3}+\frac{((A-C)d(3c^2-d^2)-B(c^3-3cd^2))\log(c\cos(e+fx)}{(c^2+d^2)^3\,f}$$

```
[Out] -(c^3*C-3*B*c^2*d-3*c*C*d^2+B*d^3-A*(c^3-3*c*d^2))*x/(c^2+d^2)^3+((A-C)*d*(
3*c^2-d^2)-B*(c^3-3*c*d^2))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)^3/f+1/2
*(-A*d^2+B*c*d-C*c^2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(-2*c*(A-C)*d+B*(c^2
-d^2))/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]

time = 0.25, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3709, 3610, 3612, 3611}

$$-\frac{Ad^2-Bcd+c^2C}{2df\,(c^2+d^2)\,(c+d\tan(e+fx))^2}-\frac{2cd(A-C)-B(c^2-d^2)}{f\,(c^2+d^2)^2\,(c+d\tan(e+fx))}+\frac{(d(A-C)\,(3c^2-d^2)-B(c^3-3cd^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f\,(c^2+d^2)^3}-\frac{x(-A(c^3-3cd^2)-3Bc^2d+Bd^3+c^3C-3cCd^2)}{(c^2+d^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -((((c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2))*x)/(c^2 + d^
2)^3) + (((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2))*Log[c*Cos[e + f*x] +
 d*Sin[e + f*x]])/((c^2 + d^2)^3*f) - (c^2*C - B*c*d + A*d^2)/(2*d*(c^2 + d
^2)*f*(c + d*Tan[e + f*x])^2) - (2*c*(A - C)*d - B*(c^2 - d^2))/((c^2 + d^2
)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx = -\frac{c^2C-Bcd+Ad^2}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}+\frac{\int \frac{Ac-cC+Bd+(Bc-(A-C)}{(c+d\tan(e+fx}}{c^2+d^2}$$

$$= -\frac{c^2C-Bcd+Ad^2}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}-\frac{2c(A-C)d-B(c}{\left(c^2+d^2\right)^2f(c+d\tan}$$

$$= \frac{\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)x}{\left(c^2+d^2\right)^3}-\frac{c}{2d\left(c^2+\right.}$$

$$= \frac{\left(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3\right)x}{\left(c^2+d^2\right)^3}+\frac{((A-C)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.49, size = 261, normalized size = 1.25

$$-\frac{\frac{C}{(c+d\tan(e+fx))^2}+B\left(\frac{i\log(i-\tan(e+fx))}{(c+id)^2}-\frac{i\log(i+\tan(e+fx))}{(c-id)^2}+\frac{2d\left(-2c\log(c+d\tan(e+fx))+\frac{c^2+d^2}{c+d\tan(e+fx)}\right)}{\left(c^2+d^2\right)^2}\right)-(Bc+(-A+C)d)\left(\frac{i\log(i-\tan(e+fx))}{(c+id)^3}-\frac{\log(i+\tan(e+fx))}{(ic+d)^3}+\frac{d\left(\left(-6c^2+2d^2\right)\log(c+d\tan(e+fx))+\frac{\left(c^2+d^2\right)\left(5c^2+d^2+4cd\tan(e+fx)\right)}{(c+d\tan(e+fx))^2}\right)}{\left(c^2+d^2\right)^3}\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*(C/(c + d*Tan[e + f*x])^2 + B*((I*Log[I - Tan[e + f*x]])/(c + I*d)^2 -
 (I*Log[I + Tan[e + f*x]])/(c - I*d)^2 + (2*d*(-2*c*Log[c + d*Tan[e + f*x]]
 + (c^2 + d^2)/(c + d*Tan[e + f*x])))/(c^2 + d^2)^2) - (B*c + (-A + C)*d)*(
(I*Log[I - Tan[e + f*x]])/(c + I*d)^3 - Log[I + Tan[e + f*x]]/(I*c + d)^3 +
 (d*((-6*c^2 + 2*d^2)*Log[c + d*Tan[e + f*x]] + ((c^2 + d^2)*(5*c^2 + d^2 +
 4*c*d*Tan[e + f*x]))/(c + d*Tan[e + f*x])^2))/(c^2 + d^2)^3))/(d*f)
```

Maple [A]
time = 0.28, size = 262, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x,method=_RETURNVERB
OSE)
```

```
[Out] 1/f*(1/(c^2+d^2)^3*(1/2*(-3*A*c^2*d+A*d^3+B*c^3-3*B*c*d^2+3*C*c^2*d-C*d^3)*
ln(1+tan(f*x+e)^2)+(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)*arctan
(tan(f*x+e)))-1/2*(A*d^2-B*c*d+C*c^2)/(c^2+d^2)/d/(c+d*tan(f*x+e))^2-(2*A*c
*d-B*c^2+B*d^2-2*C*c*d)/(c^2+d^2)^2/(c+d*tan(f*x+e))+(3*A*c^2*d-A*d^3-B*c^3
+3*B*c*d^2-3*C*c^2*d+C*d^3)/(c^2+d^2)^3*ln(c+d*tan(f*x+e)))
```

Maxima [A]
time = 0.55, size = 373, normalized size = 1.78

$$\frac{\frac{2\left((A-C)c^3+3Bc^2d-3(A-C)cd^2-Bd^3\right)(fx+e)}{c^6+3c^4d^2+3c^2d^4+d^6}-\frac{2\left(Bc^3-3(A-C)c^2d-3Bcd^2+(A-C)d^3\right)\log\left(d\tan\left(fx+e\right)+c\right)}{c^6+3c^4d^2+3c^2d^4+d^6}+\frac{\left(Bc^3-3(A-C)c^2d-3Bcd^2+(A-C)d^3\right)\log\left(\tan\left(fx+e\right)^2+1\right)}{c^6+3c^4d^2+3c^2d^4+d^6}-\frac{Cc^4-3Bc^3d+(5A-3C)c^2d^2+Bcd^3+Ad^4-2\left(Bc^2d^2-2(A-C)cd^3-Bd^4\right)\tan\left(fx+e\right)}{c^6d+2c^4d^3+c^2d^5+\left(c^4d^3+2c^2d^5+d^7\right)\tan\left(fx+e\right)^2+2\left(c^5d^2+2c^3d^4+cd^6\right)\tan\left(fx+e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] 1/2*(2*((A - C)*c^3 + 3*B*c^2*d - 3*(A - C)*c*d^2 - B*d^3)*(f*x + e)/(c^6 +
 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(B*c^3 - 3*(A - C)*c^2*d - 3*B*c*d^2 + (A
 - C)*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (B
*c^3 - 3*(A - C)*c^2*d - 3*B*c*d^2 + (A - C)*d^3)*log(tan(f*x + e)^2 + 1)/(
c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (C*c^4 - 3*B*c^3*d + (5*A - 3*C)*c^2*d
^2 + B*c*d^3 + A*d^4 - 2*(B*c^2*d^2 - 2*(A - C)*c*d^3 - B*d^4)*tan(f*x + e)
)/(c^6*d + 2*c^4*d^3 + c^2*d^5 + (c^4*d^3 + 2*c^2*d^5 + d^7)*tan(f*x + e)^2
 + 2*(c^5*d^2 + 2*c^3*d^4 + c*d^6)*tan(f*x + e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(211) = 422$.
time = 5.26, size = 575, normalized size = 2.75

$$-\frac{3Cc^4d-5Bc^3d^2+(7A-3C)c^2d^3+Bcd^4+Ad^5-2\left((A-C)c^5+3Bc^4d-3(A-C)c^3d^2-Bc^2d^3\right)fx-\left(Cc^4d-3Bc^3d^2+5(A-C)c^2d^3+3Bcd^4-Ad^5+2\left((A-C)c^3d^2+3Bc^2d^3-3(A-C)cd^4-Bd^5\right)fx\right)\tan\left(fx+e\right)^2+\left(Bc^5-3(A-C)c^4d-3Bc^3d^2+(A-C)c^2d^3+\left(Bc^3d^2-3(A-C)c^2d^3-3Bcd^4+(A-C)d^5\right)\tan\left(fx+e\right)^2+2\left(Bc^4d-3(A-C)c^3d^2-3Bc^2d^3+(A-C)cd^4\right)\tan\left(fx+e\right)\right)\log\left(\frac{d^2\tan\left(fx+e\right)^2+2cd\tan\left(fx+e\right)+c^2}{\tan\left(fx+e\right)^2+1}\right)-2\left(Cc^5-2Bc^4d+3(A-C)c^3d^2+3Bc^2d^3-(3A-2C)cd^4-Bd^5+2\left((A-C)c^4d+3Bc^3d^2-3(A-C)c^2d^3-Bcd^4\right)fx\right)\tan\left(fx+e\right)}{2\left(\left(c^6d^2+3c^4d^4+3c^2d^6+d^8\right)f\tan\left(fx+e\right)^2+2\left(c^7d+3c^5d^3+3c^3d^5+cd^7\right)f\tan\left(fx+e\right)+\left(c^8+3c^6d^2+3c^4d^4+c^2d^6\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] -1/2*(3*C*c^4*d - 5*B*c^3*d^2 + (7*A - 3*C)*c^2*d^3 + B*c*d^4 + A*d^5 - 2*(
(A - C)*c^5 + 3*B*c^4*d - 3*(A - C)*c^3*d^2 - B*c^2*d^3)*f*x - (C*c^4*d - 3
*B*c^3*d^2 + 5*(A - C)*c^2*d^3 + 3*B*c*d^4 - A*d^5 + 2*((A - C)*c^3*d^2 + 3
*B*c^2*d^3 - 3*(A - C)*c*d^4 - B*d^5)*f*x)*tan(f*x + e)^2 + (B*c^5 - 3*(A -
 C)*c^4*d - 3*B*c^3*d^2 + (A - C)*c^2*d^3 + (B*c^3*d^2 - 3*(A - C)*c^2*d^3
```

```
- 3*B*c*d^4 + (A - C)*d^5)*tan(f*x + e)^2 + 2*(B*c^4*d - 3*(A - C)*c^3*d^2
- 3*B*c^2*d^3 + (A - C)*c*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*
d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*(C*c^5 - 2*B*c^4*d + 3*(A -
 C)*c^3*d^2 + 3*B*c^2*d^3 - (3*A - 2*C)*c*d^4 - B*d^5 + 2*((A - C)*c^4*d +
3*B*c^3*d^2 - 3*(A - C)*c^2*d^3 - B*c*d^4)*f*x)*tan(f*x + e))/((c^6*d^2 + 3
*c^4*d^4 + 3*c^2*d^6 + d^8)*f*tan(f*x + e)^2 + 2*(c^7*d + 3*c^5*d^3 + 3*c^3
*d^5 + c*d^7)*f*tan(f*x + e) + (c^8 + 3*c^6*d^2 + 3*c^4*d^4 + c^2*d^6)*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(211) = 422.
time = 0.89, size = 548, normalized size = 2.62

$$\frac{\frac{2\,(Ac^3-Cc^3+3\,Bc^2d-3\,Acd^2+3\,Ccd^2-Bd^3)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}+\frac{(Bc^3-3\,Ac^2d+3\,Cc^2d-3\,Bcd^2+Ad^3-Cd^3)\log\left(\tan(fx+e)^2+1\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}-\frac{2\,(Bc^3d-3\,Ac^2d^2+3\,Cc^2d^2-3\,Bcd^3+Ad^4-Cd^4)\log(|d\tan(fx+e)+c|)}{c^6d+3\,c^4d^3+3\,c^2d^5+d^7}+\frac{3\,Bc^3d^3\tan(fx+e)^2-9\,Ac^2d^4\tan(fx+e)^2+9\,Cc^2d^4\tan(fx+e)^2-9\,Bcd^5\tan(fx+e)^2+3\,Ad^6\tan(fx+e)^2-3\,Cd^6\tan(fx+e)^2+8\,Bc^4d^2\tan(fx+e)-22\,Ac^3d^3\tan(fx+e)+22\,Cc^3d^3\tan(fx+e)-18\,Bc^2d^4\tan(fx+e)+2\,Acd^5\tan(fx+e)-2\,Ccd^5\tan(fx+e)-2\,Bd^6\tan(fx+e)-Cc^6+6\,Bc^5d-14\,Ac^4d^2+11\,Cc^4d^2-7\,Bc^3d^3-3\,Ac^2d^4-Bcd^5-Ad^6}{(c^6d+3\,c^4d^3+3\,c^2d^5+d^7)(d\tan(fx+e)+c)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] 1/2*(2*(A*c^3 - C*c^3 + 3*B*c^2*d - 3*A*c*d^2 + 3*C*c*d^2 - B*d^3)*(f*x + e
)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (B*c^3 - 3*A*c^2*d + 3*C*c^2*d - 3*
B*c*d^2 + A*d^3 - C*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d
^4 + d^6) - 2*(B*c^3*d - 3*A*c^2*d^2 + 3*C*c^2*d^2 - 3*B*c*d^3 + A*d^4 - C*
d^4)*log(abs(d*tan(f*x + e) + c))/(c^6*d + 3*c^4*d^3 + 3*c^2*d^5 + d^7) + (
3*B*c^3*d^3*tan(f*x + e)^2 - 9*A*c^2*d^4*tan(f*x + e)^2 + 9*C*c^2*d^4*tan(f
*x + e)^2 - 9*B*c*d^5*tan(f*x + e)^2 + 3*A*d^6*tan(f*x + e)^2 - 3*C*d^6*tan
(f*x + e)^2 + 8*B*c^4*d^2*tan(f*x + e) - 22*A*c^3*d^3*tan(f*x + e) + 22*C*c
^3*d^3*tan(f*x + e) - 18*B*c^2*d^4*tan(f*x + e) + 2*A*c*d^5*tan(f*x + e) -
2*C*c*d^5*tan(f*x + e) - 2*B*d^6*tan(f*x + e) - C*c^6 + 6*B*c^5*d - 14*A*c^
4*d^2 + 11*C*c^4*d^2 - 7*B*c^3*d^3 - 3*A*c^2*d^4 - B*c*d^5 - A*d^6)/((c^6*d
 + 3*c^4*d^3 + 3*c^2*d^5 + d^7)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [**B**]
time = 11.88, size = 327, normalized size = 1.56

$$-\frac{\frac{\tan(e+f\,x)\,\left(B\,d^3+2\,A\,c\,d^2-B\,c^2\,d-2\,C\,c\,d^2\right)}{c^4+2\,c^2\,d^2+d^4}+\frac{A\,d^4+C\,c^4+5\,A\,c^2\,d^2-3\,C\,c^2\,d^2+B\,c\,d^3-3\,B\,c^3\,d}{2\,d\,\left(c^4+2\,c^2\,d^2+d^4\right)}}{f\,\left(c^2+2\,c\,d\tan\left(e+f\,x\right)+d^2\tan\left(e+f\,x\right)^2\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B-A\,1\mathrm{i}+C\,1\mathrm{i}\right)}{2\,f\,\left(-c^3-c^2\,d\,3\mathrm{i}+3\,c\,d^2+d^3\,1\mathrm{i}\right)}-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(B\,c^3+\left(3\,C-3\,A\right)\,c^2\,d-3\,B\,c\,d^2+\left(A-C\right)\,d^3\right)}{f\,\left(c^6+3\,c^4\,d^2+3\,c^2\,d^4+d^6\right)}-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(C-A+B\,1\mathrm{i}\right)}{2\,f\,\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^3,x)
```

```
[Out] - ((tan(e + f*x)*(B*d^3 + 2*A*c*d^2 - B*c^2*d - 2*C*c*d^2))/(c^4 + d^4 + 2*
c^2*d^2) + (A*d^4 + C*c^4 + 5*A*c^2*d^2 - 3*C*c^2*d^2 + B*c*d^3 - 3*B*c^3*d
)/(2*d*(c^4 + d^4 + 2*c^2*d^2)))/(f*(c^2 + d^2*tan(e + f*x)^2 + 2*c*d*tan(e
 + f*x))) - (log(tan(e + f*x) - 1i)*(B - A*1i + C*1i))/(2*f*(3*c*d^2 - c^2*
d*3i - c^3 + d^3*1i)) - (log(c + d*tan(e + f*x))*(B*c^3 + d^3*(A - C) - c^2
*d*(3*A - 3*C) - 3*B*c*d^2))/(f*(c^6 + d^6 + 3*c^2*d^4 + 3*c^4*d^2)) - (log
(tan(e + f*x) + 1i)*(B*1i - A + C))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3
))
```

3.88 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=487

$$-\frac{(a(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))+b((A-C)d(3c^2-d^2)-B(c^3-3cd^2)))\,x}{(a^2+b^2)\,(c^2+d^2)^3}+\frac{b^2(Ab^2-$$

```
[Out] -(a*(c^3*C-3*B*c^2*d-3*c*C*d^2+B*d^3-A*(c^3-3*c*d^2))+b*((A-C)*d*(3*c^2-d^2
)-B*(c^3-3*c*d^2)))*x/(a^2+b^2)/(c^2+d^2)^3+b^2*(A*b^2-a*(B*b-C*a))*ln(a*co
s(f*x+e)+b*sin(f*x+e))/(a^2+b^2)/(-a*d+b*c)^3/f-(b^2*(c^6*C-3*B*c^5*d+3*c^4
*(2*A-C)*d^2+B*c^3*d^3+3*A*c^2*d^4+A*d^6)+a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*(c
^3-3*c*d^2))-a*b*d^2*(8*c^3*(A-C)*d-B*(3*c^4-6*c^2*d^2-d^4)))*ln(c*cos(f*x+
e)+d*sin(f*x+e))/(-a*d+b*c)^3/(c^2+d^2)^3/f+1/2*(A*d^2-B*c*d+C*c^2)/(-a*d+b
*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(b*(c^4*C-2*B*c^3*d+c^2*(3*A-C)*d^2+A*d^
4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/(-a*d+b*c)^2/(c^2+d^2)^2/f/(c+d*tan(f*x
+e))
```

Rubi [A]
time = 1.23, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3730, 3732, 3611}

$$-\frac{x(a(-A(c^3-3cd^2)-3Bc^2d+Bd^3+c^3C-3cCd^2)+b(d(A-C)(3c^2-d^2)-B(c^3-3cd^2)))}{(a^2+b^2)(c^2+d^2)^3}-\frac{(a^2d^3(d(A-C)(3c^2-d^2)-B(c^3-3cd^2))-abd^2(8c^3d(A-C)-B(3c^4-6c^2d^2-d^4))+b^2(3c^4d^2(2A-C)+3Ac^2d^4+Ad^6-3Bc^5d+Bc^3d^3+c^6C))\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)^3(bc-ad)^3}+\frac{b^2(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)(bc-ad)^3}+\frac{Ad^2-Bcd+c^2C}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}+\frac{b(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C)-ad^2(2cd(A-C)-B(c^2-d^2))}{f(c^2+d^2)^2(bc-ad)^2(c+d\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])^3),x]
```

```
[Out] -(((a*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) + b*((A -
 C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)^3)) +
 (b^2*(A*b^2 - a*(b*B - a*C))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 +
 b^2)*(b*c - a*d)^3*f) - ((b^2*(c^6*C - 3*B*c^5*d + 3*c^4*(2*A - C)*d^2 + B
*c^3*d^3 + 3*A*c^2*d^4 + A*d^6) + a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3
 - 3*c*d^2)) - a*b*d^2*(8*c^3*(A - C)*d - B*(3*c^4 - 6*c^2*d^2 - d^4)))*Log
[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)^3*(c^2 + d^2)^3*f) + (c^2*C
 - B*c*d + A*d^2)/(2*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) + (b
*(c^4*C - 2*B*c^3*d + c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B
*(c^2 - d^2)))/((b*c - a*d)^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx = \frac{c^2C-Bcd+Ad^2}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{\int \frac{-2(aAcd-ad(cC}{}}{}$$

$$= \frac{c^2C-Bcd+Ad^2}{2(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} + \frac{b(c^4C-2Bc^3}{}$$

$$= -\frac{(b(A-C)d(3c^2-d^2)-bB(c^3-3cd^2)-a(Ac^3-c^3C+3}{\left(a^2+b^2\right)\left(c^2+d^2\right)^3}$$

$$= -\frac{(b(A-C)d(3c^2-d^2)-bB(c^3-3cd^2)-a(Ac^3-c^3C+3}{\left(a^2+b^2\right)\left(c^2+d^2\right)^3}$$

Mathematica [A]
time = 8.12, size = 912, normalized size = 1.87

$$-\frac{Ad^2-c(-cC+Bd)}{2(-bc+ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2} - \cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])^3),x]
```

```
[Out] -1/2*(A*d^2 - c*(-(c*C) + B*d))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e
+ f*x])^2) - (-((-((b*(b*c - a*d)^2*(A*b*c^3 - a*B*c^3 - b*c^3*C + 3*a*A*c^
2*d + 3*b*B*c^2*d - 3*a*c^2*C*d - 3*A*b*c*d^2 + 3*a*B*c*d^2 + 3*b*c*C*d^2 -
 a*A*d^3 - b*B*d^3 + a*C*d^3 - (Sqrt[-b^2]*(a*(c^3*C - 3*B*c^2*d - 3*c*C*d^
2 + B*d^3 - A*(c^3 - 3*c*d^2)) + b*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*
d^2))))/b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2))) + (
2*b^3*(A*b^2 - a*(b*B - a*C))*(c^2 + d^2)^2*Log[a + b*Tan[e + f*x]])/((a^2
+ b^2)*(b*c - a*d)) - (b*(b*c - a*d)^2*(A*b*c^3 - a*B*c^3 - b*c^3*C + 3*a*A
*c^2*d + 3*b*B*c^2*d - 3*a*c^2*C*d - 3*A*b*c*d^2 + 3*a*B*c*d^2 + 3*b*c*C*d^
2 - a*A*d^3 - b*B*d^3 + a*C*d^3 + (Sqrt[-b^2]*(b*(A - C)*d*(3*c^2 - d^2) -
b*B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2
- B*d^3)))/b)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) -
 (2*b*(b^2*(c^6*C - 3*B*c^5*d + 3*c^4*(2*A - C)*d^2 + B*c^3*d^3 + 3*A*c^2*d
^4 + A*d^6) + a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - a*b*d
^2*(8*c^3*(A - C)*d - B*(3*c^4 - 6*c^2*d^2 - d^4)))*Log[c + d*Tan[e + f*x]]
)/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)*(c^2 + d^2)*f)) - (-2*d^2*(a
*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)) - c*(2*d*(b*c - a*d)*(B*c - (A
- C)*d) - 2*b*c*(c^2*C - B*c*d + A*d^2)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c
+ d*Tan[e + f*x])))/(2*(-(b*c) + a*d)*(c^2 + d^2))
```

Maple [A]
time = 1.83, size = 649, normalized size = 1.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/f*(-(2*A*a*c*d^3-3*A*b*c^2*d^2-A*b*d^4-B*a*c^2*d^2+B*a*d^4+2*B*b*c^3*d-2*
C*a*c*d^3-C*b*c^4+C*b*c^2*d^2)/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan(f*x+e))+(3*
A*a^2*c^2*d^4-A*a^2*d^6-8*A*a*b*c^3*d^3+6*A*b^2*c^4*d^2+3*A*b^2*c^2*d^4+A*b
^2*d^6-B*a^2*c^3*d^3+3*B*a^2*c*d^5+3*B*a*b*c^4*d^2-6*B*a*b*c^2*d^4-B*a*b*d^
6-3*B*b^2*c^5*d+B*b^2*c^3*d^3-3*C*a^2*c^2*d^4+C*a^2*d^6+8*C*a*b*c^3*d^3+C*b
^2*c^6-3*C*b^2*c^4*d^2)/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*x+e))-1/2*(A*d
^2-B*c*d+C*c^2)/(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))^2-(A*b^2-B*a*b+C*a^2)*
b^2/(a*d-b*c)^3/(a^2+b^2)*ln(a+b*tan(f*x+e))+1/(a^2+b^2)/(c^2+d^2)^3*(1/2*(
-3*A*a*c^2*d+A*a*d^3-A*b*c^3+3*A*b*c*d^2+B*a*c^3-3*B*a*c*d^2-3*B*b*c^2*d+B*
b*d^3+3*C*a*c^2*d-C*a*d^3+C*b*c^3-3*C*b*c*d^2)*ln(1+tan(f*x+e)^2)+(A*a*c^3-
3*A*a*c*d^2-3*A*b*c^2*d+A*b*d^3+3*B*a*c^2*d-B*a*d^3+B*b*c^3-3*B*b*c*d^2-C*a
*c^3+3*C*a*c*d^2+3*C*b*c^2*d-C*b*d^3)*arctan(tan(f*x+e))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(491) = 982$.

time = 0.60, size = 1085, normalized size = 2.23

$$\frac{\frac{2\left(\left((A-C)a+Bb\right)c^3+3\left(Ba-(A-C)b\right)c^2d-3\left((A-C)a+Bb\right)cd^2-\left(Ba-(A-C)b\right)d^3\right)(fx+e)}{(a^2+b^2)c^6+3(a^2+b^2)c^4d^2+3(a^2+b^2)c^2d^4+(a^2+b^2)d^6}+\frac{2\left(Ca^2b^2-Bab^3+Ab^4\right)\log\left(b\tan(fx+e)+a\right)}{(a^2b^3+b^5)c^3-3(a^3b^2+ab^4)c^2d+3(a^4b+a^2b^3)cd^2-(a^5+a^3b^2)d^3}-\frac{2\left(Cb^2c^6-3Bb^2c^5d+3Ba^2cd^5+3\left(Bab+(2A-C)b^2\right)c^4d^2-\left(Ba^2+8(A-C)ab-Bb^2\right)c^3d^3+3\left((A-C)a^2-2Bab+Ab^2\right)c^2d^4-\left((A-C)a^2+Bab-Ab^2\right)d^6\right)\log\left(d\tan(fx+e)+c\right)}{b^3c^9-3ab^2c^8d+3a^2bcd^8-a^3d^9+3(a^2b+b^3)c^7d^2-(a^3+9ab^2)c^6d^3+3(3a^2b+b^3)c^5d^4-3(a^3+3ab^2)c^4d^5+(9a^2b+b^3)c^3d^6-3(a^3+ab^2)c^2d^7}+\frac{\left(\left(Ba-(A-C)b\right)c^3-3\left((A-C)a+Bb\right)c^2d-3\left(Ba-(A-C)b\right)cd^2+\left((A-C)a+Bb\right)d^3\right)\log\left(\tan(fx+e)^2+1\right)}{(a^2+b^2)c^6+3(a^2+b^2)c^4d^2+3(a^2+b^2)c^2d^4+(a^2+b^2)d^6}+\frac{3Cbc^5-Aad^5-(Ca+5Bb)c^4d+\left(3Ba+(7A-C)b\right)c^3d^2-\left((5A-3C)a+Bb\right)c^2d^3-(Ba-3Ab)cd^4+2\left(Cbc^4d-2Bbc^3d^2-2(A-C)acd^4+\left(Ba+(3A-C)b\right)c^2d^3-(Ba-Ab)d^5\right)\tan(fx+e)}{b^2c^8-2abc^7d-4abc^5d^3-2abc^3d^5+a^2c^2d^6+(a^2+2b^2)c^6d^2+(2a^2+b^2)c^4d^4+\left(b^2c^6d^2-2abc^5d^3-4abc^3d^5-2abcd^7+a^2d^8+(a^2+2b^2)c^4d^4+(2a^2+b^2)c^2d^6\right)\tan(fx+e)^2+2\left(b^2c^7d-2abc^6d^2-4abc^4d^4-2abc^2d^6+a^2cd^7+(a^2+2b^2)c^5d^3+(2a^2+b^2)c^3d^5\right)\tan(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a + B*b)*c^3 + 3*(B*a - (A - C)*b)*c^2*d - 3*((A - C)*a +
B*b)*c*d^2 - (B*a - (A - C)*b)*d^3)*(f*x + e)/((a^2 + b^2)*c^6 + 3*(a^2 + b
^2)*c^4*d^2 + 3*(a^2 + b^2)*c^2*d^4 + (a^2 + b^2)*d^6) + 2*(C*a^2*b^2 - B*a
*b^3 + A*b^4)*log(b*tan(f*x + e) + a)/((a^2*b^3 + b^5)*c^3 - 3*(a^3*b^2 + a
*b^4)*c^2*d + 3*(a^4*b + a^2*b^3)*c*d^2 - (a^5 + a^3*b^2)*d^3) - 2*(C*b^2*c
^6 - 3*B*b^2*c^5*d + 3*B*a^2*c*d^5 + 3*(B*a*b + (2*A - C)*b^2)*c^4*d^2 - (B
*a^2 + 8*(A - C)*a*b - B*b^2)*c^3*d^3 + 3*((A - C)*a^2 - 2*B*a*b + A*b^2)*c
^2*d^4 - ((A - C)*a^2 + B*a*b - A*b^2)*d^6)*log(d*tan(f*x + e) + c)/(b^3*c^
9 - 3*a*b^2*c^8*d + 3*a^2*b*c*d^8 - a^3*d^9 + 3*(a^2*b + b^3)*c^7*d^2 - (a^
3 + 9*a*b^2)*c^6*d^3 + 3*(3*a^2*b + b^3)*c^5*d^4 - 3*(a^3 + 3*a*b^2)*c^4*d^
5 + (9*a^2*b + b^3)*c^3*d^6 - 3*(a^3 + a*b^2)*c^2*d^7) + ((B*a - (A - C)*b)
*c^3 - 3*((A - C)*a + B*b)*c^2*d - 3*(B*a - (A - C)*b)*c*d^2 + ((A - C)*a +
 B*b)*d^3)*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*c^6 + 3*(a^2 + b^2)*c^4*d^2
 + 3*(a^2 + b^2)*c^2*d^4 + (a^2 + b^2)*d^6) + (3*C*b*c^5 - A*a*d^5 - (C*a +
 5*B*b)*c^4*d + (3*B*a + (7*A - C)*b)*c^3*d^2 - ((5*A - 3*C)*a + B*b)*c^2*d
^3 - (B*a - 3*A*b)*c*d^4 + 2*(C*b*c^4*d - 2*B*b*c^3*d^2 - 2*(A - C)*a*c*d^4
 + (B*a + (3*A - C)*b)*c^2*d^3 - (B*a - A*b)*d^5)*tan(f*x + e))/(b^2*c^8 -
2*a*b*c^7*d - 4*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + a^2*c^2*d^6 + (a^2 + 2*b^2)*c
^6*d^2 + (2*a^2 + b^2)*c^4*d^4 + (b^2*c^6*d^2 - 2*a*b*c^5*d^3 - 4*a*b*c^3*d
^5 - 2*a*b*c*d^7 + a^2*d^8 + (a^2 + 2*b^2)*c^4*d^4 + (2*a^2 + b^2)*c^2*d^6)
*tan(f*x + e)^2 + 2*(b^2*c^7*d - 2*a*b*c^6*d^2 - 4*a*b*c^4*d^4 - 2*a*b*c^2*
d^6 + a^2*c*d^7 + (a^2 + 2*b^2)*c^5*d^3 + (2*a^2 + b^2)*c^3*d^5)*tan(f*x +
e)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3510 vs. 2(491) = 982.
time = 48.34, size = 3510, normalized size = 7.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*(C*a^2*b^2 + C*b^4)*c^6*d^2 - (8*C*a^3*b + 7*B*a^2*b^2 + 8*C*a*b^3 +
 7*B*b^4)*c^5*d^3 + (3*C*a^4 + 12*B*a^3*b + (9*A + 2*C)*a^2*b^2 + 12*B*a*b^
3 + (9*A - C)*b^4)*c^4*d^4 - (5*B*a^4 + 4*(4*A - C)*a^3*b + 6*B*a^2*b^2 + 4
```

*(4*A - C)*a*b^3 + B*b^4)*c^3*d^5 + ((7*A - 3*C)*a^4 + (10*A - 3*C)*a^2*b^2
 + 3*A*b^4)*c^2*d^6 + (B*a^4 - 4*A*a^3*b + B*a^2*b^2 - 4*A*a*b^3)*c*d^7 + (
A*a^4 + A*a^2*b^2)*d^8 + 2*(((A - C)*a*b^3 + B*b^4)*c^8 - 3*((A - C)*a^2*b^
2 + (A - C)*b^4)*c^7*d + 3*((A - C)*a^3*b - 2*B*a^2*b^2 + 2*(A - C)*a*b^3 -
 B*b^4)*c^6*d^2 - ((A - C)*a^4 - 8*B*a^3*b - 8*B*a*b^3 - (A - C)*b^4)*c^5*d
^3 - 3*(B*a^4 + 2*(A - C)*a^3*b + 2*B*a^2*b^2 + (A - C)*a*b^3)*c^4*d^4 + 3*
((A - C)*a^4 + (A - C)*a^2*b^2)*c^3*d^5 + (B*a^4 - (A - C)*a^3*b)*c^2*d^6)*
f*x - (3*(C*a^2*b^2 + C*b^4)*c^6*d^2 - (4*C*a^3*b + 5*B*a^2*b^2 + 4*C*a*b^3
 + 5*B*b^4)*c^5*d^3 + (C*a^4 + 8*B*a^3*b + (7*A - 2*C)*a^2*b^2 + 8*B*a*b^3
+ (7*A - 3*C)*b^4)*c^4*d^4 - (3*B*a^4 + 4*(3*A - 2*C)*a^3*b + 2*B*a^2*b^2 +
 4*(3*A - 2*C)*a*b^3 - B*b^4)*c^3*d^5 + (5*(A - C)*a^4 - 4*B*a^3*b + (6*A -
 5*C)*a^2*b^2 - 4*B*a*b^3 + A*b^4)*c^2*d^6 + 3*(B*a^4 + B*a^2*b^2)*c*d^7 -
(A*a^4 + A*a^2*b^2)*d^8 - 2*(((A - C)*a*b^3 + B*b^4)*c^6*d^2 - 3*((A - C)*a
^2*b^2 + (A - C)*b^4)*c^5*d^3 + 3*((A - C)*a^3*b - 2*B*a^2*b^2 + 2*(A - C)*
a*b^3 - B*b^4)*c^4*d^4 - ((A - C)*a^4 - 8*B*a^3*b - 8*B*a*b^3 - (A - C)*b^4
)*c^3*d^5 - 3*(B*a^4 + 2*(A - C)*a^3*b + 2*B*a^2*b^2 + (A - C)*a*b^3)*c^2*d
^6 + 3*((A - C)*a^4 + (A - C)*a^2*b^2)*c*d^7 + (B*a^4 - (A - C)*a^3*b)*d^8)
*f*x)*tan(f*x + e)^2 + ((C*a^2*b^2 - B*a*b^3 + A*b^4)*c^8 + 3*(C*a^2*b^2 -
B*a*b^3 + A*b^4)*c^6*d^2 + 3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c^4*d^4 + (C*a^2
*b^2 - B*a*b^3 + A*b^4)*c^2*d^6 + ((C*a^2*b^2 - B*a*b^3 + A*b^4)*c^6*d^2 +
3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c^4*d^4 + 3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c
^2*d^6 + (C*a^2*b^2 - B*a*b^3 + A*b^4)*d^8)*tan(f*x + e)^2 + 2*((C*a^2*b^2
- B*a*b^3 + A*b^4)*c^7*d + 3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c^5*d^3 + 3*(C*a
^2*b^2 - B*a*b^3 + A*b^4)*c^3*d^5 + (C*a^2*b^2 - B*a*b^3 + A*b^4)*c*d^7)*ta
n(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x +
e)^2 + 1)) - ((C*a^2*b^2 + C*b^4)*c^8 - 3*(B*a^2*b^2 + B*b^4)*c^7*d + 3*(B*
a^3*b + (2*A - C)*a^2*b^2 + B*a*b^3 + (2*A - C)*b^4)*c^6*d^2 - (B*a^4 + 8*(
A - C)*a^3*b + 8*(A - C)*a*b^3 - B*b^4)*c^5*d^3 + 3*((A - C)*a^4 - 2*B*a^3*
b + (2*A - C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c^4*d^4 + 3*(B*a^4 + B*a^2*b^2)*
c^3*d^5 - ((A - C)*a^4 + B*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c^2*d^6 + (
(C*a^2*b^2 + C*b^4)*c^6*d^2 - 3*(B*a^2*b^2 + B*b^4)*c^5*d^3 + 3*(B*a^3*b +
(2*A - C)*a^2*b^2 + B*a*b^3 + (2*A - C)*b^4)*c^4*d^4 - (B*a^4 + 8*(A - C)*a
^3*b + 8*(A - C)*a*b^3 - B*b^4)*c^3*d^5 + 3*((A - C)*a^4 - 2*B*a^3*b + (2*A
 - C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c^2*d^6 + 3*(B*a^4 + B*a^2*b^2)*c*d^7 -
((A - C)*a^4 + B*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*d^8)*tan(f*x + e)^2 +
 2*((C*a^2*b^2 + C*b^4)*c^7*d - 3*(B*a^2*b^2 + B*b^4)*c^6*d^2 + 3*(B*a^3*b
+ (2*A - C)*a^2*b^2 + B*a*b^3 + (2*A - C)*b^4)*c^5*d^3 - (B*a^4 + 8*(A - C)
*a^3*b + 8*(A - C)*a*b^3 - B*b^4)*c^4*d^4 + 3*((A - C)*a^4 - 2*B*a^3*b + (2
*A - C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c^3*d^5 + 3*(B*a^4 + B*a^2*b^2)*c^2*d^
6 - ((A - C)*a^4 + B*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d^7)*tan(f*x +
e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1
)) - 2*(2*(C*a^2*b^2 + C*b^4)*c^7*d - 3*(C*a^3*b + B*a^2*b^2 + C*a*b^3 + B*
b^4)*c^6*d^2 + (C*a^4 + 5*B*a^3*b + 2*(2*A - C)*a^2*b^2 + 5*B*a*b^3 + (4*A
- 3*C)*b^4)*c^5*d^3 - (2*B*a^4 + (7*A - 6*C)*a^3*b - B*a^2*b^2 + (7*A - 6*C
)*a*b^3 - 3*B*b^4)*c^4*d^4 + (3*(A - C)*a^4 - 6*B*a^3*b - 2*C*a^2*b^2 - 6*B

```
*a*b^3 - (3*A - C)*b^4)*c^3*d^5 + 3*(B*a^4 + (2*A - C)*a^3*b + B*a^2*b^2 +
(2*A - C)*a*b^3)*c^2*d^6 - ((3*A - 2*C)*a^4 - B*a^3*b + 2*(2*A - C)*a^2*b^2
 - B*a*b^3 + A*b^4)*c*d^7 - (B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*d^8 - 2
*(((A - C)*a*b^3 + B*b^4)*c^7*d - 3*((A - C)*a^2*b^2 + (A - C)*b^4)*c^6*d^2
 + 3*((A - C)*a^3*b - 2*B*a^2*b^2 + 2*(A - C)*a*b^3 - B*b^4)*c^5*d^3 - ((A
- C)*a^4 - 8*B*a^3*b - 8*B*a*b^3 - (A - C)*b^4)*c^4*d^4 - 3*(B*a^4 + 2*(A -
 C)*a^3*b + 2*B*a^2*b^2 + (A - C)*a*b^3)*c^3*d^5 + 3*((A - C)*a^4 + (A - C)
*a^2*b^2)*c^2*d^6 + (B*a^4 - (A - C)*a^3*b)*c*d^7)*f*x)*tan(f*x + e))/(((a^
2*b^3 + b^5)*c^9*d^2 - 3*(a^3*b^2 + a*b^4)*c^8*d^3 + 3*(a^4*b + 2*a^2*b^3 +
 b^5)*c^7*d^4 - (a^5 + 10*a^3*b^2 + 9*a*b^4)*c^6*d^5 + 3*(3*a^4*b + 4*a^2*b
^3 + b^5)*c^5*d^6 - 3*(a^5 + 4*a^3*b^2 + 3*a*b^4)*c^4*d^7 + (9*a^4*b + 10*a
^2*b^3 + b^5)*c^3*d^8 - 3*(a^5 + 2*a^3*b^2 + a*b^4)*c^2*d^9 + 3*(a^4*b + a^
2*b^3)*c*d^10 - (a^5 + a^3*b^2)*d^11)*f*tan(f*x + e)^2 + 2*((a^2*b^3 + b^5)
*c^10*d - 3*(a^3*b^2 + a*b^4)*c^9*d^2 + 3*(a^4*b + 2*a^2*b^3 + b^5)*c^8*d^3
 - (a^5 + 10*a^3*b^2 + 9*a*b^4)*c^7*d^4 + 3*(3*a^4*b + 4*a^2*b^3 + b^5)*c^6
*d^5 - 3*(a^5 + 4*a^3*b^2 + 3*a*b^4)*c^5*d^6 + (9*a^4*b + 10*a^2*b^3 + b^5)
*c^4*d^7 - 3*(a^5 + 2*a^3*b^2 + a*b^4)*c^3*d^8 + 3*(a^4*b + a^2*b^3)*c^2*d^
9 - (a^5 + a^3*b^2)*c*d^10)*f*tan(f*x + e) + ((a^2*b^3 + b^5)*c^11 - 3*(a^3
*b^2 + a*b^4)*c^10*d + 3*(a^4*b + 2*a^2*b^3 + b...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)**3,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2125 vs. 2(491) = 982.
time = 1.19, size = 2125, normalized size = 4.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a*c^3 - C*a*c^3 + B*b*c^3 + 3*B*a*c^2*d - 3*A*b*c^2*d + 3*C*b*c^2
*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 - 3*B*b*c*d^2 - B*a*d^3 + A*b*d^3 - C*b*d^3)
*(f*x + e)/(a^2*c^6 + b^2*c^6 + 3*a^2*c^4*d^2 + 3*b^2*c^4*d^2 + 3*a^2*c^2*d
^4 + 3*b^2*c^2*d^4 + a^2*d^6 + b^2*d^6) + (B*a*c^3 - A*b*c^3 + C*b*c^3 - 3*
A*a*c^2*d + 3*C*a*c^2*d - 3*B*b*c^2*d - 3*B*a*c*d^2 + 3*A*b*c*d^2 - 3*C*b*c
```

```
*d^2 + A*a*d^3 - C*a*d^3 + B*b*d^3)*log(tan(f*x + e)^2 + 1)/(a^2*c^6 + b^2*
c^6 + 3*a^2*c^4*d^2 + 3*b^2*c^4*d^2 + 3*a^2*c^2*d^4 + 3*b^2*c^2*d^4 + a^2*d
^6 + b^2*d^6) + 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*log(abs(b*tan(f*x + e) + a)
)/(a^2*b^4*c^3 + b^6*c^3 - 3*a^3*b^3*c^2*d - 3*a*b^5*c^2*d + 3*a^4*b^2*c*d^
2 + 3*a^2*b^4*c*d^2 - a^5*b*d^3 - a^3*b^3*d^3) - 2*(C*b^2*c^6*d - 3*B*b^2*c
^5*d^2 + 3*B*a*b*c^4*d^3 + 6*A*b^2*c^4*d^3 - 3*C*b^2*c^4*d^3 - B*a^2*c^3*d^
4 - 8*A*a*b*c^3*d^4 + 8*C*a*b*c^3*d^4 + B*b^2*c^3*d^4 + 3*A*a^2*c^2*d^5 - 3
*C*a^2*c^2*d^5 - 6*B*a*b*c^2*d^5 + 3*A*b^2*c^2*d^5 + 3*B*a^2*c*d^6 - A*a^2*
d^7 + C*a^2*d^7 - B*a*b*d^7 + A*b^2*d^7)*log(abs(d*tan(f*x + e) + c))/(b^3*
c^9*d - 3*a*b^2*c^8*d^2 + 3*a^2*b*c^7*d^3 + 3*b^3*c^7*d^3 - a^3*c^6*d^4 - 9
*a*b^2*c^6*d^4 + 9*a^2*b*c^5*d^5 + 3*b^3*c^5*d^5 - 3*a^3*c^4*d^6 - 9*a*b^2*
c^4*d^6 + 9*a^2*b*c^3*d^7 + b^3*c^3*d^7 - 3*a^3*c^2*d^8 - 3*a*b^2*c^2*d^8 +
 3*a^2*b*c*d^9 - a^3*d^10) + (3*C*b^2*c^6*d^2*tan(f*x + e)^2 - 9*B*b^2*c^5*
d^3*tan(f*x + e)^2 + 9*B*a*b*c^4*d^4*tan(f*x + e)^2 + 18*A*b^2*c^4*d^4*tan(
f*x + e)^2 - 9*C*b^2*c^4*d^4*tan(f*x + e)^2 - 3*B*a^2*c^3*d^5*tan(f*x + e)^
2 - 24*A*a*b*c^3*d^5*tan(f*x + e)^2 + 24*C*a*b*c^3*d^5*tan(f*x + e)^2 + 3*B
*b^2*c^3*d^5*tan(f*x + e)^2 + 9*A*a^2*c^2*d^6*tan(f*x + e)^2 - 9*C*a^2*c^2*
d^6*tan(f*x + e)^2 - 18*B*a*b*c^2*d^6*tan(f*x + e)^2 + 9*A*b^2*c^2*d^6*tan(
f*x + e)^2 + 9*B*a^2*c*d^7*tan(f*x + e)^2 - 3*A*a^2*d^8*tan(f*x + e)^2 + 3*
C*a^2*d^8*tan(f*x + e)^2 - 3*B*a*b*d^8*tan(f*x + e)^2 + 3*A*b^2*d^8*tan(f*x
 + e)^2 + 8*C*b^2*c^7*d*tan(f*x + e) - 2*C*a*b*c^6*d^2*tan(f*x + e) - 22*B*
b^2*c^6*d^2*tan(f*x + e) + 24*B*a*b*c^5*d^3*tan(f*x + e) + 42*A*b^2*c^5*d^3
*tan(f*x + e) - 18*C*b^2*c^5*d^3*tan(f*x + e) - 8*B*a^2*c^4*d^4*tan(f*x + e
) - 58*A*a*b*c^4*d^4*tan(f*x + e) + 52*C*a*b*c^4*d^4*tan(f*x + e) + 2*B*b^2
*c^4*d^4*tan(f*x + e) + 22*A*a^2*c^3*d^5*tan(f*x + e) - 22*C*a^2*c^3*d^5*ta
n(f*x + e) - 32*B*a*b*c^3*d^5*tan(f*x + e) + 26*A*b^2*c^3*d^5*tan(f*x + e)
- 2*C*b^2*c^3*d^5*tan(f*x + e) + 18*B*a^2*c^2*d^6*tan(f*x + e) - 12*A*a*b*c
^2*d^6*tan(f*x + e) + 6*C*a*b*c^2*d^6*tan(f*x + e) - 2*A*a^2*c*d^7*tan(f*x
+ e) + 2*C*a^2*c*d^7*tan(f*x + e) - 8*B*a*b*c*d^7*tan(f*x + e) + 8*A*b^2*c*
d^7*tan(f*x + e) + 2*B*a^2*d^8*tan(f*x + e) - 2*A*a*b*d^8*tan(f*x + e) + 6*
C*b^2*c^8 - 4*C*a*b*c^7*d - 14*B*b^2*c^7*d + C*a^2*c^6*d^2 + 17*B*a*b*c^6*d
^2 + 25*A*b^2*c^6*d^2 - 7*C*b^2*c^6*d^2 - 6*B*a^2*c^5*d^3 - 36*A*a*b*c^5*d^
3 + 24*C*a*b*c^5*d^3 - 3*B*b^2*c^5*d^3 + 14*A*a^2*c^4*d^4 - 11*C*a^2*c^4*d^
4 - 10*B*a*b*c^4*d^4 + 19*A*b^2*c^4*d^4 - C*b^2*c^4*d^4 + 7*B*a^2*c^3*d^5 -
 16*A*a*b*c^3*d^5 + 4*C*a*b*c^3*d^5 - B*b^2*c^3*d^5 + 3*A*a^2*c^2*d^6 - 3*B
*a*b*c^2*d^6 + 6*A*b^2*c^2*d^6 + B*a^2*c*d^7 - 4*A*a*b*c*d^7 + A*a^2*d^8)/(
(b^3*c^9 - 3*a*b^2*c^8*d + 3*a^2*b*c^7*d^2 + 3*b^3*c^7*d^2 - a^3*c^6*d^3 -
9*a*b^2*c^6*d^3 + 9*a^2*b*c^5*d^4 + 3*b^3*c^5*d^4 - 3*a^3*c^4*d^5 - 9*a*b^2
*c^4*d^5 + 9*a^2*b*c^3*d^6 + b^3*c^3*d^6 - 3*a^3*c^2*d^7 - 3*a*b^2*c^2*d^7
+ 3*a^2*b*c*d^8 - a^3*d^9)*(d*tan(f*x + e) + c)^2))/f
```

Mupad [B]
time = 24.61, size = 2500, normalized size = 5.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*ta
n(e + f*x))^3),x)
```

```
[Out] (symsum(log(- root(480*a^9*b*c^7*d^11*f^4 + 480*a*b^9*c^11*d^7*f^4 + 360*a^
9*b*c^9*d^9*f^4 + 360*a^9*b*c^5*d^13*f^4 + 360*a*b^9*c^13*d^5*f^4 + 360*a*b
^9*c^9*d^9*f^4 + 144*a^9*b*c^11*d^7*f^4 + 144*a^9*b*c^3*d^15*f^4 + 144*a*b^
9*c^15*d^3*f^4 + 144*a*b^9*c^7*d^11*f^4 + 48*a^7*b^3*c*d^17*f^4 + 48*a^3*b^
7*c^17*d*f^4 + 24*a^9*b*c^13*d^5*f^4 + 24*a^5*b^5*c^17*d*f^4 + 24*a^5*b^5*c
*d^17*f^4 + 24*a*b^9*c^5*d^13*f^4 + 24*a^9*b*c*d^17*f^4 + 24*a*b^9*c^17*d*f
^4 + 3920*a^5*b^5*c^9*d^9*f^4 - 3360*a^6*b^4*c^8*d^10*f^4 - 3360*a^4*b^6*c^
10*d^8*f^4 - 3024*a^6*b^4*c^10*d^8*f^4 + 3024*a^5*b^5*c^11*d^7*f^4 + 3024*a
^5*b^5*c^7*d^11*f^4 - 3024*a^4*b^6*c^8*d^10*f^4 + 2320*a^7*b^3*c^9*d^9*f^4
+ 2320*a^3*b^7*c^9*d^9*f^4 - 2240*a^6*b^4*c^6*d^12*f^4 - 2240*a^4*b^6*c^12*
d^6*f^4 + 2160*a^7*b^3*c^7*d^11*f^4 + 2160*a^3*b^7*c^11*d^7*f^4 - 1624*a^6*
b^4*c^12*d^6*f^4 - 1624*a^4*b^6*c^6*d^12*f^4 + 1488*a^7*b^3*c^11*d^7*f^4 +
1488*a^3*b^7*c^7*d^11*f^4 + 1344*a^5*b^5*c^13*d^5*f^4 + 1344*a^5*b^5*c^5*d^
13*f^4 - 1320*a^8*b^2*c^8*d^10*f^4 - 1320*a^2*b^8*c^10*d^8*f^4 + 1200*a^7*b
^3*c^5*d^13*f^4 + 1200*a^3*b^7*c^13*d^5*f^4 - 1060*a^8*b^2*c^6*d^12*f^4 - 1
060*a^2*b^8*c^12*d^6*f^4 - 948*a^8*b^2*c^10*d^8*f^4 - 948*a^2*b^8*c^8*d^10*
f^4 - 840*a^6*b^4*c^4*d^14*f^4 - 840*a^4*b^6*c^14*d^4*f^4 + 528*a^7*b^3*c^1
3*d^5*f^4 + 528*a^3*b^7*c^5*d^13*f^4 - 480*a^8*b^2*c^4*d^14*f^4 - 480*a^6*b
^4*c^14*d^4*f^4 - 480*a^4*b^6*c^4*d^14*f^4 - 480*a^2*b^8*c^14*d^4*f^4 - 368
*a^8*b^2*c^12*d^6*f^4 + 368*a^7*b^3*c^3*d^15*f^4 + 368*a^3*b^7*c^15*d^3*f^4
 - 368*a^2*b^8*c^6*d^12*f^4 + 304*a^5*b^5*c^15*d^3*f^4 + 304*a^5*b^5*c^3*d^
15*f^4 - 144*a^6*b^4*c^2*d^16*f^4 - 144*a^4*b^6*c^16*d^2*f^4 - 108*a^8*b^2*
c^2*d^16*f^4 - 108*a^2*b^8*c^16*d^2*f^4 + 80*a^7*b^3*c^15*d^3*f^4 + 80*a^3*
b^7*c^3*d^15*f^4 - 60*a^8*b^2*c^14*d^4*f^4 - 60*a^6*b^4*c^16*d^2*f^4 - 60*a
^4*b^6*c^2*d^16*f^4 - 60*a^2*b^8*c^4*d^14*f^4 - 80*b^10*c^12*d^6*f^4 - 60*b
^10*c^14*d^4*f^4 - 60*b^10*c^10*d^8*f^4 - 24*b^10*c^16*d^2*f^4 - 24*b^10*c^
8*d^10*f^4 - 4*b^10*c^6*d^12*f^4 - 80*a^10*c^6*d^12*f^4 - 60*a^10*c^8*d^10*
f^4 - 60*a^10*c^4*d^14*f^4 - 24*a^10*c^10*d^8*f^4 - 24*a^10*c^2*d^16*f^4 -
4*a^10*c^12*d^6*f^4 - 8*a^8*b^2*d^18*f^4 - 4*a^6*b^4*d^18*f^4 - 8*a^2*b^8*c
^18*f^4 - 4*a^4*b^6*c^18*f^4 - 4*b^10*c^18*f^4 - 4*a^10*d^18*f^4 - 12*A*C*a
^7*b*c*d^11*f^2 - 12*A*C*a*b^7*c^11*d*f^2 - 912*B*C*a^4*b^4*c^5*d^7*f^2 + 7
92*B*C*a^5*b^3*c^4*d^8*f^2 - 792*B*C*a^3*b^5*c^8*d^4*f^2 + 720*B*C*a^4*b^4*
c^7*d^5*f^2 - 480*B*C*a^6*b^2*c^5*d^7*f^2 - 408*B*C*a^2*b^6*c^5*d^7*f^2 + 3
84*B*C*a^2*b^6*c^7*d^5*f^2 - 336*B*C*a^5*b^3*c^8*d^4*f^2 + 324*B*C*a^3*b^5*
c^4*d^8*f^2 + 312*B*C*a^6*b^2*c^7*d^5*f^2 - 248*B*C*a^6*b^2*c^3*d^9*f^2 + 2
16*B*C*a^2*b^6*c^9*d^3*f^2 - 196*B*C*a^4*b^4*c^3*d^9*f^2 + 132*B*C*a^4*b^4*
c^9*d^3*f^2 + 80*B*C*a^3*b^5*c^6*d^6*f^2 - 64*B*C*a^5*b^3*c^6*d^6*f^2 - 36*
B*C*a^3*b^5*c^2*d^10*f^2 - 28*B*C*a^2*b^6*c^3*d^9*f^2 + 12*B*C*a^5*b^3*c^10
*d^2*f^2 - 12*B*C*a^5*b^3*c^2*d^10*f^2 - 12*B*C*a^3*b^5*c^10*d^2*f^2 - 4*B*
C*a^6*b^2*c^9*d^3*f^2 - 1468*A*C*a^4*b^4*c^6*d^6*f^2 + 996*A*C*a^3*b^5*c^7*
```

d^5*f^2 + 900*A*C*a^5*b^3*c^5*d^7*f^2 - 676*A*C*a^6*b^2*c^6*d^6*f^2 - 660*A
*C*a^2*b^6*c^6*d^6*f^2 + 636*A*C*a^3*b^5*c^5*d^7*f^2 + 540*A*C*a^5*b^3*c^7*
d^5*f^2 - 236*A*C*a^5*b^3*c^3*d^9*f^2 - 204*A*C*a^3*b^5*c^9*d^3*f^2 + 156*A
*C*a^2*b^6*c^10*d^2*f^2 + 132*A*C*a^6*b^2*c^2*d^10*f^2 - 72*A*C*a^6*b^2*c^4
*d^8*f^2 - 72*A*C*a^5*b^3*c^9*d^3*f^2 + 66*A*C*a^2*b^6*c^4*d^8*f^2 + 54*A*C
*a^4*b^4*c^10*d^2*f^2 + 54*A*C*a^4*b^4*c^2*d^10*f^2 - 48*A*C*a^4*b^4*c^4*d^
8*f^2 - 48*A*C*a^2*b^6*c^8*d^4*f^2 + 42*A*C*a^6*b^2*c^8*d^4*f^2 - 40*A*C*a^
3*b^5*c^3*d^9*f^2 - 36*A*C*a^4*b^4*c^8*d^4*f^2 + 24*A*C*a^2*b^6*c^2*d^10*f^
2 + 960*A*B*a^4*b^4*c^5*d^7*f^2 - 864*A*B*a^5*b^3*c^4*d^8*f^2 + 756*A*B*a^3
*b^5*c^8*d^4*f^2 - 744*A*B*a^4*b^4*c^7*d^5*f^2 - 528*A*B*a^3*b^5*c^4*d^8*f^
2 + 504*A*B*a^6*b^2*c^5*d^7*f^2 - 432*A*B*a^2*b^6*c^7*d^5*f^2 + 432*A*B*a^2
*b^6*c^5*d^7*f^2 + 348*A*B*a^5*b^3*c^8*d^4*f^2 - 312*A*B*a^6*b^2*c^7*d^5*f^
2 - 284*A*B*a^2*b^6*c^9*d^3*f^2 + 280*A*B*a^6*b^2*c^3*d^9*f^2 + 264*A*B*a^4
*b^4*c^3*d^9*f^2 - 240*A*B*a^3*b^5*c^6*d^6*f^2 - 172*A*B*a^4*b^4*c^9*d^3*f^
2 + 68*A*B*a^2*b^6*c^3*d^9*f^2 - 60*A*B*a^3*b^5*c^2*d^10*f^2 + 24*A*B*a^5*b
^3*c^6*d^6*f^2 - 24*A*B*a^5*b^3*c^2*d^10*f^2 + 12*A*B*a^3*b^5*c^10*d^2*f^2
+ 360*B*C*a^7*b*c^4*d^8*f^2 - 336*B*C*a*b^7*c^8*d^4*f^2 + 168*B*C*a*b^7*c^6
*d^6*f^2 - 136*B*C*a^7*b*c^6*d^6*f^2 + 36*B*C*a^6*b^2*c*d^11*f^2 - 36*B*C*a
^2*b^6*c^11*d*f^2 - 24*B*C*a^7*b*c^2*d^10*f^2 + 24*B*C*a*b^7*c^10*d^2*f^2 -
12*B*C*a^4*b^4*c^11*d*f^2 + 12*B*C*a^4*b^4*c*d^11*f^2 + 12*B*C*a*b^7*c^4*d
^8*f^2 + 444*A*C*a*b^7*c^7*d^5*f^2 + 348*A*C*a^7*b*c^5*d^7*f^2 - 164*A*C*a^
7*b*c^3*d^9*f^2 - 132*A*C*a*b^7*c^9*d^3*f^2 + 84*A*C*a*b^7*c^5*d^7*f^2 + 32
*A*C*a*b^7*c^3*d^9*f^2 - 12*A*C*a^7*b*c^7*d^5*f^2 - 12*A*C*a^5*b^3*c*d^11*f
^2 - 12*A*C*a^3*b^5*c^11*d*f^2 - 360*A*B*a^7*b*c^4*d^8*f^2 + 288*A*B*a*b^7*
c^8*d^4*f^2 - 288*A*B*a*b^7*c^6*d^6*f^2 - 144*A*B*a*b^7*c^4*d^8*f^2 + 136*A
*B*a^7*b*c^6*d^6*f^2 - 60*A*B*a*b^7*c^2*d^10*f^2 - 36*A*B*a*b^7*c^10*d^2*f^
2 + 24*A*B*a^7*b*c^2*d^10*f^2 - 24*A*B*a^6*b^2*...

3.89 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=861

$$-\frac{(b^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)+a^2(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))+2a}{(a^2+b^2)^2(c^2+d^2)^3}$$

```
[Out] -(b^2*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)+a^2*(c^3*C-3*B*c^2*
d-3*c*C*d^2+B*d^3-A*(c^3-3*c*d^2))+2*a*b*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^
2)))*x/(a^2+b^2)^2/(c^2+d^2)^3+b^2*(4*a^3*b*B*d-3*a^4*C*d+b^4*(-3*A*d+B*c)+
2*a*b^3*(A*c+B*d-C*c)-a^2*b^2*(B*c+(5*A+C)*d))*ln(a*cos(f*x+e)+b*sin(f*x+e)
)/(a^2+b^2)^2/(-a*d+b*c)^4/f+d*(b^2*(3*c^6*C-6*B*c^5*d+c^4*(10*A-C)*d^2-3*B
*c^3*d^3+9*A*c^2*d^4-B*c*d^5+3*A*d^6)+a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3
*c*d^2))-2*a*b*d^2*(c*(A-C)*d*(5*c^2+d^2)-B*(2*c^4-3*c^2*d^2-d^4)))*ln(c*co
s(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^4/(c^2+d^2)^3/f-1/2*d*(b^2*c*(-B*d+C*c)-2
*a*b*B*(c^2+d^2)+a^2*(-B*c*d+3*C*c^2+2*C*d^2)+A*(a^2*d^2+b^2*(2*c^2+3*d^2))
)/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(-A*b^2+a*(B*b-C*a)
)/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2-d*(b^3*c*(-3*B
*c^2*d-B*d^3+2*C*c^3)+a^2*b*(-3*B*c^3*d-B*c*d^3+3*C*c^4+2*C*c^2*d^2+C*d^4)+
a^3*d^2*(2*c*C*d+B*(c^2-d^2))+a*b^2*(2*c*C*d^3-B*(c^4+c^2*d^2+2*d^4))-A*(2*
a^3*c*d^3+2*a*b^2*c*d^3-2*a^2*b*d^2*(2*c^2+d^2)-b^3*(c^4+6*c^2*d^2+3*d^4)))
/(a^2+b^2)/(-a*d+b*c)^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 2.89, antiderivative size = 860, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3730, 3732, 3611}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^3),x]
```

```
[Out] -(((b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a^2*(
c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) + 2*a*b*((A - C)
*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/((a^2 + b^2)^2*(c^2 + d^2)^3)) +
(b^2*(4*a^3*b*B*d - 3*a^4*C*d + b^4*(B*c - 3*A*d) + 2*a*b^3*(A*c - c*C + B*
d) - a^2*b^2*(B*c + (5*A + C)*d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a
^2 + b^2)^2*(b*c - a*d)^4*f) + (d*(b^2*(3*c^6*C - 6*B*c^5*d + c^4*(10*A - C
)*d^2 - 3*B*c^3*d^3 + 9*A*c^2*d^4 - B*c*d^5 + 3*A*d^6) + a^2*d^3*((A - C)*d
*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*d^2*(c*(A - C)*d*(5*c^2 + d^2)
- B*(2*c^4 - 3*c^2*d^2 - d^4)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c
 - a*d)^4*(c^2 + d^2)^3*f) - (d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - 2*a*b*B*(c
```

```
^2 + d^2) + A*b^2*(2*c^2 + 3*d^2) + a^2*(3*c^2*C - B*c*d + 2*C*d^2)))/(2*(a
^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (A*b^2 - a*
(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e +
 f*x])^2) - (d*(b^3*c*(2*c^3*C - 3*B*c^2*d - B*d^3) + a^2*b*(3*c^4*C - 3*B*
c^3*d + 2*c^2*C*d^2 - B*c*d^3 + C*d^4) + a^3*d^2*(2*c*C*d + B*(c^2 - d^2))
+ a*b^2*(2*c*C*d^3 - B*(c^4 + c^2*d^2 + 2*d^4)) - A*(2*a^3*c*d^3 + 2*a*b^2*
c*d^3 - 2*a^2*b*d^2*(2*c^2 + d^2) - b^3*(c^4 + 6*c^2*d^2 + 3*d^4))))/((a^2
+ b^2)*(b*c - a*d)^3*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = -\frac{Ab^2-a(bB-aC)}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx$$

$$= -\frac{d(a^2Ad^2+b^2c(cC-Bd)-2abB(c^2+d^2)+Ab^2(2c^2+3}{2\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d}$$

$$= -\frac{d(a^2Ad^2+b^2c(cC-Bd)-2abB(c^2+d^2)+Ab^2(2c^2+3}{2\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+d}$$

$$= -\frac{(b^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)+a^2(c}{}$$

$$= -\frac{(b^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)+a^2(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1732 vs. $2(861) = 1722$.
time = 7.75, size = 1732, normalized size = 2.01

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^3),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(
c + d*Tan[e + f*x])^2)) - (-1/2*(-(c*(-3*c*(A*b^2 - a*(b*B - a*C))*d + (A*b
 - a*B - b*C)*d*(b*c - a*d))) + d^2*(3*A*b^2*d - a*A*(b*c - a*d) - (b*B - a
*C)*(b*c + 2*a*d)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) -
 (-((-(((b*c - a*d)^3*(-(b^2*(2*a*A*b*c^3 - a^2*B*c^3 + b^2*B*c^3 - 2*a*b*c
^3*C + 3*a^2*A*c^2*d - 3*A*b^2*c^2*d + 6*a*b*B*c^2*d - 3*a^2*c^2*C*d + 3*b^
2*c^2*C*d - 6*a*A*b*c*d^2 + 3*a^2*B*c*d^2 - 3*b^2*B*c*d^2 + 6*a*b*c*C*d^2 -
 a^2*A*d^3 + A*b^2*d^3 - 2*a*b*B*d^3 + a^2*C*d^3 - b^2*C*d^3)) + Sqrt[-b^2]
*(-(a^2*A*b*c^3) + A*b^3*c^3 - 2*a*b^2*B*c^3 + a^2*b*c^3*C - b^3*c^3*C + 6*
a*A*b^2*c^2*d - 3*a^2*b*B*c^2*d + 3*b^3*B*c^2*d - 6*a*b^2*c^2*C*d + 3*a^2*A
*b*c*d^2 - 3*A*b^3*c*d^2 + 6*a*b^2*B*c*d^2 - 3*a^2*b*c*C*d^2 + 3*b^3*c*C*d^
2 - 2*a*A*b^2*d^3 + a^2*b*B*d^3 - b^3*B*d^3 + 2*a*b^2*C*d^3))*Log[Sqrt[-b^2
] - b*Tan[e + f*x]])/(b*(a^2 + b^2)*(c^2 + d^2))) - (2*b^3*(c^2 + d^2)^2*(4
*a^3*b*B*d - 3*a^4*C*d + b^4*(B*c - 3*A*d) + 2*a*b^3*(A*c - c*C + B*d) - a^
2*b^2*(B*c + (5*A + C)*d))*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d
)) + ((b*c - a*d)^3*(b^2*(2*a*A*b*c^3 - a^2*B*c^3 + b^2*B*c^3 - 2*a*b*c^3*C
 + 3*a^2*A*c^2*d - 3*A*b^2*c^2*d + 6*a*b*B*c^2*d - 3*a^2*c^2*C*d + 3*b^2*c^
2*C*d - 6*a*A*b*c*d^2 + 3*a^2*B*c*d^2 - 3*b^2*B*c*d^2 + 6*a*b*c*C*d^2 - a^2
```

```
*A*d^3 + A*b^2*d^3 - 2*a*b*B*d^3 + a^2*C*d^3 - b^2*C*d^3) + Sqrt[-b^2]*(-(a
^2*A*b*c^3) + A*b^3*c^3 - 2*a*b^2*B*c^3 + a^2*b*c^3*C - b^3*c^3*C + 6*a*A*b
^2*c^2*d - 3*a^2*b*B*c^2*d + 3*b^3*B*c^2*d - 6*a*b^2*c^2*C*d + 3*a^2*A*b*c*
d^2 - 3*A*b^3*c*d^2 + 6*a*b^2*B*c*d^2 - 3*a^2*b*c*C*d^2 + 3*b^3*c*C*d^2 - 2
*a*A*b^2*d^3 + a^2*b*B*d^3 - b^3*B*d^3 + 2*a*b^2*C*d^3))*Log[Sqrt[-b^2] + b
*Tan[e + f*x]])/(b*(a^2 + b^2)*(c^2 + d^2)) - (2*b*(a^2 + b^2)*d*(b^2*(3*c^
6*C - 6*B*c^5*d + c^4*(10*A - C)*d^2 - 3*B*c^3*d^3 + 9*A*c^2*d^4 - B*c*d^5
+ 3*A*d^6) + a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*
d^2*(c*(A - C)*d*(5*c^2 + d^2) - B*(2*c^4 - 3*c^2*d^2 - d^4)))*Log[c + d*Ta
n[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)*(c^2 + d^2)*f)) -
 (d^2*(-2*a*d*(-3*c*(A*b^2 - a*(b*B - a*C))*d + (A*b - a*B - b*C)*d*(b*c -
a*d)) + (2*b*d^2 - 2*c*(-(b*c) + a*d))*(3*A*b^2*d - a*A*(b*c - a*d) - (b*B
- a*C)*(b*c + 2*a*d))) - c*(2*d*(-(b*c) + a*d)*(-3*(A*b^2 - a*(b*B - a*C))*
d^2 - c*(A*b - a*B - b*C)*(b*c - a*d) + d*(3*A*b^2*d - a*A*(b*c - a*d) - (b
*B - a*C)*(b*c + 2*a*d))) - 2*b*c*(-(c*(-3*c*(A*b^2 - a*(b*B - a*C))*d + (A
*b - a*B - b*C)*d*(b*c - a*d))) + d^2*(3*A*b^2*d - a*A*(b*c - a*d) - (b*B -
 a*C)*(b*c + 2*a*d)))))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
)/(2*(-(b*c) + a*d)*(c^2 + d^2)))/((a^2 + b^2)*(b*c - a*d))
```

Maple [A]
time = 3.47, size = 949, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{b^2\left(5A a^2b^2d-2Aa b^3c+3A b^4d-4a^3bBd+B a^2b^2c-2Ba b^3d-B b^4c+3a^4Cd+C a^2b^2d+2Ca b^3c\right)\ln(a+b\tan(fx+e))}{(ad-bc)^4\left(a^2+b^2\right)^2}+\frac{\left(A b^2 \cdots\right.}{(ad-bc)^3\left(a \cdots\right.}}{\cdots}$
default	$\frac{-\frac{b^2\left(5A a^2b^2d-2Aa b^3c+3A b^4d-4a^3bBd+B a^2b^2c-2Ba b^3d-B b^4c+3a^4Cd+C a^2b^2d+2Ca b^3c\right)\ln(a+b\tan(fx+e))}{(ad-bc)^4\left(a^2+b^2\right)^2}+\frac{\left(A b^2 \cdots\right.}{(ad-bc)^3\left(a \cdots\right.}}{\cdots}$
norman	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,x
,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-b^2*(5*A*a^2*b^2*d-2*A*a*b^3*c+3*A*b^4*d-4*B*a^3*b*d+B*a^2*b^2*c-2*B*
a*b^3*d-B*b^4*c+3*C*a^4*d+C*a^2*b^2*d+2*C*a*b^3*c)/(a*d-b*c)^4/(a^2+b^2)^2*
ln(a+b*tan(f*x+e))+(A*b^2-B*a*b+C*a^2)*b^2/(a*d-b*c)^3/(a^2+b^2)/(a+b*tan(f
*x+e))+1/(a^2+b^2)^2/(c^2+d^2)^3*(1/2*(-3*A*a^2*c^2*d+A*a^2*d^3-2*A*a*b*c^3
+6*A*a*b*c*d^2+3*A*b^2*c^2*d-A*b^2*d^3+B*a^2*c^3-3*B*a^2*c*d^2-6*B*a*b*c^2*
d+2*B*a*b*d^3-B*b^2*c^3+3*B*b^2*c*d^2+3*C*a^2*c^2*d-C*a^2*d^3+2*C*a*b*c^3-6
*C*a*b*c*d^2-3*C*b^2*c^2*d+C*b^2*d^3)*ln(1+tan(f*x+e)^2)+(A*a^2*c^3-3*A*a^2
*c*d^2-6*A*a*b*c^2*d+2*A*a*b*d^3-A*b^2*c^3+3*A*b^2*c*d^2+3*B*a^2*c^2*d-B*a^
2*d^3+2*B*a*b*c^3-6*B*a*b*c*d^2-3*B*b^2*c^2*d+B*b^2*d^3-C*a^2*c^3+3*C*a^2*c
```

```
*d^2+6*C*a*b*c^2*d-2*C*a*b*d^3+C*b^2*c^3-3*C*b^2*c*d^2)*arctan(tan(f*x+e)))
-d*(2*A*a*c*d^3-4*A*b*c^2*d^2-2*A*b*d^4-B*a*c^2*d^2+B*a*d^4+3*B*b*c^3*d+B*b
*c*d^3-2*C*a*c*d^3-2*C*b*c^4)/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(f*x+e))+d*(3
*A*a^2*c^2*d^4-A*a^2*d^6-10*A*a*b*c^3*d^3-2*A*a*b*c*d^5+10*A*b^2*c^4*d^2+9*
A*b^2*c^2*d^4+3*A*b^2*d^6-B*a^2*c^3*d^3+3*B*a^2*c*d^5+4*B*a*b*c^4*d^2-6*B*a
*b*c^2*d^4-2*B*a*b*d^6-6*B*b^2*c^5*d-3*B*b^2*c^3*d^3-B*b^2*c*d^5-3*C*a^2*c^
2*d^4+C*a^2*d^6+10*C*a*b*c^3*d^3+2*C*a*b*c*d^5+3*C*b^2*c^6-C*b^2*c^4*d^2)/(
a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))-1/2*(A*d^2-B*c*d+C*c^2)*d/(a*d-b*
c)^2/(c^2+d^2)/(c+d*tan(f*x+e))^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2546 vs. $2(870) = 1740$.
time = 0.71, size = 2546, normalized size = 2.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^3 + 3*(B*a^2 - 2*(A - C)*a*
b - B*b^2)*c^2*d - 3*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 -
 2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*(a^
4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4 + 2
*a^2*b^2 + b^4)*d^6) - 2*((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c + (3*C*a^
4*b^2 - 4*B*a^3*b^3 + (5*A + C)*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6)*d)*log(b*tan
(f*x + e) + a)/((a^4*b^4 + 2*a^2*b^6 + b^8)*c^4 - 4*(a^5*b^3 + 2*a^3*b^5 +
a*b^7)*c^3*d + 6*(a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*c^2*d^2 - 4*(a^7*b + 2*a^5
*b^3 + a^3*b^5)*c*d^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*d^4) + 2*(3*C*b^2*c^6*d
 - 6*B*b^2*c^5*d^2 + (4*B*a*b + (10*A - C)*b^2)*c^4*d^3 - (B*a^2 + 10*(A -
C)*a*b + 3*B*b^2)*c^3*d^4 + 3*((A - C)*a^2 - 2*B*a*b + 3*A*b^2)*c^2*d^5 + (
3*B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d^6 - ((A - C)*a^2 + 2*B*a*b - 3*A*b^2)*
d^7)*log(d*tan(f*x + e) + c)/(b^4*c^10 - 4*a*b^3*c^9*d - 4*a^3*b*c*d^9 + a^
4*d^10 + 3*(2*a^2*b^2 + b^4)*c^8*d^2 - 4*(a^3*b + 3*a*b^3)*c^7*d^3 + (a^4 +
 18*a^2*b^2 + 3*b^4)*c^6*d^4 - 12*(a^3*b + a*b^3)*c^5*d^5 + (3*a^4 + 18*a^2
*b^2 + b^4)*c^4*d^6 - 4*(3*a^3*b + a*b^3)*c^3*d^7 + 3*(a^4 + 2*a^2*b^2)*c^2
*d^8) + ((B*a^2 - 2*(A - C)*a*b - B*b^2)*c^3 - 3*((A - C)*a^2 + 2*B*a*b - (
A - C)*b^2)*c^2*d - 3*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d^2 + ((A - C)*a^2
+ 2*B*a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^2*b^2 + b
^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2
*d^4 + (a^4 + 2*a^2*b^2 + b^4)*d^6) - (2*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c^6
+ 5*(C*a^3*b + C*a*b^3)*c^5*d - (C*a^4 + 7*B*a^3*b - 3*C*a^2*b^2 + 11*B*a*b
^3 - 4*A*b^4)*c^4*d^2 + (3*B*a^4 + (9*A + C)*a^3*b + 3*B*a^2*b^2 + (9*A + C
)*a*b^3)*c^3*d^3 - ((5*A - 3*C)*a^4 + 3*B*a^3*b + 5*(A - C)*a^2*b^2 + 5*B*a
*b^3 - 2*A*b^4)*c^2*d^4 - (B*a^4 - 5*A*a^3*b + B*a^2*b^2 - 5*A*a*b^3)*c*d^5
```

```
 - (A*a^4 + A*a^2*b^2)*d^6 + 2*((3*C*a^2*b^2 - B*a*b^3 + (A + 2*C)*b^4)*c^4
*d^2 - 3*(B*a^2*b^2 + B*b^4)*c^3*d^3 + (B*a^3*b + 2*(2*A + C)*a^2*b^2 - B*a
*b^3 + 6*A*b^4)*c^2*d^4 - (2*(A - C)*a^3*b + B*a^2*b^2 + 2*(A - C)*a*b^3 +
B*b^4)*c*d^5 - (B*a^3*b - (2*A + C)*a^2*b^2 + 2*B*a*b^3 - 3*A*b^4)*d^6)*tan
(f*x + e)^2 + ((9*C*a^2*b^2 - 4*B*a*b^3 + (4*A + 5*C)*b^4)*c^5*d + (3*C*a^3
*b - 7*B*a^2*b^2 + 3*C*a*b^3 - 7*B*b^4)*c^4*d^2 - (3*B*a^3*b - 9*(A + C)*a^
2*b^2 + 11*B*a*b^3 - (17*A + C)*b^4)*c^3*d^3 + (2*B*a^4 + 3*(A + C)*a^3*b -
 B*a^2*b^2 + 3*(A + C)*a*b^3 - 3*B*b^4)*c^2*d^4 - (4*(A - C)*a^4 + 3*B*a^3*
b - (A + 8*C)*a^2*b^2 + 7*B*a*b^3 - 9*A*b^4)*c*d^5 - (2*B*a^4 - 3*A*a^3*b +
 2*B*a^2*b^2 - 3*A*a*b^3)*d^6)*tan(f*x + e))/((a^3*b^3 + a*b^5)*c^9 - 3*(a^
4*b^2 + a^2*b^4)*c^8*d + (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*c^7*d^2 - (a^6 + 7
*a^4*b^2 + 6*a^2*b^4)*c^6*d^3 + (6*a^5*b + 7*a^3*b^3 + a*b^5)*c^5*d^4 - (2*
a^6 + 5*a^4*b^2 + 3*a^2*b^4)*c^4*d^5 + 3*(a^5*b + a^3*b^3)*c^3*d^6 - (a^6 +
 a^4*b^2)*c^2*d^7 + ((a^2*b^4 + b^6)*c^7*d^2 - 3*(a^3*b^3 + a*b^5)*c^6*d^3
+ (3*a^4*b^2 + 5*a^2*b^4 + 2*b^6)*c^5*d^4 - (a^5*b + 7*a^3*b^3 + 6*a*b^5)*c
^4*d^5 + (6*a^4*b^2 + 7*a^2*b^4 + b^6)*c^3*d^6 - (2*a^5*b + 5*a^3*b^3 + 3*a
*b^5)*c^2*d^7 + 3*(a^4*b^2 + a^2*b^4)*c*d^8 - (a^5*b + a^3*b^3)*d^9)*tan(f*
x + e)^3 + (2*(a^2*b^4 + b^6)*c^8*d - 5*(a^3*b^3 + a*b^5)*c^7*d^2 + (3*a^4*
b^2 + 7*a^2*b^4 + 4*b^6)*c^6*d^3 + (a^5*b - 9*a^3*b^3 - 10*a*b^5)*c^5*d^4 -
 (a^6 - 5*a^4*b^2 - 8*a^2*b^4 - 2*b^6)*c^4*d^5 + (2*a^5*b - 3*a^3*b^3 - 5*a
*b^5)*c^3*d^6 - (2*a^6 - a^4*b^2 - 3*a^2*b^4)*c^2*d^7 + (a^5*b + a^3*b^3)*c
*d^8 - (a^6 + a^4*b^2)*d^9)*tan(f*x + e)^2 + ((a^2*b^4 + b^6)*c^9 - (a^3*b^
3 + a*b^5)*c^8*d - (3*a^4*b^2 + a^2*b^4 - 2*b^6)*c^7*d^2 + (5*a^5*b + 3*a^3
*b^3 - 2*a*b^5)*c^6*d^3 - (2*a^6 + 8*a^4*b^2 + 5*a^2*b^4 - b^6)*c^5*d^4 + (
10*a^5*b + 9*a^3*b^3 - a*b^5)*c^4*d^5 - (4*a^6 + 7*a^4*b^2 + 3*a^2*b^4)*c^3
*d^6 + 5*(a^5*b + a^3*b^3)*c^2*d^7 - 2*(a^6 + a^4*b^2)*c*d^8)*tan(f*x + e))
)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9585 vs. $2(870) = 1740$.
time = 71.53, size = 9585, normalized size = 11.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*a^2*b^5 - B*a*b^6 + A*b^7)*c^9 - 2*(C*a^3*b^4 - B*a^2*b^5 + A*a*
b^6)*c^8*d + 6*(C*a^2*b^5 - B*a*b^6 + A*b^7)*c^7*d^2 + (7*C*a^5*b^2 + 8*C*a
^3*b^4 + 6*B*a^2*b^5 - (6*A - 7*C)*a*b^6)*c^6*d^3 - (10*C*a^6*b + 9*B*a^5*b
^2 + 20*C*a^4*b^3 + 18*B*a^3*b^4 + 4*C*a^2*b^5 + 15*B*a*b^6 - 6*A*b^7)*c^5*
d^4 + (3*C*a^7 + 14*B*a^6*b + (11*A + 7*C)*a^5*b^2 + 28*B*a^4*b^3 + (22*A -
 C)*a^3*b^4 + 20*B*a^2*b^5 + (5*A + C)*a*b^6)*c^4*d^5 - (5*B*a^7 + 2*(9*A -
 C)*a^6*b + 13*B*a^5*b^2 + 4*(9*A - C)*a^4*b^3 + 11*B*a^3*b^4 + 2*(9*A - 2*
```

C)*a^2*b^5 + 5*B*a*b^6 - 2*A*b^7)*c^3*d^6 + ((7*A - 3*C)*a^7 + 2*B*a^6*b +
(19*A - 6*C)*a^5*b^2 + 4*B*a^4*b^3 + (17*A - 5*C)*a^3*b^4 + 4*B*a^2*b^5 + 3
*A*a*b^6)*c^2*d^7 + (B*a^7 - 6*A*a^6*b + 2*B*a^5*b^2 - 12*A*a^4*b^3 + B*a^3
*b^4 - 6*A*a^2*b^5)*c*d^8 + (A*a^7 + 2*A*a^5*b^2 + A*a^3*b^4)*d^9 - (2*(C*a
^3*b^4 - B*a^2*b^5 + A*a*b^6)*c^7*d^2 + (3*C*a^4*b^3 + 2*B*a^3*b^4 - 2*(A -
5*C)*a^2*b^5 + 5*C*b^7)*c^6*d^3 - (6*C*a^5*b^2 + 7*B*a^4*b^3 + 6*C*a^3*b^4
+ 20*B*a^2*b^5 - 6*(A - C)*a*b^6 + 7*B*b^7)*c^5*d^4 + (C*a^6*b + 10*B*a^5*
b^2 + (9*A - 5*C)*a^4*b^3 + 26*B*a^3*b^4 + (12*A - C)*a^2*b^5 + 10*B*a*b^6
+ (9*A - C)*b^7)*c^4*d^5 - (3*B*a^6*b + 2*(7*A - 3*C)*a^5*b^2 + 7*B*a^4*b^3
+ 2*(14*A - 9*C)*a^3*b^4 + 11*B*a^2*b^5 + 2*(4*A - 3*C)*a*b^6 + B*b^7)*c^3
d^6 + (5(A - C)*a^6*b - 2*B*a^5*b^2 + (13*A - 16*C)*a^4*b^3 + 2*B*a^3*b^4
+ 5*(A - C)*a^2*b^5 - 2*B*a*b^6 + 3*A*b^7)*c^2*d^7 + (3*B*a^6*b - 2*A*a^5*
b^2 + 6*B*a^4*b^3 - 2*(2*A - C)*a^3*b^4 + B*a^2*b^5)*c*d^8 - (A*a^6*b + 2*(
A + C)*a^4*b^3 - 2*B*a^3*b^4 + 3*A*a^2*b^5)*d^9 + 2*(((A - C)*a^2*b^5 + 2*B
*a*b^6 - (A - C)*b^7)*c^7*d^2 - (4*(A - C)*a^3*b^4 + 5*B*a^2*b^5 + 2*(A - C
)*a*b^6 + 3*B*b^7)*c^6*d^3 + 3*(2*(A - C)*a^4*b^3 + 5*(A - C)*a^2*b^5 + 2*B
*a*b^6 + (A - C)*b^7)*c^5*d^4 - (4*(A - C)*a^5*b^2 - 10*B*a^4*b^3 + 20*(A -
C)*a^3*b^4 - 5*B*a^2*b^5 + 10*(A - C)*a*b^6 - B*b^7)*c^4*d^5 + ((A - C)*a^
6*b - 10*B*a^5*b^2 + 5*(A - C)*a^4*b^3 - 20*B*a^3*b^4 + 10*(A - C)*a^2*b^5
- 4*B*a*b^6)*c^3*d^6 + 3*(B*a^6*b + 2*(A - C)*a^5*b^2 + 5*B*a^4*b^3 + 2*B*a
^2*b^5)*c^2*d^7 - (3*(A - C)*a^6*b + 2*B*a^5*b^2 + 5*(A - C)*a^4*b^3 + 4*B*
a^3*b^4)*c*d^8 - (B*a^6*b - 2*(A - C)*a^5*b^2 - B*a^4*b^3)*d^9)*f*x)*tan(f*
x + e)^3 - 2*(((A - C)*a^3*b^4 + 2*B*a^2*b^5 - (A - C)*a*b^6)*c^9 - (4*(A -
C)*a^4*b^3 + 5*B*a^3*b^4 + 2*(A - C)*a^2*b^5 + 3*B*a*b^6)*c^8*d + 3*(2*(A
- C)*a^5*b^2 + 5*(A - C)*a^3*b^4 + 2*B*a^2*b^5 + (A - C)*a*b^6)*c^7*d^2 - (
4*(A - C)*a^6*b - 10*B*a^5*b^2 + 20*(A - C)*a^4*b^3 - 5*B*a^3*b^4 + 10*(A -
C)*a^2*b^5 - B*a*b^6)*c^6*d^3 + ((A - C)*a^7 - 10*B*a^6*b + 5*(A - C)*a^5*
b^2 - 20*B*a^4*b^3 + 10*(A - C)*a^3*b^4 - 4*B*a^2*b^5)*c^5*d^4 + 3*(B*a^7 +
2*(A - C)*a^6*b + 5*B*a^5*b^2 + 2*B*a^3*b^4)*c^4*d^5 - (3*(A - C)*a^7 + 2*
B*a^6*b + 5*(A - C)*a^5*b^2 + 4*B*a^4*b^3)*c^3*d^6 - (B*a^7 - 2*(A - C)*a^6
*b - B*a^5*b^2)*c^2*d^7)*f*x - (4*(C*a^3*b^4 - B*a^2*b^5 + A*a*b^6)*c^8*d +
2*(C*a^4*b^3 + 2*B*a^3*b^4 - (2*A - 5*C)*a^2*b^5 + B*a*b^6 - (A - 3*C)*b^7
)*c^7*d^2 - (3*C*a^5*b^2 + 8*B*a^4*b^3 - 8*C*a^3*b^4 + 30*B*a^2*b^5 - (14*A
- 3*C)*a*b^6 + 8*B*b^7)*c^6*d^3 - (4*C*a^6*b - 5*B*a^5*b^2 - 2*(5*A - 13*C
)*a^4*b^3 - 22*B*a^3*b^4 - 2*(4*A - 11*C)*a^2*b^5 - 11*B*a*b^6 - 2*(2*A - 3
*C)*b^7)*c^5*d^4 + (C*a^7 + 6*B*a^6*b - (7*A - 13*C)*a^5*b^2 + 18*B*a^4*b^3
- (14*A - 41*C)*a^3*b^4 + 11*(A + C)*a*b^6 + 6*B*b^7)*c^4*d^5 - (3*B*a^7 +
8*A*a^6*b + 19*B*a^5*b^2 + 2*(11*A + 6*C)*a^4*b^3 + 17*B*a^3*b^4 + 2*(16*A
+ 3*C)*a^2*b^5 + 7*B*a*b^6 + 12*A*b^7)*c^3*d^6 + (5*(A - C)*a^7 + 4*B*a^6*
b + (25*A - 14*C)*a^5*b^2 + 10*B*a^4*b^3 + (35*A - 3*C)*a^3*b^4 - 2*B*a^2*b
^5 + (25*A - 4*C)*a*b^6 + 2*B*b^7)*c^2*d^7 + (3*B*a^7 - 4*(2*A - C)*a^6*b +
6*B*a^5*b^2 - 4*(5*A - C)*a^4*b^3 + 7*B*a^3*b^4 - 2*(10*A - C)*a^2*b^5 + 2
*B*a*b^6 - 6*A*b^7)*c*d^8 - (A*a^7 + 2*B*a^6*b - 2*A*a^5*b^2 + 4*B*a^4*b^3
- (7*A + 2*C)*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6)*d^9 + 2*(2*((A - C)*a^2*b^
5 + 2*B*a*b^6 - (A - C)*b^7)*c^8*d - (7*(A - C)*a^3*b^4 + 8*B*a^2*b^5 + 5*(

```
A - C)*a*b^6 + 6*B*b^7)*c^7*d^2 + (8*(A - C)*a^4*b^3 - 5*B*a^3*b^4 + 28*(A
- C)*a^2*b^5 + 9*B*a*b^6 + 6*(A - C)*b^7)*c^6*d^3 - (2*(A - C)*a^5*b^2 - 20
*B*a^4*b^3 + 25*(A - C)*a^3*b^4 - 16*B*a^2*b^5 + 17*(A - C)*a*b^6 - 2*B*b^7
)*c^5*d^4 - (2*(A - C)*a^6*b + 10*B*a^5*b^2 + 10*(A - C)*a^4*b^3 + 35*B*a^3
*b^4 - 10*(A - C)*a^2*b^5 + 7*B*a*b^6)*c^4*d^5 + ((A - C)*a^7 - 4*B*a^6*b +
 17*(A - C)*a^5*b^2 + 10*B*a^4*b^3 + 10*(A - C)*a^3*b^4 + 8*B*a^2*b^5)*c^3*
d^6 + (3*B*a^7 + 11*B*a^5*b^2 - 10*(A - C)*a^4*b^3 - 2*B*a^3*b^4)*c^2*d^7 -
 (3*(A - C)*a^7 + 4*B*a^6*b + (A - C)*a^5*b^2 + 2*B*a^4*b^3)*c*d^8 - (B*a^7
 - 2*(A - C)*a^6*b - B*a^5*b^2)*d^9)*f*x)*tan(f*x + e)^2 + ((B*a^3*b^4 - 2*
(A - C)*a^2*b^5 - B*a*b^6)*c^9 + (3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3
*b^4 - 2*B*a^2*b^5 + 3*A*a*b^6)*c^8*d + 3*(B*a^3*b^4 - 2*(A - C)*a^2*b^5 -
B*a*b^6)*c^7*d^2 + 3*(3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3*b^4 - 2*B*a
^2*b^5 + 3*A*a*b^6)*c^6*d^3 + 3*(B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c
^5*d^4 + 3*(3*C*a^5*b^2 - 4*B*a^4*b^3 + (5*A + C)*a^3*b^4 - 2*B*a^2*b^5 + 3
*A*a*b^6)*c^4*d^5 + (B*a^3*b^4 - 2*(A - C)*a^2*...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e))**3,x)
```

```
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3176 vs. 2(870) = 1740.
time = 1.16, size = 3176, normalized size = 3.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(A*a^2*c^3 - C*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 + 3*B*a
^2*c^2*d - 6*A*a*b*c^2*d + 6*C*a*b*c^2*d - 3*B*b^2*c^2*d - 3*A*a^2*c*d^2 +
3*C*a^2*c*d^2 - 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 - B*a^2*d^3 +
 2*A*a*b*d^3 - 2*C*a*b*d^3 + B*b^2*d^3)*(f*x + e)/(a^4*c^6 + 2*a^2*b^2*c^6
+ b^4*c^6 + 3*a^4*c^4*d^2 + 6*a^2*b^2*c^4*d^2 + 3*b^4*c^4*d^2 + 3*a^4*c^2*d
^4 + 6*a^2*b^2*c^2*d^4 + 3*b^4*c^2*d^4 + a^4*d^6 + 2*a^2*b^2*d^6 + b^4*d^6)
 + (B*a^2*c^3 - 2*A*a*b*c^3 + 2*C*a*b*c^3 - B*b^2*c^3 - 3*A*a^2*c^2*d + 3*C
*a^2*c^2*d - 6*B*a*b*c^2*d + 3*A*b^2*c^2*d - 3*C*b^2*c^2*d - 3*B*a^2*c*d^2
+ 6*A*a*b*c*d^2 - 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 + A*a^2*d^3 - C*a^2*d^3 + 2
```

*B*a*b*d^3 - A*b^2*d^3 + C*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^4*c^6 + 2*a^
2*b^2*c^6 + b^4*c^6 + 3*a^4*c^4*d^2 + 6*a^2*b^2*c^4*d^2 + 3*b^4*c^4*d^2 + 3
*a^4*c^2*d^4 + 6*a^2*b^2*c^2*d^4 + 3*b^4*c^2*d^4 + a^4*d^6 + 2*a^2*b^2*d^6
+ b^4*d^6) - 2*(B*a^2*b^5*c - 2*A*a*b^6*c + 2*C*a*b^6*c - B*b^7*c + 3*C*a^4
*b^3*d - 4*B*a^3*b^4*d + 5*A*a^2*b^5*d + C*a^2*b^5*d - 2*B*a*b^6*d + 3*A*b^
7*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^5*c^4 + 2*a^2*b^7*c^4 + b^9*c^4 -
4*a^5*b^4*c^3*d - 8*a^3*b^6*c^3*d - 4*a*b^8*c^3*d + 6*a^6*b^3*c^2*d^2 + 12*
a^4*b^5*c^2*d^2 + 6*a^2*b^7*c^2*d^2 - 4*a^7*b^2*c*d^3 - 8*a^5*b^4*c*d^3 - 4
*a^3*b^6*c*d^3 + a^8*b*d^4 + 2*a^6*b^3*d^4 + a^4*b^5*d^4) + 2*(3*C*b^2*c^6*
d^2 - 6*B*b^2*c^5*d^3 + 4*B*a*b*c^4*d^4 + 10*A*b^2*c^4*d^4 - C*b^2*c^4*d^4
- B*a^2*c^3*d^5 - 10*A*a*b*c^3*d^5 + 10*C*a*b*c^3*d^5 - 3*B*b^2*c^3*d^5 + 3
*A*a^2*c^2*d^6 - 3*C*a^2*c^2*d^6 - 6*B*a*b*c^2*d^6 + 9*A*b^2*c^2*d^6 + 3*B*
a^2*c*d^7 - 2*A*a*b*c*d^7 + 2*C*a*b*c*d^7 - B*b^2*c*d^7 - A*a^2*d^8 + C*a^2
*d^8 - 2*B*a*b*d^8 + 3*A*b^2*d^8)*log(abs(d*tan(f*x + e) + c))/(b^4*c^10*d
- 4*a*b^3*c^9*d^2 + 6*a^2*b^2*c^8*d^3 + 3*b^4*c^8*d^3 - 4*a^3*b*c^7*d^4 - 1
2*a*b^3*c^7*d^4 + a^4*c^6*d^5 + 18*a^2*b^2*c^6*d^5 + 3*b^4*c^6*d^5 - 12*a^3
*b*c^5*d^6 - 12*a*b^3*c^5*d^6 + 3*a^4*c^4*d^7 + 18*a^2*b^2*c^4*d^7 + b^4*c^
4*d^7 - 12*a^3*b*c^3*d^8 - 4*a*b^3*c^3*d^8 + 3*a^4*c^2*d^9 + 6*a^2*b^2*c^2*
d^9 - 4*a^3*b*c*d^10 + a^4*d^11) + 2*(B*a^2*b^5*c*tan(f*x + e) - 2*A*a*b^6*
c*tan(f*x + e) + 2*C*a*b^6*c*tan(f*x + e) - B*b^7*c*tan(f*x + e) + 3*C*a^4*
b^3*d*tan(f*x + e) - 4*B*a^3*b^4*d*tan(f*x + e) + 5*A*a^2*b^5*d*tan(f*x + e
) + C*a^2*b^5*d*tan(f*x + e) - 2*B*a*b^6*d*tan(f*x + e) + 3*A*b^7*d*tan(f*x
 + e) - C*a^4*b^3*c + 2*B*a^3*b^4*c - 3*A*a^2*b^5*c + C*a^2*b^5*c - A*b^7*c
 + 4*C*a^5*b^2*d - 5*B*a^4*b^3*d + 6*A*a^3*b^4*d + 2*C*a^3*b^4*d - 3*B*a^2*
b^5*d + 4*A*a*b^6*d)/((a^4*b^4*c^4 + 2*a^2*b^6*c^4 + b^8*c^4 - 4*a^5*b^3*c^
3*d - 8*a^3*b^5*c^3*d - 4*a*b^7*c^3*d + 6*a^6*b^2*c^2*d^2 + 12*a^4*b^4*c^2*
d^2 + 6*a^2*b^6*c^2*d^2 - 4*a^7*b*c*d^3 - 8*a^5*b^3*c*d^3 - 4*a^3*b^5*c*d^3
 + a^8*d^4 + 2*a^6*b^2*d^4 + a^4*b^4*d^4)*(b*tan(f*x + e) + a)) - (9*C*b^2*
c^6*d^3*tan(f*x + e)^2 - 18*B*b^2*c^5*d^4*tan(f*x + e)^2 + 12*B*a*b*c^4*d^5
*tan(f*x + e)^2 + 30*A*b^2*c^4*d^5*tan(f*x + e)^2 - 3*C*b^2*c^4*d^5*tan(f*x
 + e)^2 - 3*B*a^2*c^3*d^6*tan(f*x + e)^2 - 30*A*a*b*c^3*d^6*tan(f*x + e)^2
+ 30*C*a*b*c^3*d^6*tan(f*x + e)^2 - 9*B*b^2*c^3*d^6*tan(f*x + e)^2 + 9*A*a^
2*c^2*d^7*tan(f*x + e)^2 - 9*C*a^2*c^2*d^7*tan(f*x + e)^2 - 18*B*a*b*c^2*d^
7*tan(f*x + e)^2 + 27*A*b^2*c^2*d^7*tan(f*x + e)^2 + 9*B*a^2*c*d^8*tan(f*x
+ e)^2 - 6*A*a*b*c*d^8*tan(f*x + e)^2 + 6*C*a*b*c*d^8*tan(f*x + e)^2 - 3*B*
b^2*c*d^8*tan(f*x + e)^2 - 3*A*a^2*d^9*tan(f*x + e)^2 + 3*C*a^2*d^9*tan(f*x
 + e)^2 - 6*B*a*b*d^9*tan(f*x + e)^2 + 9*A*b^2*d^9*tan(f*x + e)^2 + 22*C*b^
2*c^7*d^2*tan(f*x + e) - 4*C*a*b*c^6*d^3*tan(f*x + e) - 42*B*b^2*c^6*d^3*ta
n(f*x + e) + 32*B*a*b*c^5*d^4*tan(f*x + e) + 68*A*b^2*c^5*d^4*tan(f*x + e)
- 2*C*b^2*c^5*d^4*tan(f*x + e) - 8*B*a^2*c^4*d^5*tan(f*x + e) - 72*A*a*b*c^
4*d^5*tan(f*x + e) + 60*C*a*b*c^4*d^5*tan(f*x + e) - 26*B*b^2*c^4*d^5*tan(f
*x + e) + 22*A*a^2*c^3*d^6*tan(f*x + e) - 22*C*a^2*c^3*d^6*tan(f*x + e) - 2
8*B*a*b*c^3*d^6*tan(f*x + e) + 66*A*b^2*c^3*d^6*tan(f*x + e) + 18*B*a^2*c^2
*d^7*tan(f*x + e) - 28*A*a*b*c^2*d^7*tan(f*x + e) + 16*C*a*b*c^2*d^7*tan(f*
x + e) - 8*B*b^2*c^2*d^7*tan(f*x + e) - 2*A*a^2*c*d^8*tan(f*x + e) + 2*C*a^

```
2*c*d^8*tan(f*x + e) - 12*B*a*b*c*d^8*tan(f*x + e) + 22*A*b^2*c*d^8*tan(f*x
 + e) + 2*B*a^2*d^9*tan(f*x + e) - 4*A*a*b*d^9*tan(f*x + e) + 14*C*b^2*c^8*
d - 6*C*a*b*c^7*d^2 - 25*B*b^2*c^7*d^2 + C*a^2*c^6*d^3 + 22*B*a*b*c^6*d^3 +
 39*A*b^2*c^6*d^3 + 3*C*b^2*c^6*d^3 - 6*B*a^2*c^5*d^4 - 44*A*a*b*c^5*d^4 +
26*C*a*b*c^5*d^4 - 19*B*b^2*c^5*d^4 + 14*A*a^2*c^4*d^5 - 11*C*a^2*c^4*d^5 -
 6*B*a*b*c^4*d^5 + 41*A*b^2*c^4*d^5 + C*b^2*c^4*d^5 + 7*B*a^2*c^3*d^6 - 26*
A*a*b*c^3*d^6 + 8*C*a*b*c^3*d^6 - 6*B*b^2*c^3*d^6 + 3*A*a^2*c^2*d^7 - 4*B*a
*b*c^2*d^7 + 14*A*b^2*c^2*d^7 + B*a^2*c*d^8 - 6*A*a*b*c*d^8 + A*a^2*d^9)/((
b^4*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 + 3*b^4*c^8*d^2 - 4*a^3*b*c^7*
d^3 - 12*a*b^3*c^7*d^3 + a^4*c^6*d^4 + 18*a^2*b^2*c^6*d^4 + 3*b^4*c^6*d^4 -
 12*a^3*b*c^5*d^5 - 12*a*b^3*c^5*d^5 + 3*a^4*c^4*d^6 + 18*a^2*b^2*c^4*d^6 +
 b^4*c^4*d^6 - 12*a^3*b*c^3*d^7 - 4*a*b^3*c^3*d^7 + 3*a^4*c^2*d^8 + 6*a^2*b
^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*(d*tan(f...
```

Mupad [B]
time = 47.93, size = 2500, normalized size = 2.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d*
tan(e + f*x))^3),x)
```

```
[Out] (((2*A*b^4*c^6 - A*a^4*d^6 - 2*B*a*b^3*c^6 - B*a^4*c*d^5 - A*a^2*b^2*d^6 -
5*A*a^4*c^2*d^4 + 2*C*a^2*b^2*c^6 + 2*A*b^4*c^2*d^4 + 4*A*b^4*c^4*d^2 + 3*B
*a^4*c^3*d^3 + 3*C*a^4*c^2*d^4 - C*a^4*c^4*d^2 + 9*A*a*b^3*c^3*d^3 + 9*A*a^
3*b*c^3*d^3 - 5*B*a*b^3*c^2*d^4 - 11*B*a*b^3*c^4*d^2 - B*a^2*b^2*c*d^5 - 3*
B*a^3*b*c^2*d^4 - 7*B*a^3*b*c^4*d^2 + C*a*b^3*c^3*d^3 + C*a^3*b*c^3*d^3 - 5
*A*a^2*b^2*c^2*d^4 + 3*B*a^2*b^2*c^3*d^3 + 5*C*a^2*b^2*c^2*d^4 + 3*C*a^2*b^
2*c^4*d^2 + 5*A*a*b^3*c*d^5 + 5*A*a^3*b*c*d^5 + 5*C*a*b^3*c^5*d + 5*C*a^3*b
*c^5*d)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^2*c^4 + a
^2*d^4 + b^2*c^4 + b^2*d^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)) + (tan(e + f*x
)*(3*A*a*b^3*d^6 - 2*B*a^4*d^6 + 3*A*a^3*b*d^6 - 4*A*a^4*c*d^5 + 9*A*b^4*c*
d^5 + 4*A*b^4*c^5*d + 4*C*a^4*c*d^5 + 5*C*b^4*c^5*d - 2*B*a^2*b^2*d^6 + 17*
A*b^4*c^3*d^3 + 2*B*a^4*c^2*d^4 - 3*B*b^4*c^2*d^4 - 7*B*b^4*c^4*d^2 + C*b^4
*c^3*d^3 + 3*A*a*b^3*c^2*d^4 + A*a^2*b^2*c*d^5 + 3*A*a^3*b*c^2*d^4 - 11*B*a
*b^3*c^3*d^3 - 3*B*a^3*b*c^3*d^3 + 3*C*a*b^3*c^2*d^4 + 3*C*a*b^3*c^4*d^2 +
8*C*a^2*b^2*c*d^5 + 9*C*a^2*b^2*c^5*d + 3*C*a^3*b*c^2*d^4 + 3*C*a^3*b*c^4*d
^2 + 9*A*a^2*b^2*c^3*d^3 - B*a^2*b^2*c^2*d^4 - 7*B*a^2*b^2*c^4*d^2 + 9*C*a^
2*b^2*c^3*d^3 - 7*B*a*b^3*c*d^5 - 4*B*a*b^3*c^5*d - 3*B*a^3*b*c*d^5))/(2*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^2*c^4 + a^2*d^4 + b^2*
c^4 + b^2*d^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)) + (tan(e + f*x)^2*(3*A*b^4*
d^6 - 2*B*a*b^3*d^6 - B*a^3*b*d^6 - B*b^4*c*d^5 + 2*A*a^2*b^2*d^6 + 6*A*b^4
*c^2*d^4 + A*b^4*c^4*d^2 + C*a^2*b^2*d^6 - 3*B*b^4*c^3*d^3 + 2*C*b^4*c^4*d^
2 - B*a*b^3*c^2*d^4 - B*a*b^3*c^4*d^2 - B*a^2*b^2*c*d^5 + B*a^3*b*c^2*d^4 +
```

4*A*a^2*b^2*c^2*d^4 - 3*B*a^2*b^2*c^3*d^3 + 2*C*a^2*b^2*c^2*d^4 + 3*C*a^2*
b^2*c^4*d^2 - 2*A*a*b^3*c*d^5 - 2*A*a^3*b*c*d^5 + 2*C*a*b^3*c*d^5 + 2*C*a^3
*b*c*d^5))/((a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^2*c^4 +
a^2*d^4 + b^2*c^4 + b^2*d^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)))/(tan(e + f*x
)*(b*c^2 + 2*a*c*d) + a*c^2 + tan(e + f*x)^2*(a*d^2 + 2*b*c*d) + b*d^2*tan(
e + f*x)^3) + symsum(log((3*A^3*a^3*b^6*d^10 - A^3*a^5*b^4*d^10 + 4*B^3*a^2
*b^7*d^10 + 6*B^3*a^4*b^5*d^10 + 24*A^3*b^9*c^3*d^7 + 27*A^3*b^9*c^5*d^5 +
C^3*a^5*b^4*d^10 + B^3*b^9*c^2*d^8 + 4*B^3*b^9*c^4*d^6 + 7*B^3*b^9*c^6*d^4
+ 9*A^2*B*b^9*d^10 + 9*A^3*b^9*c*d^9 + 26*A^3*a^2*b^7*c^3*d^7 + 31*A^3*a^2*
b^7*c^5*d^5 + 16*A^3*a^3*b^6*c^2*d^8 - 11*A^3*a^3*b^6*c^4*d^6 - 6*A^3*a^4*b
^5*c^3*d^7 + 3*A^3*a^5*b^4*c^2*d^8 + 5*B^3*a^2*b^7*c^2*d^8 - 14*B^3*a^2*b^7
*c^4*d^6 + 9*B^3*a^2*b^7*c^6*d^4 + 28*B^3*a^3*b^6*c^3*d^7 + 19*B^3*a^3*b^6*
c^5*d^5 + 6*B^3*a^4*b^5*c^2*d^8 - 20*B^3*a^4*b^5*c^4*d^6 + 7*B^3*a^5*b^4*c^
3*d^7 + C^3*a^2*b^7*c^3*d^7 - 4*C^3*a^2*b^7*c^5*d^5 - 9*C^3*a^2*b^7*c^7*d^3
 - 7*C^3*a^3*b^6*c^2*d^8 - 28*C^3*a^3*b^6*c^4*d^6 + 3*C^3*a^3*b^6*c^6*d^4 +
 15*C^3*a^4*b^5*c^3*d^7 - 9*C^3*a^4*b^5*c^7*d^3 - 3*C^3*a^5*b^4*c^2*d^8 - 2
4*C^3*a^5*b^4*c^4*d^6 + 6*C^3*a^6*b^3*c^3*d^7 - 12*A*B^2*a*b^8*d^10 - 6*A*B
^2*b^9*c*d^9 - 9*A^2*C*b^9*c*d^9 + 4*B^3*a*b^8*c*d^9 - 17*A*B^2*a^3*b^6*d^1
0 + 3*A*B^2*a^5*b^4*d^10 + 12*A^2*B*a^2*b^7*d^10 - 7*A^2*B*a^4*b^5*d^10 + 3
*A*C^2*a^3*b^6*d^10 - 3*A*C^2*a^5*b^4*d^10 - 6*A^2*C*a^3*b^6*d^10 + 3*A^2*C
*a^5*b^4*d^10 - 20*A*B^2*b^9*c^3*d^7 - 28*A*B^2*b^9*c^5*d^5 + 6*A*B^2*b^9*c
^7*d^3 - B*C^2*a^4*b^5*d^10 + 3*B*C^2*a^6*b^3*d^10 + 21*A^2*B*b^9*c^2*d^8 +
 13*A^2*B*b^9*c^4*d^6 - 27*A^2*B*b^9*c^6*d^4 - 4*B^2*C*a^3*b^6*d^10 - 9*B^2
*C*a^5*b^4*d^10 - 3*A*C^2*b^9*c^3*d^7 - 9*A*C^2*b^9*c^7*d^3 - 21*A^2*C*b^9*
c^3*d^7 - 27*A^2*C*b^9*c^5*d^5 + 9*A^2*C*b^9*c^7*d^3 + B*C^2*b^9*c^4*d^6 +
3*B*C^2*b^9*c^8*d^2 - B^2*C*b^9*c^3*d^7 - 2*B^2*C*b^9*c^5*d^5 - 9*B^2*C*b^9
*c^7*d^3 - 3*A^3*a*b^8*c^2*d^8 - 31*A^3*a*b^8*c^4*d^6 - 8*A^3*a*b^8*c^6*d^4
 + 3*A^3*a^2*b^7*c*d^9 - 10*A^3*a^4*b^5*c*d^9 + 11*B^3*a*b^8*c^3*d^7 + 5*B^
3*a*b^8*c^5*d^5 - 6*B^3*a*b^8*c^7*d^3 + B^3*a^3*b^6*c*d^9 - 5*B^3*a^5*b^4*c
*d^9 - 2*C^3*a*b^8*c^4*d^6 - C^3*a*b^8*c^6*d^4 - 3*C^3*a*b^8*c^8*d^2 - 2*C^
3*a^4*b^5*c*d^9 - 6*C^3*a^6*b^3*c*d^9 - 4*A*B^2*a^2*b^7*c^3*d^7 - 77*A*B^2*
a^2*b^7*c^5*d^5 - 6*A*B^2*a^2*b^7*c^7*d^3 - 60*A*B^2*a^3*b^6*c^2*d^8 + 25*A
*B^2*a^3*b^6*c^4*d^6 + 28*A*B^2*a^3*b^6*c^6*d^4 + 44*A*B^2*a^4*b^5*c^3*d^7
- 17*A*B^2*a^4*b^5*c^5*d^5 - 21*A*B^2*a^5*b^4*c^2*d^8 + 4*A*B^2*a^5*b^4*c^4
*d^6 + 71*A^2*B*a^2*b^7*c^2*d^8 + 86*A^2*B*a^2*b^7*c^4*d^6 - 13*A^2*B*a^2*b
^7*c^6*d^4 - 116*A^2*B*a^3*b^6*c^3*d^7 - 37*A^2*B*a^3*b^6*c^5*d^5 + 16*A^2*
B*a^4*b^5*c^2*d^8 + 35*A^2*B*a^4*b^5*c^4*d^6 - 9*A^2*B*a^5*b^4*c^3*d^7 - 30
*A*C^2*a^2*b^7*c^3*d^7 - 15*A*C^2*a^2*b^7*c^5*d^5 + 30*A*C^2*a^3*b^6*c^2*d^
8 + 45*A*C^2*a^3*b^6*c^4*d^6 - 6*A*C^2*a^3*b^6*c^6*d^4 - 63*A*C^2*a^4*b^5*c
^3*d^7 - 27*A*C^2*a^4*b^5*c^5*d^5 + 9*A*C^2*a^4*b^5*c^7*d^3 + 9*A*C^2*a^5*b
^4*c^2*d^8 + 48*A*C^2*a^5*b^4*c^4*d^6 - 12*A*C^2*a^6*b^3*c^3*d^7 + 3*A^2*C*
a^2*b^7*c^3*d^7 - 12*A^2*C*a^2*b^7*c^5*d^5 + 9*A^2*C*a^2*b^7*c^7*d^3 - 39*A
^2*C*a^3*b^6*c^2*d^8 - 6*A^2*C*a^3*b^6*c^4*d^6 + 3*A^2*C*a^3*b^6*c^6*d^4 +
54*A^2*C*a^4*b^5*c^3*d^7 + 27*A^2*C*a^4*b^5*c^5*d^5 - 9*A^2*C*a^5*b^4*c^2*d
^8 - 24*A^2*C*a^5*b^4*c^4*d^6 + 6*A^2*C*a^6*b^3...

3.90 $\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e$

Optimal. Leaf size=464

$$-\frac{(a-ib)^3(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(a+ib)^3(iA-B-iC)\sqrt{c+id}\,\tanh}{f}$$

```
[Out] -(a-I*b)^3*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d
)^(1/2)/f+(a+I*b)^3*(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))*(c+I*d)^(1/2)/f+2*(a^3*B-3*a*b^2*B+3*a^2*b*(A-C)-b^3*(A-C))*(c+d*tan(f*x
+e))^(1/2)/f+2/315*(40*a^3*C*d^3-6*a^2*b*d^2*(-45*B*d+16*C*c)+9*a*b^2*d*(8*
c^2*C-14*B*c*d+35*(A-C)*d^2)-b^3*(16*c^3*C-24*B*c^2*d+42*c*(A-C)*d^2+105*B*
d^3))*(c+d*tan(f*x+e))^(3/2)/d^4/f+2/105*b*(21*b*(A*b+B*a-C*b)*d^2+4*(-a*d+
b*c)*(-3*B*b*d-2*C*a*d+2*C*b*c))*tan(f*x+e)*(c+d*tan(f*x+e))^(3/2)/d^3/f-2/
21*(-3*B*b*d-2*C*a*d+2*C*b*c)*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)/d^2
/f+2/9*C*(a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 1.41, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3728, 3718, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(c+d\tan(e+fx))^{3/2}\left(40a^3Cd^3-6a^2bd^2(16cC-45Bd)+9ab^2d(35d^2(A-C)-14Bcd+8c^2C)-\left(b^3(42cd^2(A-C)-24Bc^2d+105Bd^3+16c^3C)\right)\right)}{315d^4f}+\frac{2(a^3B+3a^2b(A-C)-3ab^2B-b^3(A-C))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{3/2}(21bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-3bBd+2bcC))}{105d^3f}-\frac{(a-ib)^3\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(a+ib)^3\sqrt{c+id}\,(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{21d^2f}+\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c - I*d]])/f) + ((a + I*b)^3*(I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(a^3*B - 3*a*b^2*B + 3*a^2*
b*(A - C) - b^3*(A - C))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(40*a^3*C*d^3 - 6
*a^2*b*d^2*(16*c*C - 45*B*d) + 9*a*b^2*d*(8*c^2*C - 14*B*c*d + 35*(A - C)*d
^2) - b^3*(16*c^3*C - 24*B*c^2*d + 42*c*(A - C)*d^2 + 105*B*d^3))*(c + d*Ta
n[e + f*x])^(3/2))/(315*d^4*f) + (2*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c
- a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/
2))/(105*d^3*f) - (2*(2*b*c*C - 3*b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(3/2))/(21*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])^(3/2))/(9*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
```

```
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\,\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2C(a+b\tan(e+f}{}$$

$$= -\frac{2(2bcC-3bBd-}{}$$

$$= \frac{2b(21b(Ab+aB-}{}$$

$$= \frac{2(40a^3Cd^3-6a^2bd}{}$$

$$= \frac{2(a^3B-3ab^2B+3}{}$$

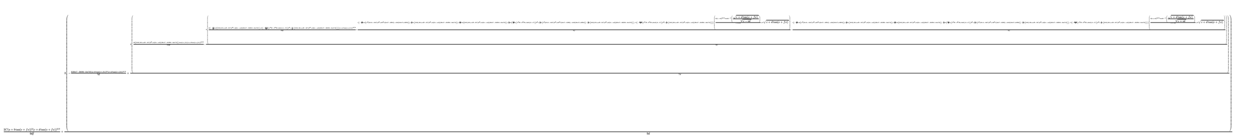
$$= \frac{2(a^3B-3ab^2B+3}{}$$

$$= \frac{2(a^3B-3ab^2B+3}{}$$

$$= \frac{2(a^3B-3ab^2B+3}{}$$

$$= -\frac{(a-ib)^3(iA+B}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1232 vs. 2(464) = 928.
time = 6.29, size = 1232, normalized size = 2.66



Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2))/(9*d*f) + (2*((-3*(
2*b*c*C - 3*b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3
/2))/(7*d*f) + (2*((3*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*
```

```
C - 3*b*B*d - 2*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(10*d*f) -
 (2*((2*((-15*a*d*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*
b*B*d - 2*a*C*d)))/8 + b*((-315*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (3
*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d
)))/4))*(c + d*Tan[e + f*x])^(3/2))/(3*d*f) + ((I/2)*((-15*a*d*(a^2*(21*A -
 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C + 9*B*d)))/8 + (3*b*c*
(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d)))
/4 + (15*a*d*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d
 - 2*a*C*d)))/8 + ((5*I)/2)*d*((63*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2)/4
 + (3*b*(a^2*(21*A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C +
9*B*d)))/4 - (3*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*
b*B*d - 2*a*C*d)))/4) - b*((-315*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (
3*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*
d)))/4))*((2*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/(-c + I*d) + 2*Sqrt[c + d*Tan[e + f*x]]))/f - ((I/2)*((-15*a*d*(a^2*(21*
A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C + 9*B*d)))/8 + (3*b
*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a*C*d
)))/4 + (15*a*d*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*
B*d - 2*a*C*d)))/8 - ((5*I)/2)*d*((63*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2
)/4 + (3*b*(a^2*(21*A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C
 + 9*B*d)))/4 - (3*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C -
 3*b*B*d - 2*a*C*d)))/4) - b*((-315*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8
+ (3*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d - 2*a
*C*d)))/4))*((2*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I
*d]])/(-c - I*d) + 2*Sqrt[c + d*Tan[e + f*x]]))/f))/(5*d)))/(7*d)))/(9*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3065 vs. $2(424) = 848$.
time = 0.68, size = 3066, normalized size = 6.61

method	result	size
derivativedivides	Expression too large to display	3066
default	Expression too large to display	3066

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^4*(C*a*b^2*c^2*d*(c+d*tan(f*x+e))^(3/2)+1/7*B*b^3*d*(c+d*tan(f*x+e))^
(7/2)+C*b^3*d^4*(c+d*tan(f*x+e))^(1/2)-A*b^3*d^4*(c+d*tan(f*x+e))^(1/2)-1/3
*B*b^3*d^3*(c+d*tan(f*x+e))^(3/2)+3/5*C*b^3*c^2*(c+d*tan(f*x+e))^(5/2)-1/5*
C*b^3*d^2*(c+d*tan(f*x+e))^(5/2)+1/3*C*b^3*c*d^2*(c+d*tan(f*x+e))^(3/2)-3*B
*a*b^2*d^4*(c+d*tan(f*x+e))^(1/2)-3*C*a^2*b*d^4*(c+d*tan(f*x+e))^(1/2)+1/3*
B*b^3*c^2*d*(c+d*tan(f*x+e))^(3/2)-1/3*A*b^3*c*d^2*(c+d*tan(f*x+e))^(3/2)+3
/7*C*a*b^2*d*(c+d*tan(f*x+e))^(7/2)+3/5*B*a*b^2*d^2*(c+d*tan(f*x+e))^(5/2)-
```

C*a^2*b*c*d^2*(c+d*tan(f*x+e))^(3/2)-6/5*C*a*b^2*c*d*(c+d*tan(f*x+e))^(5/2)
-B*a*b^2*c*d^2*(c+d*tan(f*x+e))^(3/2)+B*a^3*d^4*(c+d*tan(f*x+e))^(1/2)+1/5*
A*b^3*d^2*(c+d*tan(f*x+e))^(5/2)+d^4*(1/4/d*(1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^3-3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*a*b^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a^2*b*d+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-A*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b^3*d-3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b+B
(c^2+d^2)^(1/2)(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3+B*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^3*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c-3*B*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*b^2*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c-C*(c^2+d^2)^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a*b^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*C*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a^2*b*d-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+C*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-6*A*(c^2+d^2)^(1/2)*a^2*b*d+
2*A*(c^2+d^2)^(1/2)*b^3*d-2*B*(c^2+d^2)^(1/2)*a^3*d+6*B*(c^2+d^2)^(1/2)*a*b
^2*d+6*C*(c^2+d^2)^(1/2)*a^2*b*d-2*C*(c^2+d^2)^(1/2)*b^3*d+1/2*(A*(c^2+d^2)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a*b^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c+3*A*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a^2*b*d+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-A*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d-3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a^2*b+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3+B*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^3*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c-3*B*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c-
C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*C*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*C*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b
^2*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)
^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(-A*(c^2+d^2)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a*b^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*A*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a^2*b*d-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+A*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d+3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*a^2*b-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3-B*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^3*d-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c+3*B*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c+
C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*C*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c+3*C*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b
^2*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+
e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-6*A*(c^2+d^2)^
(1/2)*a^2*b*d+2*A*(c^2+d^2)^(1/2)*b^3*d-2*B*(c^2+d^2)^(1/2)*a^3*d+6*B*(c^2+
d^2)^(1/2)*a*b^2*d+6*C*(c^2+d^2)^(1/2)*a^2*b*d-2*C*(c^2+d^2)^(1/2)*b^3*d-1/
2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*A*(c^2+d^2)^(1/2)

```
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-
3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*b^2*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d+3*B*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^2*b-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*b^3-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*a^2*b*c+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d+B*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b^3*c+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*C*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a^3*c+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*b^2*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/
2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))+1/3*C*a^
3*d^3*(c+d*tan(f*x+e))^(3/2)-1/3*C*b^3*c^3*(c+d*tan(f*x+e))^(3/2)-3/7*C*b^3
*c*(c+d*tan(f*x+e))^(7/2)+1/9*C*b^3*(c+d*tan(f*x+e))^(9/2)-2/5*B*b^3*c*d*(c
+d*tan(f*x+e))^(5/2)+3/5*C*a^2*b*d^2*(c+d*tan(f...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) +
 C*tan(e + f*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.91 $\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e$

Optimal. Leaf size=325

$$-\frac{(a-ib)^2(B+i(A-C))\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(a+ib)^2(B-i(A-C))\sqrt{c+id}\,\tan}{f}$$

```
[Out] -(a-I*b)^2*(B+I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d
)^(1/2)/f-(a+I*b)^2*(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))*(c+I*d)^(1/2)/f+2*(a^2*B-b^2*B+2*a*b*(A-C))*(c+d*tan(f*x+e))^(1/2)/f+2/1
05*(20*a^2*C*d^2-14*a*b*d*(-5*B*d+2*C*c)+b^2*(8*c^2*C-14*B*c*d+35*(A-C)*d^2
))*(c+d*tan(f*x+e))^(3/2)/d^3/f-2/35*b*(-7*B*b*d-4*C*a*d+4*C*b*c)*tan(f*x+e
)*(c+d*tan(f*x+e))^(3/2)/d^2/f+2/7*C*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3
/2)/d/f
```

Rubi [A]
time = 0.86, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3728, 3718, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(c+d\tan(e+fx))^{3/2}\left(20a^2Cd^2-14abd(2cC-5Bd)+b^2(35d^2(A-C)-14Bcd+8c^2C)\right)}{105d^3f}+\frac{2(a^2B+2ab(A-C)-b^2B)\sqrt{c+d\tan(e+fx)}}{f}-\frac{(a-ib)^2\sqrt{c-id}\,(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(a+ib)^2\sqrt{c+id}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-\frac{2b\tan(e+fx)(-4aCd-7bBd+4bcC)(c+d\tan(e+fx))^{3/2}}{35d^2f}+\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^2*(B + I*(A - C))*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c - I*d]])/f) - ((a + I*b)^2*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[
Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(a^2*B - b^2*B + 2*a*b*(A -
 C))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(20*a^2*C*d^2 - 14*a*b*d*(2*c*C - 5*B
*d) + b^2*(8*c^2*C - 14*B*c*d + 35*(A - C)*d^2))*(c + d*Tan[e + f*x])^(3/2)
)/(105*d^3*f) - (2*b*(4*b*c*C - 7*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[
e + f*x])^(3/2))/(35*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*
x])^(3/2))/(7*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2C(a+b\tan(e+ \cdots}{\cdots}$$

$$= -\frac{2b(4bcC-7bB \cdots}{\cdots}$$

$$= \frac{2(20a^2Cd^2-14a \cdots}{\cdots}$$

$$= \frac{2(a^2B-b^2B+2a \cdots}{\cdots}$$

$$= \frac{2(a^2B-b^2B+2a \cdots}{\cdots}$$

$$= \frac{2(a^2B-b^2B+2a \cdots}{\cdots}$$

$$= \frac{2(a^2B-b^2B+2a \cdots}{\cdots}$$

$$= -\frac{(a-ib)^2(B+i \cdots}{\cdots}$$

Mathematica [A]
time = 3.18, size = 314, normalized size = 0.97

$$\frac{2\left(\left(20a^2Cd^2+14abd(-2cC+5Bd)+b^2(8c^2C-14Bcd+35(A-C)d^2)\right)(c+d\tan(e+fx))^{3/2}+3bd(-4bcC+7bBd+4aCd)\tan(e+fx)(c+d\tan(e+fx))^{3/2}+15Cd^2(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}+\frac{105}{2}(a-ib)^2(iA+B-iC)d^3\left(-\sqrt{c-id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\right)+\frac{105}{2}(a+ib)^2(-iA+B+iC)d^3\left(-\sqrt{c+id}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\right)\right)}{105d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*((20*a^2*C*d^2 + 14*a*b*d*(-2*c*C + 5*B*d) + b^2*(8*c^2*C - 14*B*c*d + 3
5*(A - C)*d^2))*(c + d*Tan[e + f*x])^(3/2) + 3*b*d*(-4*b*c*C + 7*b*B*d + 4*
a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2) + 15*C*d^2*(a + b*Tan[e + f*
x])^2*(c + d*Tan[e + f*x])^(3/2) + (105*(a - I*b)^2*(I*A + B - I*C)*d^3*(-(
Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]) + Sqrt[c + d
*Tan[e + f*x]]))/2 + (105*(a + I*b)^2*((-I)*A + B + I*C)*d^3*(-(Sqrt[c + I*
d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]) + Sqrt[c + d*Tan[e + f*
x]]))/2))/(105*d^3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2207 vs. $2(291) = 582$.
time = 0.49, size = 2208, normalized size = 6.79

method	result	size
derivativedivides	Expression too large to display	2208
default	Expression too large to display	2208

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^3*(1/7*b^2*C*(c+d*tan(f*x+e))^(7/2)+1/5*B*b^2*d*(c+d*tan(f*x+e))^(5/2
)+2/5*C*a*b*d*(c+d*tan(f*x+e))^(5/2)-2/5*C*b^2*c*(c+d*tan(f*x+e))^(5/2)+1/3
*A*b^2*d^2*(c+d*tan(f*x+e))^(3/2)+2/3*B*a*b*d^2*(c+d*tan(f*x+e))^(3/2)-1/3*
B*b^2*c*d*(c+d*tan(f*x+e))^(3/2)+1/3*C*a^2*d^2*(c+d*tan(f*x+e))^(3/2)-2/3*C
*a*b*c*d*(c+d*tan(f*x+e))^(3/2)+1/3*C*b^2*c^2*(c+d*tan(f*x+e))^(3/2)-1/3*C*
b^2*d^2*(c+d*tan(f*x+e))^(3/2)+2*A*a*b*d^3*(c+d*tan(f*x+e))^(1/2)+B*a^2*d^3
*(c+d*tan(f*x+e))^(1/2)-B*b^2*d^3*(c+d*tan(f*x+e))^(1/2)-2*C*a*b*d^3*(c+d*t
an(f*x+e))^(1/2)+d^3*(1/4/d*(1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^2+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+A*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a^2*c-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-A*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*b^2*c+2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d-2*B*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d+C*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
b^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*b*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*ln(d*tan(f*x+e)+c+(c+d*tan(f*
x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-4*A*(c^2+d^2
)^(1/2)*a*b*d-2*B*(c^2+d^2)^(1/2)*a^2*d+2*B*(c^2+d^2)^(1/2)*b^2*d+4*C*(c^2+
d^2)^(1/2)*a*b*d-1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2+
A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+A*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a^2*c-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-A*(2*(c^2+d^2)^(1/2)
```

```
+2*c)^(1/2)*b^2*c+2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b-B*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c+
B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2-C*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d+C*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^
2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c
)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(A*(c^2+d^2)^(1/2)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a^2-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*b*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-2*B*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*b+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d+2*B*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*b*c-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d-C*(c^2+
d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*b^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*C*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a*b*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*ln(d*tan(f*
x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2)
)+2*(-4*A*(c^2+d^2)^(1/2)*a*b*d-2*B*(c^2+d^2)^(1/2)*a^2*d+2*B*(c^2+d^2)^(1/
2)*b^2*d+4*C*(c^2+d^2)^(1/2)*a*b*d+1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a^2-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2-A*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d+A*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*b+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d+2*B*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*b*c-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d-C*(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*C*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*b*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^2*sq
rt(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^2 \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) +
 C*tan(e + f*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.92 $\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e$

Optimal. Leaf size=224

$$-\frac{(ia+b)(A-iB-C)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(ia-b)(A+iB-C)\sqrt{c+id}\,\tanh^{-1}\left(\right.}{f}$$

```
[Out] -(I*a+b)*(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1
/2)/f+(I*a-b)*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*
d)^(1/2)/f+2*(A*b+B*a-C*b)*(c+d*tan(f*x+e))^(1/2)/f-2/15*(-5*B*b*d-5*C*a*d+
2*C*b*c)*(c+d*tan(f*x+e))^(3/2)/d^2/f+2/5*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^(
3/2)/d/f
```

Rubi [A]
time = 0.42, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3718, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(aB+Ab-bC)\sqrt{c+d\tan(e+fx)}}{f}-\frac{(b+ia)\sqrt{c-id}\,(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(-b+ia)\sqrt{c+id}\,(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-\frac{2(-5aCd-5bBd+2bcC)(c+d\tan(e+fx))^{3/2}}{15d^2f}+\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c +
 d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(A*b + a*B - b*C)*Sqrt[c + d*Tan[e
+ f*x]])/f - (2*(2*b*c*C - 5*b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^(3/2))/(
15*d^2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\,\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2bC\tan(e+fx)(c}{5}$$

$$= -\frac{2(2bcC-5bBd}{}$$

$$= \frac{2(Ab+aB-bC)}{}$$

$$= \frac{2(Ab+aB-bC)}{}$$

$$= \frac{2(Ab+aB-bC)}{}$$

$$= \frac{2(Ab+aB-bC)}{}$$

$$= -\frac{(ia+b)(A-iB}{}$$

Mathematica [A]
time = 1.32, size = 220, normalized size = 0.98

$$\frac{\frac{2(-2bcC+5bBd+5aCd)(c+d\tan(e+fx))^{3/2}}{d}+6bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}+15(ia+b)(A-iB-C)d\left(-\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\right)+15(-ia+b)(A+iB-C)d\left(-\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\right)}{15df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] ((2*(-2*b*c*C + 5*b*B*d + 5*a*C*d)*(c + d*Tan[e + f*x])^(3/2))/d + 6*b*C*Ta
n[e + f*x]*(c + d*Tan[e + f*x])^(3/2) + 15*(I*a + b)*(A - I*B - C)*d*(-(Sqr
t[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]) + Sqrt[c + d*Ta
n[e + f*x]]) + 15*((-I)*a + b)*(A + I*B - C)*d*(-(Sqrt[c + I*d]*ArcTanh[Sqr
t[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]) + Sqrt[c + d*Tan[e + f*x]]))/(15*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. $2(194) = 388$.
time = 0.48, size = 1398, normalized size = 6.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^2*(1/5*C*b*(c+d*tan(f*x+e))^(5/2)+1/3*B*b*d*(c+d*tan(f*x+e))^(3/2)+1/
3*C*a*d*(c+d*tan(f*x+e))^(3/2)-1/3*C*b*c*(c+d*tan(f*x+e))^(3/2)+A*b*d^2*(c+
d*tan(f*x+e))^(1/2)+B*a*d^2*(c+d*tan(f*x+e))^(1/2)-C*b*d^2*(c+d*tan(f*x+e))
^(1/2)+d^2*(1/4/d*(1/2*(-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+
A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+B*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*(c^2+d^2)^(1/2)*a-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*A*(c^2+d^2)^(1/2)*b*d-2*B*(c^2+d^2)^(1/
2)*a*d+2*C*(c^2+d^2)^(1/2)*b*d-1/2*(-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d
^2)^(1/2)*a+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-B*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+C*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+C*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*b*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1
/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/
2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(-1/2*(-A*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*(c^2+d^2)^(1/2)*a+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-A*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*b*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-B*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+C*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(2*A*(c^2+d^2)^(1/
2)*b*d+2*B*(c^2+d^2)^(1/2)*a*d-2*C*(c^2+d^2)^(1/2)*b*d+1/2*(-A*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-A*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^
(1/2)*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*b*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a-C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2
)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)*sqrt
(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x)
+ C*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 60.11, size = 2500, normalized size = 11.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C
*tan(e + f*x)^2),x)
```

```
[Out] ((2*B*a*d - 4*C*a*c)/(d*f) + (4*C*a*c)/(d*f))*(c + d*tan(e + f*x))^(1/2) +
((2*B*b*d - 6*C*b*c)/(3*d^2*f) + (4*C*b*c)/(3*d^2*f))*(c + d*tan(e + f*x))^
(3/2) + (c + d*tan(e + f*x))^(1/2)*(2*c*((2*B*b*d - 6*C*b*c)/(d^2*f) + (4*C
*b*c)/(d^2*f)) + (2*A*b*d^2 + 6*C*b*c^2 - 4*B*b*c*d)/(d^2*f) - (2*C*b*(d^4*
f + c^2*d^2*f))/(d^4*f^2)) - atan(((((8*(4*A*b*d^4*f^2 - 4*C*b*d^4*f^2 + 4*
A*b*c^2*d^2*f^2 - 4*C*b*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(
1/2)*((A^2*b^2*c)/(4*f^2) - (4*A*C^3*b^4*d^2*f^4 - B^4*b^4*d^2*f^4 - C^4*b^
4*d^2*f^4 - A^4*b^4*d^2*f^4 + 4*A^3*C*b^4*d^2*f^4 - 4*A^2*B^2*b^4*c^2*f^4 +
 2*A^2*B^2*b^4*d^2*f^4 - 6*A^2*C^2*b^4*d^2*f^4 - 4*B^2*C^2*b^4*c^2*f^4 + 2*
B^2*C^2*b^4*d^2*f^4 + 4*A*B^3*b^4*c*d*f^4 - 4*A^3*B*b^4*c*d*f^4 + 4*B*C^3*b
^4*c*d*f^4 - 4*B^3*C*b^4*c*d*f^4 + 8*A*B^2*C*b^4*c^2*f^4 - 4*A*B^2*C*b^4*d^
2*f^4 - 12*A*B*C^2*b^4*c*d*f^4 + 12*A^2*B*C*b^4*c*d*f^4)^(1/2)/(4*f^4) - (B
^2*b^2*c)/(4*f^2) + (C^2*b^2*c)/(4*f^2) - (A*B*b^2*d)/(2*f^2) - (A*C*b^2*c)
/(2*f^2) + (B*C*b^2*d)/(2*f^2))^(1/2))*((A^2*b^2*c)/(4*f^2) - (4*A*C^3*b^4*
d^2*f^4 - B^4*b^4*d^2*f^4 - C^4*b^4*d^2*f^4 - A^4*b^4*d^2*f^4 + 4*A^3*C*b^4
*d^2*f^4 - 4*A^2*B^2*b^4*c^2*f^4 + 2*A^2*B^2*b^4*d^2*f^4 - 6*A^2*C^2*b^4*d^
2*f^4 - 4*B^2*C^2*b^4*c^2*f^4 + 2*B^2*C^2*b^4*d^2*f^4 + 4*A*B^3*b^4*c*d*f^4
 - 4*A^3*B*b^4*c*d*f^4 + 4*B*C^3*b^4*c*d*f^4 - 4*B^3*C*b^4*c*d*f^4 + 8*A*B^
2*C*b^4*c^2*f^4 - 4*A*B^2*C*b^4*d^2*f^4 - 12*A*B*C^2*b^4*c*d*f^4 + 12*A^2*B
*C*b^4*c*d*f^4)^(1/2)/(4*f^4) - (B^2*b^2*c)/(4*f^2) + (C^2*b^2*c)/(4*f^2) -
 (A*B*b^2*d)/(2*f^2) - (A*C*b^2*c)/(2*f^2) + (B*C*b^2*d)/(2*f^2))^(1/2) - (
16*(c + d*tan(e + f*x))^(1/2)*(A^2*b^2*d^4 - B^2*b^2*d^4 + C^2*b^2*d^4 - A^
2*b^2*c^2*d^2 + B^2*b^2*c^2*d^2 - C^2*b^2*c^2*d^2 - 2*A*C*b^2*d^4 + 2*A*C*b
^2*c^2*d^2 + 4*A*B*b^2*c*d^3 - 4*B*C*b^2*c*d^3))/f^2)*((A^2*b^2*c)/(4*f^2)
- (4*A*C^3*b^4*d^2*f^4 - B^4*b^4*d^2*f^4 - C^4*b^4*d^2*f^4 - A^4*b^4*d^2*f^
4 + 4*A^3*C*b^4*d^2*f^4 - 4*A^2*B^2*b^4*c^2*f^4 + 2*A^2*B^2*b^4*d^2*f^4 - 6
*A^2*C^2*b^4*d^2*f^4 - 4*B^2*C^2*b^4*c^2*f^4 + 2*B^2*C^2*b^4*d^2*f^4 + 4*A*
B^3*b^4*c*d*f^4 - 4*A^3*B*b^4*c*d*f^4 + 4*B*C^3*b^4*c*d*f^4 - 4*B^3*C*b^4*c
*d*f^4 + 8*A*B^2*C*b^4*c^2*f^4 - 4*A*B^2*C*b^4*d^2*f^4 - 12*A*B*C^2*b^4*c*d
*f^4 + 12*A^2*B*C*b^4*c*d*f^4)^(1/2)/(4*f^4) - (B^2*b^2*c)/(4*f^2) + (C^2*b
^2*c)/(4*f^2) - (A*B*b^2*d)/(2*f^2) - (A*C*b^2*c)/(2*f^2) + (B*C*b^2*d)/(2*
f^2))^(1/2)*1i - (((8*(4*A*b*d^4*f^2 - 4*C*b*d^4*f^2 + 4*A*b*c^2*d^2*f^2 -
4*C*b*c^2*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((A^2*b^2*c)/
(4*f^2) - (4*A*C^3*b^4*d^2*f^4 - B^4*b^4*d^2*f^4 - C^4*b^4*d^2*f^4 - A^4*b^
4*d^2*f^4 + 4*A^3*C*b^4*d^2*f^4 - 4*A^2*B^2*b^4*c^2*f^4 + 2*A^2*B^2*b^4*d^2
*f^4 - 6*A^2*C^2*b^4*d^2*f^4 - 4*B^2*C^2*b^4*c^2*f^4 + 2*B^2*C^2*b^4*d^2*f^
4 + 4*A*B^3*b^4*c*d*f^4 - 4*A^3*B*b^4*c*d*f^4 + 4*B*C^3*b^4*c*d*f^4 - 4*B^3
*C*b^4*c*d*f^4 + 8*A*B^2*C*b^4*c^2*f^4 - 4*A*B^2*C*b^4*d^2*f^4 - 12*A*B*C^2
*b^4*c*d*f^4 + 12*A^2*B*C*b^4*c*d*f^4)^(1/2)/(4*f^4) - (B^2*b^2*c)/(4*f^2)
+ (C^2*b^2*c)/(4*f^2) - (A*B*b^2*d)/(2*f^2) - (A*C*b^2*c)/(2*f^2) + (B*C*b^
2*d)/(2*f^2))^(1/2))*((A^2*b^2*c)/(4*f^2) - (4*A*C^3*b^4*d^2*f^4 - B^4*b^4*
d^2*f^4 - C^4*b^4*d^2*f^4 - A^4*b^4*d^2*f^4 + 4*A^3*C*b^4*d^2*f^4 - 4*A^2*B
^2*b^4*c^2*f^4 + 2*A^2*B^2*b^4*d^2*f^4 - 6*A^2*C^2*b^4*d^2*f^4 - 4*B^2*C^2*
b^4*c^2*f^4 + 2*B^2*C^2*b^4*d^2*f^4 + 4*A*B^3*b^4*c*d*f^4 - 4*A^3*B*b^4*c*d
*f^4 + 4*B*C^3*b^4*c*d*f^4 - 4*B^3*C*b^4*c*d*f^4 + 8*A*B^2*C*b^4*c^2*f^4 -
```

4*A*B^2*C*b^4*d^2*f^4 - 12*A*B*C^2*b^4*c*d*f^4 + 12*A^2*B*C*b^4*c*d*f^4)^(1/2)/(4*f^4) - (B^2*b^2*c)/(4*f^2) + (C^2*b^2*c)/(4*f^2) - (A*B*b^2*d)/(2*f^2) - (A*C*b^2*c)/(2*f^2) + (B*C*b^2*d)/(2*f^2))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(A^2*b^2*d^4 - B^2*b^2*d^4 + C^2*b^2*d^4 - A^2*b^2*c^2*d^2 + B^2*b^2*c^2*d^2 - C^2*b^2*c^2*d^2 - 2*A*C*b^2*d^4 + 2*A*C*b^2*c^2*d^2 + 4*A*B*b^2*c*d^3 - 4*B*C*b^2*c*d^3))/f^2)*((A^2*b^2*c)/(4*f^2) - (4*A*C^3*b^4*d^2*f^4 - B^4*b^4*d^2*f^4 - C^4*b^4*d^2*f^4 - A^4*b^4*d^2*f^4 + 4*A^3*C*b^4*d^2*f^4 - 4*A^2*B^2*b^4*c^2*f^4 + 2*A^2*B^2*b^4*d^2*f^4 - 6*A^2*C^2*b^4*d^2*f^4 - 4*B^2*C^2*b^4*c^2*f^4 + 2*B^2*C^2*b^4*d^2*f^4 + 4*A*B^3*b^4*c*d*f^4 - 4*A^3*B*b^4*c*d*f^4 + 4*B*C^3*b^4*c*d*f^4 - 4*B^3*C*b^4*c*d*f^4 + 8*A*B^2*C*b^4*c^2*f^4 - 4*A*B^2*C*b^4*d^2*f^4 - 12*A*B*C^2*b^4*c*d*f^4 + 12*A^2*B*C*b^4*c*d*f^4)^(1/2)/(4*f^4) - (B^2*b^2*c)/(4*f^2) + (C^2*b^2*c)/(4*f^2) - (A*B*b^2*d)/(2*f^2) - (A*C*b^2*c)/(2*f^2) + (B*C*b^2*d)/(2*f^2))^(1/2)*1i)/((16*(B^3*b^3*d^5 - A^3*b^3*c^3*d^2 + B^3*b^3*c^2*d^3 + C^3*b^3*c^3*d^2 + A^2*B*b^3*d^5 + B*C^2*b^3*d^5 - A^3*b^3*c*d^4 + C^3*b^3*c*d^4 - A*B^2*b^3*c*d^4 - 3*A*C^2*b^3*c*d^4 + 3*A^2*C*b^3*c*d^4 + B^2*C*b^3*c*d^4 - A*B^2*b^3*c^3*d^2 + A^2*B*b^3*c^2*d^3 - 3*A*C^2*b^3*c^3*d^2 + 3*A^2*C*b^3*c^3*d^2 + B*C^2*b^3*c^2*d^3 + B^2*C*b^3*c^3*d^2 - 2*A*B*C*b^3*d^5 - 2*A*B*C*b^3*c^2*d^3))/f^3 + (((8*(4*A*b*d^4*f^2 - 4*C*b*d^4*f^2 + 4*A*b*c^2*d^2*f^2 - 4*C*b*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((A^2*b^2*c)/(4*f^2) - (4*A*C^3*b^4*d^2*f^4 - B^4*b^4*d^2*f^4 - C^4*b^4*d^2*f^4 - A^4*b^4*d^2*f^4 + 4*A^3*C*b^4*d^2*f^4 - 4*A^2*B^2*b^4*c^2*f^4 +...

3.93 $\int \sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e$

Optimal. Leaf size=155

$$-\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(B-i(A-C))\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e}{\sqrt{c+i}}\right.}{f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/f-
(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/f+2
*B*(c+d*tan(f*x+e))^(1/2)/f+2/3*C*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.21, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3711, 3609, 3620, 3618, 65, 214}

$$-\frac{\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{\sqrt{c+id}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/f) - ((B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]])/f + (2*B*Sqrt[c + d*Tan[e + f*x]])/f + (2*C*(c + d*Tan[e +
 f*x])^(3/2))/(3*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
```

```
0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sqrt{c+d\tan(e+fx)}\,\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2C(c+d\tan(e+fx))^{3/2}}{3df}+\int (A-C$$

$$= \frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\,\mathrm{t}}{}$$

$$= \frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\,\mathrm{t}}{}$$

$$= \frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\,\mathrm{t}}{}$$

$$= \frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\,\mathrm{t}}{}$$

$$= -\frac{(B+i(A-C))\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{f}$$

Mathematica [A]
time = 0.39, size = 150, normalized size = 0.97

$$\frac{-3i(A-iB-C)\sqrt{c-id}\,d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+3i(A+iB-C)\sqrt{c+id}\,d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+2\sqrt{c+d\tan(e+fx)}\,(cC+3Bd+Cd\tan(e+fx))}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),
x]
```

```
[Out] ((-3*I)*(A - I*B - C)*Sqrt[c - I*d]*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c - I*d]] + (3*I)*(A + I*B - C)*Sqrt[c + I*d]*d*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c + I*d]] + 2*Sqrt[c + d*Tan[e + f*x]]*(c*C + 3*B*d + C*d*Tan[e
 + f*x]))/(3*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(130) = 260$.
time = 0.42, size = 846, normalized size = 5.46

method	result
derivativedivides	$\frac{\frac{2C(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2Bd\sqrt{c+d\tan(fx+e)}+2d\left(\frac{\left(A\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}-A\sqrt{2\sqrt{c^2}\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$
default	$\frac{\frac{2C(c+d\tan(fx+e))^{\frac{3}{2}}}{3}+2Bd\sqrt{c+d\tan(fx+e)}+2d\left(\frac{\left(A\sqrt{c^2+d^2}\,\sqrt{2\sqrt{c^2+d^2}+2c}-A\sqrt{2\sqrt{c^2}\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/d*(1/3*C*(c+d*tan(f*x+e))^(3/2)+B*d*(c+d*tan(f*x+e))^(1/2)+d*(1/4/d*(1/
2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-A*(2*(c^2+d^2)^(1/2)+2*c
```

```
)^(1/2)*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c)*ln(d*tan(f*x+e)+c-(c+d*
tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*B*(c
^2+d^2)^(1/2)*d+1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-A*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d-C*(c^2+d^2)^(
1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(
f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
)+1/4/d*(1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+A*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d+C*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c)*ln(d*tan(f*
x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2)
)+2*(-2*B*(c^2+d^2)^(1/2)*d-1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d+
C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-C*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*c)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan
((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2
)-2*c)^(1/2)))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + d \tan{\left(e + fx \right)}} \left(A + B \tan{\left(e + fx \right)} + C \tan^{2}{\left(e + fx \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2),
 x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 17.40, size = 1199, normalized size = 7.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

```
[Out] 2*atanh((32*B^2*d^4*((B^2*c)/(4*f^2) - (-B^4*d^2*f^4)^(1/2)/(4*f^4))^(1/2)*
(c + d*tan(e + f*x))^(1/2))/((16*B*d^4*(-B^4*d^2*f^4)^(1/2))/f^3 + (16*B*c^
2*d^2*(-B^4*d^2*f^4)^(1/2))/f^3) - (32*c*d^2*((B^2*c)/(4*f^2) - (-B^4*d^2*f
^4)^(1/2)/(4*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-B^4*d^2*f^4)^(1/2))/(
(16*B*d^4*(-B^4*d^2*f^4)^(1/2))/f + (16*B*c^2*d^2*(-B^4*d^2*f^4)^(1/2))/f))
*(-((-B^4*d^2*f^4)^(1/2) - B^2*c*f^2)/(4*f^4))^(1/2) - 2*atanh((32*B^2*d^4*
((-B^4*d^2*f^4)^(1/2)/(4*f^4) + (B^2*c)/(4*f^2))^(1/2)*(c + d*tan(e + f*x))
^(1/2))/((16*B*d^4*(-B^4*d^2*f^4)^(1/2))/f^3 + (16*B*c^2*d^2*(-B^4*d^2*f^4)
^(1/2))/f^3) + (32*c*d^2*((-B^4*d^2*f^4)^(1/2)/(4*f^4) + (B^2*c)/(4*f^2))^(
1/2)*(c + d*tan(e + f*x))^(1/2)*(-B^4*d^2*f^4)^(1/2))/((16*B*d^4*(-B^4*d^2*
f^4)^(1/2))/f + (16*B*c^2*d^2*(-B^4*d^2*f^4)^(1/2))/f))*(((-B^4*d^2*f^4)^(1
/2) + B^2*c*f^2)/(4*f^4))^(1/2) - atanh((f^3*(-((-A^4*d^2*f^4)^(1/2) + A^2*
c*f^2)/f^4)^(1/2)*((16*(A^2*d^4 - A^2*c^2*d^2)*(c + d*tan(e + f*x))^(1/2))/
f^2 + (16*c*d^2*((-A^4*d^2*f^4)^(1/2) + A^2*c*f^2)*(c + d*tan(e + f*x))^(1/
2))/f^4))/(16*(A^3*d^5 + A^3*c^2*d^3)))*(-((-A^4*d^2*f^4)^(1/2) + A^2*c*f^2
)/f^4)^(1/2) - atanh((f^3*(((-A^4*d^2*f^4)^(1/2) - A^2*c*f^2)/f^4)^(1/2)*((
16*(A^2*d^4 - A^2*c^2*d^2)*(c + d*tan(e + f*x))^(1/2))/f^2 - (16*c*d^2*((-A
```

```
^4*d^2*f^4)^(1/2) - A^2*c*f^2)*(c + d*tan(e + f*x))^(1/2))/f^4))/(16*(A^3*d
^5 + A^3*c^2*d^3)))*(((-A^4*d^2*f^4)^(1/2) - A^2*c*f^2)/f^4)^(1/2) + atanh(
(f^3*(-((-C^4*d^2*f^4)^(1/2) + C^2*c*f^2)/f^4)^(1/2)*((16*(C^2*d^4 - C^2*c^
2*d^2)*(c + d*tan(e + f*x))^(1/2))/f^2 + (16*c*d^2*((-C^4*d^2*f^4)^(1/2) +
C^2*c*f^2)*(c + d*tan(e + f*x))^(1/2))/f^4))/(16*(C^3*d^5 + C^3*c^2*d^3)))*
(-((-C^4*d^2*f^4)^(1/2) + C^2*c*f^2)/f^4)^(1/2) + atanh((f^3*(((-C^4*d^2*f^
4)^(1/2) - C^2*c*f^2)/f^4)^(1/2)*((16*(C^2*d^4 - C^2*c^2*d^2)*(c + d*tan(e
+ f*x))^(1/2))/f^2 - (16*c*d^2*((-C^4*d^2*f^4)^(1/2) - C^2*c*f^2)*(c + d*ta
n(e + f*x))^(1/2))/f^4))/(16*(C^3*d^5 + C^3*c^2*d^3)))*(((-C^4*d^2*f^4)^(1/
2) - C^2*c*f^2)/f^4)^(1/2) + (2*B*(c + d*tan(e + f*x))^(1/2))/f + (2*C*(c +
 d*tan(e + f*x))^(3/2))/(3*d*f)
```

3.94 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=234

$$-\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)f}+\frac{(iA-B-iC)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/(a
-I*b)/f+(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(
1/2)/(a+I*b)/f-2*(A*b^2-a*(B*b-C*a))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)
/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(3/2)/(a^2+b^2)/f+2*C*(c+d*tan(f*x+e)
)^(1/2)/b/f
```

Rubi [A]
time = 0.72, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3728, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2\sqrt{bc-ad}\,(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}f(a^2+b^2)}-\frac{\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)}+\frac{\sqrt{c+id}\,(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)}+\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x]),x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((a - I*b)*f)) + ((I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)*f) - (2*(A*b^2 - a*(b*B - a*C))*Sqr
t[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(
b^(3/2)*(a^2 + b^2)*f) + (2*C*Sqrt[c + d*Tan[e + f*x]])/(b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \frac{2C\sqrt{c+d\tan(e+fx)}}{bf} + \frac{2\int \frac{\frac{1}{2}(Abc-aC}{}}{}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{bf} + \frac{2\int \frac{\frac{1}{2}b(bBc+b(}{}}{}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{bf} + \frac{((A-iB-}{}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{bf} - \frac{(i(A+iB-}{}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{bc-ad}\,\tanh}{b^{3/2}(a^2+}$$

$$= \frac{(A-iB-C)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+}}{}\right.}{(ia+b)f}$$

Mathematica [A]
time = 0.46, size = 233, normalized size = 1.00

$$\frac{b^{3/2}(-ia+b)(A-iB-C)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+b^{3/2}(ia+b)(A+iB-C)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)-2(Ab^2+a(-bB+aC))\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)+2\sqrt{b}\,(a^2+b^2)C\sqrt{c+d\tan(e+fx)}}{b^{3/2}(a^2+b^2)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (b^(3/2)*((-I)*a + b)*(A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e
+ f*x]]/Sqrt[c - I*d]] + b^(3/2)*(I*a + b)*(A + I*B - C)*Sqrt[c + I*d]*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] - 2*(A*b^2 + a*(-(b*B) + a*C))*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]
 + 2*Sqrt[b]*(a^2 + b^2)*C*Sqrt[c + d*Tan[e + f*x]])/(b^(3/2)*(a^2 + b^2)*f
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1393 vs. $2(200) = 400$.
time = 0.62, size = 1394, normalized size = 5.96 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*C/b*(c+d*tan(f*x+e))^(1/2)+2/(a^2+b^2)*(1/4/d*(-1/2*(-A*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+A*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^
(1/2)*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*b*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a-C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d)*ln((c+d*tan(f*x+e))^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-2*A*(
c^2+d^2)^(1/2)*b*d+2*B*(c^2+d^2)^(1/2)*a*d+2*C*(c^2+d^2)^(1/2)*b*d+1/2*(-A*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+A*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*(c^2+d^2)^(1/2)*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d+B*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*b*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a-C*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/
2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4
/d*(1/2*(-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-B*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d+B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2
)*a-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d
)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c
^2+d^2)^(1/2))+2*(2*A*(c^2+d^2)^(1/2)*b*d-2*B*(c^2+d^2)^(1/2)*a*d-2*C*(c^2+
d^2)^(1/2)*b*d-1/2*(-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+A*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^
2+d^2)^(1/2)*a-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-C*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*a
rctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)
^(1/2)-2*c)^(1/2))))+2*(-A*a*b^2*d+A*b^3*c+B*a^2*b*d-B*a*b^2*c-C*a^3*d+C*a^
2*b*c)/b/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*
d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan{(e+fx)}}\,(A+B\tan{(e+fx)}+C\tan^2{(e+fx)})}{a+b\tan{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e)),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 36.22, size = 2500, normalized size = 10.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b*tan(e + f*x)),x)

[Out] atan(((((((32*(4*C*a*b^8*d^11*f^4 - 4*C*b^9*c*d^10*f^4 + 8*C*a^3*b^6*d^11*f^4 + 4*C*a^5*b^4*d^11*f^4 - 4*C*b^9*c^3*d^8*f^4 + 4*C*a*b^8*c^2*d^9*f^4 - 8*C*a^2*b^7*c*d^10*f^4 - 4*C*a^4*b^5*c*d^10*f^4 - 8*C*a^2*b^7*c^3*d^8*f^4 + 8*C*a^3*b^6*c^2*d^9*f^4 - 4*C*a^4*b^5*c^3*d^8*f^4 + 4*C*a^5*b^4*c^2*d^9*f^4))/(b*f^5) - (32*(c + d*tan(e + f*x))^(1/2)*((((8*C^2*a^2*c*f^2 - 8*C^2*b^2*c*f^2 + 16*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2)*(16*b^10*d^10*f^4 + 16*a^2*b^8*d^10*f^4 - 16*a^4*b^6*d^10*f^4 - 16*a^6*b^4*d^10*f^4 + 24*b^10*c^2*d^8*f^4 + 40*a^2*b^8*c^2*d^8*f^4 + 8*a^4*b^6*c^2*d^8*f^4 - 8*a^6*b^4*c^2*d^8*f^4 + 8*a*b^9*c*d^9*f^4 + 24*a^3*b^7*c*d^9*f^4 + 24*a^5*b^5*c*d^9*f^4 + 8*a^7*b^3*c*d^9*f^4))/(b*f^4))*((((8*C^2*a^2*c*f^2 - 8*C^2*b^2*c*f^2 + 16*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(14*C^2*a*b^7*d^11*f^2 - 2*C^2*a^5*b^3*d^11*f^2 - 10*C^2*b^8*c^3*d^8*f^2 - 4*C^2*a^3*b^5*d^11*f^2 - 16*C^2*a^7*b*d^11*f^2 + 8*C^2*a^8*c*d^10*f^2 - 6*C^2*b^8*c*d^10*f^2 + 18*C^2*a*b^7*c^2*d^9*f^2 + 12*C^2*a^2*b^6*c*d^10*f^2 + 2*C^2*a^4*b^4*c*d^10*f^2 + 24*C^2*a^6*b^2*c*d^10*f^2 - 16*C^2*a^7*b*c^2*d^9*f^2 + 4*C^2*a^2*b^6*c^3*d^8*f^2 + 4*C^2*a^3*b^5*c^2*d^9*f^2 - 10*C^2*a^4*b^4*c^3*d^8*f^2 + 2*C^2*a^5*b^3*c^2*d^9*f^2 + 8*C^2*a^6*b^2*c^3*d^8*f^2))/(b*f^4))*((((8*C^2*a^2*c*f^2 - 8*C^2*b^2*c*f^2 + 16*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2) + (32*(15*C^3*a^4*b^3*d^12*f^2 - C^3*a^2*b^5*d^12*f^2 + C^3*b^7*c^2*d^10*f^2 + C^3*b^7*c^4*d^8*f^2 - 12*C^3*a^6*b*d^12*f^2 - 24*C^3*a^3*b^4*c*d^11*f^2 + 24*C^3*a^5*b^2*c*d^11*f^2 - 12*C^3*a^6*b*c^2*d^10*f^2 + 8*C^3*a^2*b^5*c^2*d^10*f^2 + 9*C^3*a^2*b^5*c^4*d^8*f^2 - 24*C^3*a^3*b^4*c^3*d^9*f^2 + 3*C^3*a^4*b^3*c^2*d^10*f^2 - 12*C^3*a^4*b^3*c^4*d^8*f^2 + 24*C^3*a^5*b^2*c^3*d^9*f^2))/(b*f^5))*((((8*C^2*a^2*c*f^2 - 8*C^2*b^2*c*f^2 + 16*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(C^4*b^6*d^12 - 2*C^4*a^6*d^12 + 2*C^4*a^6*c^2*d^10 + 2*C^4*b^6*c^2*d^10 + C^4*b^6*c^4*d^8 - 2*C^4*a^4*b^2*c^2*d^10 + 2*C^4*a^4*b^2*c^4*d^8 + 4*C^4*a^5*b*c*d^11 - 4*C^4*a^5*b*c^3*d^9))/(b*f^4))*((((8*C^2*a^2*c*f^2 - 8*C^2*b^2*c*f^2 + 16*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2)*1i - (((((32*(4*C*a*b^8*d^11*f^4 - 4*C*b^9*c*d^10*f^4 + 8*C*a^3*b^6*d^11*f^4 + 4*C*a^5*b^4*d^11*f^4 - 4*C*b^9*c^3*d^8*f^4 + 4*C*a*b^8*c^2*d^9*f^4 - 8*C*a^2*b^7*c*d^10*f^4 - 4*C*a^4*b^5*c*d^10*f^4 - 8*C*a^2*b^7*c^3*d^8*f^4 + 8*C*a^3*b^6

```
*c^2*d^9*f^4 - 4*C*a^4*b^5*c^3*d^8*f^4 + 4*C*a^5*b^4*c^2*d^9*f^4))/(b*f^5)
+ (32*(c + d*tan(e + f*x))^(1/2)*((((8*C^2*a^2*c*f^2 - 8*C^2*b^2*c*f^2 + 16
*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16*b^4*f^4 + 32*a^2
*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(16
*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2)*(16*b^10*d^10*f^4 + 16*a^2*b^8
*d^10*f^4 - 16*a^4*b^6*d^10*f^4 - 16*a^6*b^4*d^10*f^4 + 24*b^10*c^2*d^8*f^4
 + 40*a^2*b^8*c^2*d^8*f^4 + 8*a^4*b^6*c^2*d^8*f^4 - 8*a^6*b^4*c^2*d^8*f^4 +
 8*a*b^9*c*d^9*f^4 + 24*a^3*b^7*c*d^9*f^4 + 24*a^5*b^5*c*d^9*f^4 + 8*a^7*b^
3*c*d^9*f^4))/(b*f^4))*((((8*C^2*a^2*c*f^2 - 8*C^2*b^2*c*f^2 + 16*C^2*a*b*d
*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4))
^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(16*(a^4*f^4
+ b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2) + (32*(c + d*tan(e + f*x))^(1/2)*(14*C^2
*a*b^7*d^11*f^2 - 2*C^2*a^5*b^3*d^11*f^2 - 10*C^2*b^8*c^3*d^8*f^2 - 4*C^2*a
^3*b^5*d^11*f^2 - 16*C^2*a^7*b*d^11*f^2 + 8*C^2*a^8*c*d^10*f^2 - 6*C^2*b^8*
c*d^10*f^2 + 18*C^2*a*b^7*c^2*d^9*f^2 + 12*C^2*a^2*b^6*c*d^10*f^2 + 2*C^2*a
^4*b^4*c*d^10*f^2 + 24*C^2*a^6*b^2*c*d^10*f^2 - 16*C^2*a^7*b*c^2*d^9*f^2 +
4*C^2*a^2*b^6*c^3*d^8*f^2 + 4*C^2*a^3*b^5*c^2*d^9*f^2 - 10*C^2*a^4*b^4*c^3*
d^8*f^2 + 2*C^2*a^5*b^3*c^2*d^9*f^2 + 8*C^2*a^6*b^2*c^3*d^8*f^2))/(b*f^4))*
((((8*C^2*a^2*c*f^2 - 8*C^2*b^2*c*f^2 + 16*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 +
C^4*d^2)*(16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^
2 + 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f
^4)))^(1/2) + (32*(15*C^3*a^4*b^3*d^12*f^2 - C^3*a^2*b^5*d^12*f^2 + C^3*b^7
*c^2*d^10*f^2 + C^3*b^7*c^4*d^8*f^2 - 12*C^3*a^6*b*d^12*f^2 - 24*C^3*a^3*b^
4*c*d^11*f^2 + 24*C^3*a^5*b^2*c*d^11*f^2 - 12*C^3*a^6*b*c^2*d^10*f^2 + 8*C^
3*a^2*b^5*c^2*d^10*f^2 + 9*C^3*a^2*b^5*c^4*d^8*...
```

3.95 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=317

$$-\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f}-\frac{(B-i(A-C))\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan}}{\sqrt{c+}}\right.}{(a+ib)^2f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/(a
-I*b)^2/f-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)
^(1/2)/(a+I*b)^2/f-(a^3*b*B*d+a^4*C*d+b^4*(A*d+2*B*c)+a*b^3*(4*A*c-3*B*d-4*
C*c)-a^2*b^2*(3*A*d+2*B*c-5*C*d))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-
a*d+b*c)^(1/2))/b^(3/2)/(a^2+b^2)^2/f/(-a*d+b*c)^(1/2)-(A*b^2-a*(B*b-C*a))*
(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.98, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3726, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{bf(a^2+b^2)(a+b\tan(e+fx))}-\frac{(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}f(a^2+b^2)^2\sqrt{bc-ad}}-\frac{\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2}-\frac{\sqrt{c+id}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^2,x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) - ((a^3*b*B*d + a^4*C*d + b^
4*(2*B*c + A*d) + a*b^3*(4*A*c - 4*c*C - 3*B*d) - a^2*b^2*(2*B*c + 3*A*d -
5*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(b^(3/
2)*(a^2 + b^2)^2*Sqrt[b*c - a*d]*f) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*T
an[e + f*x]])/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

```
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{(a^3bBd+a^4Cd+b^4(2Bc+Ad)+a}{}$$

$$= -\frac{(B+i(A-C))\sqrt{c-id}\,\tanh^{-1}\Big(\frac{\sqrt{}}{}}{(a-ib)^2f}$$

Mathematica [A]
time = 4.11, size = 362, normalized size = 1.14

$$\frac{2\left(\frac{\frac{ib\left(-(a+ib)^2(A-iB-C)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+(a-ib)^2(A+iB-C)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)\right)}{2f}-\frac{\left(a^3bBd+a^4Cd+b^4(2Bc+Ad)+ab^3(4Ac-4cC-3Bd)+a^2b^2(-2Bc-3Ad+5Cd)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}\,f}}{(a^2+b^2)^2}-\frac{C\sqrt{c+d\tan(e+fx)}}{f(a+b\tan(e+fx))}+\frac{(-Ab^2+abB+a^2C+2b^2C)\sqrt{c+d\tan(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*((((I/2)*b*(-((a + I*b)^2*(A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c - I*d]]) + (a - I*b)^2*(A + I*B - C)*Sqrt[c + I*d]*Ar
cTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]))/f - ((a^3*b*B*d + a^4*C*d +
 b^4*(2*B*c + A*d) + a*b^3*(4*A*c - 4*c*C - 3*B*d) + a^2*b^2*(-2*B*c - 3*A*
d + 5*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(2
```

```
*Sqrt[b]*Sqrt[b*c - a*d]*f))/(a^2 + b^2)^2 - (C*Sqrt[c + d*Tan[e + f*x]])/(
f*(a + b*Tan[e + f*x])) + ((-(A*b^2) + a*b*B + a^2*C + 2*b^2*C)*Sqrt[c + d*
Tan[e + f*x]])/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))))/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2142 vs. $2(284) = 568$.

time = 0.58, size = 2143, normalized size = 6.76

method	result	size
derivativedivides	Expression too large to display	2143
default	Expression too large to display	2143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d*(1/d/(a^2+b^2)^2*(1/4/d*(-1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a^2+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+A*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-A*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d+2*B*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*b*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d+C*(c^2+d^2)^(1/2)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*b*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*ln((c+d*tan(f*x+e))^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-4*A*(c^2+
d^2)^(1/2)*a*b*d+2*B*(c^2+d^2)^(1/2)*a^2*d-2*B*(c^2+d^2)^(1/2)*b^2*d+4*C*(c
^2+d^2)^(1/2)*a*b*d+1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+A*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a^2*c+2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d-A*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b^2*c-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b-
B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b
*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2-C*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d+C*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2
+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x
+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(-A*(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*b^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*A*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*b*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-2*B*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d+2*B*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d+C*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2-C*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*C*(2*(c^2
```

```
+d^2)^(1/2)+2*c)^(1/2)*a*b*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*ln(d*ta
n(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(
1/2))+2*(4*A*(c^2+d^2)^(1/2)*a*b*d-2*B*(c^2+d^2)^(1/2)*a^2*d+2*B*(c^2+d^2)^
(1/2)*b^2*d-4*C*(c^2+d^2)^(1/2)*a*b*d-1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a^2+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2+A*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c+2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d
-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*b-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d+2*B*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*b*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d+C*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c-2*C*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*b*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))-1/d/(a^2+b^2
)^2*(1/2*d*(A*a^2*b^2+A*b^4-B*a^3*b-B*a*b^3+C*a^4+C*a^2*b^2)/b*(c+d*tan(f*x
+e))^(1/2)/(b*(c+d*tan(f*x+e))+a*d-b*c)+1/2*(3*A*a^2*b^2*d-4*A*a*b^3*c-A*b^
4*d-B*a^3*b*d+2*B*a^2*b^2*c+3*B*a*b^3*d-2*B*b^4*c-C*a^4*d-5*C*a^2*b^2*d+4*C
*a*b^3*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*
b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\tan(e + fx)}\,(A + B\tan(e + fx) + C\tan^2(e + fx))}{(a + b\tan(e + fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 45.42, size = 2500, normalized size = 7.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^2,x)
```

```
[Out] atan(((((8*(156*B^3*a^2*b^9*d^12*f^2 - 16*B^3*a^4*b^7*d^12*f^2 - 120*B^3*a^
6*b^5*d^12*f^2 + 48*B^3*a^8*b^3*d^12*f^2 + 12*B^3*b^11*c^2*d^10*f^2 + 12*B^
3*b^11*c^4*d^8*f^2 - 4*B^3*a^10*b*d^12*f^2 - 124*B^3*a*b^10*c*d^11*f^2 - 12
4*B^3*a*b^10*c^3*d^9*f^2 + 224*B^3*a^3*b^8*c*d^11*f^2 + 200*B^3*a^5*b^6*c*d
^11*f^2 - 128*B^3*a^7*b^4*c*d^11*f^2 + 20*B^3*a^9*b^2*c*d^11*f^2 - 4*B^3*a^
10*b*c^2*d^10*f^2 + 44*B^3*a^2*b^9*c^2*d^10*f^2 - 112*B^3*a^2*b^9*c^4*d^8*f
^2 + 224*B^3*a^3*b^8*c^3*d^9*f^2 - 40*B^3*a^4*b^7*c^2*d^10*f^2 - 24*B^3*a^4
*b^7*c^4*d^8*f^2 + 200*B^3*a^5*b^6*c^3*d^9*f^2 - 40*B^3*a^6*b^5*c^2*d^10*f^
2 + 80*B^3*a^6*b^5*c^4*d^8*f^2 - 128*B^3*a^7*b^4*c^3*d^9*f^2 + 28*B^3*a^8*b
```

^3*c^2*d^10*f^2 - 20*B^3*a^8*b^3*c^4*d^8*f^2 + 20*B^3*a^9*b^2*c^3*d^9*f^2))
/(a^8*f^5 + b^8*f^5 + 4*a^2*b^6*f^5 + 6*a^4*b^4*f^5 + 4*a^6*b^2*f^5) + (((8
*(80*B*a*b^14*d^11*f^4 - 48*B*b^15*c*d^10*f^4 + 384*B*a^3*b^12*d^11*f^4 + 7
20*B*a^5*b^10*d^11*f^4 + 640*B*a^7*b^8*d^11*f^4 + 240*B*a^9*b^6*d^11*f^4 -
16*B*a^13*b^2*d^11*f^4 - 48*B*b^15*c^3*d^8*f^4 + 80*B*a*b^14*c^2*d^9*f^4 -
224*B*a^2*b^13*c*d^10*f^4 - 400*B*a^4*b^11*c*d^10*f^4 - 320*B*a^6*b^9*c*d^1
0*f^4 - 80*B*a^8*b^7*c*d^10*f^4 + 32*B*a^10*b^5*c*d^10*f^4 + 16*B*a^12*b^3*
c*d^10*f^4 - 224*B*a^2*b^13*c^3*d^8*f^4 + 384*B*a^3*b^12*c^2*d^9*f^4 - 400*
B*a^4*b^11*c^3*d^8*f^4 + 720*B*a^5*b^10*c^2*d^9*f^4 - 320*B*a^6*b^9*c^3*d^8
*f^4 + 640*B*a^7*b^8*c^2*d^9*f^4 - 80*B*a^8*b^7*c^3*d^8*f^4 + 240*B*a^9*b^6
*c^2*d^9*f^4 + 32*B*a^10*b^5*c^3*d^8*f^4 + 16*B*a^12*b^3*c^3*d^8*f^4 - 16*B
*a^13*b^2*c^2*d^9*f^4))/(a^8*f^5 + b^8*f^5 + 4*a^2*b^6*f^5 + 6*a^4*b^4*f^5
+ 4*a^6*b^2*f^5) - (16*(c + d*tan(e + f*x))^(1/2)*(-(((8*B^2*a^4*c*f^2 + 8*
B^2*b^4*c*f^2 - 32*B^2*a*b^3*d*f^2 + 32*B^2*a^3*b*d*f^2 - 48*B^2*a^2*b^2*c*
f^2)^2/4 - (B^4*c^2 + B^4*d^2)*(16*a^8*f^4 + 16*b^8*f^4 + 64*a^2*b^6*f^4 +
96*a^4*b^4*f^4 + 64*a^6*b^2*f^4))^(1/2) - 4*B^2*a^4*c*f^2 - 4*B^2*b^4*c*f^2
+ 16*B^2*a*b^3*d*f^2 - 16*B^2*a^3*b*d*f^2 + 24*B^2*a^2*b^2*c*f^2)/(16*(a^8
*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4)))^(1/2)*(32
*b^17*d^10*f^4 + 160*a^2*b^15*d^10*f^4 + 288*a^4*b^13*d^10*f^4 + 160*a^6*b^
11*d^10*f^4 - 160*a^8*b^9*d^10*f^4 - 288*a^10*b^7*d^10*f^4 - 160*a^12*b^5*d
^10*f^4 - 32*a^14*b^3*d^10*f^4 + 48*b^17*c^2*d^8*f^4 + 272*a^2*b^15*c^2*d^8
*f^4 + 624*a^4*b^13*c^2*d^8*f^4 + 720*a^6*b^11*c^2*d^8*f^4 + 400*a^8*b^9*c^
2*d^8*f^4 + 48*a^10*b^7*c^2*d^8*f^4 - 48*a^12*b^5*c^2*d^8*f^4 - 16*a^14*b^3
*c^2*d^8*f^4 + 16*a*b^16*c*d^9*f^4 + 112*a^3*b^14*c*d^9*f^4 + 336*a^5*b^12*
c*d^9*f^4 + 560*a^7*b^10*c*d^9*f^4 + 560*a^9*b^8*c*d^9*f^4 + 336*a^11*b^6*c
*d^9*f^4 + 112*a^13*b^4*c*d^9*f^4 + 16*a^15*b^2*c*d^9*f^4))/(a^8*f^4 + b^8*
f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4))*(-(((8*B^2*a^4*c*f^2
+ 8*B^2*b^4*c*f^2 - 32*B^2*a*b^3*d*f^2 + 32*B^2*a^3*b*d*f^2 - 48*B^2*a^2*b^
2*c*f^2)^2/4 - (B^4*c^2 + B^4*d^2)*(16*a^8*f^4 + 16*b^8*f^4 + 64*a^2*b^6*f^
4 + 96*a^4*b^4*f^4 + 64*a^6*b^2*f^4))^(1/2) - 4*B^2*a^4*c*f^2 - 4*B^2*b^4*c
*f^2 + 16*B^2*a*b^3*d*f^2 - 16*B^2*a^3*b*d*f^2 + 24*B^2*a^2*b^2*c*f^2)/(16*
(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4)))^(1/2)
- (16*(c + d*tan(e + f*x))^(1/2)*(44*B^2*a^9*b^4*d^11*f^2 - 168*B^2*a^5*b^
8*d^11*f^2 - 40*B^2*a^7*b^6*d^11*f^2 - 20*B^2*a^3*b^10*d^11*f^2 - 4*B^2*a^1
1*b^2*d^11*f^2 - 36*B^2*b^13*c^3*d^8*f^2 + 60*B^2*a*b^12*d^11*f^2 - 12*B^2*
b^13*c*d^10*f^2 + 4*B^2*a^12*b*c*d^10*f^2 + 100*B^2*a*b^12*c^2*d^9*f^2 + 12
0*B^2*a^2*b^11*c*d^10*f^2 + 156*B^2*a^4*b^9*c*d^10*f^2 - 112*B^2*a^6*b^7*c*
d^10*f^2 - 148*B^2*a^8*b^5*c*d^10*f^2 - 8*B^2*a^10*b^3*c*d^10*f^2 + 68*B^2*
a^2*b^11*c^3*d^8*f^2 + 124*B^2*a^3*b^10*c^2*d^9*f^2 + 184*B^2*a^4*b^9*c^3*d
^8*f^2 + 8*B^2*a^5*b^8*c^2*d^9*f^2 + 40*B^2*a^6*b^7*c^3*d^8*f^2 + 24*B^2*a^
7*b^6*c^2*d^9*f^2 - 20*B^2*a^8*b^5*c^3*d^8*f^2 + 20*B^2*a^9*b^4*c^2*d^9*f^2
+ 20*B^2*a^10*b^3*c^3*d^8*f^2 - 20*B^2*a^11*b^2*c^2*d^9*f^2))/(a^8*f^4 + b
^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4))*(-(((8*B^2*a^4*c*f
^2 + 8*B^2*b^4*c*f^2 - 32*B^2*a*b^3*d*f^2 + 32*B^2*a^3*b*d*f^2 - 48*B^2*a^2
*b^2*c*f^2)^2/4 - (B^4*c^2 + B^4*d^2)*(16*a^8*f^4 + 16*b^8*f^4 + 64*a^2*b^6

```
*f^4 + 96*a^4*b^4*f^4 + 64*a^6*b^2*f^4))^(1/2) - 4*B^2*a^4*c*f^2 - 4*B^2*b^
4*c*f^2 + 16*B^2*a*b^3*d*f^2 - 16*B^2*a^3*b*d*f^2 + 24*B^2*a^2*b^2*c*f^2)/(
16*(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^4 + 4*a^6*b^2*f^4)))^(1
/2))*(-(((8*B^2*a^4*c*f^2 + 8*B^2*b^4*c*f^2 - 32*B^2*a*b^3*d*f^2 + 32*B^2*a
^3*b*d*f^2 - 48*B^2*a^2*b^2*c*f^2)^2/4 - (B^4*c^2 + B^4*d^2)*(16*a^8*f^4 +
16*b^8*f^4 + 64*a^2*b^6*f^4 + 96*a^4*b^4*f^4 + 64*a^6*b^2*f^4))^(1/2) - 4*B
^2*a^4*c*f^2 - 4*B^2*b^4*c*f^2 + 16*B^2*a*b^3*d*f^2 - 16*B^2*a^3*b*d*f^2 +
24*B^2*a^2*b^2*c*f^2)/(16*(a^8*f^4 + b^8*f^4 + 4*a^2*b^6*f^4 + 6*a^4*b^4*f^
4 + 4*a^6*b^2*f^4)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(2*B^4*b^9*d^12
 - 5*B^4*a^2*b^7*d^12 + 17*B^4*a^4*b^5*d^12 - 7*B^4*a^6*b^3*d^12 + 6*B^4*b^
9*c^4*d^8 + B^4*a^8*b*d^12 + 77*B^4*a^2*b^7*c^2*d^10 - 8*B^4*a^2*b^7*c^4*d^
8 + 60*B^4*a^3*b^6*c^3*d^9 - 87*B^4*a^4*b^5*c^2*d^10 + 14*B^4*a^4*b^5*c^4*d
^8 - 36*B^4*a^5*b^4*c^3*d^9 + 27*B^4*a^6*b^3*c^...
```

3.96 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=543

$$-\frac{(A-iB-C)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}+\frac{(A+iB-C)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)^3f}$$

```
[Out] 1/4*(3*a^5*b*B*d^2+a^6*C*d^2-3*a^4*b^2*d*(5*A*d+4*B*c-6*C*d)-3*a^2*b^4*(8*A
*c^2-6*A*d^2-16*B*c*d-8*C*c^2+5*C*d^2)+2*a^3*b^3*(20*c*(A-C)*d+B*(4*c^2-13*
d^2))-3*a*b^5*(8*c*(A-C)*d+B*(8*c^2-d^2))-b^6*(4*c*(B*d+2*C*c)-A*(8*c^2+d^2
)))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(a^2+b
^2)^3/(-a*d+b*c)^(3/2)/f-(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(
1/2))*(c-I*d)^(1/2)/(I*a+b)^3/f+(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c
+I*d)^(1/2))*(c+I*d)^(1/2)/(I*a-b)^3/f-1/2*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x
+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2-1/4*(3*a^3*b*B*d+a^4*C*d+b^4*(A
*d+4*B*c)+a*b^3*(8*A*c-5*B*d-8*C*c)-a^2*b^2*(7*A*d+4*B*c-9*C*d))*(c+d*tan(f
*x+e))^(1/2)/b/(a^2+b^2)^2/(-a*d+b*c)/f/(a+b*tan(f*x+e))
```

Rubi [A]

time = 2.87, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3726, 3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{2bf(a^2+b^2)(a+b\tan(e+fx))^2}-\frac{\sqrt{c+d\tan(e+fx)}\,(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc))}{4bf(a^2+b^2)^2(bc-ad)(a+b\tan(e+fx))}+\frac{(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd)+2a^3b^3(20cd(A-C)+B(4c^2-13d^2))-3a^2b^4(8Ac^2-6Ad^2-16Bcd-8c^2C+5Cd^2)-3ab^5(8cd(A-C)+B(8c^2-d^2))-b^6(4c(Bd+2cC)-A(8c^2+d^2)))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{4b^{3/2}f(a^2+b^2)^3(bc-ad)^{3/2}}-\frac{\sqrt{c-id}\,(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)^3}+\frac{\sqrt{c+id}\,(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^3,x]
```

```
[Out] -(((A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((I*a + b)^3*f)) + ((A + I*B - C)*Sqrt[c + I*d]*ArcTanh[Sqrt[c + d*Tan
[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) + ((3*a^5*b*B*d^2 + a^6*C*d^2 -
3*a^4*b^2*d*(4*B*c + 5*A*d - 6*C*d) - 3*a^2*b^4*(8*A*c^2 - 8*c^2*C - 16*B*c
*d - 6*A*d^2 + 5*C*d^2) + 2*a^3*b^3*(20*c*(A - C)*d + B*(4*c^2 - 13*d^2)) -
 3*a*b^5*(8*c*(A - C)*d + B*(8*c^2 - d^2)) - b^6*(4*c*(2*c*C + B*d) - A*(8*
c^2 + d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(
4*b^(3/2)*(a^2 + b^2)^3*(b*c - a*d)^(3/2)*f) - ((A*b^2 - a*(b*B - a*C))*Sqr
t[c + d*Tan[e + f*x]])/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - ((3*a^3
*b*B*d + a^4*C*d + b^4*(4*B*c + A*d) + a*b^3*(8*A*c - 8*c*C - 5*B*d) - a^2*
b^2*(4*B*c + 7*A*d - 9*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b*(a^2 + b^2)^2*(
b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

$$= -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

$$= -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

$$= -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

$$= -\frac{(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e+}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

$$= \frac{(3a^5bBd^2+a^6Cd^2-3a^4b^2d(4Bc+5A}{}$$

$$= -\frac{(A-iB-C)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.}{(ia+b)^3f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2819 vs. 2(543) = 1086.
time = 6.28, size = 2819, normalized size = 5.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*C*Sqrt[c + d*Tan[e + f*x]])/(3*b*f*(a + b*Tan[e + f*x])^2) - (2*(-1/2*(
((b^2*(-3*A*b*c + 4*b*c*C - a*C*d))/2 - a*((-3*b^2*(B*c + (A - C)*d))/2 - (
a*(b*c*C - 3*b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*
c - a*d)*f*(a + b*Tan[e + f*x])^2) - (-((((I*Sqrt[c - I*d]*(b*(b*c - a*d)*(
(3*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 + 3*a*b*(b*c -
a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d) + (3*b*(b*c - a*d)*(a^
2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4) + a*((3*(b*c - a
```

d)((b^2*d)/2 - a*(b*c - a*d))*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c -
 4*c*C - B*d)))/4 + (-(b*c) + (a*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4
*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d
 - b*B*d + a*C*d)) - (d*((3*b^2*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) +
a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*
C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*
B*d + a*C*d))))/2) - I*(a*(b*c - a*d)*((3*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*
b^2*C)*d*(b*c - a*d))/4 + 3*a*b*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d
- b*B*d + a*C*d) + (3*b*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A
*c - 4*c*C - B*d)))/4) - b*((3*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(a^2
*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 + (-(b*c) + (a*d)/
2)*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*
c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d)) - (d*((3*b^2*(b*c
 - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((
3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a
d)(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))))/2)))*ArcTanh[Sqrt[c
+ d*Tan[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*(b*(b*c
 - a*d)*((3*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 + 3*a*
b*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d) + (3*b*(b*c -
 a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4) + a*((3
*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(a^2*C*d + b^2*(4*B*c + A*d) + a*b
*(4*A*c - 4*c*C - B*d)))/4 + (-(b*c) + (a*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B -
a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*
C - a*A*d - b*B*d + a*C*d)) - (d*((3*b^2*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c
+ A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C
 - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a
*A*d - b*B*d + a*C*d))))/2) + I*(a*(b*c - a*d)*((3*b*(3*A*b^2 - 3*a*b*B - a
^2*C - 4*b^2*C)*d*(b*c - a*d))/4 + 3*a*b*(b*c - a*d)*(A*b*c - a*B*c - b*c*C
 - a*A*d - b*B*d + a*C*d) + (3*b*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) +
 a*b*(4*A*c - 4*c*C - B*d)))/4) - b*((3*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a
d))(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 + (-(b*c)
 + (a*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 -
3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d)) - (d*((3
b^2(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))
/4 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2
*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))))/2)))*ArcTan
h[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2
*Sqrt[b*c - a*d]*(-(a*b*(b*c - a*d)*((3*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^
2*C)*d*(b*c - a*d))/4 + 3*a*b*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d -
b*B*d + a*C*d) + (3*b*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c
 - 4*c*C - B*d)))/4)) + (a^2*d*((3*b^2*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c +
A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C -
 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A
*d - b*B*d + a*C*d))))/2 + b^2*((3*(b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*
(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 + (-(b*c) + (a

```
*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2
*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))))*ArcTanh[(Sq
rt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b
*c) + a*d)*f))/((a^2 + b^2)*(b*c - a*d))) - (((3*b^2*(b*c - a*d)*(a^2*C*d +
 b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a
*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c
 - b*c*C - a*A*d - b*B*d + a*C*d)))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*
(b*c - a*d)*f*(a + b*Tan[e + f*x])))/(2*(a^2 + b^2)*(b*c - a*d))))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3101 vs. $2(503) = 1006$.

time = 0.63, size = 3102, normalized size = 5.71

method	result	size
derivativedivides	Expression too large to display	3102
default	Expression too large to display	3102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^2*(-1/d^2/(a^2+b^2)^3*((1/8*d*(7*A*a^4*b^2*d-8*A*a^3*b^3*c+6*A*a^2*b^
4*d-8*A*a*b^5*c-A*b^6*d-3*B*a^5*b*d+4*B*a^4*b^2*c+2*B*a^3*b^3*d+5*B*a*b^5*d
-4*B*b^6*c-C*a^6*d-10*C*a^4*b^2*d+8*C*a^3*b^3*c-9*C*a^2*b^4*d+8*C*a*b^5*c)/
(a*d-b*c)*(c+d*tan(f*x+e))^(3/2)+1/8*d*(9*A*a^4*b^2*d-8*A*a^3*b^3*c+10*A*a^
2*b^4*d-8*A*a*b^5*c+A*b^6*d-5*B*a^5*b*d+4*B*a^4*b^2*c-2*B*a^3*b^3*d+3*B*a*b
^5*d-4*B*b^6*c+C*a^6*d-6*C*a^4*b^2*d+8*C*a^3*b^3*c-7*C*a^2*b^4*d+8*C*a*b^5*
c)/b*(c+d*tan(f*x+e))^(1/2))/(b*(c+d*tan(f*x+e))+a*d-b*c)^2+1/8*(15*A*a^4*b
^2*d^2-40*A*a^3*b^3*c*d+24*A*a^2*b^4*c^2-18*A*a^2*b^4*d^2+24*A*a*b^5*c*d-8*
A*b^6*c^2-A*b^6*d^2-3*B*a^5*b*d^2+12*B*a^4*b^2*c*d-8*B*a^3*b^3*c^2+26*B*a^3
*b^3*d^2-48*B*a^2*b^4*c*d+24*B*a*b^5*c^2-3*B*a*b^5*d^2+4*B*b^6*c*d-C*a^6*d^
2-18*C*a^4*b^2*d^2+40*C*a^3*b^3*c*d-24*C*a^2*b^4*c^2+15*C*a^2*b^4*d^2-24*C*
a*b^5*c*d+8*C*b^6*c^2)/(a*d-b*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*
x+e))^(1/2)/((a*d-b*c)*b)^(1/2)))+1/d^2/(a^2+b^2)^3*(1/4/d*(-1/2*(-A*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a*b^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c+3*A*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-A*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d-3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a^2*b+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3-B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c+3*
B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3
*c+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*C*(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-3
*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a*b^2*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*ln((c+d*tan(f*x+e))^(1/2)*(2
```

```
*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-6*A*(c^2+d^
2)^(1/2)*a^2*b*d+2*A*(c^2+d^2)^(1/2)*b^3*d+2*B*(c^2+d^2)^(1/2)*a^3*d-6*B*(c
^2+d^2)^(1/2)*a*b^2*d+6*C*(c^2+d^2)^(1/2)*a^2*b*d-2*C*(c^2+d^2)^(1/2)*b^3*d
+1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*A*(c^2+d^2)^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3
*c+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*b^2*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d-3*B*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b^3-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a^2*b*c+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d-B*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b^3*c+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*C*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2-C*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^3*c-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*C*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*b^2*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+
2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*
(1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*A*(c^2+d^2)^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3
*c+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*b^2*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d-3*B*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b^3-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a^2*b*c+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d-B*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b^3*c+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3-3*C*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2-C*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^3*c-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*C*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*b^2*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)*ln(d*tan(f*x+
e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+
2*(6*A*(c^2+d^2)^(1/2)*a^2*b*d-2*A*(c^2+d^2)^(1/2)*b^3*d-2*B*(c^2+d^2)^(1/2
)*a^3*d+6*B*(c^2+d^2)^(1/2)*a*b^2*d-6*C*(c^2+d^2)^(1/2)*a^2*b*d+2*C*(c^2+d^
2)^(1/2)*b^3*d-1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3+3*
A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2+A*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a^3*c+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*A*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*b^2*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d-3*B*(c^2+d^
2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*b^3-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d+3*B*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a^2*b*c+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d-B*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a^3-3*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2-C*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^3*c-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*C*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+
d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^3,x)
```

```
[Out] \text{Hanged}
```

3.97 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+$

Optimal. Leaf size=550

$$\frac{(ia+b)^3(A-iB-C)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(a+ib)^3(iA-B-iC)(c+id)^{3/2}\tanh^{-}}{f}$$

```
[Out] (I*a+b)^3*(A-I*B-C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1
/2))/f+(a+I*b)^3*(I*A-B-I*C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(
c+I*d)^(1/2))/f+2*(3*a^2*b*(A*c-B*d-C*c)-b^3*(A*c-B*d-C*c)+a^3*(B*c+(A-C)*d
)-3*a*b^2*(B*c+(A-C)*d))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(a^3*B-3*a*b^2*B+3*a^
2*b*(A-C)-b^3*(A-C))*(c+d*tan(f*x+e))^(3/2)/f+2/3465*(168*a^3*C*d^3-2*a^2*b
*d^2*(-847*B*d+192*C*c)+33*a*b^2*d*(8*c^2*C-18*B*c*d+63*(A-C)*d^2)-b^3*(48*
c^3*C-88*B*c^2*d+198*c*(A-C)*d^2+693*B*d^3))*(c+d*tan(f*x+e))^(5/2)/d^4/f+2
/693*b*(99*b*(A*b+B*a-C*b)*d^2+4*(-a*d+b*c)*(-11*B*b*d-6*C*a*d+6*C*b*c))*ta
n(f*x+e)*(c+d*tan(f*x+e))^(5/2)/d^3/f-2/99*(-11*B*b*d-6*C*a*d+6*C*b*c)*(a+b
*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)/d^2/f+2/11*C*(a+b*tan(f*x+e))^3*(c+d*
tan(f*x+e))^(5/2)/d/f
```

Rubi [A]

time = 1.84, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3728, 3718, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(c+d\tan(e+fx))^{5/2}\left(168a^3Cd^3-2a^2bd^2(192cC-847Bd)+33ab^2d(63d^2(A-C)-18Bcd+8c^2C)-\left(b^3(198cd^2(A-C)-88Bc^2d+693Bd^3+48c^3C)\right)\right)}{3465d^4f}+\frac{2(a^3B+3a^2b(A-C)-3ab^2B-b^3(A-C))(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2\sqrt{c+d\tan(e+fx)}\left(a^3(d(A-C)+Bc)+3a^2b(Ac-Bd-cC)-3ab^2(d(A-C)+Bc)-b^3(Ac-Bd-cC)\right)}{f}+\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{5/2}\left(99bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-11bBd+6bcC)\right)}{693d^3f}+\frac{(a+ib)^3(c+id)^{3/2}(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{(b+ia)^3(c-id)^{3/2}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{99d^2f}+\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] ((I*a + b)^3*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f + ((a + I*b)^3*(I*A - B - I*C)*(c + I*d)^(3/2)*ArcTanh[S
qrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(3*a^2*b*(A*c - c*C - B*d) -
 b^3*(A*c - c*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))
*Sqrt[c + d*Tan[e + f*x]])/f + (2*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^
3*(A - C))*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*(168*a^3*C*d^3 - 2*a^2*b*
d^2*(192*c*C - 847*B*d) + 33*a*b^2*d*(8*c^2*C - 18*B*c*d + 63*(A - C)*d^2)
- b^3*(48*c^3*C - 88*B*c^2*d + 198*c*(A - C)*d^2 + 693*B*d^3))*(c + d*Tan[e
 + f*x])^(5/2))/(3465*d^4*f) + (2*b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c -
a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2
))/(693*d^3*f) - (2*(6*b*c*C - 11*b*B*d - 6*a*C*d)*(a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(5/2))/(99*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*(c + d*
Tan[e + f*x])^(5/2))/(11*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{2C(a+b\tan(e\ldots}{}$$

$$=-\frac{2(6bcC-11b\ldots}{}$$

$$=\frac{2b(99b(Ab+aB\ldots}{}$$

$$=\frac{2(168a^3Cd^3-2\ldots}{}$$

$$=\frac{2(a^3B-3ab^2B\ldots}{}$$

$$=\frac{2(3a^2b(Ac-cC\ldots}{}$$

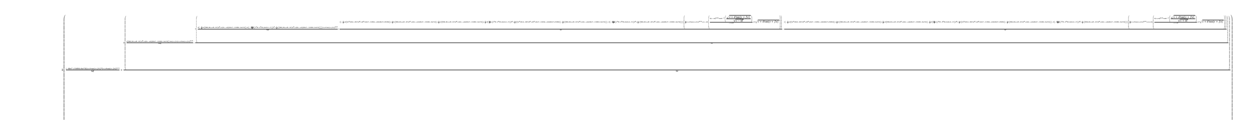
$$=\frac{2(3a^2b(Ac-cC\ldots}{}$$

$$=\frac{2(3a^2b(Ac-cC\ldots}{}$$

$$=\frac{2(3a^2b(Ac-cC\ldots}{}$$

$$=-\frac{(a-ib)^3(iA+\ldots}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1290 vs. $2(550)=1100$.
time = 6.29, size = 1290, normalized size = 2.35



Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e +
f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2))/(11*d*f) + (2*(((-6
*b*c*C + 11*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5
```

```
/2))/(9*d*f) + (2*((b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C
- 11*b*B*d - 6*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(14*d*f) -
(2*((2*((-7*a*d*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b
*B*d - 6*a*C*d)))/8 + b*((-693*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (c*
(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d))
)/4))*(c + d*Tan[e + f*x])^(5/2))/(5*d*f) + ((I/2)*((-7*a*d*(3*a^2*(33*A -
25*C)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*b*d*(48*c*C + 55*B*d)))/8 + (b*c*(
99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))
/4 + (7*a*d*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d
 - 6*a*C*d)))/8 + ((7*I)/2)*d*((99*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2)/4
 + (b*(3*a^2*(33*A - 25*C)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*b*d*(48*c*C +
 55*B*d)))/4 - (b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11
*b*B*d - 6*a*C*d)))/4) - b*((-693*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 +
(c*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*
d)))/4))*((2*(c + d*Tan[e + f*x])^(3/2))/3 + (c - I*d)*((2*(c - I*d)^(3/2)*
ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(-c + I*d) + 2*Sqrt[c + d*
Tan[e + f*x]])))/f - ((I/2)*((-7*a*d*(3*a^2*(33*A - 25*C)*d^2 + 4*b^2*c*(6*
c*C - 11*B*d) - a*b*d*(48*c*C + 55*B*d)))/8 + (b*c*(99*b*(A*b + a*B - b*C)*
d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/4 + (7*a*d*(99*b*(A*b
+ a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/8 - ((7*I
)/2)*d*((99*a*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2)/4 + (b*(3*a^2*(33*A - 25
*C)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*b*d*(48*c*C + 55*B*d)))/4 - (b*(99*b
*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/4)
- b*((-693*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8 + (c*(99*b*(A*b + a*B - b
*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/4))*((2*(c + d*Tan
[e + f*x])^(3/2))/3 + (c + I*d)*((2*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[
e + f*x]]/Sqrt[c + I*d]])/(-c - I*d) + 2*Sqrt[c + d*Tan[e + f*x]])))/f))/(7
*d)))/(9*d)))/(11*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5094 vs. $2(507) = 1014$.
time = 0.56, size = 5095, normalized size = 9.26

method	result	size
derivativedivides	Expression too large to display	5095
default	Expression too large to display	5095

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan (e+fx))^3 (c+d\tan (e+fx))^{\frac{3}{2}} \left(A+B\tan (e+fx)+C\tan^2 (e+fx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e +
 f*x) + C*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]

time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) +
 C*tan(e + f*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.98 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e\right.$

Optimal. Leaf size=396

$$-\frac{(a-ib)^2(B+i(A-C))(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(a+ib)^2(iA-B-iC)(c+id)^{3/2}\mathrm{t}}{f}$$

```
[Out] -(a-I*b)^2*(B+I*(A-C))*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)
^(1/2))/f+(a+I*b)^2*(I*A-B-I*C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2
)/(c+I*d)^(1/2))/f+2*(2*a*b*(A*c-B*d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*
d))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(a^2*B-b^2*B+2*a*b*(A-C))*(c+d*tan(f*x+e))
^(3/2)/f+2/315*(28*a^2*C*d^2-18*a*b*d*(-7*B*d+2*C*c)+b^2*(8*c^2*C-18*B*c*d+
63*(A-C)*d^2))*(c+d*tan(f*x+e))^(5/2)/d^3/f-2/63*b*(-9*B*b*d-4*C*a*d+4*C*b*
c)*tan(f*x+e)*(c+d*tan(f*x+e))^(5/2)/d^2/f+2/9*C*(a+b*tan(f*x+e))^2*(c+d*ta
n(f*x+e))^(5/2)/d/f
```

Rubi [A]

time = 1.13, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3728, 3718, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(c+d\tan(e+fx))^{5/2}\left(28a^2Cd^2-18abd(2cC-7Bd)+b^2(63d^2(A-C)-18Bcd+8c^2C)\right)}{315d^3f}+\frac{2\left(a^2B+2ab(A-C)-b^2B\right)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2\sqrt{c+d\tan(e+fx)}\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}-\frac{(a-ib)^2(c-id)^{3/2}(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(a+ib)^2(c+id)^{3/2}(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-\frac{2b\tan(e+fx)(-4aCd-9bBd+4bcC)(c+d\tan(e+fx))^{5/2}}{63d^2f}+\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^2*(B + I*(A - C))*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c - I*d]])/f) + ((a + I*b)^2*(I*A - B - I*C)*(c + I*d)^(3/2)*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(2*a*b*(A*c - c*C - B*d
) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*Sqrt[c + d*Tan[e + f*x]]
)/f + (2*(a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x])^(3/2))/(3*f)
+ (2*(28*a^2*C*d^2 - 18*a*b*d*(2*c*C - 7*B*d) + b^2*(8*c^2*C - 18*B*c*d + 6
3*(A - C)*d^2))*(c + d*Tan[e + f*x])^(5/2))/(315*d^3*f) - (2*b*(4*b*c*C - 9
*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(63*d^2*f) + (2*
C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2))/(9*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{2C(a+b\tan(e\cdots}{}$$

$$=-\frac{2b(4bcC-9bI\cdots}{}$$

$$=\frac{2(28a^2Cd^2-18\cdots}{}$$

$$=\frac{2(a^2B-b^2B+\cdots}{}$$

$$=\frac{2(2ab(Ac-cC\cdots}{}$$

$$=\frac{2(2ab(Ac-cC\cdots}{}$$

$$=\frac{2(2ab(Ac-cC\cdots}{}$$

$$=\frac{2(2ab(Ac-cC\cdots}{}$$

$$=-\frac{(a-ib)^2(iA+\cdots}{}$$

Mathematica [A]

time = 3.96, size = 350, normalized size = 0.88

$$\frac{2\left((28a^2Cd^2+18abd(-2cC+7Bd)+b^2(8c^2C-18Bcd+63(A-C)d^2))\,(c+d\tan(e+fx))^{5/2}+5bd(-4bcC+9bBd+4aCd)\tan(e+fx)(c+d\tan(e+fx))^{5/2}+35Cd^2(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}+\frac{105}{2}(a-ib)^2(iA+B-iC)d^3\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c-3id+d\tan(e+fx))\right)+\frac{105}{2}(a+ib)^2(-iA+B+iC)d^3\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c+3id+d\tan(e+fx))\right)\right)}{315d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e +
f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*((28*a^2*C*d^2 + 18*a*b*d*(-2*c*C + 7*B*d) + b^2*(8*c^2*C - 18*B*c*d + 6
3*(A - C)*d^2))*(c + d*Tan[e + f*x])^(5/2) + 5*b*d*(-4*b*c*C + 9*b*B*d + 4*
a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2) + 35*C*d^2*(a + b*Tan[e + f*
x])^2*(c + d*Tan[e + f*x])^(5/2) + (105*(a - I*b)^2*(I*A + B - I*C)*d^3*(-3
*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c +
 d*Tan[e + f*x]]*(4*c - (3*I)*d + d*Tan[e + f*x])))/2 + (105*(a + I*b)^2*((
-I)*A + B + I*C)*d^3*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))
/2))/(315*d^3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3688 vs. $2(357) = 714$.
time = 0.52, size = 3689, normalized size = 9.32

method	result	size
derivativedivides	Expression too large to display	3689
default	Expression too large to display	3689

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^3*(2*A*a*b*c*d^3*(c+d*tan(f*x+e))^(1/2)-2*B*a*b*d^4*(c+d*tan(f*x+e))^
(1/2)-B*b^2*c*d^3*(c+d*tan(f*x+e))^(1/2)-2/3*C*a*b*d^3*(c+d*tan(f*x+e))^(3/
2)+B*a^2*c*d^3*(c+d*tan(f*x+e))^(1/2)-1/5*B*b^2*c*d*(c+d*tan(f*x+e))^(5/2)+
2/3*A*a*b*d^3*(c+d*tan(f*x+e))^(3/2)+2/7*C*a*b*d*(c+d*tan(f*x+e))^(7/2)+2/5
*B*a*b*d^2*(c+d*tan(f*x+e))^(5/2)-d^3*(1/4/d*(1/2*(A*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*(c^2+d^2)^(1/2)*a^2*c-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(
1/2)*a*b*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*c-A*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2+4*A*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^
2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(
c^2+d^2)^(1/2)*a^2*d-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*
c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*d+2*B*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a^2*c*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2-2*B*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^2-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d-C
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c+2*C*(2*(c^2+d^2)^(1/2)
```

+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)
^(1/2)*b^2*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-C*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a^2*d^2-4*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d-C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b^2*c^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2)*ln(d*tan(
f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/
2))+2*(2*A*(c^2+d^2)^(1/2)*a^2*d^2+4*A*(c^2+d^2)^(1/2)*a*b*c*d-2*A*(c^2+d^2
)^(1/2)*b^2*d^2+2*B*(c^2+d^2)^(1/2)*a^2*c*d-4*B*(c^2+d^2)^(1/2)*a*b*d^2-2*B
*(c^2+d^2)^(1/2)*b^2*c*d-2*C*(c^2+d^2)^(1/2)*a^2*d^2-4*C*(c^2+d^2)^(1/2)*a*
b*c*d+2*C*(c^2+d^2)^(1/2)*b^2*d^2-1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*a^2*c-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d-A
(2(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*c-A*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a^2*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2+4*A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*b*c*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-A*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b^2*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/
2)*a^2*d-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*c+B*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a^2*c*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2-2*B*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*b*d^2-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d-C*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c+2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
(c^2+d^2)^(1/2)*a*b*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*c
+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^
2*d^2-4*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b^2*c^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2
*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(-1/2*(A
(2(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c-2*A*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)
^(1/2)*b^2*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+A*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a^2*d^2+4*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b^2*c^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2-B*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*d-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*(c^2+d^2)^(1/2)*a*b*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^
2*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*b*c^2-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^2-2*B*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*b^2*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c
+2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d+C*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2
-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a*b*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b^2*d^2)*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan
(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-2*A*(c^2+d^2)^(1/2)*a^2*d^2-4*A*(c^2+d^2)^(1
/2)*a*b*c*d+2*A*(c^2+d^2)^(1/2)*b^2*d^2-2*B*(c^2+d^2)^(1/2)*a^2*c*d+4*B*(c^
2+d^2)^(1/2)*a*b*d^2+2*B*(c^2+d^2)^(1/2)*b^2*c*d+2*C*(c^2+d^2)^(1/2)*a^2*d^
2+4*C*(c^2+d^2)^(1/2)*a*b*c*d-2*C*(c^2+d^2)^(1/2)*b^2*d^2+1/2*(A*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)

```
*(c^2+d^2)^(1/2)*a*b*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*
c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
^2*d^2+4*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+A*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b^2*c^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2-B*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*d-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2
)^(1/2)*a*b*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*d+2*B*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d+2*B*(2*(c^2+...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^2 (c+d\tan(e+fx))^{\frac{3}{2}} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e +
 f*x) + C*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) +
 C*tan(e + f*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.99 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2} (A+B\tan(e+$

Optimal. Leaf size=273

$$-\frac{(ia+b)(A-iB-C)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(ia-b)(A+iB-C)(c+id)^{3/2}\tanh^{-1}}{f}$$

```
[Out] -(I*a+b)*(A-I*B-C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/
2))/f+(I*a-b)*(A+I*B-C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d
)^(1/2))/f+2*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*(c+d*tan(f*x+e))^(1/2)/f
+2/3*(A*b+B*a-C*b)*(c+d*tan(f*x+e))^(3/2)/f-2/35*(-7*B*b*d-7*C*a*d+2*C*b*c)
*(c+d*tan(f*x+e))^(5/2)/d^2/f+2/7*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.60, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3718, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(aB+Ab-bC)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2\sqrt{c+d\tan(e+fx)}\,(aAd+aBc-aCd+Abc-bBd-bcC)}{f}-\frac{(b+ia)(c-id)^{3/2}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(-b+ia)(c+id)^{3/2}(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-\frac{2(-7aCd-7bBd+2bcC)(c+d\tan(e+fx))^{5/2}}{35d^2f}+\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt
[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(A*b*c + a*B*c - b*c*C + a*A*d
- b*B*d - a*C*d)*Sqrt[c + d*Tan[e + f*x]])/f + (2*(A*b + a*B - b*C)*(c + d*
Tan[e + f*x])^(3/2))/(3*f) - (2*(2*b*c*C - 7*b*B*d - 7*a*C*d)*(c + d*Tan[e
+ f*x])^(5/2))/(35*d^2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))
/(7*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2bC\tan(e+fx)(c}{7}$$

$$= -\frac{2(2bcC-7bBd}{}$$

$$= \frac{2(Ab+aB-bC)(}{}$$

$$= \frac{2(Abc+aBc-bc}{}$$

$$= \frac{2(Abc+aBc-bc}{}$$

$$= \frac{2(Abc+aBc-bc}{}$$

$$= \frac{2(Abc+aBc-bc}{}$$

$$= -\frac{(a-ib)(iA+B}{}$$

Mathematica [A]
time = 3.03, size = 260, normalized size = 0.95

$$\frac{\frac{2(-2bcC+7bBd+7aCd)(c+d\tan(e+fx))^{5/2}}{d}+10bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}+\frac{35}{3}(ia+b)(A-iB-C)d\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c-3id+d\tan(e+fx))\right)+\frac{35}{3}(-ia+b)(A+iB-C)d\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c+3id+d\tan(e+fx))\right)}{35df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] ((2*(-2*b*c*C + 7*b*B*d + 7*a*C*d)*(c + d*Tan[e + f*x])^(5/2))/d + 10*b*C*T
an[e + f*x]*(c + d*Tan[e + f*x])^(5/2) + (35*(I*a + b)*(A - I*B - C)*d*(-3*
(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c +
d*Tan[e + f*x]]*(4*c - (3*I)*d + d*Tan[e + f*x])))/3 + (35*((-I)*a + b)*(A
+ I*B - C)*d*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/3)/(35*
d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2314 vs. 2(239) = 478.
time = 0.49, size = 2315, normalized size = 8.48

method	result	size
derivativedivides	Expression too large to display	2315
default	Expression too large to display	2315

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^2*(1/7*C*b*(c+d*tan(f*x+e))^(7/2)+1/5*B*b*d*(c+d*tan(f*x+e))^(5/2)+1/
5*C*a*d*(c+d*tan(f*x+e))^(5/2)-1/5*C*b*c*(c+d*tan(f*x+e))^(5/2)+1/3*A*b*d^2
*(c+d*tan(f*x+e))^(3/2)+1/3*B*a*d^2*(c+d*tan(f*x+e))^(3/2)-1/3*C*b*d^2*(c+d
*tan(f*x+e))^(3/2)+A*a*d^3*(c+d*tan(f*x+e))^(1/2)+A*b*c*d^2*(c+d*tan(f*x+e)
)^(1/2)+B*a*c*d^2*(c+d*tan(f*x+e))^(1/2)-B*b*d^3*(c+d*tan(f*x+e))^(1/2)-C*a
*d^3*(c+d*tan(f*x+e))^(1/2)-C*b*c*d^2*(c+d*tan(f*x+e))^(1/2)-d^2*(1/4/d*(1/
2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+A*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-A*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d+B
*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d+B*(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*b*c-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d-B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2+C*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+C*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*d^2+2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d)*ln(d*tan(f
*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2
))+2*(2*A*(c^2+d^2)^(1/2)*a*d^2+2*A*(c^2+d^2)^(1/2)*b*c*d+2*B*(c^2+d^2)^(1/
2)*a*c*d-2*B*(c^2+d^2)^(1/2)*b*d^2-2*C*(c^2+d^2)^(1/2)*a*d^2-2*C*(c^2+d^2)^
(1/2)*b*c*d+1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+A*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*c^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*A*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*b*c*d+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d+B*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b*d^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-C*(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+
C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c
*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*
(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2)))+1/4/d*(1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-
A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-A*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*A*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b*c*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d-B*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+2*B*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*c*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2-B*(2*(c^2+d^2)^(1/2)+2*c)
```

```
^(1/2)*b*d^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+C*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b*c*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)+(c^2+d^2)^(1/2))+2*(2*A*(c^2+d^2)^(1/2)*a*d^2+2*A*(c^2+d^2)^(1/2)*b*c*
d+2*B*(c^2+d^2)^(1/2)*a*c*d-2*B*(c^2+d^2)^(1/2)*b*d^2-2*C*(c^2+d^2)^(1/2)*a
*d^2-2*C*(c^2+d^2)^(1/2)*b*c*d-1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*c-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-A*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*A*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+2*B*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*a*c*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2-B*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b*d^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*c+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*C*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c
)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*
(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))\,(c+d\tan(e+fx))^{\frac{3}{2}}\,\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C
*tan(e + f*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.100 $\int (c+d\tan(e+fx))^{3/2} (A + B\tan(e + fx) + C\tan^2(e$

Optimal. Leaf size=187

$$-\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(B-i(A-C))(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan}}{\sqrt{c+}}\right.}{f}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f-
(B-I*(A-C))*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f+2
*(B*c+(A-C)*d)*(c+d*tan(f*x+e))^(1/2)/f+2/3*B*(c+d*tan(f*x+e))^(3/2)/f+2/5*
C*(c+d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.29, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(c+id)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/f) - ((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/f + (2*(B*c + (A - C)*d)*Sqrt[c + d*Tan[e + f*x]])/f +
 (2*B*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*C*(c + d*Tan[e + f*x])^(5/2))/
(5*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2C(c+d\tan(e+fx))^{5/2}}{5df} + \int (A-C$$

$$= \frac{2B(c+d\tan(e+fx))^{3/2}}{3f} + \frac{2C(c+d}{}$$

$$= \frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx}}{f}$$

$$= \frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx}}{f}$$

$$= \frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx}}{f}$$

$$= \frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx}}{f}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\Big(}{f}$$

Mathematica [A]
time = 0.87, size = 202, normalized size = 1.08

$$\frac{\frac{6C(c+d\tan(e+fx))^{5/2}}{d}+5(iA+B-iC)\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c-3id+d\tan(e+fx))\right)+5(-iA+B+iC)\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c+3id+d\tan(e+fx))\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
),x]
```

```
[Out] ((6*C*(c + d*Tan[e + f*x])^(5/2))/d + 5*(I*A + B - I*C)*(-3*(c - I*d)^(3/2)
*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]
*(4*c - (3*I)*d + d*Tan[e + f*x])) + 5*((-I)*A + B + I*C)*(-3*(c + I*d)^(3/
2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x
]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/(15*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(158) = 316$.
time = 0.50, size = 1293, normalized size = 6.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/d*(1/5*C*(c+d*tan(f*x+e))^(5/2)+1/3*B*d*(c+d*tan(f*x+e))^(3/2)+A*d^2*(c
+d*tan(f*x+e))^(1/2)+B*c*d*(c+d*tan(f*x+e))^(1/2)-C*d^2*(c+d*tan(f*x+e))^(1
/2)-d*(1/4/d*(-1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c-A*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-B*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c+C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2)*ln((c+d*tan(f*x+e))^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))+2*(-2*A*(
c^2+d^2)^(1/2)*d^2-2*B*(c^2+d^2)^(1/2)*c*d+2*C*(c^2+d^2)^(1/2)*d^2+1/2*(A*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*(c^2+d^2)^(1/2)*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d-C*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-C*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*d^2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(
1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1
/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*(c^2+d^2)^(1/2)*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*d+2*B*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)
^(1/2)*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*d^2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)+(c^2+d^2)^(1/2))+2*(2*A*(c^2+d^2)^(1/2)*d^2+2*B*(c^2+d^2)^(1/2)*c*d-2*C*
(c^2+d^2)^(1/2)*d^2-1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c-
A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-B*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c+C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2
)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/2
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+d\tan(e+fx))^{\frac{3}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 44.87, size = 2500, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

```
[Out] ((2*C*c^2)/(d*f) - (2*C*(d^3*f + c^2*d*f))/(d^2*f^2))*(c + d*tan(e + f*x))^
(1/2) - log((((16*c*d^2*(((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + B^2*c^3*f^
2 - 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(B*c^2 + B*d^2 + f*(((-B^4*d^2*f^4*(3*c^2 -
 d^2)^2)^(1/2) + B^2*c^3*f^2 - 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f
*x))^(1/2)))/f - (16*B^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*
d^2))/f^2)*(((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + B^2*c^3*f^2 - 3*B^2*c*d
```

^2*f^2)/f^4)^(1/2))/2 - (8*B^3*d^2*(c^2 - d^2)*(c^2 + d^2)^2)/f^3)*(((6*B^4
*c^2*d^4*f^4 - B^4*d^6*f^4 - 9*B^4*c^4*d^2*f^4)^(1/2) + B^2*c^3*f^2 - 3*B^2
*c*d^2*f^2)/(4*f^4))^(1/2) - log((((16*c*d^2*(-((-B^4*d^2*f^4*(3*c^2 - d^2)
^2)^(1/2) - B^2*c^3*f^2 + 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(B*c^2 + B*d^2 + f*(-
((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - B^2*c^3*f^2 + 3*B^2*c*d^2*f^2)/f^4)
^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (16*B^2*d^2*(c + d*tan(e + f*x))^(1
/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2)*(-((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) -
B^2*c^3*f^2 + 3*B^2*c*d^2*f^2)/f^4)^(1/2))/2 - (8*B^3*d^2*(c^2 - d^2)*(c^2
+ d^2)^2)/f^3)*(-((6*B^4*c^2*d^4*f^4 - B^4*d^6*f^4 - 9*B^4*c^4*d^2*f^4)^(1
/2) - B^2*c^3*f^2 + 3*B^2*c*d^2*f^2)/(4*f^4))^(1/2) + log((((16*c*d^2*(((-B
^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + B^2*c^3*f^2 - 3*B^2*c*d^2*f^2)/f^4)^(1/
2)*(B*c^2 + B*d^2 - f*(((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + B^2*c^3*f^2
- 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (16*B^2*d^2*
(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2)*(((-B^4*d^2*f^4*(3
*c^2 - d^2)^2)^(1/2) + B^2*c^3*f^2 - 3*B^2*c*d^2*f^2)/f^4)^(1/2))/2 - (8*B^
3*d^2*(c^2 - d^2)*(c^2 + d^2)^2)/f^3)*((6*B^4*c^2*d^4*f^4 - B^4*d^6*f^4 - 9
*B^4*c^4*d^2*f^4)^(1/2)/(4*f^4) + (B^2*c^3)/(4*f^2) - (3*B^2*c*d^2)/(4*f^2)
)^(1/2) + log((((16*c*d^2*(-((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - B^2*c^3
*f^2 + 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(B*c^2 + B*d^2 - f*(-((-B^4*d^2*f^4*(3*c
^2 - d^2)^2)^(1/2) - B^2*c^3*f^2 + 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e
+ f*x))^(1/2)))/f + (16*B^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*
c^2*d^2))/f^2)*(-((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - B^2*c^3*f^2 + 3*B^
2*c*d^2*f^2)/f^4)^(1/2))/2 - (8*B^3*d^2*(c^2 - d^2)*(c^2 + d^2)^2)/f^3)*((B
^2*c^3)/(4*f^2) - (6*B^4*c^2*d^4*f^4 - B^4*d^6*f^4 - 9*B^4*c^4*d^2*f^4)^(1/
2)/(4*f^4) - (3*B^2*c*d^2)/(4*f^2))^(1/2) - log((((16*d^2*(-((-A^4*d^2*f^4*
(3*c^2 - d^2)^2)^(1/2) + A^2*c^3*f^2 - 3*A^2*c*d^2*f^2)/f^4)^(1/2)*(A*d^3 +
A*c^2*d + c*f*(-((-A^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + A^2*c^3*f^2 - 3*A^
2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (16*A^2*d^2*(c + d
*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2)*(-((-A^4*d^2*f^4*(3*c^2
- d^2)^2)^(1/2) + A^2*c^3*f^2 - 3*A^2*c*d^2*f^2)/f^4)^(1/2))/2 - (16*A^3*c*
d^3*(c^2 + d^2)^2)/f^3)*(-((6*A^4*c^2*d^4*f^4 - A^4*d^6*f^4 - 9*A^4*c^4*d^2
*f^4)^(1/2) + A^2*c^3*f^2 - 3*A^2*c*d^2*f^2)/(4*f^4))^(1/2) - log((((16*d^2
*(((-A^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - A^2*c^3*f^2 + 3*A^2*c*d^2*f^2)/f^
4)^(1/2)*(A*d^3 + A*c^2*d + c*f*(((-A^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - A^
2*c^3*f^2 + 3*A^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f + (1
6*A^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2)*(((-A^4*
d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - A^2*c^3*f^2 + 3*A^2*c*d^2*f^2)/f^4)^(1/2))
/2 - (16*A^3*c*d^3*(c^2 + d^2)^2)/f^3)*(((6*A^4*c^2*d^4*f^4 - A^4*d^6*f^4 -
9*A^4*c^4*d^2*f^4)^(1/2) - A^2*c^3*f^2 + 3*A^2*c*d^2*f^2)/(4*f^4))^(1/2) +
log((((16*d^2*(((-A^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - A^2*c^3*f^2 + 3*A^2
*c*d^2*f^2)/f^4)^(1/2)*(A*d^3 + A*c^2*d - c*f*(((-A^4*d^2*f^4*(3*c^2 - d^2)
^2)^(1/2) - A^2*c^3*f^2 + 3*A^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^
(1/2)))/f - (16*A^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))
/f^2)*(((-A^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - A^2*c^3*f^2 + 3*A^2*c*d^2*f^
2)/f^4)^(1/2))/2 - (16*A^3*c*d^3*(c^2 + d^2)^2)/f^3)*((6*A^4*c^2*d^4*f^4 -

```
A^4*d^6*f^4 - 9*A^4*c^4*d^2*f^4)^(1/2)/(4*f^4) - (A^2*c^3)/(4*f^2) + (3*A^2
*c*d^2)/(4*f^2))^(1/2) + log((((16*d^2*(-((-A^4*d^2*f^4*(3*c^2 - d^2)^2)^(1
/2) + A^2*c^3*f^2 - 3*A^2*c*d^2*f^2)/f^4)^(1/2)*(A*d^3 + A*c^2*d - c*f*(-((
-A^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + A^2*c^3*f^2 - 3*A^2*c*d^2*f^2)/f^4)^(
1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (16*A^2*d^2*(c + d*tan(e + f*x))^(1/2
)*(c^4 + d^4 - 6*c^2*d^2))/f^2)*(-((-A^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + A
^2*c^3*f^2 - 3*A^2*c*d^2*f^2)/f^4)^(1/2))/2 - (16*A^3*c*d^3*(c^2 + d^2)^2)/
f^3)*((3*A^2*c*d^2)/(4*f^2) - (A^2*c^3)/(4*f^2) - (6*A^4*c^2*d^4*f^4 - A^4*
d^6*f^4 - 9*A^4*c^4*d^2*f^4)^(1/2)/(4*f^4))^(1/2) - log((16*C^3*c*d^3*(c^2
+ d^2)^2)/f^3 - (((16*d^2*(-((-C^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + C^2*c^3
*f^2 - 3*C^2*c*d^2*f^2)/f^4)^(1/2)*(C*d^3 + C*c^2*d - c*f*(-((-C^4*d^2*f^4*
(3*c^2 - d^2)^2)^(1/2) + C^2*c^3*f^2 - 3*C^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*t
an(e + f*x))^(1/2)))/f - (16*C^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4
- 6*c^2*d^2))/f^2)*(-((-C^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + C^2*c^3*f^2 -
3*C^2*c*d^2*f^2)/f^4)^(1/2))/2)*(-((6*C^4*c^2*d^4*f^4 - C^4*d^6*f^4 - 9*C^4
*c^4*d^2*f^4)^(1/2) + C^2*c^3*f^2 - 3*C^2*c*d^2...
```

3.101 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=271

$$-\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)f}-\frac{(A+iB-C)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)f}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a
-I*b)/f-(A+I*B-C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))/(I*a-b)/f-2*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(c+d*ta
n(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(a^2+b^2)/f+2*(B*b*d-C*a*d+C*b*c)
*(c+d*tan(f*x+e))^(1/2)/b^2/f+2/3*C*(c+d*tan(f*x+e))^(3/2)/b/f
```

Rubi [A]

time = 1.21, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3728, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2(bc-ad)^{3/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{5/2}f\left(a^2+b^2\right)}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)}-\frac{(c+id)^{3/2}(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)}+\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{b^2f}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x]),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)*f)) - ((A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)*f) - (2*(A*b^2 - a*(b*B - a*C))*(
b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]
])/(b^(5/2)*(a^2 + b^2)*f) + (2*(b*c*C + b*B*d - a*C*d)*Sqrt[c + d*Tan[e +
f*x]])/(b^2*f) + (2*C*(c + d*Tan[e + f*x])^(3/2))/(3*b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{2C(c+d\tan(e+fx))^{3/2}}{3bf} + \frac{2\int \sqrt{c}}{}$$

$$= \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+}}{b^2f}$$

$$= \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+}}{b^2f}$$

$$= \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+}}{b^2f}$$

$$= \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+}}{b^2f}$$

$$= -\frac{2(Ab^2-a(bB-aC))\,(bc-ad)^{3/2}}{b^{5/2}\,(a}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\Big(}{(a-ib)f}$$

Mathematica [A]
time = 1.65, size = 266, normalized size = 0.98

$$\frac{\frac{3ib\left(-\left((a+ib)(A-iB-C)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)\right)+(a-ib)(A+iB-C)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)\right)}{a^2+b^2}-\frac{6\left(Ab^2+a(-bB+aC)\right)(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}(a^2+b^2)}+\frac{6(bcC+bBd-aCd)\sqrt{c+d\tan(e+fx)}}{b}+2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x]),x]
```

```
[Out] (((3*I)*b*(-((a + I*b)*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan
[e + f*x]]/Sqrt[c - I*d]]) + (a - I*b)*(A + I*B - C)*(c + I*d)^(3/2)*ArcTan
h[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]))/(a^2 + b^2) - (6*(A*b^2 + a*(-(
b*B) + a*C))*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/S
qrt[b*c - a*d]])/(b^(3/2)*(a^2 + b^2)) + (6*(b*c*C + b*B*d - a*C*d)*Sqrt[c
+ d*Tan[e + f*x]])/b + 2*C*(c + d*Tan[e + f*x])^(3/2))/(3*b*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. $2(234) = 468$.

time = 0.64, size = 2291, normalized size = 8.45

method	result	size
derivativedivides	Expression too large to display	2291
default	Expression too large to display	2291

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/b^2*(1/3*C*(c+d*tan(f*x+e))^(3/2)*b+B*b*d*(c+d*tan(f*x+e))^(1/2)-C*a
*d*(c+d*tan(f*x+e))^(1/2)+C*b*c*(c+d*tan(f*x+e))^(1/2))+2/(a^2+b^2)*(1/4/d*
(1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-A*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-A
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*
d+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d-B*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d+B*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2+C*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+C*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+C*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*d^2-2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d)*ln(d*ta
n(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(
1/2))+2*(-2*A*(c^2+d^2)^(1/2)*a*d^2+2*A*(c^2+d^2)^(1/2)*b*c*d-2*B*(c^2+d^2)
^(1/2)*a*c*d-2*B*(c^2+d^2)^(1/2)*b*d^2+2*C*(c^2+d^2)^(1/2)*a*d^2-2*C*(c^2+d
^2)^(1/2)*b*c*d-1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-A
*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+A*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*c^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*A*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*b*c*d+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d-B*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*c*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2-B*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b*d^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+C*(c^2+d^2)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*
c^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*b*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan
((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2
)-2*c)^(1/2)))+1/4/d*(1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
a*c+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-A*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*A*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b*c*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d+B
*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+2*B*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*c*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2+B*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*b*d^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-C*(c^2
```

```
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*C*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b*c*d)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*A*(c^2+d^2)^(1/2)*a*d^2+2*A*(c^2+d^2)^(1/2)
*b*c*d-2*B*(c^2+d^2)^(1/2)*a*c*d-2*B*(c^2+d^2)^(1/2)*b*d^2+2*C*(c^2+d^2)^(1
/2)*a*d^2-2*C*(c^2+d^2)^(1/2)*b*c*d+1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*c+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d-A*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*A*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a*d+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+2*B*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*c*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2+B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*c-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*C*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*b*c*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2
)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))+2/b^2*(A*a^2*b^2*d^2-2*A*a*b^3*c*d+A*b^4
*c^2-B*a^3*b*d^2+2*B*a^2*b^2*c*d-B*a*b^3*c^2+C*a^4*d^2-2*C*a^3*b*c*d+C*a^2*
b^2*c^2)/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*
d-b*c)*b)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{3}{2}}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e)),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 58.88, size = 2500, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x)),x)
```

```
[Out] atan((((((((32*(4*B*a*b^8*d^12*f^4 - 4*B*b^9*c*d^11*f^4 + 8*B*a^3*b^6*d^12*f
^4 + 4*B*a^5*b^4*d^12*f^4 - 4*B*b^9*c^3*d^9*f^4 + 8*B*a*b^8*c^2*d^10*f^4 +
4*B*a*b^8*c^4*d^8*f^4 - 12*B*a^2*b^7*c*d^11*f^4 - 12*B*a^4*b^5*c*d^11*f^4 -
 4*B*a^6*b^3*c*d^11*f^4 - 12*B*a^2*b^7*c^3*d^9*f^4 + 16*B*a^3*b^6*c^2*d^10*
f^4 + 8*B*a^3*b^6*c^4*d^8*f^4 - 12*B*a^4*b^5*c^3*d^9*f^4 + 8*B*a^5*b^4*c^2*
d^10*f^4 + 4*B*a^5*b^4*c^4*d^8*f^4 - 4*B*a^6*b^3*c^3*d^9*f^4))/(b*f^5) - (3
2*(c + d*tan(e + f*x))^(1/2)*(-(((8*B^2*a^2*c^3*f^2 - 8*B^2*b^2*c^3*f^2 - 1
6*B^2*a*b*d^3*f^2 - 24*B^2*a^2*c*d^2*f^2 + 24*B^2*b^2*c*d^2*f^2 + 48*B^2*a*
b*c^2*d*f^2)^2/4 - (16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4)*(B^4*c^6 + B^
```

4*d^6 + 3*B^4*c^2*d^4 + 3*B^4*c^4*d^2))^(1/2) - 4*B^2*a^2*c^3*f^2 + 4*B^2*b
^2*c^3*f^2 + 8*B^2*a*b*d^3*f^2 + 12*B^2*a^2*c*d^2*f^2 - 12*B^2*b^2*c*d^2*f^
2 - 24*B^2*a*b*c^2*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2)*(
16*b^10*d^10*f^4 + 16*a^2*b^8*d^10*f^4 - 16*a^4*b^6*d^10*f^4 - 16*a^6*b^4*d
^10*f^4 + 24*b^10*c^2*d^8*f^4 + 40*a^2*b^8*c^2*d^8*f^4 + 8*a^4*b^6*c^2*d^8*
f^4 - 8*a^6*b^4*c^2*d^8*f^4 + 8*a*b^9*c*d^9*f^4 + 24*a^3*b^7*c*d^9*f^4 + 24
*a^5*b^5*c*d^9*f^4 + 8*a^7*b^3*c*d^9*f^4))/(b*f^4))*(-(((8*B^2*a^2*c^3*f^2
- 8*B^2*b^2*c^3*f^2 - 16*B^2*a*b*d^3*f^2 - 24*B^2*a^2*c*d^2*f^2 + 24*B^2*b^
2*c*d^2*f^2 + 48*B^2*a*b*c^2*d*f^2)^2/4 - (16*a^4*f^4 + 16*b^4*f^4 + 32*a^2
*b^2*f^4)*(B^4*c^6 + B^4*d^6 + 3*B^4*c^2*d^4 + 3*B^4*c^4*d^2))^(1/2) - 4*B^
2*a^2*c^3*f^2 + 4*B^2*b^2*c^3*f^2 + 8*B^2*a*b*d^3*f^2 + 12*B^2*a^2*c*d^2*f^
2 - 12*B^2*b^2*c*d^2*f^2 - 24*B^2*a*b*c^2*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2
*a^2*b^2*f^4)))^(1/2) + (32*(c + d*tan(e + f*x))^(1/2)*(4*B^2*a^3*b^5*d^13*
f^2 + 2*B^2*a^5*b^3*d^13*f^2 + 28*B^2*b^8*c^3*d^10*f^2 - 10*B^2*b^8*c^5*d^8
*f^2 - 14*B^2*a*b^7*d^13*f^2 + 16*B^2*a^7*b*d^13*f^2 - 8*B^2*a^8*c*d^12*f^2
 + 22*B^2*b^8*c*d^12*f^2 + 20*B^2*a*b^7*c^2*d^11*f^2 + 50*B^2*a*b^7*c^4*d^9
*f^2 - 28*B^2*a^2*b^6*c*d^12*f^2 - 2*B^2*a^4*b^4*c*d^12*f^2 - 56*B^2*a^6*b^
2*c*d^12*f^2 + 32*B^2*a^7*b*c^2*d^11*f^2 + 8*B^2*a^2*b^6*c^3*d^10*f^2 + 12*
B^2*a^2*b^6*c^5*d^8*f^2 - 24*B^2*a^3*b^5*c^2*d^11*f^2 - 12*B^2*a^3*b^5*c^4*
d^9*f^2 - 4*B^2*a^4*b^4*c^3*d^10*f^2 - 10*B^2*a^4*b^4*c^5*d^8*f^2 + 52*B^2*
a^5*b^3*c^2*d^11*f^2 + 34*B^2*a^5*b^3*c^4*d^9*f^2 - 48*B^2*a^6*b^2*c^3*d^10
*f^2))/(b*f^4))*(-(((8*B^2*a^2*c^3*f^2 - 8*B^2*b^2*c^3*f^2 - 16*B^2*a*b*d^3
*f^2 - 24*B^2*a^2*c*d^2*f^2 + 24*B^2*b^2*c*d^2*f^2 + 48*B^2*a*b*c^2*d*f^2)^
2/4 - (16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4)*(B^4*c^6 + B^4*d^6 + 3*B^4
*c^2*d^4 + 3*B^4*c^4*d^2))^(1/2) - 4*B^2*a^2*c^3*f^2 + 4*B^2*b^2*c^3*f^2 +
8*B^2*a*b*d^3*f^2 + 12*B^2*a^2*c*d^2*f^2 - 12*B^2*b^2*c*d^2*f^2 - 24*B^2*a*
b*c^2*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2) + (32*(15*B^3*
a^4*b^3*d^15*f^2 - B^3*a^2*b^5*d^15*f^2 - 4*B^3*a^7*c^3*d^12*f^2 + 2*B^3*b^
7*c^2*d^13*f^2 + 4*B^3*b^7*c^4*d^11*f^2 + 2*B^3*b^7*c^6*d^9*f^2 - 12*B^3*a^
6*b*d^15*f^2 - 4*B^3*a^7*c*d^14*f^2 - B^3*a*b^6*c*d^14*f^2 - 27*B^3*a*b^6*c
^3*d^12*f^2 - 19*B^3*a*b^6*c^5*d^10*f^2 + 7*B^3*a*b^6*c^7*d^8*f^2 - 57*B^3*
a^3*b^4*c*d^14*f^2 + 64*B^3*a^5*b^2*c*d^14*f^2 + 4*B^3*a^6*b*c^2*d^13*f^2 +
 16*B^3*a^6*b*c^4*d^11*f^2 + 65*B^3*a^2*b^5*c^2*d^13*f^2 + 9*B^3*a^2*b^5*c^
4*d^11*f^2 - 57*B^3*a^2*b^5*c^6*d^9*f^2 + 77*B^3*a^3*b^4*c^3*d^12*f^2 + 129
*B^3*a^3*b^4*c^5*d^10*f^2 - 5*B^3*a^3*b^4*c^7*d^8*f^2 - 121*B^3*a^4*b^3*c^2
*d^13*f^2 - 119*B^3*a^4*b^3*c^4*d^11*f^2 + 17*B^3*a^4*b^3*c^6*d^9*f^2 + 40*
B^3*a^5*b^2*c^3*d^12*f^2 - 24*B^3*a^5*b^2*c^5*d^10*f^2))/(b*f^5))*(-(((8*B^
2*a^2*c^3*f^2 - 8*B^2*b^2*c^3*f^2 - 16*B^2*a*b*d^3*f^2 - 24*B^2*a^2*c*d^2*f
^2 + 24*B^2*b^2*c*d^2*f^2 + 48*B^2*a*b*c^2*d*f^2)^2/4 - (16*a^4*f^4 + 16*b^
4*f^4 + 32*a^2*b^2*f^4)*(B^4*c^6 + B^4*d^6 + 3*B^4*c^2*d^4 + 3*B^4*c^4*d^2)
)^(1/2) - 4*B^2*a^2*c^3*f^2 + 4*B^2*b^2*c^3*f^2 + 8*B^2*a*b*d^3*f^2 + 12*B^
2*a^2*c*d^2*f^2 - 12*B^2*b^2*c*d^2*f^2 - 24*B^2*a*b*c^2*d*f^2)/(16*(a^4*f^4
 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2) - (32*(c + d*tan(e + f*x))^(1/2)*(B^4*b
^6*d^16 - 2*B^4*a^6*d^16 + 12*B^4*a^6*c^2*d^14 - 2*B^4*a^6*c^4*d^12 + 4*B^4
*b^6*c^2*d^14 + 6*B^4*b^6*c^4*d^12 + 4*B^4*b^6*c^6*d^10 + B^4*b^6*c^8*d^8 -

```
 2*B^4*a^2*b^4*c^4*d^12 + 12*B^4*a^2*b^4*c^6*d^10 - 2*B^4*a^2*b^4*c^8*d^8 +
 8*B^4*a^3*b^3*c^3*d^13 - 48*B^4*a^3*b^3*c^5*d^11 + 8*B^4*a^3*b^3*c^7*d^9 -
 12*B^4*a^4*b^2*c^2*d^14 + 72*B^4*a^4*b^2*c^4*d^12 - 12*B^4*a^4*b^2*c^6*d^1
0 + 8*B^4*a^5*b*c*d^15 - 48*B^4*a^5*b*c^3*d^13 + 8*B^4*a^5*b*c^5*d^11))/(b*
f^4))*(-(((8*B^2*a^2*c^3*f^2 - 8*B^2*b^2*c^3*f^2 - 16*B^2*a*b*d^3*f^2 - 24*
B^2*a^2*c*d^2*f^2 + 24*B^2*b^2*c*d^2*f^2 + 48*B^2*a*b*c^2*d*f^2)^2/4 - (16*
a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4)*(B^4*c^6 + B^4*d^6 + 3*B^4*c^2*d^4 +
 3*B^4*c^4*d^2))^(1/2) - 4*B^2*a^2*c^3*f^2 + 4*B^2*b^2*c^3*f^2 + 8*B^2*a*b*
d^3*f^2 + 12*B^2*a^2*c*d^2*f^2 - 12*B^2*b^2*c*d^2*f^2 - 24*B^2*a*b*c^2*d*f^
2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2)*1i - (((((32*(4*B*a*b^8*
d^12*f^4 - 4*B*b^9*c*d^11*f^4 + 8*B*a^3*b^6*d^12*f^4 + 4*B*a^5*b^4*d^12*f^4
 - 4*B*b^9*c^3*d^9*f^4 + 8*B*a*b^8*c^2*d^10*f^4 + 4*B*a*b^8*c^4*d^8*f^4 - 1
2*B*a^2*b^7*c*d^11*f^4 - 12*B*a^4*b^5*c*d^11*f^...
```

3.102 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=372

$$-\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2 f}-\frac{(B-i(A-C))(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{t}}}{\sqrt{c}}\right.}{(a+ib)^2 f}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a
-I*b)^2/f-(B-I*(A-C))*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^
(1/2))/(a+I*b)^2/f+(a^3*b*B*d-3*a^4*C*d-b^4*(3*A*d+2*B*c)-a*b^3*(4*A*c-5*B*
d-4*C*c)+a^2*b^2*(2*B*c+(A-7*C)*d))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/
(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(5/2)/(a^2+b^2)^2/f+(A*b^2-B*a*b+3*C*a
^2+2*C*b^2)*d*(c+d*tan(f*x+e))^(1/2)/b^2/(a^2+b^2)/f-(A*b^2-a*(B*b-C*a))*(c
+d*tan(f*x+e))^(3/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 1.68, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3726, 3728, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{d(3a^2C-abB+Ab^2+2b^2C)\sqrt{c+d\tan(e+fx)}}{b^2f(a^2+b^2)}+\frac{\sqrt{bc-ad}\,(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{5/2}f(a^2+b^2)^2}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2}-\frac{(c+id)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^2,x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[Sqrt[c
 + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) + (Sqrt[b*c - a*d]*(a^3*
b*B*d - 3*a^4*C*d - b^4*(2*B*c + 3*A*d) - a*b^3*(4*A*c - 4*c*C - 5*B*d) + a
^2*b^2*(2*B*c + (A - 7*C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sq
rt[b*c - a*d]])/(b^(5/2)*(a^2 + b^2)^2*f) + ((A*b^2 - a*b*B + 3*a^2*C + 2*b
^2*C)*d*Sqrt[c + d*Tan[e + f*x]])/(b^2*(a^2 + b^2)*f) - ((A*b^2 - a*(b*B -
a*C))*(c + d*Tan[e + f*x])^(3/2))/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
```

```
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+}{b\,(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= \frac{(Ab^2-abB+3a^2C+2b^2C)\,d\sqrt{c+}}{b^2\,(a^2+b^2)\,f}$$

$$= \frac{(Ab^2-abB+3a^2C+2b^2C)\,d\sqrt{c+}}{b^2\,(a^2+b^2)\,f}$$

$$= \frac{(Ab^2-abB+3a^2C+2b^2C)\,d\sqrt{c+}}{b^2\,(a^2+b^2)\,f}$$

$$= \frac{(Ab^2-abB+3a^2C+2b^2C)\,d\sqrt{c+}}{b^2\,(a^2+b^2)\,f}$$

$$= \frac{\sqrt{bc-ad}\,(a^3bBd-3a^4Cd-b^4(2B}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\Big(}{(a-ib)^2f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than

twice the leaf count of optimal. 1732 vs. 2(372) = 744.
time = 4.14, size = 1732, normalized size = 4.66

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-4*a^2*A*b^3*c*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/
Sqrt[b*c - a*d]] + 2*a^3*b^2*B*c*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c +
d*Tan[e + f*x]])/Sqrt[b*c - a*d]] - 2*a*b^4*B*c*Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]] + 4*a^2*b^3*c*C*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]] + a^3*A*b
^2*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
a*d]] - 3*a*A*b^4*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x
]])/Sqrt[b*c - a*d]] + a^4*b*B*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c +
d*Tan[e + f*x]])/Sqrt[b*c - a*d]] + 5*a^2*b^3*B*d*Sqrt[b*c - a*d]*ArcTanh[(
Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]] - 3*a^5*C*d*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]] - 7*a^3*b^2
*C*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
a*d]] - 4*a*A*b^4*c*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x
]])/Sqrt[b*c - a*d]]*Tan[e + f*x] + 2*a^2*b^3*B*c*Sqrt[b*c - a*d]*ArcTanh[(
Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] - 2*b^5*B*c
*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]
]*Tan[e + f*x] + 4*a*b^4*c*C*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] + a^2*A*b^3*d*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] - 3*
A*b^5*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c
 - a*d]]*Tan[e + f*x] + a^3*b^2*B*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] + 5*a*b^4*B*d*Sqrt[b*c -
a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*
x] - 3*a^4*b*C*d*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])
/Sqrt[b*c - a*d]]*Tan[e + f*x] - 7*a^2*b^3*C*d*Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Tan[e + f*x] + b^(5/2)*((-I
)*a + b)^2*(I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]]*(a + b*Tan[e + f*x]) + b^(5/2)*(I*a + b)^2*((-I)*A + B + I*
C)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]*(a + b*T
an[e + f*x]) - a^2*A*b^(7/2)*c*Sqrt[c + d*Tan[e + f*x]] - A*b^(11/2)*c*Sqrt
[c + d*Tan[e + f*x]] + a^3*b^(5/2)*B*c*Sqrt[c + d*Tan[e + f*x]] + a*b^(9/2)
*B*c*Sqrt[c + d*Tan[e + f*x]] - a^4*b^(3/2)*c*C*Sqrt[c + d*Tan[e + f*x]] -
a^2*b^(7/2)*c*C*Sqrt[c + d*Tan[e + f*x]] + a^3*A*b^(5/2)*d*Sqrt[c + d*Tan[e
 + f*x]] + a*A*b^(9/2)*d*Sqrt[c + d*Tan[e + f*x]] - a^4*b^(3/2)*B*d*Sqrt[c
+ d*Tan[e + f*x]] - a^2*b^(7/2)*B*d*Sqrt[c + d*Tan[e + f*x]] + 3*a^5*Sqrt[b
]*C*d*Sqrt[c + d*Tan[e + f*x]] + 5*a^3*b^(5/2)*C*d*Sqrt[c + d*Tan[e + f*x]]
```

```
 + 2*a*b^(9/2)*C*d*Sqrt[c + d*Tan[e + f*x]] + 2*a^4*b^(3/2)*C*d*Tan[e + f*x
]*Sqrt[c + d*Tan[e + f*x]] + 4*a^2*b^(7/2)*C*d*Tan[e + f*x]*Sqrt[c + d*Tan[
e + f*x]] + 2*b^(11/2)*C*d*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(b^(5/2)*
(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3604 vs. $2(337) = 674$.
time = 0.68, size = 3605, normalized size = 9.69

method	result	size
derivativedivides	Expression too large to display	3605
default	Expression too large to display	3605

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d*(C/b^2*(c+d*tan(f*x+e))^(1/2)+1/d/(a^2+b^2)^2*(1/4/d*(1/2*(-A*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*(c^2+d^2)^(1/2)*a*b*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b
^2*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*a^2*d^2+4*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d-A*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*b^2*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2+B*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*d-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+
d^2)^(1/2)*a*b*c-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*d-2*B*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b
*c^2-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^2+2*B*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b^2*c*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c+2*C*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d-C*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*(c^2+d^2)^(1/2)*b^2*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+C*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+
C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2
*d^2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)+(c^2+d^2)^(1/2))+2*(-2*A*(c^2+d^2)^(1/2)*a^2*d^2+4*A*(c^2+d^2)^(1/2)*a*b*
c*d+2*A*(c^2+d^2)^(1/2)*b^2*d^2-2*B*(c^2+d^2)^(1/2)*a^2*c*d-4*B*(c^2+d^2)^(
1/2)*a*b*d^2+2*B*(c^2+d^2)^(1/2)*b^2*c*d+2*C*(c^2+d^2)^(1/2)*a^2*d^2-4*C*(c
^2+d^2)^(1/2)*a*b*c*d-2*C*(c^2+d^2)^(1/2)*b^2*d^2-1/2*(-A*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d
^2)^(1/2)*a*b*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*c+A*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2+
4*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b
^2*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*(c^2+d^2)^(1/2)*a^2*d-2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)
*a*b*c-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*d-2*B*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a^2*c*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c^2-2*B*(
```

```
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^2+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*
c*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c+2*C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*b^2*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+C*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a^2*d^2-4*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+C*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan
(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)
))+1/4/d*(1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c+2*A*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d-A*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*(c^2+d^2)^(1/2)*b^2*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+A*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2-4*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d
+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^
2*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*d+2*B*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c
^2+d^2)^(1/2)*b^2*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d-2*B*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*b*c^2+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^2-2*B*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+
d^2)^(1/2)*a^2*c-2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d+C*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*c+C*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*a^2*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*d^2+4*C*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a*b*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*c^2+C*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b^2*d^2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*A*(c^2+d^2)^(1/2)*a^2*d^2+
4*A*(c^2+d^2)^(1/2)*a*b*c*d+2*A*(c^2+d^2)^(1/2)*b^2*d^2-2*B*(c^2+d^2)^(1/2)
*a^2*c*d-4*B*(c^2+d^2)^(1/2)*a*b*d^2+2*B*(c^2+d^2)^(1/2)*b^2*c*d+2*C*(c^2+d
^2)^(1/2)*a^2*d^2-4*C*(c^2+d^2)^(1/2)*a*b*c*d-2*C*(c^2+d^2)^(1/2)*b^2*d^2+1
/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*c+2*A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*b^2*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c^2+A*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a^2*d^2-4*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c*d+A*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b^2*c^2-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^2*d^2-B*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^2*d+2*B*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*(c^2+d^2)^(1/2)*a*b*c+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/
2)*b^2*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*c*d-2*B*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*b*c^2+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b*d^2-2*B*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b^2*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*
a^2*c-2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b*d+C*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^2*c+C*(2*(...
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{3}{2}}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**2,x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^2,x)
```

```
[Out] \text{Hanged}
```

3.103 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=532

$$-\frac{(A-iB-C)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3 f}+\frac{(A+iB-C)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan}}{\sqrt{c+}}\right.}{(ia-b)^3 f}$$

```
[Out] -(A-I*B-C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a
+b)^3/f+(A+I*B-C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))/(I*a-b)^3/f-1/4*(a^5*b*B*d^2+3*a^6*C*d^2+a^4*b^2*d*(4*B*c+3*(A+2*C)*d)-b
^6*(8*A*c^2-3*A*d^2-12*B*c*d-8*C*c^2)+a^2*b^4*(24*A*c^2-26*A*d^2-48*B*c*d-2
4*C*c^2+35*C*d^2)-2*a^3*b^3*(12*c*(A-C)*d+B*(4*c^2-9*d^2))+a*b^5*(40*c*(A-C
)*d+3*B*(8*c^2-5*d^2)))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(
1/2))/b^(5/2)/(a^2+b^2)^3/f/(-a*d+b*c)^(1/2)-1/4*(a^3*b*B*d+3*a^4*C*d+b^4*(
3*A*d+4*B*c)+a*b^3*(8*A*c-7*B*d-8*C*c)-a^2*b^2*(5*A*d+4*B*c-11*C*d))*(c+d*t
an(f*x+e))^(1/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))-1/2*(A*b^2-a*(B*b-C*a))
*(c+d*tan(f*x+e))^(3/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2
```

Rubi [A]

time = 2.85, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3726, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{2bf(a^2+b^2)(a+b\tan(e+fx))^2}-\frac{\sqrt{c+d\tan(e+fx)}\,(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc))}{4b^2f(a^2+b^2)^2(a+b\tan(e+fx))}-\frac{(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc)-2a^3b^3(12cd(A-C)+B(4c^2-9d^2))+a^2b^4(24Ac^2-26Ad^2-48Bcd-24c^2C+35Cd^2)+ab^5(40cd(A-C)+3B(8c^2-5d^2))-b^6(8Ac^2-3Ad^2-12Bcd-8c^2C))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{4b^{5/2}f(a^2+b^2)^3\sqrt{bc-ad}}-\frac{(c-id)^{3/2}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)^3}+\frac{(c+id)^{3/2}(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^3,x]
```

```
[Out] -(((A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((I*a + b)^3*f)) + ((A + I*B - C)*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) - ((a^5*b*B*d^2 + 3*a^6*C*d^
2 + a^4*b^2*d*(4*B*c + 3*(A + 2*C)*d) - b^6*(8*A*c^2 - 8*c^2*C - 12*B*c*d -
 3*A*d^2) + a^2*b^4*(24*A*c^2 - 24*c^2*C - 48*B*c*d - 26*A*d^2 + 35*C*d^2)
- 2*a^3*b^3*(12*c*(A - C)*d + B*(4*c^2 - 9*d^2)) + a*b^5*(40*c*(A - C)*d +
3*B*(8*c^2 - 5*d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
 a*d]])/(4*b^(5/2)*(a^2 + b^2)^3*Sqrt[b*c - a*d]*f) - ((a^3*b*B*d + 3*a^4*C
*d + b^4*(4*B*c + 3*A*d) + a*b^3*(8*A*c - 8*c*C - 7*B*d) - a^2*b^2*(4*B*c +
 5*A*d - 11*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b^2*(a^2 + b^2)^2*f*(a + b*T
an[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(2*b*(
a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+}{2b\,(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= -\frac{(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)}{}$$

$$= -\frac{(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)}{}$$

$$= -\frac{(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)}{}$$

$$= -\frac{(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)}{}$$

$$= -\frac{(a^5bBd^2+3a^6Cd^2+a^4b^2d(4Bc+}{}$$

$$= -\frac{(A-iB-C)(c-id)^{3/2}\tanh^{-1}\Big(}{(ia+b)^3f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 7678 vs. 2(532) = 1064.
time = 6.35, size = 7678, normalized size = 14.43

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4968 vs. $2(492) = 984$.
time = 0.65, size = 4969, normalized size = 9.34

method	result	size
derivativedivides	Expression too large to display	4969
default	Expression too large to display	4969

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d^2*(1/d^2/(a^2+b^2)^3*(1/4/d*(1/2*(3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*b^2*d^2-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d+B*(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a^3*d+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b^3*c+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2-3*C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*b^2*d^2+2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d-2*B*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2-
3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a^3*c-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*
c+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d-3*A*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*a*b^2*c^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^
3*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*b^3*d^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2+C*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a^3*d^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2-A*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a^3*d^2+6*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d+3*C*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*C*(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*b^2*c-6*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d-3*
A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*A*(c^2+d^2)^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+6*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^
2*b*c*d-3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c-3*B*(c^2+
d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d)*ln(d*tan(f*x+e)+c+(c+d*ta
n(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-2*A*(c^2
+d^2)^(1/2)*a^3*d^2+6*A*(c^2+d^2)^(1/2)*a^2*b*c*d+6*A*(c^2+d^2)^(1/2)*a*b^2
*d^2-2*A*(c^2+d^2)^(1/2)*b^3*c*d-2*B*(c^2+d^2)^(1/2)*a^3*c*d-6*B*(c^2+d^2)^
(1/2)*a^2*b*d^2+6*B*(c^2+d^2)^(1/2)*a*b^2*c*d+2*B*(c^2+d^2)^(1/2)*b^3*d^2+2
*C*(c^2+d^2)^(1/2)*a^3*d^2-6*C*(c^2+d^2)^(1/2)*a^2*b*c*d-6*C*(c^2+d^2)^(1/2
)*a*b^2*d^2+2*C*(c^2+d^2)^(1/2)*b^3*c*d-1/2*(3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1
```

```
/2)*a*b^2*d^2-2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d+B*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b^3*c+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2-3*C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*b^2*d^2+2*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c*d-2*B*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b
*c^2-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d^2+C*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a^3*c-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^3*c+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d-3*A*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*b^2*c^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b^3*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c^2+B*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*b^3*d^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2+C*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a^3*d^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c^2-A*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a^3*d^2+6*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c*d+3*C*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d-3*C*(c^2+d^2)^(1/2)*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-6*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c
*d-3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*d+3*A*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+6*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a^2*b*c*d-3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c-3*B*
(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d*(1/2*(-
3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^2+2*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b^3*c*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d-B*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*c-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*b^2*c^2+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*d^2-2*C*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b^3*c*d+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c*d-3*B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c^2+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*
d^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c+A*(c^2+d^2)^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*b^3*d+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c^2+C*(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*
c^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b^3*d^2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a^3*c^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d^2-A*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a^3*c^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*d^2-6*B*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a*b^2*c*d-3*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^
2*b*d+3*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c+6*C*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d+3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a^2*b*d-3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*b^2*c-6
*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^2*b*c*d+3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a^2*b*c+3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*b^2*d)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan{(e + fx)})^{\frac{3}{2}} (A + B\tan{(e + fx)} + C\tan^{2}{(e + fx)})}{(a + b\tan{(e + fx)})^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**3,x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x))**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^3,x)
```

```
[Out] \text{Hanged}
```

3.104 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e$

Optimal. Leaf size=503

$$-\frac{(a-ib)^2(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(a+ib)^2(iA-B-iC)(c+id)^{5/2}\tan}{f}$$

```
[Out] -(a-I*b)^2*(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)
^(1/2))/f+(a+I*b)^2*(I*A-B-I*C)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2
)/(c+I*d)^(1/2))/f-2*(2*a*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-a^2*(2*c*(A-C
)*d+B*(c^2-d^2))+b^2*(2*c*(A-C)*d+B*(c^2-d^2)))*(c+d*tan(f*x+e))^(1/2)/f+2/
3*(2*a*b*(A*c-B*d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*d))*(c+d*tan(f*x+e)
)^(3/2)/f+2/5*(a^2*B-b^2*B+2*a*b*(A-C))*(c+d*tan(f*x+e))^(5/2)/f+2/693*(36*
a^2*C*d^2-22*a*b*d*(-9*B*d+2*C*c)+b^2*(8*c^2*C-22*B*c*d+99*(A-C)*d^2))*(c+d
*tan(f*x+e))^(7/2)/d^3/f-2/99*b*(-11*B*b*d-4*C*a*d+4*C*b*c)*tan(f*x+e)*(c+d
*tan(f*x+e))^(7/2)/d^2/f+2/11*C*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(7/2)/d
/f
```

Rubi [A]
time = 1.55, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3728, 3718, 3711, 3609, 3620, 3618, 65, 214}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c - I*d]])/f) + ((a + I*b)^2*(I*A - B - I*C)*(c + I*d)^(5/2)*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f - (2*(2*a*b*(c^2*C + 2*B*c*d
 - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*
(A - C)*d + B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(2*a*b*(A*c -
c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*(c + d*Tan[e +
f*x])^(3/2))/(3*f) + (2*(a^2*B - b^2*B + 2*a*b*(A - C))*(c + d*Tan[e + f*x]
)^(5/2))/(5*f) + (2*(36*a^2*C*d^2 - 22*a*b*d*(2*c*C - 9*B*d) + b^2*(8*c^2*C
 - 22*B*c*d + 99*(A - C)*d^2))*(c + d*Tan[e + f*x])^(7/2))/(693*d^3*f) - (2
*b*(4*b*c*C - 11*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/
(99*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(7/2))/(11*d*
f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
```

```
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2C(a+b\tan(e\ldots}{}$$

$$= -\frac{2b(4bcC-11b\ldots}{}$$

$$= \frac{2(36a^2Cd^2-22\ldots}{}$$

$$= \frac{2(a^2B-b^2B+\ldots}{}$$

$$= \frac{2(2ab(Ac-cC\ldots}{}$$

$$= -\frac{2(2ab(c^2C+2\ldots}{}$$

$$= -\frac{2(2ab(c^2C+2\ldots}{}$$

$$= -\frac{2(2ab(c^2C+2\ldots}{}$$

$$= -\frac{2(2ab(c^2C+2\ldots}{}$$

$$= -\frac{(a-ib)^2(iA+\ldots}{}$$

Mathematica [A]
time = 6.32, size = 564, normalized size = 1.12

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e +
f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(7/2))/(11*d*f) + (2*((b*(
-4*b*c*C + 11*b*B*d + 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/(9*
```

```
d*f) - (2*(((-36*a^2*C*d^2 + 22*a*b*d*(2*c*C - 9*B*d) - b^2*(8*c^2*C - 22*B
*c*d + 99*(A - C)*d^2))*(c + d*Tan[e + f*x])^(7/2))/(14*d*f) + ((I/2)*(((99
*I)/4)*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2 + (99*(2*a*b*B - a^2*(A - C) + b
^2*(A - C))*d^2)/4)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (c - I*d)*((2*(c +
d*Tan[e + f*x])^(3/2))/3 + (c - I*d)*((2*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c - I*d]])/(-c + I*d) + 2*Sqrt[c + d*Tan[e + f*x]]))))/
f - ((I/2)*(((-99*I)/4)*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2 + (99*(2*a*b*B
- a^2*(A - C) + b^2*(A - C))*d^2)/4)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (c
 + I*d)*((2*(c + d*Tan[e + f*x])^(3/2))/3 + (c + I*d)*((2*(c + I*d)^(3/2)*A
rcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(-c - I*d) + 2*Sqrt[c + d*T
an[e + f*x]]))))/f))/(9*d)))/(11*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5352 vs. $2(459) = 918$.
time = 0.58, size = 5353, normalized size = 10.64

method	result	size
derivativedivides	Expression too large to display	5353
default	Expression too large to display	5353

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^2 (c+d\tan(e+fx))^{\frac{5}{2}} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan
(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(5/2)*(A + B*tan(e +
 f*x) + C*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) +
 C*tan(e + f*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.105 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e\right.$

Optimal. Leaf size=353

$$-\frac{(ia+b)(A-iB-C)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(ia-b)(A+iB-C)(c+id)^{5/2}\tanh^{-1}}{f}$$

```
[Out] -(I*a+b)*(A-I*B-C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/
2))/f+(I*a-b)*(A+I*B-C)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d
)^(1/2))/f+2*(a*(B*c^2-B*d^2-2*C*c*d)-b*(2*B*c*d+C*c^2-C*d^2)+A*(2*a*c*d+b*
(c^2-d^2)))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b
*c)*(c+d*tan(f*x+e))^(3/2)/f+2/5*(A*b+B*a-C*b)*(c+d*tan(f*x+e))^(5/2)/f-2/6
3*(-9*B*b*d-9*C*a*d+2*C*b*c)*(c+d*tan(f*x+e))^(7/2)/d^2/f+2/9*b*C*tan(f*x+e
)*(c+d*tan(f*x+e))^(7/2)/d/f
```

Rubi [A]
time = 0.83, antiderivative size = 351, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3718, 3711, 3609, 3620, 3618, 65, 214}

$$\frac{2\sqrt{c+d\tan(e+fx)}\,(2aAcd+aB(c^2-d^2)-2acCd+Ab(c^2-d^2)-b(2Bcd+c^2C-Cd^2))}{f}+\frac{2(aB+Ab-bC)(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2(c+d\tan(e+fx))^{3/2}(aAd+aBc-aCd+Abc-bBd-bcC)}{3f}-\frac{(b+ia)(c-id)^{5/2}(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}+\frac{(-b+ia)(c+id)^{5/2}(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}-\frac{2(-9aCd-9bBd+2bcC)(c+d\tan(e+fx))^{7/2}}{63d^2f}+\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C
*Tan[e + f*x]^2),x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/f) + ((I*a - b)*(A + I*B - C)*(c + I*d)^(5/2)*ArcTanh[Sqrt
[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/f + (2*(2*a*A*c*d - 2*a*c*C*d + A*b*(c
^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C + 2*B*c*d - C*d^2))*Sqrt[c + d*Tan[e
 + f*x]])/f + (2*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*(c + d*Tan
[e + f*x])^(3/2))/(3*f) + (2*(A*b + a*B - b*C)*(c + d*Tan[e + f*x])^(5/2))/
(5*f) - (2*(2*b*c*C - 9*b*B*d - 9*a*C*d)*(c + d*Tan[e + f*x])^(7/2))/(63*d^
2*f) + (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/(9*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2bC\tan(e+fx)(c}{9}$$

$$= -\frac{2(2bcC-9bBd}{}$$

$$= \frac{2(Ab+aB-bC)(}{}$$

$$= \frac{2(Abc+aBc-bc}{}$$

$$= \frac{2(2aAcd-2acCd}{}$$

$$= \frac{2(2aAcd-2acCd}{}$$

$$= \frac{2(2aAcd-2acCd}{}$$

$$= \frac{2(2aAcd-2acCd}{}$$

$$= -\frac{(ia+b)(A-iB}{}$$

<u>Mathematica</u> [A]
time = 3.50, size = 324, normalized size = 0.92

$$\frac{\frac{2(-2bcC+9bBd+9aCd)(c+d\tan(e+fx))^{7/2}}{d}+14bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}+\frac{63}{2}i(a-ib)(A-iB-C)d\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c-id)\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}(4c-3id+d\tan(e+fx))\right)\right)-\frac{63}{2}i(a+ib)(A+iB-C)d\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c+id)\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}(4c+3id+d\tan(e+fx))\right)\right)}{63df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] ((2*(-2*b*c*C + 9*b*B*d + 9*a*C*d)*(c + d*Tan[e + f*x])^(7/2))/d + 14*b*C*T
an[e + f*x]*(c + d*Tan[e + f*x])^(7/2) + ((63*I)/2)*(a - I*b)*(A - I*B - C)
*d*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (2*(c - I*d)*(-3*(c - I*d)^(3/2)*Arc
Tanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*
c - (3*I)*d + d*Tan[e + f*x])))/3) - ((63*I)/2)*(a + I*b)*(A + I*B - C)*d*(
(2*(c + d*Tan[e + f*x])^(5/2))/5 + (2*(c + I*d)*(-3*(c + I*d)^(3/2)*ArcTanh
```

```
[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c +
(3*I)*d + d*Tan[e + f*x]))))/3))/(63*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3367 vs. $2(315) = 630$.

time = 0.53, size = 3368, normalized size = 9.54

method	result	size
derivativedivides	Expression too large to display	3368
default	Expression too large to display	3368

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)
,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^2*(1/9*C*b*(c+d*tan(f*x+e))^(9/2)+1/7*B*b*d*(c+d*tan(f*x+e))^(7/2)+1/
7*C*a*d*(c+d*tan(f*x+e))^(7/2)-1/7*C*b*c*(c+d*tan(f*x+e))^(7/2)+1/5*A*b*d^2
*(c+d*tan(f*x+e))^(5/2)+1/5*B*a*d^2*(c+d*tan(f*x+e))^(5/2)-1/5*C*b*d^2*(c+d
*tan(f*x+e))^(5/2)+1/3*A*a*d^3*(c+d*tan(f*x+e))^(3/2)+1/3*A*b*c*d^2*(c+d*ta
n(f*x+e))^(3/2)+1/3*B*a*c*d^2*(c+d*tan(f*x+e))^(3/2)-1/3*B*b*d^3*(c+d*tan(f
*x+e))^(3/2)-1/3*C*a*d^3*(c+d*tan(f*x+e))^(3/2)-1/3*C*b*c*d^2*(c+d*tan(f*x+
e))^(3/2)+2*A*a*c*d^3*(c+d*tan(f*x+e))^(1/2)+A*b*c^2*d^2*(c+d*tan(f*x+e))^(
1/2)-A*b*d^4*(c+d*tan(f*x+e))^(1/2)+B*a*c^2*d^2*(c+d*tan(f*x+e))^(1/2)-B*a*
d^4*(c+d*tan(f*x+e))^(1/2)-2*B*b*c*d^3*(c+d*tan(f*x+e))^(1/2)-2*C*a*c*d^3*(
c+d*tan(f*x+e))^(1/2)-C*b*c^2*d^2*(c+d*tan(f*x+e))^(1/2)+C*b*d^4*(c+d*tan(f
*x+e))^(1/2)-d^2*(1/4/d*(1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*c^2-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*A*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c^3+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2+3*A*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b*c^2*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3-2*B*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b*c^2+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2+3*B*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3+B*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d
^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+C*(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*b*c*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*C*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*c*d^2-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d+C*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(4*A*(c^2+d^2)^(1/2)*a*c*d^2+2*A
*(c^2+d^2)^(1/2)*b*c^2*d-2*A*(c^2+d^2)^(1/2)*b*d^3+2*B*(c^2+d^2)^(1/2)*a*c^
2*d-2*B*(c^2+d^2)^(1/2)*a*d^3-4*B*(c^2+d^2)^(1/2)*b*c*d^2-4*C*(c^2+d^2)^(1/
2)*a*c*d^2-2*C*(c^2+d^2)^(1/2)*b*c^2*d+2*C*(c^2+d^2)^(1/2)*b*d^3-1/2*(A*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-A*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
```

```
2)*b*c*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3+3*A*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*c*d^2+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-A*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*b*d^3-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d-
B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2+B*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d-
B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3
-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a*c^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2
+2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d+C*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*c^3-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2-3*C*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b*c^2*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*
x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+
1/4/d*(1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+A*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*A*(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b*c*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*A*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d+A
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3+2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*a*c*d+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2-B*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2-3*B*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*c^2*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3-B*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*b*c^3+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2+C*(c^2+d^2)^(1
/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*a*d^2-2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d-
C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c
*d^2+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b*d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)+(c^2+d^2)^(1/2))+2*(4*A*(c^2+d^2)^(1/2)*a*c*d^2+2*A*(c^2+d^2)^(1/2)*
b*c^2*d-2*A*(c^2+d^2)^(1/2)*b*d^3+2*B*(c^2+d^2)^(1/2)*a*c^2*d-2*B*(c^2+d^2)
^(1/2)*a*d^3-4*B*(c^2+d^2)^(1/2)*b*c*d^2-4*C*(c^2+d^2)^(1/2)*a*c*d^2-2*C*(c
^2+d^2)^(1/2)*b*c^2*d+2*C*(c^2+d^2)^(1/2)*b*d^3+1/2*(-A*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*a*d^2+2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d+A*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2-3*
A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d
^3+2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d+B*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2-B*(c^2+d^...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))\,(c+d\tan(e+fx))^{\frac{5}{2}}\,\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C
*tan(e + f*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.106 $\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e\right.$

Optimal. Leaf size=229

$$-\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(B-i(A-C))(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{ta}}}{\sqrt{c\cdot}}\right.}{f}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f-
(B-I*(A-C))*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f+2
*(2*c*(A-C)*d+B*(c^2-d^2))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(B*c+(A-C)*d)*(c+d*
tan(f*x+e))^(3/2)/f+2/5*B*(c+d*tan(f*x+e))^(5/2)/f+2/7*C*(c+d*tan(f*x+e))^(
7/2)/d/f
```

Rubi [A]
time = 0.42, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3711, 3609, 3620, 3618, 65, 214}

$$\frac{2(2cd(A-C)+B(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}-\frac{(c-id)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}-\frac{(c+id)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/f) - ((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/f + (2*(2*c*(A - C)*d + B*(c^2 - d^2))*Sqrt[c + d*Tan[
e + f*x]])/f + (2*(B*c + (A - C)*d)*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*
B*(c + d*Tan[e + f*x])^(5/2))/(5*f) + (2*C*(c + d*Tan[e + f*x])^(7/2))/(7*d
*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2C(c+d\tan(e+fx))^{7/2}}{7df}+\int (A-C$$
$$= \frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d$$
$$= \frac{2(Bc+(A-C)d)(c+d\tan(e+fx))}{3f}$$
$$= \frac{2(2c(A-C)d+B(c^2-d^2))\sqrt{c+d\,\mathrm{t}}}{f}$$
$$= \frac{2(2c(A-C)d+B(c^2-d^2))\sqrt{c+d\,\mathrm{t}}}{f}$$
$$= \frac{2(2c(A-C)d+B(c^2-d^2))\sqrt{c+d\,\mathrm{t}}}{f}$$
$$= \frac{2(2c(A-C)d+B(c^2-d^2))\sqrt{c+d\,\mathrm{t}}}{f}$$
$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\Big(}{f}$$

Mathematica [A]
time = 1.39, size = 262, normalized size = 1.14

$$\frac{\frac{4C(c+d\tan(e+fx))^{7/2}}{d}+7i(A-iB-C)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c-id)\left(-3(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c-3id+d\tan(e+fx))\right)\right)-7i(A+iB-C)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c+id)\left(-3(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+\sqrt{c+d\tan(e+fx)}\,(4c+3id+d\tan(e+fx))\right)\right)}{14f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
),x]
```

```
[Out] ((4*C*(c + d*Tan[e + f*x])^(7/2))/d + (7*I)*(A - I*B - C)*((2*(c + d*Tan[e
+ f*x])^(5/2))/5 + (2*(c - I*d)*(-3*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[
e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c - (3*I)*d + d*Tan[
e + f*x])))/3) - (7*I)*(A + I*B - C)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (2
*(c + I*d)*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*
d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/3))/(14*f
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. 2(196) = 392.
time = 0.48, size = 1787, normalized size = 7.80

method	result	size
derivativedivides	Expression too large to display	1787
default	Expression too large to display	1787

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/d*(-d*(1/4/d*(1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2
+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2+A*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*c^3-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+2*B*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+B*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*c^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-C*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c^3+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2)*ln(d*tan(f
*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2
))+2*(4*A*(c^2+d^2)^(1/2)*c*d^2+2*B*(c^2+d^2)^(1/2)*c^2*d-2*B*(c^2+d^2)^(1/
2)*d^3-4*C*(c^2+d^2)^(1/2)*c*d^2+1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*c^2+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2+A*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*c^3-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+2*B*(c
^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d-3*B*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*c^2*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+C*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d
^2-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*
d^2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2
*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2
*c)^(1/2)))+1/4/d*(1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2
-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-A*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*c^3+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2-2*B*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d-B*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*c^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2+C*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c^3-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2)*ln(d*tan(f
*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2
))+2*(4*A*(c^2+d^2)^(1/2)*c*d^2+2*B*(c^2+d^2)^(1/2)*c^2*d-2*B*(c^2+d^2)^(1/
2)*d^3-4*C*(c^2+d^2)^(1/2)*c*d^2-1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*c^2-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^2-A*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*c^3+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2-2*B*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d+3*B*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*c^2*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3-C*(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*c^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^
2+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d
^2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*
```

```
(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2))))+1/7*C*(c+d*tan(f*x+e))^(7/2)+1/5*B*d*(c+d*tan(f*x+e))^(5/2)+1/3
*A*d^2*(c+d*tan(f*x+e))^(3/2)+1/3*B*c*d*(c+d*tan(f*x+e))^(3/2)-1/3*C*d^2*(c
+d*tan(f*x+e))^(3/2)+2*A*c*d^2*(c+d*tan(f*x+e))^(1/2)+B*c^2*d*(c+d*tan(f*x+
e))^(1/2)-B*d^3*(c+d*tan(f*x+e))^(1/2)-2*C*c*d^2*(c+d*tan(f*x+e))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(5/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + d\tan(e + fx))^{\frac{5}{2}} \left(A + B\tan(e + fx) + C\tan^2(e + fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 117.31, size = 2500, normalized size = 10.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

```
[Out] ((2*C*c^2)/(3*d*f) - (2*C*(d^3*f + c^2*d*f))/(3*d^2*f^2))*(c + d*tan(e + f*
x))^(3/2) - log(((((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*
c^5*f^2 - 10*B^2*c^3*d^2*f^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(((((-B^4*d^2*f^
4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*c^5*f^2 - 10*B^2*c^3*d^2*f^2 +
5*B^2*c*d^4*f^2)/f^4)^(1/2)*(32*B*c^4*d^2 - 32*B*d^6 + 32*c*d^2*f*(((-B^4*d
^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*c^5*f^2 - 10*B^2*c^3*d^2*f
^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) - (16*B
^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^
2))/2 - (8*B^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*(((20*B^4*c^2*d^8*f^
4 - B^4*d^10*f^4 - 110*B^4*c^4*d^6*f^4 + 100*B^4*c^6*d^4*f^4 - 25*B^4*c^8*d
^2*f^4)^(1/2) + B^2*c^5*f^2 - 10*B^2*c^3*d^2*f^2 + 5*B^2*c*d^4*f^2)/(4*f^4)
)^(1/2) + log(- ((((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*
c^5*f^2 - 10*B^2*c^3*d^2*f^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(((((-B^4*d^2*f^
4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*c^5*f^2 - 10*B^2*c^3*d^2*f^2 +
5*B^2*c*d^4*f^2)/f^4)^(1/2)*(32*B*d^6 - 32*B*c^4*d^2 + 32*c*d^2*f*(((-B^4*d
^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*c^5*f^2 - 10*B^2*c^3*d^2*f
^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) - (16*B
^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^
2))/2 - (8*B^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*((20*B^4*c^2*d^8*f^4
 - B^4*d^10*f^4 - 110*B^4*c^4*d^6*f^4 + 100*B^4*c^6*d^4*f^4 - 25*B^4*c^8*d^
2*f^4)^(1/2)/(4*f^4) + (B^2*c^5)/(4*f^2) - (5*B^2*c^3*d^2)/(2*f^2) + (5*B^2
*c*d^4)/(4*f^2))^(1/2) - log(((-((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2
)^(1/2) - B^2*c^5*f^2 + 10*B^2*c^3*d^2*f^2 - 5*B^2*c*d^4*f^2)/f^4)^(1/2)*((
(-((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - B^2*c^5*f^2 + 10*B^2
*c^3*d^2*f^2 - 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(32*B*c^4*d^2 - 32*B*d^6 + 32*c*
d^2*f*(-((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - B^2*c^5*f^2 +
10*B^2*c^3*d^2*f^2 - 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)
))/(2*f) - (16*B^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 -
 15*c^4*d^2))/f^2))/2 - (8*B^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*(-((
20*B^4*c^2*d^8*f^4 - B^4*d^10*f^4 - 110*B^4*c^4*d^6*f^4 + 100*B^4*c^6*d^4*f
```

^4 - 25*B^4*c^8*d^2*f^4)^(1/2) - B^2*c^5*f^2 + 10*B^2*c^3*d^2*f^2 - 5*B^2*c
*d^4*f^2)/(4*f^4))^(1/2) + log(- ((-((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^
2)^2)^(1/2) - B^2*c^5*f^2 + 10*B^2*c^3*d^2*f^2 - 5*B^2*c*d^4*f^2)/f^4)^(1/2
)*(((-((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - B^2*c^5*f^2 + 10
*B^2*c^3*d^2*f^2 - 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(32*B*d^6 - 32*B*c^4*d^2 + 3
2*c*d^2*f*(-((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - B^2*c^5*f^
2 + 10*B^2*c^3*d^2*f^2 - 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(
1/2)))/(2*f) - (16*B^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d
^4 - 15*c^4*d^2))/f^2))/2 - (8*B^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*
((B^2*c^5)/(4*f^2) - (20*B^4*c^2*d^8*f^4 - B^4*d^10*f^4 - 110*B^4*c^4*d^6*f
^4 + 100*B^4*c^6*d^4*f^4 - 25*B^4*c^8*d^2*f^4)^(1/2)/(4*f^4) - (5*B^2*c^3*d
^2)/(2*f^2) + (5*B^2*c*d^4)/(4*f^2))^(1/2) + ((4*B*c^2)/f - (2*B*(c^2*f + d
^2*f))/f^2)*(c + d*tan(e + f*x))^(1/2) - log(((((-((-A^4*d^2*f^4*(5*c^4 + d
^4 - 10*c^2*d^2)^2)^(1/2) + A^2*c^5*f^2 - 10*A^2*c^3*d^2*f^2 + 5*A^2*c*d^4*
f^2)/f^4)^(1/2)*(64*A*c^3*d^3 + 64*A*c*d^5 + 32*c*d^2*f*(-((-A^4*d^2*f^4*(5
*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + A^2*c^5*f^2 - 10*A^2*c^3*d^2*f^2 + 5*A^
2*c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) + (16*A^2*d^2*(c
 + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^2)*(-((-A
^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + A^2*c^5*f^2 - 10*A^2*c^3*d
^2*f^2 + 5*A^2*c*d^4*f^2)/f^4)^(1/2))/2 - (8*A^3*d^3*(3*c^2 - d^2)*(c^2 + d
^2)^3)/f^3)*(-((20*A^4*c^2*d^8*f^4 - A^4*d^10*f^4 - 110*A^4*c^4*d^6*f^4 + 1
00*A^4*c^6*d^4*f^4 - 25*A^4*c^8*d^2*f^4)^(1/2) + A^2*c^5*f^2 - 10*A^2*c^3*d
^2*f^2 + 5*A^2*c*d^4*f^2)/(4*f^4))^(1/2) - log((((((-A^4*d^2*f^4*(5*c^4 +
d^4 - 10*c^2*d^2)^2)^(1/2) - A^2*c^5*f^2 + 10*A^2*c^3*d^2*f^2 - 5*A^2*c*d^4
f^2)/f^4)^(1/2)(64*A*c^3*d^3 + 64*A*c*d^5 + 32*c*d^2*f*(((-A^4*d^2*f^4*(5
*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - A^2*c^5*f^2 + 10*A^2*c^3*d^2*f^2 - 5*A^
2*c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f) + (16*A^2*d^2*(c
 + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*d^2))/f^2)*(((-A^
4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - A^2*c^5*f^2 + 10*A^2*c^3*d^
2*f^2 - 5*A^2*c*d^4*f^2)/f^4)^(1/2))/2 - (8*A^3*d^3*(3*c^2 - d^2)*(c^2 + d^
2)^3)/f^3)*(((20*A^4*c^2*d^8*f^4 - A^4*d^10*f^4 - 110*A^4*c^4*d^6*f^4 + 100
*A^4*c^6*d^4*f^4 - 25*A^4*c^8*d^2*f^4)^(1/2) - A^2*c^5*f^2 + 10*A^2*c^3*d^2
*f^2 - 5*A^2*c*d^4*f^2)/(4*f^4))^(1/2) + log(((((((-A^4*d^2*f^4*(5*c^4 + d^
4 - 10*c^2*d^2)^2)^(1/2) - A^2*c^5*f^2 + 10*A^2*c^3*d^2*f^2 - 5*A^2*c*d^4*f
^2)/f^4)^(1/2)*(64*A*c^3*d^3 + 64*A*c*d^5 - 32*c*d^2*f*(((-A^4*d^2*f^4*(5*c
^4 + d^4 - 10*c^2*d^2)^2)^(1/2) - A^2*c^5*f^2 + 10*A^2*c^3*d^2*f^2 - 5*A^2*
c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2...

3.107 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=336

$$-\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)f}+\frac{(iA-B-iC)(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\,\mathrm{ta}}}{\sqrt{c}}\right.}{(a+ib)f}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a
-I*b)/f+(I*A-B-I*C)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))/(a+I*b)/f-2*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^(5/2)*arctanh(b^(1/2)*(c+d*
tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(a^2+b^2)/f+2*(b^2*d*(B*c+(A-C)
*d)+(-a*d+b*c)*(B*b*d-C*a*d+C*b*c))*(c+d*tan(f*x+e))^(1/2)/b^3/f+2/3*(B*b*d
-C*a*d+C*b*c)*(c+d*tan(f*x+e))^(3/2)/b^2/f+2/5*C*(c+d*tan(f*x+e))^(5/2)/b/f
```

Rubi [A]

time = 1.85, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3728, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2(bc-ad)^{5/2}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{7/2}f\left(a^2+b^2\right)}+\frac{2\sqrt{c+d\tan(e+fx)}\,((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc))}{b^3f}-\frac{(c-id)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)}+\frac{(c+id)^{5/2}(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)}+\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3b^2f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x]),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)*f)) + ((I*A - B - I*C)*(c + I*d)^(5/2)*ArcTanh[Sqrt[c +
 d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)*f) - (2*(A*b^2 - a*(b*B - a*C))
*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*
d]])/(b^(7/2)*(a^2 + b^2)*f) + (2*(b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(b
*c*C + b*B*d - a*C*d))*Sqrt[c + d*Tan[e + f*x]])/(b^3*f) + (2*(b*c*C + b*B*
d - a*C*d)*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*f) + (2*C*(c + d*Tan[e + f*x]
)^(5/2))/(5*b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
```

```
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{2C(c+d\tan(e+fx))^{5/2}}{5bf} + \frac{2\int \frac{(c+d}{}}{}$$

$$= \frac{2(bcC+bBd-aCd)(c+d\tan(e+}{3b^2f}$$

$$= \frac{2(b^2d(Bc+(A-C)d)+(bc-ad)(b}{}$$

$$= \frac{2(b^2d(Bc+(A-C)d)+(bc-ad)(b}{}$$

$$= \frac{2(b^2d(Bc+(A-C)d)+(bc-ad)(b}{}$$

$$= \frac{2(b^2d(Bc+(A-C)d)+(bc-ad)(b}{}$$

$$= -\frac{2\left(Ab^2-a(bB-aC)\right)(bc-ad)^{5/2}}{b^{7/2}\,(a}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\Big(}{(a-ib)f}$$

Mathematica [A]
time = 3.44, size = 322, normalized size = 0.96

$$\frac{\frac{15\left(b^{7/2}(-ia+b)(A-iB-C)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+b^{7/2}(ia+b)(A+iB-C)(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)-2\left(Ab^2+a(-bB+aC)\right)(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)\right)}{b^{5/2}(a^2+b^2)}+\frac{30\left(b^2d(Bc+(A-C)d)+(bc-ad)(bcC+bBd-aCd)\right)\sqrt{c+d\tan(e+fx)}}{b^2}+\frac{10(bcC+bBd-aCd)(c+d\tan(e+fx))^{3/2}}{b}+6C(c+d\tan(e+fx))^{5/2}}{15bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((15*(b^(7/2)*((-I)*a + b)*(A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c - I*d]] + b^(7/2)*(I*a + b)*(A + I*B - C)*(c + I*d)^(
```

```
5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] - 2*(A*b^2 + a*(-(b*B)
 + a*C))*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[
b*c - a*d]]))/(b^(5/2)*(a^2 + b^2)) + (30*(b^2*d*(B*c + (A - C)*d) + (b*c -
 a*d)*(b*c*C + b*B*d - a*C*d))*Sqrt[c + d*Tan[e + f*x]])/b^2 + (10*(b*c*C +
 b*B*d - a*C*d)*(c + d*Tan[e + f*x])^(3/2))/b + 6*C*(c + d*Tan[e + f*x])^(5
/2))/(15*b*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3355 vs. $2(294) = 588$.
time = 0.64, size = 3356, normalized size = 9.99

method	result	size
derivativedivides	Expression too large to display	3356
default	Expression too large to display	3356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/b^3*(1/5*C*(c+d*tan(f*x+e))^(5/2)*b^2+1/3*B*b^2*d*(c+d*tan(f*x+e))^(
3/2)-1/3*C*a*b*d*(c+d*tan(f*x+e))^(3/2)+1/3*C*b^2*c*(c+d*tan(f*x+e))^(3/2)+
A*b^2*d^2*(c+d*tan(f*x+e))^(1/2)-B*a*b*d^2*(c+d*tan(f*x+e))^(1/2)+2*B*b^2*c
*d*(c+d*tan(f*x+e))^(1/2)+C*a^2*d^2*(c+d*tan(f*x+e))^(1/2)-2*C*a*b*c*d*(c+d
*tan(f*x+e))^(1/2)+C*b^2*c^2*(c+d*tan(f*x+e))^(1/2)-C*b^2*d^2*(c+d*tan(f*x+
e))^(1/2))+2/(a^2+b^2)*(1/4/d*(1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*c^2+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*A*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d+A*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*c^3-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2+3*A*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b*c^2*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3+2*B*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*b*c^2+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2-
3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a
*d^3+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b*c*d^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-C*(c^2+d^2)
^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*b*c*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3+3*C*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*c*d^2-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d+C*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-4*A*(c^2+d^2)^(1/2)*a*c
*d^2+2*A*(c^2+d^2)^(1/2)*b*c^2*d-2*A*(c^2+d^2)^(1/2)*b*d^3-2*B*(c^2+d^2)^(1
/2)*a*c^2*d+2*B*(c^2+d^2)^(1/2)*a*d^3-4*B*(c^2+d^2)^(1/2)*b*c*d^2+4*C*(c^2+
d^2)^(1/2)*a*c*d^2-2*C*(c^2+d^2)^(1/2)*b*c^2*d+2*C*(c^2+d^2)^(1/2)*b*d^3-1/
2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+A*(c^2+d^2)^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b*c*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*A*(2*(c^2+d^2)^(1
```

```
/2)+2*c)^(1/2)*a*c*d^2+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-A*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b*d^3+2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*a*c*d-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2+B*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c^2*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3+B*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b*c^3-3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2+C*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*a*d^2+2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d-C*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*c^3+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2-3*C*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c
+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)
^(1/2)))+1/4/d*(1/2*(A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-
A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2+2*A*(c^2+d^2)^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3+3*
A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^2-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b
*c^2*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^3-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*a*c*d+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b
*c^2-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^2+3*B*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*c^2*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^3-B*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*b*c^3+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2-C*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*d^2-2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*b*c*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*a*c*d^2+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d-C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b*d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(-4*A*(c^2+d^2)^(1/2)*a*c*d^2+2*A*(c^2+d^
2)^(1/2)*b*c^2*d-2*A*(c^2+d^2)^(1/2)*b*d^3-2*B*(c^2+d^2)^(1/2)*a*c^2*d+2*B*
(c^2+d^2)^(1/2)*a*d^3-4*B*(c^2+d^2)^(1/2)*b*c*d^2+4*C*(c^2+d^2)^(1/2)*a*c*d
^2-2*C*(c^2+d^2)^(1/2)*b*c^2*d+2*C*(c^2+d^2)^(1/2)*b*d^3+1/2*(A*(c^2+d^2)^(
1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*d^2+2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d
-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*
c*d^2-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b*d^3-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d+B*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*b*d^2+3*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d-B*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*a*d^3-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3+3*B*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^2-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*a*c^2+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^2-2*C*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x)),x)
```

```
[Out] \text{Hanged}
```

3.108 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=473

$$-\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f}-\frac{(B-i(A-C))(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-\ldots}}\right)}{(a+ib)^2f}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a
-I*b)^2/f-(B-I*(A-C))*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^
(1/2))/(a+I*b)^2/f+(-a*d+b*c)^(3/2)*(3*a^3*b*B*d-5*a^4*C*d-b^4*(5*A*d+2*B*c
)-a*b^3*(4*A*c-7*B*d-4*C*c)+a^2*b^2*(2*B*c-(A+9*C)*d))*arctanh(b^(1/2)*(c+d
*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(a^2+b^2)^2/f-d*(5*a^3*C*d-A*b
^2*(-a*d+b*c)-2*b^3*(B*d+2*C*c)-a^2*b*(3*B*d+5*C*c)+a*b^2*(B*c+4*C*d))*(c+d
*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)/f+1/3*(3*A*b^2-3*B*a*b+5*C*a^2+2*C*b^2)*d*
(c+d*tan(f*x+e))^(3/2)/b^2/(a^2+b^2)/f-(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))
^(5/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 2.64, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3726, 3728, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3b^2f(a^2+b^2)}-\frac{d\sqrt{c+d\tan(e+fx)}\,(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC))}{b^3f(a^2+b^2)}+\frac{(bc-ad)^{3/2}(-5a^4Cd+3a^3bBd+a^2b^2(2Bc-d(A+9C))-ab^3(4Ac-7Bd-4cC)-b^4(5Ad+2Bc))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{7/2}f(a^2+b^2)^2}-\frac{(c-id)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2}-\frac{(c+id)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^2,x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
- I*d]])/((a - I*b)^2*f)) - ((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[Sqrt[c
 + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((a + I*b)^2*f) + ((b*c - a*d)^(3/2)*(3*
a^3*b*B*d - 5*a^4*C*d - b^4*(2*B*c + 5*A*d) - a*b^3*(4*A*c - 4*c*C - 7*B*d)
 + a^2*b^2*(2*B*c - (A + 9*C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]]
)/Sqrt[b*c - a*d]])/(b^(7/2)*(a^2 + b^2)^2*f) - (d*(5*a^3*C*d - A*b^2*(b*c
- a*d) - 2*b^3*(2*c*C + B*d) - a^2*b*(5*c*C + 3*B*d) + a*b^2*(B*c + 4*C*d))
*Sqrt[c + d*Tan[e + f*x]])/(b^3*(a^2 + b^2)*f) + ((3*A*b^2 - 3*a*b*B + 5*a^
2*C + 2*b^2*C)*d*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*(a^2 + b^2)*f) - ((A*b^
2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(b*(a^2 + b^2)*f*(a + b*Tan[
e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+}{b\left(a^2+b^2\right)f(a+b\tan(e+f}$$

$$= \frac{\left(3Ab^2-3abB+5a^2C+2b^2C\right)d(c+}{3b^2\left(a^2+b^2\right)f}$$

$$= -\frac{d(5a^3Cd-Ab^2(bc-ad)-2b^3(2c}{}$$

$$= -\frac{d(5a^3Cd-Ab^2(bc-ad)-2b^3(2c}{}$$

$$= -\frac{d(5a^3Cd-Ab^2(bc-ad)-2b^3(2c}{}$$

$$= -\frac{d(5a^3Cd-Ab^2(bc-ad)-2b^3(2c}{}$$

$$= \frac{(bc-ad)^{3/2}\left(3a^3bBd-5a^4Cd-b^4(\right.}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\right.}{(a-ib)^2f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6112 vs. 2(473) = 946.
time = 6.35, size = 6112, normalized size = 12.92

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5213 vs. 2(434) = 868.
time = 0.69, size = 5214, normalized size = 11.02

method	result	size
derivativedivides	Expression too large to display	5214
default	Expression too large to display	5214

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^2,x)
```

```
[Out] \text{Hanged}
```

3.109 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=643

$$-\frac{(A-iB-C)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}+\frac{(A+iB-C)(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e}{\sqrt{c+i}}\right.}{(ia-b)^3f}$$

```
[Out] -(A-I*B-C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a
+b)^3/f+(A+I*B-C)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))/(I*a-b)^3/f+1/4*(3*a^5*b*B*d^2-15*a^6*C*d^2+a^4*b^2*d*(4*B*c+(A-46*C)*d)
-3*a^2*b^4*(8*A*c^2-6*A*d^2-16*B*c*d-8*C*c^2+21*C*d^2)-a*b^5*(56*c*(A-C)*d+
B*(24*c^2-35*d^2))-b^6*(4*c*(5*B*d+2*C*c)-A*(8*c^2-15*d^2))+2*a^3*b^3*(4*c*
(A-C)*d+B*(4*c^2+3*d^2)))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)
^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)/(a^2+b^2)^3/f-1/4*d*(3*a^3*b*B*d-15*a^4*C*
d-a*b^3*(8*A*c-11*B*d-8*C*c)+a^2*b^2*(4*B*c+(A-31*C)*d)-b^4*(7*A*d+4*B*c+8*
C*d))*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)^2/f+1/4*(a^3*b*B*d-5*a^4*C*d-b^4
*(5*A*d+4*B*c)-a*b^3*(8*A*c-9*B*d-8*C*c)+a^2*b^2*(3*A*d+4*B*c-13*C*d))*(c+d
*tan(f*x+e))^(3/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))-1/2*(A*b^2-a*(B*b-C*a
))*(c+d*tan(f*x+e))^(5/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2
```

Rubi [A]

time = 4.32, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3726, 3728, 3734, 3620, 3618, 65, 214, 3715}

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^3,x]
```

```
[Out] -(((A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c -
I*d]])/((I*a + b)^3*f)) + ((A + I*B - C)*(c + I*d)^(5/2)*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((I*a - b)^3*f) + (Sqrt[b*c - a*d]*(3*a^5*b*
B*d^2 - 15*a^6*C*d^2 + a^4*b^2*d*(4*B*c + (A - 46*C)*d) - 3*a^2*b^4*(8*A*c^
2 - 8*c^2*C - 16*B*c*d - 6*A*d^2 + 21*C*d^2) - a*b^5*(56*c*(A - C)*d + B*(2
4*c^2 - 35*d^2)) - b^6*(4*c*(2*c*C + 5*B*d) - A*(8*c^2 - 15*d^2)) + 2*a^3*b
^3*(4*c*(A - C)*d + B*(4*c^2 + 3*d^2)))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e +
 f*x]])/Sqrt[b*c - a*d]])/(4*b^(7/2)*(a^2 + b^2)^3*f) - (d*(3*a^3*b*B*d - 1
5*a^4*C*d - a*b^3*(8*A*c - 8*c*C - 11*B*d) + a^2*b^2*(4*B*c + (A - 31*C)*d)
 - b^4*(4*B*c + 7*A*d + 8*C*d))*Sqrt[c + d*Tan[e + f*x]])/(4*b^3*(a^2 + b^2
)^2*f) + ((a^3*b*B*d - 5*a^4*C*d - b^4*(4*B*c + 5*A*d) - a*b^3*(8*A*c - 8*c
*C - 9*B*d) + a^2*b^2*(4*B*c + 3*A*d - 13*C*d))*(c + d*Tan[e + f*x])^(3/2))
```

```
/(4*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])) - ((A*b^2 - a*(b*B - a*C))*(c
 + d*Tan[e + f*x])^(5/2))/(2*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
```

```
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = -\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+}{2b\,(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= \frac{(a^3bBd-5a^4Cd-b^4(4Bc+5Ad)-}{}$$

$$= -\frac{d(3a^3bBd-15a^4Cd-ab^3(8Ac-}{}$$

$$= -\frac{d(3a^3bBd-15a^4Cd-ab^3(8Ac-}{}$$

$$= -\frac{d(3a^3bBd-15a^4Cd-ab^3(8Ac-}{}$$

$$= -\frac{d(3a^3bBd-15a^4Cd-ab^3(8Ac-}{}$$

$$= \frac{\sqrt{bc-ad}\,(3a^5bBd^2-15a^6Cd^2+a^4}{}$$

$$= -\frac{(A-iB-C)(c-id)^{5/2}\tanh^{-1}\Big(}{(ia+b)^3f}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 18214 vs. 2(643) = 1286.
time = 6.60, size = 18214, normalized size = 28.33

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7282 vs. 2(599) = 1198.
time = 0.78, size = 7283, normalized size = 11.33

method	result	size
derivativedivides	Expression too large to display	7283
default	Expression too large to display	7283

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^3,x)
```

```
[Out] \text{Hanged}
```

3.110 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=407

$$\frac{(ia+b)^3(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}-\frac{(ia-b)^3(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+f}}{\sqrt{c+id}}\right.}{\sqrt{c+id}\,f}$$

```
[Out] (I*a+b)^3*(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)
^(1/2)-(I*a-b)^3*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/
(c+I*d)^(1/2)+2/105*(72*a^3*C*d^3-6*a^2*b*d^2*(-49*B*d+32*C*c)+21*a*b^2*d*(
8*c^2*C-10*B*c*d+15*(A-C)*d^2)-b^3*(48*c^3*C-56*B*c^2*d+70*c*(A-C)*d^2+105*
B*d^3))*(c+d*tan(f*x+e))^(1/2)/d^4/f+2/105*b*(35*b*(A*b+B*a-C*b)*d^2+4*(-a*
d+b*c)*(-7*B*b*d-6*C*a*d+6*C*b*c))*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^3/f-
2/35*(-7*B*b*d-6*C*a*d+6*C*b*c)*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2/d
^2/f+2/7*C*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3/d/f
```

Rubi [A]

time = 1.11, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3728, 3718, 3711, 3620, 3618, 65, 214}

$$\frac{2\sqrt{c+d\tan(e+fx)}\left(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+21ab^2d(15d^2(A-C)-10Bcd+8c^2C)-(b^3(70cd^2(A-C)-56Bc^2d+105Bd^3+48c^3C))\right)}{105d^4f}+\frac{2b\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(35bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-7bBd+6bcC)\right)}{105d^3f}-\frac{(-b+ia)^3(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}+\frac{(b+ia)^3(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}-\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{35d^2f}+\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[c
 + d*Tan[e + f*x]],x]
```

```
[Out] ((I*a + b)^3*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/(Sqrt[c - I*d]*f) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e +
f*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*(72*a^3*C*d^3 - 6*a^2*b*d^2*(3
2*c*C - 49*B*d) + 21*a*b^2*d*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2) - b^3*(4
8*c^3*C - 56*B*c^2*d + 70*c*(A - C)*d^2 + 105*B*d^3))*Sqrt[c + d*Tan[e + f*
x]])/(105*d^4*f) + (2*b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*
C - 7*b*B*d - 6*a*C*d))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(105*d^3*f)
- (2*(6*b*c*C - 7*b*B*d - 6*a*C*d)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e
+ f*x]])/(35*d^2*f) + (2*C*(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]])
/(7*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
```

```
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+}{7df}$$

$$= -\frac{2(6bcC-7bBd-6aCd)(a+b\tan(e}{35d^2f}$$

$$= \frac{2b(35b(Ab+aB-bC)d^2+4(bc-ad)(}{}$$

$$= \frac{2(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+2}{}$$

$$= \frac{2(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+2}{}$$

$$= \frac{2(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+2}{}$$

$$= \frac{2(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+2}{}$$

$$= -\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.}{\sqrt{c-id}\,f}$$

Mathematica [A]
time = 5.77, size = 392, normalized size = 0.96

$$\frac{-\frac{105(a-ib)^3(iA+B-iC)d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{105i(a+ib)^3(A+iB-C)d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}-\frac{2\left(-72a^3Cd^3+6a^2bd^2(32cC-49Bd)-21ab^2d\left(8c^2C-10Bcd+15(A-C)d^2\right)+b^3\left(48c^3C-56Bc^2d+70c(A-C)d^2+105Bd^3\right)\right)\sqrt{c+d\tan(e+fx)}}{d^3}+\frac{2b\left(35b(Ab+aB-bC)d^2+4(bc-ad)(6bcC-7bBd-6aCd)\right)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{d^2}+\frac{6(-6bcC+7bBd+6aCd)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{d}+30C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{105df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-105*(a - I*b)^3*(I*A + B - I*C)*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[
c - I*d]])/Sqrt[c - I*d] + ((105*I)*(a + I*b)^3*(A + I*B - C)*d*ArcTanh[Sqr
t[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d] - (2*(-72*a^3*C*d^3 + 6
*a^2*b*d^2*(32*c*C - 49*B*d) - 21*a*b^2*d*(8*c^2*C - 10*B*c*d + 15*(A - C)*
d^2) + b^3*(48*c^3*C - 56*B*c^2*d + 70*c*(A - C)*d^2 + 105*B*d^3))*Sqrt[c +
 d*Tan[e + f*x]])/d^3 + (2*b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6
*b*c*C - 7*b*B*d - 6*a*C*d))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/d^2 + (
6*(-6*b*c*C + 7*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e +
f*x]])/d + 30*C*(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]])/(105*d*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8227 vs. $2(371) = 742$.
time = 0.46, size = 8228, normalized size = 20.22

method	result	size
derivativedivides	Expression too large to display	8228
default	Expression too large to display	8228

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/s
qrt(c + d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 122.08, size = 2500, normalized size = 6.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^(1/2),x)
```

```
[Out] atan(((((8*(4*A*a^3*d^3*f^2 - 12*A*a*b^2*d^3*f^2 + 4*A*b^3*c*d^2*f^2 - 12*A
*a^2*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^6*
c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2
*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c
^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8
 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a^6*c*f
^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*A^2*a^2
*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*f^4 + d^
```

2*f^4)))^(1/2))*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*A^2*a^2*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(A^2*a^6*d^2 - A^2*b^6*d^2 + 15*A^2*a^2*b^4*d^2 - 15*A^2*a^4*b^2*d^2))/f^2)*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*A^2*a^2*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*1i - (((8*(4*A*a^3*d^3*f^2 - 12*A*a*b^2*d^3*f^2 + 4*A*b^3*c*d^2*f^2 - 12*A*a^2*b*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*A^2*a^2*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2))*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*A^2*a^2*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(A^2*a^6*d^2 - A^2*b^6*d^2 + 15*A^2*a^2*b^4*d^2 - 15*A^2*a^4*b^2*d^2))/f^2)*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*A^2*a^2*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*1i)/((16*(6*A^3*a^4*b^5*d^2 - A^3*b^9*d^2 + 8*A^3*a^6*b^3*d^2 + 3*A^3*a^8*b*d^2))/f^3 + (((8*(4*A*a^3*d^3*f^2 - 12*A*a*b^2*d^3*f^2 + 4*A*b^3*c*d^2*f^2 - 12*A*a^2*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*A^2*a^2*b^

```
4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*f^4 + d^2*f
^4)))^(1/2))*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 4
8*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2
*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^
4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*
b^2))^(1/2) - 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A
^2*a^5*b*d*f^2 - 60*A^2*a^2*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b
^3*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*
(A^2*a^6*d^2 - A^2*b^6*d^2 + 15*A^2*a^2*b^4*d^2 - 15*A^2*a^4*b^2*d^2))/f^2)
*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*
d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f
^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 +
 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2)
- 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*...
```

3.111 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=287

$$-\frac{(a-ib)^2(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}+\frac{(a+ib)^2(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}\,f}$$

```
[Out] -(a-I*b)^2*(B+I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I
*d)^(1/2)+(a+I*b)^2*(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))/f/(c+I*d)^(1/2)+2/15*(12*a^2*C*d^2-10*a*b*d*(-3*B*d+2*C*c)+b^2*(8*c^2*C-
10*B*c*d+15*(A-C)*d^2))*(c+d*tan(f*x+e))^(1/2)/d^3/f-2/15*b*(-5*B*b*d-4*C*a
*d+4*C*b*c)*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^2/f+2/5*C*(c+d*tan(f*x+e))^
(1/2)*(a+b*tan(f*x+e))^2/d/f
```

Rubi [A]
time = 0.65, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3728, 3718, 3711, 3620, 3618, 65, 214}

$$\frac{2\sqrt{c+d\tan(e+fx)}\,(12a^2Cd^2-10abd(2cC-3Bd)+b^2(15d^2(A-C)-10Bcd+8c^2C))}{15d^3f}-\frac{(a-ib)^2(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a+ib)^2(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}-\frac{2b\tan(e+fx)(-4aCd-5bBd+4bcC)\sqrt{c+d\tan(e+fx)}}{15d^2f}+\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[c
 + d*Tan[e + f*x]],x]
```

```
[Out] -(((a - I*b)^2*(B + I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/(Sqrt[c - I*d]*f)) + ((a + I*b)^2*(I*A - B - I*C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*(12*a^2*C*d^2 - 10*a*b*
d*(2*c*C - 3*B*d) + b^2*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2))*Sqrt[c + d*T
an[e + f*x]])/(15*d^3*f) - (2*b*(4*b*c*C - 5*b*B*d - 4*a*C*d)*Tan[e + f*x]*
Sqrt[c + d*Tan[e + f*x]])/(15*d^2*f) + (2*C*(a + b*Tan[e + f*x])^2*Sqrt[c +
 d*Tan[e + f*x]])/(5*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e\ldots}}{5df}$$

$$= -\frac{2b(4bcC-5bBd-4aCd)\tan(e+f\ldots}{15d^2f}$$

$$= \frac{2(12a^2Cd^2-10abd(2cC-3Bd)+b^2(\ldots}{}$$

$$= \frac{2(12a^2Cd^2-10abd(2cC-3Bd)+b^2(\ldots}{}$$

$$= \frac{2(12a^2Cd^2-10abd(2cC-3Bd)+b^2(\ldots}{}$$

$$= \frac{2(12a^2Cd^2-10abd(2cC-3Bd)+b^2(\ldots}{}$$

$$= -\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{\ldots}}{\ldots}\right.}{\sqrt{c-id}\,f}$$

Mathematica [A]
time = 3.92, size = 275, normalized size = 0.96

$$\frac{-\frac{15(a-ib)^2(iA+B-iC)d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{15i(a+ib)^2(A+iB-C)d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}+\frac{2(12a^2Cd^2+10abd(-2cC+3Bd)+b^2(8c^2C-10Bcd+15(A-C)d^2))\sqrt{c+d\tan(e+fx)}}{d^2}+\frac{2b(-4bcC+5bBd+4aCd)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{d}+6C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{15df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((-15*(a - I*b)^2*(I*A + B - I*C)*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 - I*d]])/Sqrt[c - I*d] + ((15*I)*(a + I*b)^2*(A + I*B - C)*d*ArcTanh[Sqrt[
c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d] + (2*(12*a^2*C*d^2 + 10*a
```

```
*b*d*(-2*c*C + 3*B*d) + b^2*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2))*Sqrt[c +
 d*Tan[e + f*x]])/d^2 + (2*b*(-4*b*c*C + 5*b*B*d + 4*a*C*d)*Tan[e + f*x]*Sq
rt[c + d*Tan[e + f*x]])/d + 6*C*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f
*x]])/(15*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5859 vs. 2(254) = 508.
time = 0.46, size = 5860, normalized size = 20.42

method	result	size
derivativedivides	Expression too large to display	5860
default	Expression too large to display	5860

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/s
qrt(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 47.98, size = 2500, normalized size = 8.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^(1/2),x)
```

```
[Out] atan(((((16*(2*C*b^2*d^3*f^2 - 2*C*a^2*d^3*f^2 + 4*C*a*b*c*d^2*f^2))/f^3 -
64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 -
 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*
c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 +
4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*
f^2 - 16*C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^
(1/2))*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 32*C^2*
a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^
8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2
*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24
*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (16*(c + d*tan(e + f*
x))^(1/2)*(C^2*a^4*d^2 + C^2*b^4*d^2 - 6*C^2*a^2*b^2*d^2))/f^2)*((((8*C^2*a
^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C
^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^
4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2
```

*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2
)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*1i - (((16*(2*C*b^2*d^3*f^2 - 2*C*a^2*d^3
*f^2 + 4*C*a*b*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*
C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 -
48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 + C^4*b^8 +
4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^4*c*f^2 -
4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*
c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2))*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f
^2 - 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 -
(16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^
4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^
3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4
)))^(1/2) + (16*(c + d*tan(e + f*x))^(1/2)*(C^2*a^4*d^2 + C^2*b^4*d^2 - 6*C
^2*a^2*b^2*d^2))/f^2)*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*
d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d
^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2)
)^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a
^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*1i)/((((
16*(2*C*b^2*d^3*f^2 - 2*C*a^2*d^3*f^2 + 4*C*a*b*c*d^2*f^2))/f^3 - 64*c*d^2*
(c + d*tan(e + f*x))^(1/2)*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a
*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 +
16*d^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6
*b^2))^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*
C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2))*((
((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b*d*f
^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 + C^4*b
^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^4*c*f^
2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24*C^2*a^2*
b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)
*(C^2*a^4*d^2 + C^2*b^4*d^2 - 6*C^2*a^2*b^2*d^2))/f^2)*((((8*C^2*a^4*c*f^2
+ 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^
2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b^6
+ 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^
2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(c^
2*f^4 + d^2*f^4)))^(1/2) + (((16*(2*C*b^2*d^3*f^2 - 2*C*a^2*d^3*f^2 + 4*C*a
*b*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*C^2*a^4*c*f^
2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*
b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b
^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*
f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(
c^2*f^4 + d^2*f^4)))^(1/2))*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*
a*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4
+ 16*d^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^
6*b^2))^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16
*C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) +

```
(16*(c + d*tan(e + f*x))^(1/2)*(C^2*a^4*d^2 + C^2*b^4*d^2 - 6*C^2*a^2*b^2*d
^2))/f^2)*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 32*C
^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(C^4
*a^8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2) - 4*
C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*d*f^2 +
 24*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (32*(2*C^3*a^3*b^3
*d^2 + C^3*a*b^5*d^2 + C^3*a^5*b*d^2))/f^3))*((...
```

3.112 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=194

$$-\frac{(ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}+\frac{(ia-b)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+f}}{\sqrt{c+id}}\right.}{\sqrt{c+id}\,f}$$

```
[Out] -(I*a+b)*(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^
(1/2)+(I*a-b)*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/(c+
I*d)^(1/2)-2/3*(-3*B*b*d-3*C*a*d+2*C*b*c)*(c+d*tan(f*x+e))^(1/2)/d^2/f+2/3*
b*C*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d/f
```

Rubi [A]
time = 0.33, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3718, 3711, 3620, 3618, 65, 214}

$$-\frac{(b+ia)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(-b+ia)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}-\frac{2(-3aCd-3bBd+2bcC)\sqrt{c+d\tan(e+fx)}}{3d^2f}+\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[c +
 d*Tan[e + f*x]],x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/(Sqrt[c - I*d]*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) - (2*(2*b*c*C - 3*b*B*d - 3*a*C*d)*S
qrt[c + d*Tan[e + f*x]])/(3*d^2*f) + (2*b*C*Tan[e + f*x]*Sqrt[c + d*Tan[e +
 f*x]])/(3*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df} -$$

$$= -\frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e}}{3d^2f}$$

$$= -\frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e}}{3d^2f}$$

$$= -\frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e}}{3d^2f}$$

$$= -\frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e}}{3d^2f}$$

$$= -\frac{(ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+}}{}\right.}{\sqrt{c-id}\,f}$$

Mathematica [A]
time = 0.98, size = 192, normalized size = 0.99

$$\frac{2\left(-\frac{3i(a-ib)(A-iB-C)d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2\sqrt{c-id}}+\frac{3i(a+ib)(A+iB-C)d\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2\sqrt{c+id}}+\frac{(-2bcC+3bBd+3aCd)\sqrt{c+d\tan(e+fx)}}{d}+bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[c + d*Tan[e + f*x]],x]
```

```
[Out] (2*((((-3*I)/2)*(a - I*b)*(A - I*B - C)*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/
Sqrt[c - I*d]])/Sqrt[c - I*d] + (((3*I)/2)*(a + I*b)*(A + I*B - C)*d*ArcTan
h[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d] + ((-2*b*c*C + 3*b
*B*d + 3*a*C*d)*Sqrt[c + d*Tan[e + f*x]])/d + b*C*Tan[e + f*x]*Sqrt[c + d*T
an[e + f*x]]))/(3*d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. $2(166) = 332$.
time = 0.44, size = 1349, normalized size = 6.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)
,x,method=_RETURNVERBOSE)
```

```
[Out] 2/f/d^2*(1/3*C*(c+d*tan(f*x+e))^(3/2)*b+B*b*d*(c+d*tan(f*x+e))^(1/2)+C*a*d*
(c+d*tan(f*x+e))^(1/2)-C*b*c*(c+d*tan(f*x+e))^(1/2)+d^2*(1/4/(c^2+d^2)^(1/2
)/d*(1/2*(-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+A*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*a*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+B*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d-B*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*b*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/
2)*a-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*
d)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(
c^2+d^2)^(1/2))+2*(2*A*a*d^2-2*A*b*c*d-2*B*a*c*d-2*B*b*d^2-2*C*a*d^2+2*C*b*
c*d+1/2*(-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d+B*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d-B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*b*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2
)*a-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c
+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)
^(1/2)))+1/4/(c^2+d^2)^(1/2)/d*(1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d
^2)^(1/2)*a-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*b*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-B*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*a*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-C*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+C*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*b*d)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*A*a*d^2-2*A*b*c*d-2*B*a*c*d-
2*B*b*d^2-2*C*a*d^2+2*C*b*c*d-1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2
)^(1/2)*a-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-B*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*a*d+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c-C*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*(c^2+d^2)^(1/2)*a+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+C*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b*d)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2
)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)/sqrt
(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/sqrt
(c + d*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 23.48, size = 2500, normalized size = 12.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*t
an(e + f*x))^(1/2),x)
```

```
[Out] ((2*B*b*d - 6*C*b*c)/(d^2*f) + (4*C*b*c)/(d^2*f))*(c + d*tan(e + f*x))^(1/2
) - atan(((((8*(4*C*a*d^3*f^2 - 4*A*a*d^3*f^2 + 4*B*a*c*d^2*f^2))/f^3 - 64*
c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*
C^2*a^2*c*f^2 + 16*A*B*a^2*d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^2/4
 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4
*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4)
)^(1/2) - 4*A^2*a^2*c*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d
*f^2 + 8*A*C*a^2*c*f^2 + 8*B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2))*
((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c*f^2 + 16*A*B*a^2*d*f^2
- 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4
*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^
2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c*f^2 + 4*B^2
*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d*f^2 + 8*A*C*a^2*c*f^2 + 8*B*C*a^
2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(
A^2*a^2*d^2 - B^2*a^2*d^2 + C^2*a^2*d^2 - 2*A*C*a^2*d^2))/f^2)*((((8*A^2*a^
2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c*f^2 + 16*A*B*a^2*d*f^2 - 16*A*C*a^2
*c*f^2 - 16*B*C*a^2*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^4 + B^4*a
^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 +
2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c*f^2 + 4*B^2*a^2*c*f^2 -
 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d*f^2 + 8*A*C*a^2*c*f^2 + 8*B*C*a^2*d*f^2)/(16
*(c^2*f^4 + d^2*f^4)))^(1/2)*1i - (((8*(4*C*a*d^3*f^2 - 4*A*a*d^3*f^2 + 4*B
*a*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^2*c*f^
2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c*f^2 + 16*A*B*a^2*d*f^2 - 16*A*C*a^2*c*f^2
 - 16*B*C*a^2*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C
^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*
C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2
*a^2*c*f^2 - 8*A*B*a^2*d*f^2 + 8*A*C*a^2*c*f^2 + 8*B*C*a^2*d*f^2)/(16*(c^2*
f^4 + d^2*f^4)))^(1/2))*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c
*f^2 + 16*A*B*a^2*d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^2/4 - (16*c^
2*f^4 + 16*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^
4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) -
 4*A^2*a^2*c*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d*f^2 + 8*
A*C*a^2*c*f^2 + 8*B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) + (16*(c +
 d*tan(e + f*x))^(1/2)*(A^2*a^2*d^2 - B^2*a^2*d^2 + C^2*a^2*d^2 - 2*A*C*a^2
*d^2))/f^2)*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c*f^2 + 16*A*
B*a^2*d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^2/4 - (16*c^2*f^4 + 16*d
^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^
2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c
*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d*f^2 + 8*A*C*a^2*c*f^
2 + 8*B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*1i)/((((8*(4*C*a*d^3*f
^2 - 4*A*a*d^3*f^2 + 4*B*a*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^
(1/2)*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c*f^2 + 16*A*B*a^2*
d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4
)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4
+ 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c*f^2 +
```

```
 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d*f^2 + 8*A*C*a^2*c*f^2 + 8*
B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2))*((((8*A^2*a^2*c*f^2 - 8*B^2
*a^2*c*f^2 + 8*C^2*a^2*c*f^2 + 16*A*B*a^2*d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C
*a^2*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 -
4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 -
 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^
2 - 8*A*B*a^2*d*f^2 + 8*A*C*a^2*c*f^2 + 8*B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2
*f^4)))^(1/2) - (16*(c + d*tan(e + f*x))^(1/2)*(A^2*a^2*d^2 - B^2*a^2*d^2 +
 C^2*a^2*d^2 - 2*A*C*a^2*d^2))/f^2)*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 +
 8*C^2*a^2*c*f^2 + 16*A*B*a^2*d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^
2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4
- 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a
^4))^(1/2) - 4*A^2*a^2*c*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^
2*d*f^2 + 8*A*C*a^2*c*f^2 + 8*B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2
) + (((8*(4*C*a*d^3*f^2 - 4*A*a*d^3*f^2 + 4*B*a*c*d^2*f^2))/f^3 + 64*c*d^2*
(c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^
2*c*f^2 + 16*A*B*a^2*d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^2/4 - (16
*c^2*f^4 + 16*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C
*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2
) - 4*A^2*a^2*c*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d*f^2 +
 8*A*C*a^2*c*f^2 + 8*B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2))*((((8*
A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c*f...
```

3.113 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=133

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}\,f}+\frac{2C\sqrt{}}{}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)-
(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/(c+I*d)^(1/2)+2
*C*(c+d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.15, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3711, 3620, 3618, 65, 214}

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(Sqrt[c
 - I*d]*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*
d]])/(Sqrt[c + I*d]*f) + (2*C*Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
```

```
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{2C\sqrt{c+d\tan(e+fx)}}{df} + \int \frac{A-C+B\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{df} + \frac{1}{2}(A-iB-C)\int \frac{1+i\tan(e+}{\sqrt{c+d\tan(e+}}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{df} + \frac{(iA+B-iC)\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c-}}\right.}{2f}$$

$$= \frac{2C\sqrt{c+d\tan(e+fx)}}{df} - \frac{(A-iB-C)\text{Subst}\left(\int \frac{1}{-1-\frac{ic}{d}+\frac{ix^2}{d}}\,d\right.}{df}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}\,f} - \frac{(B-i(A}{}$$

Mathematica [A]
time = 0.15, size = 129, normalized size = 0.97

$$\frac{-\frac{i(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{i(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}+\frac{2C\sqrt{c+d\tan(e+fx)}}{d}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/Sqrt[c + d*Tan[e + f*x]],
x]
```

```
[Out] (((-I)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[
c - I*d] + (I*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]
)/Sqrt[c + I*d] + (2*C*Sqrt[c + d*Tan[e + f*x]])/d)/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1893 vs. $2(112) = 224$.
time = 0.43, size = 1894, normalized size = 14.24

method	result	size
derivativedivides	Expression too large to display	1894
default	Expression too large to display	1894

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/d*(C*(c+d*tan(f*x+e))^(1/2)+d*(1/4/(c^2+d^2)^(3/2)/d^2*(1/2*(A*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*d^3-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d-A*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*c*d^3+B*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-B*(
c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-B*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^2-B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*d^4-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*c^2*d-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*c^3*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3)*ln(d*tan(f*x+
e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+
2*(2*A*c^2*d^3+2*A*d^5-2*B*c^3*d^2-2*B*c*d^4-2*C*c^2*d^3-2*C*d^5-1/2*(A*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+A*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*d^3-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d-A*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*c*d^3+B*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
c-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-B*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^2-B*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*c^2*d-C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+C*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c^3*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x
+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1
/4/(c^2+d^2)^(3/2)/d^2*(1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*c^2*d-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+A*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*c^3*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3-B*(c^2+d^2)^(
```

```
3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*c^3+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4+C*(c^2
+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+C*(c^2+d^2)^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*d^3-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d-C*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c*d^3)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*A*c^2*d^3+2*A*d^5-2*B*c^3*d^2-
2*B*c*d^4-2*C*c^2*d^3-2*C*d^5+1/2*(-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*c^2*d-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3+A*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*c^3*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3-B*(c^2+
d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*c^3+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^2+B
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4+
C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d+C*(c^2+d^2)^(1/2)*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*d^3-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d-C*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^3)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2
)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)
^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/sqrt(d*tan(f*x + e) + c),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan\left(e + fx\right) + C\tan^{2}\left(e + fx\right)}{\sqrt{c + d\tan\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/sqrt(c + d*tan(e + f*x)),
 x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 14.21, size = 2500, normalized size = 18.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^(1/2),x)
```

```
[Out] 2*atanh((32*C^2*d^2*((-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (C^
2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*C^
3*c*d^3*f^3)/(c^2*f^4 + d^2*f^4) - (4*C*d^3*f^2*(-16*C^4*d^2*f^4)^(1/2))/(c
^2*f^5 + d^2*f^5)) + (8*c*d^2*((-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f
^4)) - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2
)*(-16*C^4*d^2*f^4)^(1/2))/((16*C^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) - (4*C*d
^5*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*C^3*c^3*d^3*f^5)/
(c^2*f^4 + d^2*f^4) - (4*C*c^2*d^3*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 +
d^2*f^5)) - (32*C^2*c^2*d^2*f^2*((-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2
*f^4)) - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1
/2))/((16*C^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) - (4*C*d^5*f^4*(-16*C^4*d^2*f^
4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*C^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f^4) -
(4*C*c^2*d^3*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)))*((-16*C^4*d
^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)
))^(1/2) - 2*atanh((8*c*d^2*(- (-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f
^4)) - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2
)*(-16*C^4*d^2*f^4)^(1/2))/((16*C^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) + (4*C*d
```

^5*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*C^3*c^3*d^3*f^5)/
(c^2*f^4 + d^2*f^4) + (4*C*c^2*d^3*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 +
d^2*f^5)) - (32*C^2*d^2*(- (-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4))
 - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/(
(16*C^3*c*d^3*f^3)/(c^2*f^4 + d^2*f^4) + (4*C*d^3*f^2*(-16*C^4*d^2*f^4)^(1/
2))/(c^2*f^5 + d^2*f^5)) + (32*C^2*c^2*d^2*f^2*(- (-16*C^4*d^2*f^4)^(1/2)/(
16*(c^2*f^4 + d^2*f^4)) - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d
*tan(e + f*x))^(1/2))/((16*C^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) + (4*C*d^5*f^
4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*C^3*c^3*d^3*f^5)/(c^2*
f^4 + d^2*f^4) + (4*C*c^2*d^3*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f
^5)))*(- (-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (C^2*c*f^2)/(4*
(c^2*f^4 + d^2*f^4)))^(1/2) - 2*atanh((32*A^2*d^2*((-16*A^4*d^2*f^4)^(1/2)/
(16*(c^2*f^4 + d^2*f^4)) - (A^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c +
d*tan(e + f*x))^(1/2))/((16*A^3*c*d^3*f^3)/(c^2*f^4 + d^2*f^4) - (4*A*d^3*f
^2*(-16*A^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) + (8*c*d^2*((-16*A^4*d^2*f
^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (A^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(
1/2)*(c + d*tan(e + f*x))^(1/2)*(-16*A^4*d^2*f^4)^(1/2))/((16*A^3*c*d^5*f^5
)/(c^2*f^4 + d^2*f^4) - (4*A*d^5*f^4*(-16*A^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^
2*f^5) + (16*A^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f^4) - (4*A*c^2*d^3*f^4*(-16*A
^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) - (32*A^2*c^2*d^2*f^2*((-16*A^4*d^2
f^4)^(1/2)/(16(c^2*f^4 + d^2*f^4)) - (A^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))
^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*A^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4)
- (4*A*d^5*f^4*(-16*A^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*A^3*c^3*d
^3*f^5)/(c^2*f^4 + d^2*f^4) - (4*A*c^2*d^3*f^4*(-16*A^4*d^2*f^4)^(1/2))/(c^
2*f^5 + d^2*f^5)))*((-16*A^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (A^2
*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2) + 2*atanh((8*c*d^2*(- (-16*A^4*d^2*f
^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (A^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(
1/2)*(c + d*tan(e + f*x))^(1/2)*(-16*A^4*d^2*f^4)^(1/2))/((16*A^3*c*d^5*f^5
)/(c^2*f^4 + d^2*f^4) + (4*A*d^5*f^4*(-16*A^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^
2*f^5) + (16*A^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f^4) + (4*A*c^2*d^3*f^4*(-16*A
^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) - (32*A^2*d^2*(- (-16*A^4*d^2*f^4)^
(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (A^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)
*(c + d*tan(e + f*x))^(1/2))/((16*A^3*c*d^3*f^3)/(c^2*f^4 + d^2*f^4) + (4*A
*d^3*f^2*(-16*A^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) + (32*A^2*c^2*d^2*f^
2*(- (-16*A^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (A^2*c*f^2)/(4*(c^2
*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*A^3*c*d^5*f^5)/(c^
2*f^4 + d^2*f^4) + (4*A*d^5*f^4*(-16*A^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5
) + (16*A^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f^4) + (4*A*c^2*d^3*f^4*(-16*A^4*d^
2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)))*(- (-16*A^4*d^2*f^4)^(1/2)/(16*(c^2*f^4
 + d^2*f^4)) - (A^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2) - 2*atanh((32*B^2
d^2((B^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)) - (-16*B^4*d^2*f^4)^(1/2)/(16*(c^
2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/((16*B^3*d^2)/f - (16*
B^3*c^2*d^2*f^3)/(c^2*f^4 + d^2*f^4) + (4*B*c*d^2*f^2*(-16*B^4*d^2*f^4)^(1/
2))/(c^2*f^5 + d^2*f^5)) + (8*c*d^2*((B^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)) -
(-16*B^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x)

```
)^(1/2)*(-16*B^4*d^2*f^4)^(1/2))/(16*B^3*d^4*f + 16*B^3*c^2*d^2*f - (16*B^3
*c^2*d^4*f^5)/(c^2*f^4 + d^2*f^4) - (16*B^3*c^4*d^2*f^5)/(c^2*f^4 + d^2*f^4
) + (4*B*c*d^4*f^4*(-16*B^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (4*B*c^3*
d^2*f^4*(-16*B^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5)) - (32*B^2*c^2*d^2*f^2
*((B^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)) - (-16*B^...
```

3.114 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=210

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)\sqrt{c-id}\,f}-\frac{(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)\sqrt{c+id}\,f}-\frac{2(Ab^2-\dots}{}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)/f/(c-I*d
)^(1/2)-(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)/f/(
c+I*d)^(1/2)-2*(A*b^2-a*(B*b-C*a))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(
-a*d+b*c)^(1/2))/(a^2+b^2)/f/b^(1/2)/(-a*d+b*c)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,f\,(a^2+b^2)\sqrt{bc-ad}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)\sqrt{c-id}}-\frac{(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(-b+ia)\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*Sqrt[c +
d*Tan[e + f*x]]),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)*Sqrt[c - I*d]*f)) - ((A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqr
t[c + I*d]])/((I*a - b)*Sqrt[c + I*d]*f) - (2*(A*b^2 - a*(b*B - a*C))*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2
)*Sqrt[b*c - a*d]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\int \frac{bB+a(A-C)-(Ab-aB-bC)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{a^2+b^2} + \frac{(Ab^2-abB+a^2C)\int}{}$$

$$= \frac{(A-iB-C)\int \frac{1+i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx}{2(a-ib)} + \frac{(A+iB-C)}{}$$

$$= -\frac{(i(A+iB-C))\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{c+idx}}\,dx, x, -i\tan(\right.}{2(a+ib)f}$$

$$= -\frac{2(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right.}{\sqrt{b}\,(a^2+b^2)\sqrt{bc-ad}\,f}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)\sqrt{c-id}\,f} - \frac{(A+}{}$$

Mathematica [A]
time = 0.29, size = 194, normalized size = 0.92

$$\frac{\frac{(-ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{(ia+b)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}} - \frac{2(Ab^2+a(-bB+aC))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}}{(a^2+b^2)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*Sqr
t[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((((-I)*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]
])/Sqrt[c - I*d] + ((I*a + b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]
]/Sqrt[c + I*d]])/Sqrt[c + I*d] - (2*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[(Sq
rt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d])
)/((a^2 + b^2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3760 vs. $2(179) = 358$.
time = 0.53, size = 3761, normalized size = 17.91

method	result	size
derivativedivides	Expression too large to display	3761
default	Expression too large to display	3761

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2/(a^2+b^2)*(1/4/d^2/(c^2+d^2)^(3/2)*(-1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*(c^2+d^2)^(1/2)*a*c^2*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/
2)*b*c*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c^3+B*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*d^3-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*a*c^2*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d-B*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*b*c*d^3-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*b*c+A*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*d^3+A*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*(c^2+d^2)^(1/2)*b*c^3-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d-A*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+B*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*a*c+C*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*(c^2+d^2)^(3/2)*b*c-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*
a*d^3-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c^3+C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*a*c^3*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3-C*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4-B*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^4-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^
(1/2)*a*c^2*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c*d^2-B*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c*d^2+B*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*(c^2+d^2)^(1/2)*b*c^2*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*ln((
c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2
)^(1/2))+2*(-2*A*a*c^2*d^3-2*A*a*d^5-2*A*b*c^3*d^2-2*A*b*c*d^4+2*B*a*c^3*d^
2+2*B*a*c*d^4-2*B*b*c^2*d^3-2*B*b*d^5+2*C*a*c^2*d^3+2*C*a*d^5+2*C*b*c^3*d^2
+2*C*b*c*d^4+1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c^2*d+A
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c*d^2-B*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c^3+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)
^(1/2)*b*d^3-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^2-B*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*b*c^3*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^3-A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*b*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d
^2)^(1/2)*a*d^3+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c^3-A*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3+
A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(
c^2+d^2)^(3/2)*a*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*b*c-C*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*d^3-C*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)*(c^2+d^2)^(1/2)*b*c^3+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d+C*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2
+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^
4-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c^2*d-C*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*a*c*d^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c^2*d
-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*d^4)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(
```

```
c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(
f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d^2/(c^2+d^2)^(3/2)*(1/2
*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c^2*d+A*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2
+d^2)^(1/2)*a*c^3+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*d^3-B*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^2*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^
3*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c*d^3-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*(c^2+d^2)^(3/2)*b*c+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*d^3+
A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c^3-A*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*c^3*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c*d^3+A*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*b*c^2*d^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*a
*c+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(3/2)*b*c-C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*d^3-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)
^(1/2)*b*c^3+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c^3*d+C*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*a*c*d^3-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^2*d^2+A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b*d^4-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*d^4-C*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c^2*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(
c^2+d^2)^(1/2)*b*c*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c*
d^2+B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*c^2*d-C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*b*d^4)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(2*A*a*c^2*d^3+2*A*a*d^5+2*A*b*c^3*d
^2+2*A*b*c*d^4-2*B*a*c^3*d^2-2*B*a*c*d^4+2*B*b*c^2*d^3+2*B*b*d^5-2*C*a*c^2*
d^3-2*C*a*d^5-2*C*b*c^3*d^2-2*C*b*c*d^4-1/2*(A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*(c^2+d^2)^(1/2)*a*c^2*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b
*c*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*c^3+B*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b*d^3-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c
^2*d^2-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*b*c^3*d-B*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b*c*d^3-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^...
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*tan
(f*x+e)),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))*sqr
t(c + d*tan(e + f*x))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 69.14, size = 2500, normalized size = 11.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*ta
n(e + f*x))^(1/2)),x)
```

```
[Out] (log(((((((((128*C*b^2*d^8*(a*d + b*c)^2*(a^2 + b^2)^2)/f - 64*b^2*d^8*(a^
2 + b^2)^2*(c + d*tan(e + f*x))^(1/2)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*
c)^2)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^
2 + b^2)^2*(c^2 + d^2)))^(1/2)*(3*b^3*c^2 + 2*b^3*d^2 - a^2*b*c^2 - 2*a^2*b
```

*d^2 + a^3*c*d + a*b^2*c*d))*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/
2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^
2*(c^2 + d^2)))^(1/2))/4 + (64*C^2*b*d^8*(c + d*tan(e + f*x))^(1/2)*(5*b^6*
c - 4*a^6*c - 2*a^2*b^4*c + 5*a^4*b^2*c - 2*a^3*b^3*d + 7*a*b^5*d + 7*a^5*b
d))/f^2)((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) - 4*C^2*a^2*c*f^
2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 + d^2)))^(1/
2))/4 + (32*C^3*b*d^8*(4*a^5*d - b^5*c - 9*a^2*b^3*c - 15*a^3*b^2*d + 12*a^
4*b*c + a*b^4*d))/f^3)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) - 4
*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2
 + d^2)))^(1/2))/4 - (32*C^4*b*d^8*(2*a^4 + b^4)*(c + d*tan(e + f*x))^(1/2)
)/f^4)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) - 4*C^2*a^2*c*f^2 +
 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 + d^2)))^(1/2))
/4 + (32*C^5*a^2*b^2*d^8)/f^5)*(((32*C^4*a^2*b^2*d^2*f^4 - 16*C^4*b^4*d^2*f
^4 - 64*C^4*a^2*b^2*c^2*f^4 - 16*C^4*a^4*d^2*f^4 + 64*C^4*a*b^3*c*d*f^4 - 6
4*C^4*a^3*b*c*d*f^4)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*
d*f^2)/(a^4*c^2*f^4 + a^4*d^2*f^4 + b^4*c^2*f^4 + b^4*d^2*f^4 + 2*a^2*b^2*c
^2*f^4 + 2*a^2*b^2*d^2*f^4))^(1/2))/4 + (log(((((((((128*C*b^2*d^8*(a*d +
b*c)^2*(a^2 + b^2)^2)/f - 64*b^2*d^8*(a^2 + b^2)^2*(c + d*tan(e + f*x))^(1/
2)*(-(4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) + 4*C^2*a^2*c*f^2 - 4*
C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 + d^2)))^(1/2)*(3*
b^3*c^2 + 2*b^3*d^2 - a^2*b*c^2 - 2*a^2*b*d^2 + a^3*c*d + a*b^2*c*d))*(-(4*
(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) + 4*C^2*a^2*c*f^2 - 4*C^2*b^2*
c*f^2 - 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 + d^2)))^(1/2))/4 + (64*C^
2*b*d^8*(c + d*tan(e + f*x))^(1/2)*(5*b^6*c - 4*a^6*c - 2*a^2*b^4*c + 5*a^4
*b^2*c - 2*a^3*b^3*d + 7*a*b^5*d + 7*a^5*b*d))/f^2)*(-(4*(-C^4*f^4*(a^2*d -
 b^2*d + 2*a*b*c)^2)^(1/2) + 4*C^2*a^2*c*f^2 - 4*C^2*b^2*c*f^2 - 8*C^2*a*b*
d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 + d^2)))^(1/2))/4 + (32*C^3*b*d^8*(4*a^5*d -
 b^5*c - 9*a^2*b^3*c - 15*a^3*b^2*d + 12*a^4*b*c + a*b^4*d))/f^3)*(-(4*(-C^
4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) + 4*C^2*a^2*c*f^2 - 4*C^2*b^2*c*f^
2 - 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 + d^2)))^(1/2))/4 - (32*C^4*b*
d^8*(2*a^4 + b^4)*(c + d*tan(e + f*x))^(1/2))/f^4)*(-(4*(-C^4*f^4*(a^2*d -
b^2*d + 2*a*b*c)^2)^(1/2) + 4*C^2*a^2*c*f^2 - 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d
f^2)/(f^4(a^2 + b^2)^2*(c^2 + d^2)))^(1/2))/4 + (32*C^5*a^2*b^2*d^8)/f^5)
*(-((32*C^4*a^2*b^2*d^2*f^4 - 16*C^4*b^4*d^2*f^4 - 64*C^4*a^2*b^2*c^2*f^4 -
 16*C^4*a^4*d^2*f^4 + 64*C^4*a*b^3*c*d*f^4 - 64*C^4*a^3*b*c*d*f^4)^(1/2) +
4*C^2*a^2*c*f^2 - 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(a^4*c^2*f^4 + a^4*d^2
*f^4 + b^4*c^2*f^4 + b^4*d^2*f^4 + 2*a^2*b^2*c^2*f^4 + 2*a^2*b^2*d^2*f^4))^
(1/2))/4 - log(((((((((128*C*b^2*d^8*(a*d + b*c)^2*(a^2 + b^2)^2)/f + 64*b
^2*d^8*(a^2 + b^2)^2*(c + d*tan(e + f*x))^(1/2)*((4*(-C^4*f^4*(a^2*d - b^2*
d + 2*a*b*c)^2)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2
)/(f^4*(a^2 + b^2)^2*(c^2 + d^2)))^(1/2)*(3*b^3*c^2 + 2*b^3*d^2 - a^2*b*c^2
 - 2*a^2*b*d^2 + a^3*c*d + a*b^2*c*d))*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b
*c)^2)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a
^2 + b^2)^2*(c^2 + d^2)))^(1/2))/4 - (64*C^2*b*d^8*(c + d*tan(e + f*x))^(1/
2)*(5*b^6*c - 4*a^6*c - 2*a^2*b^4*c + 5*a^4*b^2*c - 2*a^3*b^3*d + 7*a*b^5*d

```
 + 7*a^5*b*d))/f^2)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) - 4*C^
2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 +
d^2)))^(1/2))/4 + (32*C^3*b*d^8*(4*a^5*d - b^5*c - 9*a^2*b^3*c - 15*a^3*b^2
*d + 12*a^4*b*c + a*b^4*d))/f^3)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)
^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b
^2)^2*(c^2 + d^2)))^(1/2))/4 + (32*C^4*b*d^8*(2*a^4 + b^4)*(c + d*tan(e + f
*x))^(1/2))/f^4)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) - 4*C^2*a
^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 + d^2
)))^(1/2))/4 + (32*C^5*a^2*b^2*d^8)/f^5)*(((32*C^4*a^2*b^2*d^2*f^4 - 16*C^4
*b^4*d^2*f^4 - 64*C^4*a^2*b^2*c^2*f^4 - 16*C^4*a^4*d^2*f^4 + 64*C^4*a*b^3*c
*d*f^4 - 64*C^4*a^3*b*c*d*f^4)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 +
8*C^2*a*b*d*f^2)/(16*a^4*c^2*f^4 + 16*a^4*d^2*f^4 + 16*b^4*c^2*f^4 + 16*b^4
*d^2*f^4 + 32*a^2*b^2*c^2*f^4 + 32*a^2*b^2*d^2*f^4))^(1/2) - log(((((((((1
28*C*b^2*d^8*(a*d + b*c)^2*(a^2 + b^2)^2)/f + 64*b^2*d^8*(a^2 + b^2)^2*(c +
 d*tan(e + f*x))^(1/2)*(-(4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) +
4*C^2*a^2*c*f^2 - 4*C^2*b^2*c*f^2 - 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^
2 + d^2)))^(1/2)*(3*b^3*c^2 + 2*b^3*d^2 - a^2*b*c^2 - 2*a^2*b*d^2 + a^3*c*d
 + a*b^2*c*d))*(-(4*(-C^4*f^4*(a^2*d - b^2*d + ...
```

3.115 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=327

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2\sqrt{c-id}\,f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2\sqrt{c+id}\,f}-\frac{(3a^3b}{}$$

```
[Out] -(3*a^3*b*B*d-a^4*C*d+b^4*(-A*d+2*B*c)+a*b^3*(4*A*c-B*d-4*C*c)-a^2*b^2*(5*A
*d+2*B*c-3*C*d))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/(
a^2+b^2)^2/(-a*d+b*c)^(3/2)/f/b^(1/2)-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^
(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/f/(c-I*d)^(1/2)-(B-I*(A-C))*arctanh((c+d*tan
(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)^2/f/(c+I*d)^(1/2)-(A*b^2-a*(B*b-C*a))
*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))
```

Rubi [A]
time = 0.94, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}-\frac{(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}f(a^2+b^2)^2(bc-ad)^{3/2}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2\sqrt{c-id}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*Sqrt[c
+ d*Tan[e + f*x]]),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*Sqrt[c - I*d]*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c + I*d]])/((a + I*b)^2*Sqrt[c + I*d]*f) - ((3*a^3*b*B*d - a^4*C*d +
b^4*(2*B*c - A*d) + a*b^3*(4*A*c - 4*c*C - B*d) - a^2*b^2*(2*B*c + 5*A*d -
3*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[
b]*(a^2 + b^2)^2*(b*c - a*d)^(3/2)*f) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d
*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
```

```
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{\frac{1}{2}\left(Ab^2d-2\right.}{}}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))} - \frac{\int \frac{-\left(2abB+a\right.}{}}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))} + \frac{(A-iB-}{}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))} - \frac{(i(A+iB}{}$$

$$= -\frac{(3a^3bBd-a^4Cd+b^4(2Bc-Ad)+ab^3(4Ac-4cC-B}{\sqrt{b}\,(a^2}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2\sqrt{c-id}\,f} - \frac{(B-}{}$$

Mathematica [A]
time = 4.72, size = 338, normalized size = 1.03

$$-\frac{\frac{i\left(\frac{(a+ib)^2(A-iB-C)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{(a-ib)^2(A+iB-C)(-bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)}{a^2+b^2}+\frac{(-3a^3bBd+a^4Cd+b^4(-2Bc+Ad)+ab^3(-4Ac+4cC+Bd)+a^2b^2(2Bc+5Ad-3Cd))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(a^2+b^2)\sqrt{bc-ad}}-\frac{(Ab^2+a(-bB+aC))\sqrt{c+d\tan(e+fx)}}{a+b\tan(e+fx)}}{(a^2+b^2)(bc-ad)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*S
qrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (((-I)*(((a + I*b)^2*(A - I*B - C)*(b*c - a*d)*ArcTanh[Sqrt[c + d*Tan[e + f
*x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + ((a - I*b)^2*(A + I*B - C)*(-(b*c) + a
*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d]))/(a^2 +
 b^2) + ((-3*a^3*b*B*d + a^4*C*d + b^4*(-2*B*c + A*d) + a*b^3*(-4*A*c + 4*c
```

```
*C + B*d) + a^2*b^2*(2*B*c + 5*A*d - 3*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*Sqrt[b*c - a*d]) - ((A*
b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*Tan[e + f*x]])/(a + b*Tan[e + f*x]))/((a
^2 + b^2)*(b*c - a*d)*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5886 vs. $2(294) = 588$.
time = 0.58, size = 5887, normalized size = 18.00

method	result	size
derivativedivides	Expression too large to display	5887
default	Expression too large to display	5887

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2\sqrt{c + d\tan(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*tan
(f*x+e))**2,x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**2*
sqrt(c + d*tan(e + f*x))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 57.65, size = 2500, normalized size = 7.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d*
tan(e + f*x))^(1/2)),x)
```

```
[Out] (atan(((((((16*(8*C^3*a^6*b^7*d^11*f^2 - 78*C^3*a^4*b^9*d^11*f^2 + 60*C^3*a
^8*b^5*d^11*f^2 - 24*C^3*a^10*b^3*d^11*f^2 + 2*C^3*a^12*b*d^11*f^2 - 32*C^3
*a*b^12*c^3*d^8*f^2 + 152*C^3*a^3*b^10*c*d^10*f^2 + 128*C^3*a^5*b^8*c*d^10*
f^2 - 64*C^3*a^7*b^6*c*d^10*f^2 - 32*C^3*a^9*b^4*c*d^10*f^2 + 8*C^3*a^11*b^
2*c*d^10*f^2 - 40*C^3*a^2*b^11*c^2*d^9*f^2 + 64*C^3*a^3*b^10*c^3*d^8*f^2 -
216*C^3*a^4*b^9*c^2*d^9*f^2 + 96*C^3*a^5*b^8*c^3*d^8*f^2 - 120*C^3*a^6*b^7*
c^2*d^9*f^2 + 56*C^3*a^8*b^5*c^2*d^9*f^2))/(a^10*d^2*f^5 + b^10*c^2*f^5 + 4
*a^2*b^8*c^2*f^5 + 6*a^4*b^6*c^2*f^5 + 4*a^6*b^4*c^2*f^5 + a^8*b^2*c^2*f^5
+ a^2*b^8*d^2*f^5 + 4*a^4*b^6*d^2*f^5 + 6*a^6*b^4*d^2*f^5 + 4*a^8*b^2*d^2*f
^5 - 2*a*b^9*c*d*f^5 - 2*a^9*b*c*d*f^5 - 8*a^3*b^7*c*d*f^5 - 12*a^5*b^5*c*d
*f^5 - 8*a^7*b^3*c*d*f^5) - (((((16*(40*C*a^3*b^14*d^12*f^4 + 192*C*a^5*b^1
2*d^12*f^4 + 360*C*a^7*b^10*d^12*f^4 + 320*C*a^9*b^8*d^12*f^4 + 120*C*a^11*
b^6*d^12*f^4 - 8*C*a^15*b^2*d^12*f^4 + 8*C*b^17*c^3*d^9*f^4 + 40*C*a*b^16*c
^2*d^10*f^4 + 32*C*a*b^16*c^4*d^8*f^4 - 88*C*a^2*b^15*c*d^11*f^4 - 448*C*a^
4*b^13*c*d^11*f^4 - 920*C*a^6*b^11*c*d^11*f^4 - 960*C*a^8*b^9*c*d^11*f^4 -
```

520*C*a^10*b^7*c*d^11*f^4 - 128*C*a^12*b^5*c*d^11*f^4 - 8*C*a^14*b^3*c*d^11
*f^4 - 32*C*a^2*b^15*c^3*d^9*f^4 + 256*C*a^3*b^14*c^2*d^10*f^4 + 160*C*a^3*
b^14*c^4*d^8*f^4 - 280*C*a^4*b^13*c^3*d^9*f^4 + 680*C*a^5*b^12*c^2*d^10*f^4
 + 320*C*a^5*b^12*c^4*d^8*f^4 - 640*C*a^6*b^11*c^3*d^9*f^4 + 960*C*a^7*b^10
*c^2*d^10*f^4 + 320*C*a^7*b^10*c^4*d^8*f^4 - 680*C*a^8*b^9*c^3*d^9*f^4 + 76
0*C*a^9*b^8*c^2*d^10*f^4 + 160*C*a^9*b^8*c^4*d^8*f^4 - 352*C*a^10*b^7*c^3*d
^9*f^4 + 320*C*a^11*b^6*c^2*d^10*f^4 + 32*C*a^11*b^6*c^4*d^8*f^4 - 72*C*a^1
2*b^5*c^3*d^9*f^4 + 56*C*a^13*b^4*c^2*d^10*f^4))/(a^10*d^2*f^5 + b^10*c^2*f
^5 + 4*a^2*b^8*c^2*f^5 + 6*a^4*b^6*c^2*f^5 + 4*a^6*b^4*c^2*f^5 + a^8*b^2*c^
2*f^5 + a^2*b^8*d^2*f^5 + 4*a^4*b^6*d^2*f^5 + 6*a^6*b^4*d^2*f^5 + 4*a^8*b^2
*d^2*f^5 - 2*a*b^9*c*d*f^5 - 2*a^9*b*c*d*f^5 - 8*a^3*b^7*c*d*f^5 - 12*a^5*b
^5*c*d*f^5 - 8*a^7*b^3*c*d*f^5) - (16*((C^2*a^8*d^2 + 16*C^2*a^2*b^6*c^2 +
9*C^2*a^4*b^4*d^2 - 6*C^2*a^6*b^2*d^2 - 24*C^2*a^3*b^5*c*d + 8*C^2*a^5*b^3*
c*d)*(b^12*c^3*f^2 - a^11*b*d^3*f^2 + 4*a^2*b^10*c^3*f^2 + 6*a^4*b^8*c^3*f^
2 + 4*a^6*b^6*c^3*f^2 + a^8*b^4*c^3*f^2 - a^3*b^9*d^3*f^2 - 4*a^5*b^7*d^3*f
^2 - 6*a^7*b^5*d^3*f^2 - 4*a^9*b^3*d^3*f^2 - 3*a*b^11*c^2*d*f^2 + 3*a^2*b^1
0*c*d^2*f^2 - 12*a^3*b^9*c^2*d*f^2 + 12*a^4*b^8*c*d^2*f^2 - 18*a^5*b^7*c^2*
d*f^2 + 18*a^6*b^6*c*d^2*f^2 - 12*a^7*b^5*c^2*d*f^2 + 12*a^8*b^4*c*d^2*f^2
- 3*a^9*b^3*c^2*d*f^2 + 3*a^10*b^2*c*d^2*f^2))^(1/2)*(c + d*tan(e + f*x))^(
1/2)*(32*a^2*b^17*d^12*f^4 + 160*a^4*b^15*d^12*f^4 + 288*a^6*b^13*d^12*f^4
+ 160*a^8*b^11*d^12*f^4 - 160*a^10*b^9*d^12*f^4 - 288*a^12*b^7*d^12*f^4 - 1
60*a^14*b^5*d^12*f^4 - 32*a^16*b^3*d^12*f^4 + 32*b^19*c^2*d^10*f^4 + 48*b^1
9*c^4*d^8*f^4 + 176*a^2*b^17*c^2*d^10*f^4 + 272*a^2*b^17*c^4*d^8*f^4 - 432*
a^3*b^16*c^3*d^9*f^4 + 336*a^4*b^15*c^2*d^10*f^4 + 624*a^4*b^15*c^4*d^8*f^4
 - 912*a^5*b^14*c^3*d^9*f^4 + 112*a^6*b^13*c^2*d^10*f^4 + 720*a^6*b^13*c^4*
d^8*f^4 - 880*a^7*b^12*c^3*d^9*f^4 - 560*a^8*b^11*c^2*d^10*f^4 + 400*a^8*b^
11*c^4*d^8*f^4 - 240*a^9*b^10*c^3*d^9*f^4 - 1008*a^10*b^9*c^2*d^10*f^4 + 48
*a^10*b^9*c^4*d^8*f^4 + 240*a^11*b^8*c^3*d^9*f^4 - 784*a^12*b^7*c^2*d^10*f^
4 - 48*a^12*b^7*c^4*d^8*f^4 + 208*a^13*b^6*c^3*d^9*f^4 - 304*a^14*b^5*c^2*d
^10*f^4 - 16*a^14*b^5*c^4*d^8*f^4 + 48*a^15*b^4*c^3*d^9*f^4 - 48*a^16*b^3*c
^2*d^10*f^4 - 64*a*b^18*c*d^11*f^4 - 80*a*b^18*c^3*d^9*f^4 - 304*a^3*b^16*c
*d^11*f^4 - 464*a^5*b^14*c*d^11*f^4 + 16*a^7*b^12*c*d^11*f^4 + 880*a^9*b^10
*c*d^11*f^4 + 1136*a^11*b^8*c*d^11*f^4 + 656*a^13*b^6*c*d^11*f^4 + 176*a^15
*b^4*c*d^11*f^4 + 16*a^17*b^2*c*d^11*f^4))/((b^10*(8*a^2*c^3*f^2 + 6*a^2*c*
d^2*f^2) + b^4*(2*a^8*c^3*f^2 + 24*a^8*c*d^2*f^2) + b^8*(12*a^4*c^3*f^2 + 2
4*a^4*c*d^2*f^2) + b^6*(8*a^6*c^3*f^2 + 36*a^6*c*d^2*f^2) - b^3*(8*a^9*d^3*
f^2 + 6*a^9*c^2*d*f^2) - b^9*(2*a^3*d^3*f^2 + 24*a^3*c^2*d*f^2) - b^5*(12*a
^7*d^3*f^2 + 24*a^7*c^2*d*f^2) - b^7*(8*a^5*d^3*f^2 + 36*a^5*c^2*d*f^2) + 2
*b^12*c^3*f^2 - 2*a^11*b*d^3*f^2 - 6*a*b^11*c^2*d*f^2 + 6*a^10*b^2*c*d^2*f^
2)*(a^10*d^2*f^4 + b^10*c^2*f^4 + 4*a^2*b^8*c^2*f^4 + 6*a^4*b^6*c^2*f^4 + 4
*a^6*b^4*c^2*f^4 + a^8*b^2*c^2*f^4 + a^2*b^8*d^2*f^4 + 4*a^4*b^6*d^2*f^4 +
6*a^6*b^4*d^2*f^4 + 4*a^8*b^2*d^2*f^4 - 2*a*b^9*c*d*f^4 - 2*a^9*b*c*d*f^4 -
 8*a^3*b^7*c*d*f^4 - 12*a^5*b^5*c*d*f^4 - 8*a^7*b^3*c*d*f^4)))*((C^2*a^8*d^
2 + 16*C^2*a^2*b^6*c^2 + 9*C^2*a^4*b^4*d^2 - 6*C^2*a^6*b^2*d^2 - 24*C^2*a^3
*b^5*c*d + 8*C^2*a^5*b^3*c*d)*(b^12*c^3*f^2 - a^11*b*d^3*f^2 + 4*a^2*b^10*c

```
^3*f^2 + 6*a^4*b^8*c^3*f^2 + 4*a^6*b^6*c^3*f^2 + a^8*b^4*c^3*f^2 - a^3*b^9*
d^3*f^2 - 4*a^5*b^7*d^3*f^2 - 6*a^7*b^5*d^3*f^2 - 4*a^9*b^3*d^3*f^2 - 3*a*b
^11*c^2*d*f^2 + 3*a^2*b^10*c*d^2*f^2 - 12*a^3*b^9*c^2*d*f^2 + 12*a^4*b^8*c*
d^2*f^2 - 18*a^5*b^7*c^2*d*f^2 + 18*a^6*b^6*c*d^2*f^2 - 12*a^7*b^5*c^2*d*f^
2 + 12*a^8*b^4*c*d^2*f^2 - 3*a^9*b^3*c^2*d*f^2 + 3*a^10*b^2*c*d^2*f^2))^(1/
2))/(b^10*(8*a^2*c^3*f^2 + 6*a^2*c*d^2*f^2) + b^4*(2*a^8*c^3*f^2 + 24*a^8*c
*d^2*f^2) + b^8*(12*a^4*c^3*f^2 + 24*a^4*c*d^2*...
```

3.116 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=511

$$-\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}-\frac{(ia-b)^3(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e}{\sqrt{c+id}}\right.}{(c+id)^{3/2}f}$$

```
[Out] -(a-I*b)^3*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d
)^(3/2)/f-(I*a-b)^3*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))
/(c+I*d)^(3/2)/f+2/15*b*(6*a^2*d^2*(12*c^2*C-5*B*c*d+(5*A+7*C)*d^2)-15*a*b*
d*(8*c^3*C-6*B*c^2*d+c*(3*A+5*C)*d^2-3*B*d^3)+b^2*(48*c^4*C-40*B*c^3*d+6*c^
2*(5*A+3*C)*d^2-25*B*c*d^3+15*(A-C)*d^4))*(c+d*tan(f*x+e))^(1/2)/d^4/(c^2+d
^2)/f-2/15*b^2*(4*(-a*d+b*c)*(6*c^2*C-5*B*c*d+(5*A+C)*d^2)-5*d^2*((A-C)*(-a
*d+b*c)+B*(a*c+b*d)))*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^3/(c^2+d^2)/f+2/5
*b*(6*c^2*C-5*B*c*d+(5*A+C)*d^2)*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2/
d^2/(c^2+d^2)/f-2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^3/d/(c^2+d^2)/f/(c+d
*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 1.66, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {3726, 3728, 3718, 3711, 3620, 3618, 65, 214}

$$\frac{2b\sqrt{c+d\tan(e+fx)}\left(6a^2d^2\left(d^2(5A+7C)-5Bcd+12c^2C\right)-15abd\left(cd^2(3A+5C)-6Bc^2d-3Bd^3+8c^3C\right)+b^2\left(6c^2d^2(5A+3C)+15d^4(A-C)-40Bc^3d-25Bcd^3+48c^4C\right)\right)}{15d^4f\left(c^2+d^2\right)}-\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2\left((A-C)(bc-ad)+B(ac+bd)\right)\right)}{15d^3f\left(c^2+d^2\right)}+\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5d^2f\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(-b+ia)^3(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}-\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(3/2)*f)) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^
2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (2*
b*(6*a^2*d^2*(12*c^2*C - 5*B*c*d + (5*A + 7*C)*d^2) - 15*a*b*d*(8*c^3*C - 6
*B*c^2*d + c*(3*A + 5*C)*d^2 - 3*B*d^3) + b^2*(48*c^4*C - 40*B*c^3*d + 6*c^
2*(5*A + 3*C)*d^2 - 25*B*c*d^3 + 15*(A - C)*d^4))*Sqrt[c + d*Tan[e + f*x]])
/(15*d^4*(c^2 + d^2)*f) - (2*b^2*(4*(b*c - a*d)*(6*c^2*C - 5*B*c*d + (5*A +
 C)*d^2) - 5*d^2*((A - C)*(b*c - a*d) + B*(a*c + b*d)))*Tan[e + f*x]*Sqrt[c
 + d*Tan[e + f*x]])/(15*d^3*(c^2 + d^2)*f) + (2*b*(6*c^2*C - 5*B*c*d + (5*A
 + C)*d^2)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])/(5*d^2*(c^2 + d
^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.}{(c-id)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.50, size = 920, normalized size = 1.80

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3)/(5*d*f*Sqrt[c + d*Tan[e + f*x]]) + (2*(((-6*b*
c*C + 5*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2)/(3*d*f*Sqrt[c + d*Tan[e +
```

```
f*x]]) + (2*(((15*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 5*b*B*
d - 6*a*C*d))*(a + b*Tan[e + f*x]))/(2*d*f*Sqrt[c + d*Tan[e + f*x]]) + ((-2
*(-48*b^3*c^3*C + 40*b^3*B*c^2*d + 144*a*b^2*c^2*C*d - 30*A*b^3*c*d^2 - 110
*a*b^2*B*c*d^2 - 144*a^2*b*c*C*d^2 + 30*b^3*c*C*d^2 + 60*a*A*b^2*d^3 + 85*a
^2*b*B*d^3 - 15*b^3*B*d^3 + 48*a^3*C*d^3 - 60*a*b^2*C*d^3))/(d*Sqrt[c + d*T
an[e + f*x]]) + (2*(((45*a^2*A*b*d^3 - 15*A*b^3*d^3 + 15*a^3*B*d^3 - 45*a*b
^2*B*d^3 - 45*a^2*b*C*d^3 + 15*b^3*C*d^3)*(((-I)*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/
Sqrt[c + I*d]])/Sqrt[c + I*d]))/2 + ((-1/2*(c*d*(45*a^2*A*b*d^3 - 15*A*b^3*
d^3 + 15*a^3*B*d^3 - 45*a*b^2*B*d^3 - 45*a^2*b*C*d^3 + 15*b^3*C*d^3)) + d^2
*((-48*b^3*c^3*C + 40*b^3*B*c^2*d + 144*a*b^2*c^2*C*d - 30*A*b^3*c*d^2 - 11
0*a*b^2*B*c*d^2 - 144*a^2*b*c*C*d^2 + 30*b^3*c*C*d^2 + 15*a^3*A*d^3 + 15*a*
A*b^2*d^3 + 40*a^2*b*B*d^3 + 33*a^3*C*d^3 - 15*a*b^2*C*d^3)/2 + (48*b^3*c^3
*C - 40*b^3*B*c^2*d - 144*a*b^2*c^2*C*d + 30*A*b^3*c*d^2 + 110*a*b^2*B*c*d^
2 + 144*a^2*b*c*C*d^2 - 30*b^3*c*C*d^2 - 60*a*A*b^2*d^3 - 85*a^2*b*B*d^3 +
15*b^3*B*d^3 - 48*a^3*C*d^3 + 60*a*b^2*C*d^3)/2))*(-(Hypergeometric2F1[-1/2
, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e + f*x
]])) + Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/((I*
c - d)*Sqrt[c + d*Tan[e + f*x]])))/d))/d)/(4*d*f)))/(3*d)))/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 13577 vs. $2(476) = 952$.
time = 0.66, size = 13578, normalized size = 26.57

method	result	size
derivativedivides	Expression too large to display	13578
default	Expression too large to display	13578

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.117 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=343

$$-\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}-\frac{(a+ib)^2(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f}$$

```
[Out] -(a-I*b)^2*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d
)^(3/2)/f-(a+I*b)^2*(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))/(c+I*d)^(3/2)/f+2/3*b*(6*a*d*(2*c^2*C-B*c*d+(A+C)*d^2)-b*(8*c^3*C-6*B*c^
2*d+c*(3*A+5*C)*d^2-3*B*d^3))*(c+d*tan(f*x+e))^(1/2)/d^3/(c^2+d^2)/f+2/3*b^
2*(4*c^2*C-3*B*c*d+(3*A+C)*d^2)*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^2/(c^2+
d^2)/f-2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(c+d*tan(f*x+
e))^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3726, 3718, 3711, 3620, 3618, 65, 214}

$$-\frac{2(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^2}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{2b\sqrt{c+d\tan(e+fx)}\,(6ad(d^2(A+C)-Bcd+2c^2C)-b(cd^2(3A+5C)-6Bc^2d-3Bd^3+8c^3C))}{3d^3f(c^2+d^2)}-\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}-\frac{(a+ib)^2(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}+\frac{2b^2\tan(e+fx)(d^2(3A+C)-3Bcd+4c^2C)\sqrt{c+d\tan(e+fx)}}{3d^2f(c^2+d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(3/2)*f)) - ((a + I*b)^2*(B - I*(A - C))*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*
d^2)*(a + b*Tan[e + f*x])^2)/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (
2*b*(6*a*d*(2*c^2*C - B*c*d + (A + C)*d^2) - b*(8*c^3*C - 6*B*c^2*d + c*(3*
A + 5*C)*d^2 - 3*B*d^3))*Sqrt[c + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)*f) +
(2*b^2*(4*c^2*C - 3*B*c*d + (3*A + C)*d^2)*Tan[e + f*x]*Sqrt[c + d*Tan[e +
f*x]])/(3*d^2*(c^2 + d^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
```

```
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{(c-id)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.35, size = 476, normalized size = 1.39

$$\frac{2C(a+b\tan(e+fx))^2}{3df\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\frac{(-4bcC+3bBd+4aCd)(a+b\tan(e+fx))}{df\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{2\left(8b^2c^2C-6b^2Bcd-16abcCd+3Ab^2d^2+9abBd^2+8a^2Cd^2-3b^2Cd^2\right)}{d\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\frac{3}{2}\left(a^2B-b^2B+2ab(A-C)\right)d^2\left(-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)+\frac{\left(-\frac{3}{2}c\left(a^2B-b^2B+2ab(A-C)\right)d^3-\frac{3}{2}\left(2abB-a^2(A-C)+b^2(A-C)\right)d^4\right)\left(-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)}{(ic+d)\sqrt{c+d\tan(e+fx)}}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)}{(ic-d)\sqrt{c+d\tan(e+fx)}}\right)}{d}\right)}{d}}{2df}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^2)/(3*d*f*Sqrt[c + d*Tan[e + f*x]]) + (2*(((-4*b*
c*C + 3*b*B*d + 4*a*C*d)*(a + b*Tan[e + f*x]))/(d*f*Sqrt[c + d*Tan[e + f*x]
]) + ((-2*(8*b^2*c^2*C - 6*b^2*B*c*d - 16*a*b*c*C*d + 3*A*b^2*d^2 + 9*a*b*B
*d^2 + 8*a^2*C*d^2 - 3*b^2*C*d^2))/(d*Sqrt[c + d*Tan[e + f*x]]) + (2*((3*(a
^2*B - b^2*B + 2*a*b*(A - C))*d^2*(((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c - I*d]])/Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
 I*d]])/Sqrt[c + I*d]))/2 + (((-3*c*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/2
- (3*(2*a*b*B - a^2*(A - C) + b^2*(A - C))*d^4)/2)*(-(Hypergeometric2F1[-1/
2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e + f*
x]])) + Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/((I
*c - d)*Sqrt[c + d*Tan[e + f*x]])))/d))/d)/(2*d*f)))/(3*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9978 vs. $2(312) = 624$.
time = 0.53, size = 9979, normalized size = 29.09

method	result	size
derivativedivides	Expression too large to display	9979
default	Expression too large to display	9979

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^2 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 66.25, size = 2500, normalized size = 7.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^(3/2),x)
```

```
[Out] (2*(B*b^2*c^3 + B*a^2*c*d^2 - 2*B*a*b*c^2*d))/(d^2*f*(c^2 + d^2)*(c + d*tan
(e + f*x))^(1/2)) - atan((((-(((8*B^2*a^4*c^3*f^2 + 8*B^2*b^4*c^3*f^2 - 48*
B^2*a^2*b^2*c^3*f^2 + 32*B^2*a*b^3*d^3*f^2 - 32*B^2*a^3*b*d^3*f^2 - 24*B^2*
a^4*c*d^2*f^2 - 24*B^2*b^4*c*d^2*f^2 - 96*B^2*a*b^3*c^2*d*f^2 + 96*B^2*a^3*
b*c^2*d*f^2 + 144*B^2*a^2*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 4
8*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(B^4*a^8 + B^4*b^8 + 4*B^4*a^2*b^6 + 6*B^4*
a^4*b^4 + 4*B^4*a^6*b^2))^(1/2) - 4*B^2*a^4*c^3*f^2 - 4*B^2*b^4*c^3*f^2 + 2
4*B^2*a^2*b^2*c^3*f^2 - 16*B^2*a*b^3*d^3*f^2 + 16*B^2*a^3*b*d^3*f^2 + 12*B^
```

2*a^4*c*d^2*f^2 + 12*B^2*b^4*c*d^2*f^2 + 48*B^2*a*b^3*c^2*d*f^2 - 48*B^2*a^3*b*c^2*d*f^2 - 72*B^2*a^2*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(-(((8*B^2*a^4*c^3*f^2 + 8*B^2*b^4*c^3*f^2 - 48*B^2*a^2*b^2*c^3*f^2 + 32*B^2*a*b^3*d^3*f^2 - 32*B^2*a^3*b*d^3*f^2 - 24*B^2*a^4*c*d^2*f^2 - 24*B^2*b^4*c*d^2*f^2 - 96*B^2*a*b^3*c^2*d*f^2 + 96*B^2*a^3*b*c^2*d*f^2 + 144*B^2*a^2*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(B^4*a^8 + B^4*b^8 + 4*B^4*a^2*b^6 + 6*B^4*a^4*b^4 + 4*B^4*a^6*b^2))^(1/2) - 4*B^2*a^4*c^3*f^2 - 4*B^2*b^4*c^3*f^2 + 24*B^2*a^2*b^2*c^3*f^2 - 16*B^2*a*b^3*d^3*f^2 + 16*B^2*a^3*b*d^3*f^2 + 12*B^2*a^4*c*d^2*f^2 + 12*B^2*b^4*c*d^2*f^2 + 48*B^2*a*b^3*c^2*d*f^2 - 48*B^2*a^3*b*c^2*d*f^2 - 72*B^2*a^2*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) - 32*B*a^2*d^12*f^4 + 32*B*b^2*d^12*f^4 - 96*B*a^2*c^2*d^10*f^4 - 64*B*a^2*c^4*d^8*f^4 + 64*B*a^2*c^6*d^6*f^4 + 96*B*a^2*c^8*d^4*f^4 + 32*B*a^2*c^10*d^2*f^4 + 96*B*b^2*c^2*d^10*f^4 + 64*B*b^2*c^4*d^8*f^4 - 64*B*b^2*c^6*d^6*f^4 - 96*B*b^2*c^8*d^4*f^4 - 32*B*b^2*c^10*d^2*f^4 + 128*B*a*b*c*d^11*f^4 + 512*B*a*b*c^3*d^9*f^4 + 768*B*a*b*c^5*d^7*f^4 + 512*B*a*b*c^7*d^5*f^4 + 128*B*a*b*c^9*d^3*f^4) + (c + d*tan(e + f*x))^(1/2)*(16*B^2*a^4*d^10*f^3 + 16*B^2*b^4*d^10*f^3 - 96*B^2*a^2*b^2*d^10*f^3 + 32*B^2*a^4*c^2*d^8*f^3 - 32*B^2*a^4*c^6*d^4*f^3 - 16*B^2*a^4*c^8*d^2*f^3 + 32*B^2*b^4*c^2*d^8*f^3 - 32*B^2*b^4*c^6*d^4*f^3 - 16*B^2*b^4*c^8*d^2*f^3 + 128*B^2*a*b^3*c*d^9*f^3 - 128*B^2*a^3*b*c*d^9*f^3 + 384*B^2*a*b^3*c^3*d^7*f^3 + 384*B^2*a*b^3*c^5*d^5*f^3 + 128*B^2*a*b^3*c^7*d^3*f^3 - 384*B^2*a^3*b*c^3*d^7*f^3 - 384*B^2*a^3*b*c^5*d^5*f^3 - 128*B^2*a^3*b*c^7*d^3*f^3 - 192*B^2*a^2*b^2*c^2*d^8*f^3 + 192*B^2*a^2*b^2*c^6*d^4*f^3 + 96*B^2*a^2*b^2*c^8*d^2*f^3))*(-(((8*B^2*a^4*c^3*f^2 + 8*B^2*b^4*c^3*f^2 - 48*B^2*a^2*b^2*c^3*f^2 + 32*B^2*a*b^3*d^3*f^2 - 32*B^2*a^3*b*d^3*f^2 - 24*B^2*a^4*c*d^2*f^2 - 24*B^2*b^4*c*d^2*f^2 - 96*B^2*a*b^3*c^2*d*f^2 + 96*B^2*a^3*b*c^2*d*f^2 + 144*B^2*a^2*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(B^4*a^8 + B^4*b^8 + 4*B^4*a^2*b^6 + 6*B^4*a^4*b^4 + 4*B^4*a^6*b^2))^(1/2) - 4*B^2*a^4*c^3*f^2 - 4*B^2*b^4*c^3*f^2 + 24*B^2*a^2*b^2*c^3*f^2 - 16*B^2*a*b^3*d^3*f^2 + 16*B^2*a^3*b*d^3*f^2 + 12*B^2*a^4*c*d^2*f^2 + 12*B^2*b^4*c*d^2*f^2 + 48*B^2*a*b^3*c^2*d*f^2 - 48*B^2*a^3*b*c^2*d*f^2 - 72*B^2*a^2*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*1i - ((-(((8*B^2*a^4*c^3*f^2 + 8*B^2*b^4*c^3*f^2 - 48*B^2*a^2*b^2*c^3*f^2 + 32*B^2*a*b^3*d^3*f^2 - 32*B^2*a^3*b*d^3*f^2 - 24*B^2*a^4*c*d^2*f^2 - 24*B^2*b^4*c*d^2*f^2 - 96*B^2*a*b^3*c^2*d*f^2 + 96*B^2*a^3*b*c^2*d*f^2 + 144*B^2*a^2*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(B^4*a^8 + B^4*b^8 + 4*B^4*a^2*b^6 + 6*B^4*a^4*b^4 + 4*B^4*a^6*b^2))^(1/2) - 4*B^2*a^4*c^3*f^2 - 4*B^2*b^4*c^3*f^2 + 24*B^2*a^2*b^2*c^3*f^2 - 16*B^2*a*b^3*d^3*f^2 + 16*B^2*a^3*b*d^3*f^2 + 12*B^2*a^4*c*d^2*f^2 + 12*B^2*b^4*c*d^2*f^2 + 48*B^2*a*b^3*c^2*d*f^2 - 48*B^2*a^3*b*c^2*d*f^2 - 72*B^2*a^2*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*(32*B*b^2*d^12*f^4 - 32*B*a^2*d^12*f^4 - (c + d*tan(e + f*x))^(

1/2)*(-(((8*B^2*a^4*c^3*f^2 + 8*B^2*b^4*c^3*f^2 - 48*B^2*a^2*b^2*c^3*f^2 +
32*B^2*a*b^3*d^3*f^2 - 32*B^2*a^3*b*d^3*f^2 - 24*B^2*a^4*c*d^2*f^2 - 24*B^2
*b^4*c*d^2*f^2 - 96*B^2*a*b^3*c^2*d*f^2 + 96*B^2*a^3*b*c^2*d*f^2 + 144*B^2*
a^2*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4
*d^2*f^4)*(B^4*a^8 + B^4*b^8 + 4*B^4*a^2*b^6 + 6*B^4*a^4*b^4 + 4*B^4*a^6*b^
2))^(1/2) - 4*B^2*a^4*c^3*f^2 - 4*B^2*b^4*c^3*f^2 + 24*B^2*a^2*b^2*c^3*f^2
- 16*B^2*a*b^3*d^3*f^2 + 16*B^2*a^3*b*d^3*f^2 + 12*B^2*a^4*c*d^2*f^2 + 12*B
^2*b^4*c*d^2*f^2 + 48*B^2*a*b^3*c^2*d*f^2 - 48*B^2*a^3*b*c^2*d*f^2 - 72*B^2
*a^2*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)
))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*
f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) - 96*B*a^2*c^2*d^10*f^4 - 64*B*a^2
*c^4*d^8*f^4 + 64*B*a^2*c^6*d^6*f^4 + 96*B*a^2*c^8*d^4*f^4 + 32*B*a^2*c^10*
d^2*f^4 + 96*B*b^2*c^2*d^10*f^4 + 64*B*b^2*c^4*d^8*f^4 - 64*B*b^2*c^6*d^6*f
^4 - 96*B*b^2*c^8*d^4*f^4 - 32*B*b^2*c^10*d^2*f...

3.118 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=201

$$-\frac{(ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}+\frac{(ia-b)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+f}}{\sqrt{c+id}}\right.}{(c+id)^{3/2}f}$$

```
[Out] -(I*a+b)*(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3
/2)/f+(I*a-b)*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*
d)^(3/2)/f+2*(-a*d+b*c)*(A*d^2-B*c*d+C*c^2)/d^2/(c^2+d^2)/f/(c+d*tan(f*x+e)
)^(1/2)+2*b*C*(c+d*tan(f*x+e))^(1/2)/d^2/f
```

Rubi [A]
time = 0.37, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3716, 3711, 3620, 3618, 65, 214}

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(b+ia)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}+\frac{(-b+ia)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}+\frac{2bC\sqrt{c+d\tan(e+fx)}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x])^(3/2),x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/((c - I*d)^(3/2)*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c + I*d]])/((c + I*d)^(3/2)*f) + (2*(b*c - a*d)*(c^2*C - B*c*d
+ A*d^2))/(d^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + (2*b*C*Sqrt[c + d*
Tan[e + f*x]])/(d^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{2}{}}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC}{}$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC}{}$$

$$= -\frac{(ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{}}\right.}{(c-id)^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.71, size = 290, normalized size = 1.44

$$\frac{(Ab+aB-bC)\left(-\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{i\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)-\frac{2(-2bcC+bBd+2aCd)}{d\sqrt{c+d\tan(e+fx)}}+\frac{(Abc+aBc-bcC-aAd+bBd+aCd)\left((-ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)+(ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)}{(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{2C(a+b\tan(e+fx))}{\sqrt{c+d\tan(e+fx)}}}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((A*b + a*B - b*C)*(((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/
Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c
+ I*d]) - (2*(-2*b*c*C + b*B*d + 2*a*C*d))/(d*Sqrt[c + d*Tan[e + f*x]]) + (
(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*(((-I)*c + d)*Hypergeometri
c2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] + (I*c + d)*Hypergeometr
ic2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]))/((c^2 + d^2)*Sqrt[c +
 d*Tan[e + f*x]]) + (2*C*(a + b*Tan[e + f*x]))/Sqrt[c + d*Tan[e + f*x]])/(d
*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6236 vs. 2(177) = 354.
time = 0.48, size = 6237, normalized size = 31.03

method	result	size
derivativedivides	Expression too large to display	6237
default	Expression too large to display	6237

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2)
,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c +
 d*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 41.07, size = 2500, normalized size = 12.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*t
an(e + f*x))^(3/2),x)
```

```
[Out] atan((((c + d*tan(e + f*x))^(1/2)*(16*A^2*a^2*d^10*f^3 - 16*B^2*a^2*d^10*f^
3 + 16*C^2*a^2*d^10*f^3 + 32*A^2*a^2*c^2*d^8*f^3 - 32*A^2*a^2*c^6*d^4*f^3 -
 16*A^2*a^2*c^8*d^2*f^3 - 32*B^2*a^2*c^2*d^8*f^3 + 32*B^2*a^2*c^6*d^4*f^3 +
 16*B^2*a^2*c^8*d^2*f^3 + 32*C^2*a^2*c^2*d^8*f^3 - 32*C^2*a^2*c^6*d^4*f^3 -
 16*C^2*a^2*c^8*d^2*f^3 - 32*A*C*a^2*d^10*f^3 - 64*A*B*a^2*c*d^9*f^3 + 64*B
*C*a^2*c*d^9*f^3 - 192*A*B*a^2*c^3*d^7*f^3 - 192*A*B*a^2*c^5*d^5*f^3 - 64*A
*B*a^2*c^7*d^3*f^3 - 64*A*C*a^2*c^2*d^8*f^3 + 64*A*C*a^2*c^6*d^4*f^3 + 32*A
*C*a^2*c^8*d^2*f^3 + 192*B*C*a^2*c^3*d^7*f^3 + 192*B*C*a^2*c^5*d^5*f^3 + 64
*B*C*a^2*c^7*d^3*f^3) - ((((8*A^2*a^2*c^3*f^2 - 8*B^2*a^2*c^3*f^2 + 8*C^2*a
^2*c^3*f^2 - 16*A*B*a^2*d^3*f^2 - 16*A*C*a^2*c^3*f^2 + 16*B*C*a^2*d^3*f^2 -
 24*A^2*a^2*c*d^2*f^2 + 24*B^2*a^2*c*d^2*f^2 - 24*C^2*a^2*c*d^2*f^2 + 48*A*
B*a^2*c^2*d*f^2 + 48*A*C*a^2*c*d^2*f^2 - 48*B*C*a^2*c^2*d*f^2)^2/4 - (16*c^
6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(A^4*a^4 + B^4*a^4 +
C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2
*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c^3*f^2 + 4*B^2*a^2*c^3*f^2 -
4*C^2*a^2*c^3*f^2 + 8*A*B*a^2*d^3*f^2 + 8*A*C*a^2*c^3*f^2 - 8*B*C*a^2*d^3*f
^2 + 12*A^2*a^2*c*d^2*f^2 - 12*B^2*a^2*c*d^2*f^2 + 12*C^2*a^2*c*d^2*f^2 - 2
4*A*B*a^2*c^2*d*f^2 - 24*A*C*a^2*c*d^2*f^2 + 24*B*C*a^2*c^2*d*f^2)/(16*(c^6
*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*((c + d*tan(e + f*x
))^(1/2)*((((8*A^2*a^2*c^3*f^2 - 8*B^2*a^2*c^3*f^2 + 8*C^2*a^2*c^3*f^2 - 16
*A*B*a^2*d^3*f^2 - 16*A*C*a^2*c^3*f^2 + 16*B*C*a^2*d^3*f^2 - 24*A^2*a^2*c*d
^2*f^2 + 24*B^2*a^2*c*d^2*f^2 - 24*C^2*a^2*c*d^2*f^2 + 48*A*B*a^2*c^2*d*f^2
```

+ 48*A*C*a^2*c*d^2*f^2 - 48*B*C*a^2*c^2*d*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*
f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C
^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*
B^2*C*a^4))^(1/2) - 4*A^2*a^2*c^3*f^2 + 4*B^2*a^2*c^3*f^2 - 4*C^2*a^2*c^3*f
^2 + 8*A*B*a^2*d^3*f^2 + 8*A*C*a^2*c^3*f^2 - 8*B*C*a^2*d^3*f^2 + 12*A^2*a^2
*c*d^2*f^2 - 12*B^2*a^2*c*d^2*f^2 + 12*C^2*a^2*c*d^2*f^2 - 24*A*B*a^2*c^2*d
*f^2 - 24*A*C*a^2*c*d^2*f^2 + 24*B*C*a^2*c^2*d*f^2)/(16*(c^6*f^4 + d^6*f^4
+ 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5
+ 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) -
32*B*a*d^12*f^4 - 256*A*a*c^3*d^9*f^4 - 384*A*a*c^5*d^7*f^4 - 256*A*a*c^7*d
^5*f^4 - 64*A*a*c^9*d^3*f^4 - 96*B*a*c^2*d^10*f^4 - 64*B*a*c^4*d^8*f^4 + 64
*B*a*c^6*d^6*f^4 + 96*B*a*c^8*d^4*f^4 + 32*B*a*c^10*d^2*f^4 + 256*C*a*c^3*d
^9*f^4 + 384*C*a*c^5*d^7*f^4 + 256*C*a*c^7*d^5*f^4 + 64*C*a*c^9*d^3*f^4 - 6
4*A*a*c*d^11*f^4 + 64*C*a*c*d^11*f^4))*((((8*A^2*a^2*c^3*f^2 - 8*B^2*a^2*c^
3*f^2 + 8*C^2*a^2*c^3*f^2 - 16*A*B*a^2*d^3*f^2 - 16*A*C*a^2*c^3*f^2 + 16*B*
C*a^2*d^3*f^2 - 24*A^2*a^2*c*d^2*f^2 + 24*B^2*a^2*c*d^2*f^2 - 24*C^2*a^2*c*
d^2*f^2 + 48*A*B*a^2*c^2*d*f^2 + 48*A*C*a^2*c*d^2*f^2 - 48*B*C*a^2*c^2*d*f^
2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(A^4*a
^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*
C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c^3*f^2 + 4*B^2
*a^2*c^3*f^2 - 4*C^2*a^2*c^3*f^2 + 8*A*B*a^2*d^3*f^2 + 8*A*C*a^2*c^3*f^2 -
8*B*C*a^2*d^3*f^2 + 12*A^2*a^2*c*d^2*f^2 - 12*B^2*a^2*c*d^2*f^2 + 12*C^2*a^
2*c*d^2*f^2 - 24*A*B*a^2*c^2*d*f^2 - 24*A*C*a^2*c*d^2*f^2 + 24*B*C*a^2*c^2*
d*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*1i +
((c + d*tan(e + f*x))^(1/2)*(16*A^2*a^2*d^10*f^3 - 16*B^2*a^2*d^10*f^3 + 1
6*C^2*a^2*d^10*f^3 + 32*A^2*a^2*c^2*d^8*f^3 - 32*A^2*a^2*c^6*d^4*f^3 - 16*A
^2*a^2*c^8*d^2*f^3 - 32*B^2*a^2*c^2*d^8*f^3 + 32*B^2*a^2*c^6*d^4*f^3 + 16*B
^2*a^2*c^8*d^2*f^3 + 32*C^2*a^2*c^2*d^8*f^3 - 32*C^2*a^2*c^6*d^4*f^3 - 16*C
^2*a^2*c^8*d^2*f^3 - 32*A*C*a^2*d^10*f^3 - 64*A*B*a^2*c*d^9*f^3 + 64*B*C*a^
2*c*d^9*f^3 - 192*A*B*a^2*c^3*d^7*f^3 - 192*A*B*a^2*c^5*d^5*f^3 - 64*A*B*a^
2*c^7*d^3*f^3 - 64*A*C*a^2*c^2*d^8*f^3 + 64*A*C*a^2*c^6*d^4*f^3 + 32*A*C*a^
2*c^8*d^2*f^3 + 192*B*C*a^2*c^3*d^7*f^3 + 192*B*C*a^2*c^5*d^5*f^3 + 64*B*C*
a^2*c^7*d^3*f^3) - ((((8*A^2*a^2*c^3*f^2 - 8*B^2*a^2*c^3*f^2 + 8*C^2*a^2*c^
3*f^2 - 16*A*B*a^2*d^3*f^2 - 16*A*C*a^2*c^3*f^2 + 16*B*C*a^2*d^3*f^2 - 24*A
^2*a^2*c*d^2*f^2 + 24*B^2*a^2*c*d^2*f^2 - 24*C^2*a^2*c*d^2*f^2 + 48*A*B*a^2
*c^2*d*f^2 + 48*A*C*a^2*c*d^2*f^2 - 48*B*C*a^2*c^2*d*f^2)^2/4 - (16*c^6*f^4
+ 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a
^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*
a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c^3*f^2 + 4*B^2*a^2*c^3*f^2 - 4*C^2
*a^2*c^3*f^2 + 8*A*B*a^2*d^3*f^2 + 8*A*C*a^2*c^3*f^2 - 8*B*C*a^2*d^3*f^2 +
12*A^2*a^2*c*d^2*f^2 - 12*B^2*a^2*c*d^2*f^2 + 12*C^2*a^2*c*d^2*f^2 - 24*A*B
*a^2*c^2*d*f^2 - 24*A*C*a^2*c*d^2*f^2 + 24*B*C*a^2*c^2*d*f^2)/(16*(c^6*f^4
+ d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*((c + d*tan(e + f*x))^(1
/2)*((((8*A^2*a^2*c^3*f^2 - 8*B^2*a^2*c^3*f^2 + 8*C^2*a^2*c^3*f^2 - 16*A*B*
a^2*d^3*f^2 - 16*A*C*a^2*c^3*f^2 + 16*B*C*a^2*d...

3.119 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f}-\frac{}{d\,(c^2 -}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f-
(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(3/2)/f-2
*(A*d^2-B*c*d+C*c^2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {3709, 3620, 3618, 65, 214}

$$-\frac{2(Ad^2-Bcd+c^2C)}{df\,(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{3/2}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I
*d)^(3/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]])/((c + I*d)^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^2))/(d*(c^2 + d^2)*f*S
qrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{\int \frac{Ac-cC+Bd+(Bc-(A-C)}{\sqrt{c+d\tan(e+}}}{c^2+d^2}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(A-iB-C)\int \frac{}{\sqrt{c+}}}{2(c-}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(iA+B-iC)\mathrm{Subst}\Big(}{}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} - \frac{(A-iB-C)\mathrm{Subst}\Big(}{}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} - \frac{(B-i(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.70, size = 218, normalized size = 1.39

$$\frac{-iB\left(\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)-\frac{2C}{\sqrt{c+d\tan(e+fx)}}+\frac{(Bc+(-A+C)d)\left((-ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)+(ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)}{(c^2+d^2)\sqrt{c+d\tan(e+fx)}}}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(3/2
),x]
```

```
[Out] ((-I)*B*(ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]/Sqrt[c - I*d] - Ar
cTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]/Sqrt[c + I*d]) - (2*C)/Sqrt[c
 + d*Tan[e + f*x]] + ((B*c + (-A + C)*d)*(((-I)*c + d)*Hypergeometric2F1[-1
/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] + (I*c + d)*Hypergeometric2F1[-
1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]))/((c^2 + d^2)*Sqrt[c + d*Tan[
e + f*x]]))/(d*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3123 vs. 2(136) = 272.
time = 0.45, size = 3124, normalized size = 19.90

method	result	size
derivativedivides	Expression too large to display	3124
default	Expression too large to display	3124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/d*(d/(c^2+d^2)*(1/4/d^2/(3*c^2-d^2)/(c^2+d^2)^(3/2)*(1/2*(-3*A*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d-2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)*c^3*d^3+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
c*d^5+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*c^4*d^3-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^5+A*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*d^7-B*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4+B*(c^2+d^
2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*c^6-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d
^2-3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^4+6*B*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c^5*d^2+4*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^4-2*B*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^6+3*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*c^5*d+2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^3-C
*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^5-3*C*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*c^6*d+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^3+3*C*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*c^2*d^5-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^7)*ln(d*tan(f*x+
e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+
```

2*(12*A*c^5*d^3+8*A*c^3*d^5-4*A*c*d^7-6*B*c^6*d^2+2*B*c^4*d^4+6*B*c^2*d^6-2
*B*d^8-12*C*c^5*d^3-8*C*c^3*d^5+4*C*c*d^7+1/2*(-3*A*(c^2+d^2)^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c^5*d-2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*c^3*d^3+A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^5+3*A*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^3-3*A*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^5+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^7-B
(c^2+d^2)^(3/2)(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4+B*(c^2+d^2)^(3/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*d^4+B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)
*c^6-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^2-3*B*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^4+6*B*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*c^5*d^2+4*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^4-2*B*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*c*d^6+3*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d
+2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^3-C*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^5-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6
d+C(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^3+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*c^2*d^5-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^7)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+1/4/d^2/(3*c^2-d^2)/(
c^2+d^2)^(3/2)*(1/2*(3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*
d+2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^3-A*(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^5-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^
6*d+A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^3+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*c^2*d^5-A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^7+B*(c^2+d^2)^(3/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c^4-B*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4
-B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6+2*B*(c^2+d^2)^(1/2)*(2
*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^2+3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*c^2*d^4-6*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d^2-4*B*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c^3*d^4+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^6-3*C*(c^
2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d-2*C*(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^3+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*c*d^5+3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d-C*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c^4*d^3-3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^5+C*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*d^7)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+2*(12*A*c^5*d^3+8*A*c^3*d^5-4*A*c*d^7-6*B*
c^6*d^2+2*B*c^4*d^4+6*B*c^2*d^6-2*B*d^8-12*C*c^5*d^3-8*C*c^3*d^5+4*C*c*d^7-
1/2*(3*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d+2*A*(c^2+d^2)^
(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^3-A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*c*d^5-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d+A*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c^4*d^3+3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^5-A*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*d^7+B*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*c^4-B*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^4-B*(c^2+d^2)^(1/
2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6+2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*c^4*d^2+3*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d
^4-6*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d^2-4*B*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*c^3*d^4+2*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^6-3*C*(c^2+d^2)^(1/2)*(2*

```
(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d-2*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c^3*d^3+C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^5+3*C*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d-C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^3-
3*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^5+C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d
^7)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*
(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)}{\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c + d*tan(e + f*x))**(3/
2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 19.61, size = 2500, normalized size = 15.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^(3/2),x)
```

```
[Out] (log((((((((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2)
 - 4*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3
*c^4*d^2*f^4))^(1/2)*(64*C*c*d^11*f^4 - ((c + d*tan(e + f*x))^(1/2)*(((96*C
^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) - 4*C^2*c^3*f^
2 + 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^
(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5
 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5))/4 + 256*C*c^3*d^9*f^4 + 384*C*c^5*d^
7*f^4 + 256*C*c^7*d^5*f^4 + 64*C*c^9*d^3*f^4))/4 + (c + d*tan(e + f*x))^(1/
2)*(16*C^2*d^10*f^3 + 32*C^2*c^2*d^8*f^3 - 32*C^2*c^6*d^4*f^3 - 16*C^2*c^8*
d^2*f^3))*(((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/
2) - 4*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 +
 3*c^4*d^2*f^4))^(1/2))/4 - 8*C^3*d^9*f^2 - 24*C^3*c^2*d^7*f^2 - 24*C^3*c^4
*d^5*f^2 - 8*C^3*c^6*d^3*f^2)*(((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*
C^4*c^4*d^2*f^4)^(1/2) - 4*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f
^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 + (log(((((-((96*C^4*c^2*d^4*
f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) + 4*C^2*c^3*f^2 - 12*C^2*
c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(64*C
*c*d^11*f^4 - ((c + d*tan(e + f*x))^(1/2)*(-((96*C^4*c^2*d^4*f^4 - 16*C^4*d
^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) + 4*C^2*c^3*f^2 - 12*C^2*c*d^2*f^2)/(c^
6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(64*c*d^12*f^5 + 32
0*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c
^11*d^2*f^5))/4 + 256*C*c^3*d^9*f^4 + 384*C*c^5*d^7*f^4 + 256*C*c^7*d^5*f^4
 + 64*C*c^9*d^3*f^4))/4 + (c + d*tan(e + f*x))^(1/2)*(16*C^2*d^10*f^3 + 32*
C^2*c^2*d^8*f^3 - 32*C^2*c^6*d^4*f^3 - 16*C^2*c^8*d^2*f^3))*(-((96*C^4*c^2*
d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) + 4*C^2*c^3*f^2 - 12*
C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/
4 - 8*C^3*d^9*f^2 - 24*C^3*c^2*d^7*f^2 - 24*C^3*c^4*d^5*f^2 - 8*C^3*c^6*d^3
*f^2)*(-((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2)
+ 4*C^2*c^3*f^2 - 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*
c^4*d^2*f^4))^(1/2))/4 - log(((((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*
```

C^4*c^4*d^2*f^4)^(1/2) - 4*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(16*c^6*f^4 + 16
*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2)*((c + d*tan(e + f*x))^(1
/2)*(((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) - 4
*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4
+ 48*c^4*d^2*f^4))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^
5 + 640*c^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) + 64*C*c*d^11*f^4
+ 256*C*c^3*d^9*f^4 + 384*C*c^5*d^7*f^4 + 256*C*c^7*d^5*f^4 + 64*C*c^9*d^3*
f^4) - (c + d*tan(e + f*x))^(1/2)*(16*C^2*d^10*f^3 + 32*C^2*c^2*d^8*f^3 - 3
2*C^2*c^6*d^4*f^3 - 16*C^2*c^8*d^2*f^3))*(((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6
*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) - 4*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(16*c
^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2) - 8*C^3*d^9*f
^2 - 24*C^3*c^2*d^7*f^2 - 24*C^3*c^4*d^5*f^2 - 8*C^3*c^6*d^3*f^2)*(((96*C^4
*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) - 4*C^2*c^3*f^2
+ 12*C^2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*
f^4))^(1/2) - log(((-((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^
2*f^4)^(1/2) + 4*C^2*c^3*f^2 - 12*C^2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4 +
48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(-((96
*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) + 4*C^2*c^3*
f^2 - 12*C^2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*
d^2*f^4))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c
^7*d^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5) + 64*C*c*d^11*f^4 + 256*C*c
^3*d^9*f^4 + 384*C*c^5*d^7*f^4 + 256*C*c^7*d^5*f^4 + 64*C*c^9*d^3*f^4) - (c
+ d*tan(e + f*x))^(1/2)*(16*C^2*d^10*f^3 + 32*C^2*c^2*d^8*f^3 - 32*C^2*c^6
*d^4*f^3 - 16*C^2*c^8*d^2*f^3))*(-((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 1
44*C^4*c^4*d^2*f^4)^(1/2) + 4*C^2*c^3*f^2 - 12*C^2*c*d^2*f^2)/(16*c^6*f^4 +
16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(1/2) - 8*C^3*d^9*f^2 - 24*
C^3*c^2*d^7*f^2 - 24*C^3*c^4*d^5*f^2 - 8*C^3*c^6*d^3*f^2)*(-((96*C^4*c^2*d^
4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) + 4*C^2*c^3*f^2 - 12*C^
2*c*d^2*f^2)/(16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4))^(
1/2) + (log(8*A^3*d^9*f^2 - ((((((96*A^4*c^2*d^4*f^4 - 16*A^4*d^6*f^4 - 144
*A^4*c^4*d^2*f^4)^(1/2) - 4*A^2*c^3*f^2 + 12*A^2*c*d^2*f^2)/(c^6*f^4 + d^6*
f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(((((96*A^4*c^2*d^4*f^4 - 16*A^
4*d^6*f^4 - 144*A^4*c^4*d^2*f^4)^(1/2) - 4*A^2*c^3*f^2 + 12*A^2*c*d^2*f^2)/
(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2)*(c + d*tan(e + f
x))^(1/2)(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d^
6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5))/4 + 64*A*c*d^11*f^4 + 256*A*c^3
*d^9*f^4 + 384*A*c^5*d^7*f^4 + 256*A*c^7*d^5*f^...

3.120 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{3/2}f}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)(c+id)^{3/2}f}-\frac{2\sqrt{b}\,(A\ldots}{}$$

```
[Out] (A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/(c-I*d)^(3/
2)/f+(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)/(c+I
*d)^(3/2)/f-2*(A*b^2-a*(B*b-C*a))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-
a*d+b*c)^(1/2))*b^(1/2)/(a^2+b^2)/(-a*d+b*c)^(3/2)/f+2*(A*d^2-B*c*d+C*c^2)/
(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.87, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2\sqrt{b}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)(bc-ad)^{3/2}}+\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)(c-id)^{3/2}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])^(3/2)),x]
```

```
[Out] ((A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b)*
(c - I*d)^(3/2)*f) + ((I*A - B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c + I*d]])/((a + I*b)*(c + I*d)^(3/2)*f) - (2*Sqrt[b]*(A*b^2 - a*(b*B - a*
C))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^
2)*(b*c - a*d)^(3/2)*f) + (2*(c^2*C - B*c*d + A*d^2))/((b*c - a*d)*(c^2 + d
^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2(c^2C-Bcd+Ad^2)}{(bc-ad)\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{\frac{1}{2}(-aAcd-}{}}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)}{(bc-ad)\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(b(Ab^2-a(}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)}{(bc-ad)\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(A-iB-}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)}{(bc-ad)\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{(iA+B-}{}$$

$$= -\frac{2\sqrt{b}\,(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+}}{\sqrt{bc-ad}}\right.}{(a^2+b^2)\,(bc-ad)^{3/2}f}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)(c-id)^{3/2}f} - \frac{(A}{}$$

Mathematica [A]
time = 3.28, size = 296, normalized size = 1.13

$$-\frac{\frac{i\left(\frac{(a+ib)(A-iB-C)(c+id)(-bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{(a-ib)(A+iB-C)(c-id)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right)}{a^2+b^2}+\frac{2\sqrt{b}\,(Ab^2+a(-bB+aC))(c^2+d^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)\sqrt{bc-ad}}-\frac{2(c^2C-Bcd+Ad^2)}{\sqrt{c+d\tan(e+fx)}}}{(-bc+ad)\,(c^2+d^2)\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (((-I)*(((a + I*b)*(A - I*B - C)*(c + I*d)*(-(b*c) + a*d)*ArcTanh[Sqrt[c +
d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + ((a - I*b)*(A + I*B - C)*(c
 - I*d)*(b*c - a*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c
 + I*d]))/(a^2 + b^2) + (2*Sqrt[b]*(A*b^2 + a*(-(b*B) + a*C))*(c^2 + d^2)*A
rcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)*Sq
rt[b*c - a*d]) - (2*(c^2*C - B*c*d + A*d^2))/Sqrt[c + d*Tan[e + f*x]])/((-(
b*c) + a*d)*(c^2 + d^2)*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6343 vs. $2(229) = 458$.
time = 0.63, size = 6344, normalized size = 24.21

method	result	size
derivativedivides	Expression too large to display	6344
default	Expression too large to display	6344

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2)
,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan{(e + fx)} + C\tan^{2}{(e + fx)}}{(a + b\tan{(e + fx)})\,(c + d\tan{(e + fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))*(c
+ d*tan(e + f*x))**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*ta
n(e + f*x))^(3/2)),x)
```

```
[Out] \text{Hanged}
```

3.121 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=447

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{3/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2(c+id)^{3/2}f}-\frac{\sqrt{b}\,(5}{}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/(c-I*d
)^(3/2)/f-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)
^2/(c+I*d)^(3/2)/f-(5*a^3*b*B*d-3*a^4*C*d+b^4*(-3*A*d+2*B*c)+a*b^3*(4*A*c+B
*d-4*C*c)-a^2*b^2*(2*B*c+(7*A-C)*d))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)
/(-a*d+b*c)^(1/2))*b^(1/2)/(a^2+b^2)^2/(-a*d+b*c)^(5/2)/f-d*(2*b^2*c*(-B*d+
C*c)-a*b*B*(c^2+d^2)+a^2*(-2*B*c*d+3*C*c^2+C*d^2)+A*(2*a^2*d^2+b^2*(c^2+3*d
^2)))/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)+(-A*b^2+a*(
B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

Rubi [A]
time = 1.92, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f\left(a^2+b^2\right)\left(c^2+d^2\right)(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-\frac{\sqrt{b}\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4Ac+Bd-4cC)+b^4(2Bc-3Ad)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)^2(bc-ad)^{5/2}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(a-ib)^2(c-id)^{3/2}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)^2(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*(c - I*d)^(3/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/((a + I*b)^2*(c + I*d)^(3/2)*f) - (Sqrt[b]*(5*a^3*b*B*d
- 3*a^4*C*d + b^4*(2*B*c - 3*A*d) + a*b^3*(4*A*c - 4*c*C + B*d) - a^2*b^2*(
2*B*c + (7*A - C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c -
 a*d]])/((a^2 + b^2)^2*(b*c - a*d)^(5/2)*f) - (d*(2*a^2*A*d^2 + 2*b^2*c*(c*
C - B*d) - a*b*B*(c^2 + d^2) + A*b^2*(c^2 + 3*d^2) + a^2*(3*c^2*C - 2*B*c*d
 + C*d^2)))/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x
]]) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x
])*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
```

```
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{Ab^2-a(bB-aC)}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))\sqrt{c+d\tan(e+}}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-abB(c^2+d^2)+Ab^2(c^2+}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+}}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-abB(c^2+d^2)+Ab^2(c^2+}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+}}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-abB(c^2+d^2)+Ab^2(c^2+}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+}}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-abB(c^2+d^2)+Ab^2(c^2+}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+}}$$

$$= -\frac{\sqrt{b}\,(5a^3bBd-3a^4Cd+b^4(2Bc-3Ad)+ab^3(4Ac-4}{(a^2}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{3/2}f} - \frac{(B-}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2078 vs. 2(447) = 894.
time = 6.22, size = 2078, normalized size = 4.65

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*S
qrt[c + d*Tan[e + f*x]])) - ((-2*(((I*Sqrt[c - I*d]*((b*(-(b*c) + a*d)*((-3
*(A*b^2 - a*(b*B - a*C))*d^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A
*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 + a*(-1/2*(a
*d*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d)))
+ (((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B
 - a*C)*(2*b*c + a*d)))/2 - (b*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (
A*b - a*B - b*C)*d*(b*c - a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b
*B - a*C)*(2*b*c + a*d)))/2))/2) - I*((a*(-(b*c) + a*d)*((-3*(A*b^2 - a*(b*
B - a*C))*d^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d - 2*a*A*
(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 - b*(-1/2*(a*d*((-3*c*(A*b^
2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (((b*d^2)/2 -
 (c*(-(b*c) + a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c
+ a*d)))/2 - (b*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C
)*d*(b*c - a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*
c + a*d)))/2))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((-c +
 I*d)*f) - (I*Sqrt[c + I*d]*((b*(-(b*c) + a*d)*((-3*(A*b^2 - a*(b*B - a*C))
*d^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d - 2*a*A*(b*c - a*
d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 + a*(-1/2*(a*d*((-3*c*(A*b^2 - a*(b*
B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (((b*d^2)/2 - (c*(-(b*
c) + a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/
2 - (b*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c
- a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d))
)/2))/2) + I*((a*(-(b*c) + a*d)*((-3*(A*b^2 - a*(b*B - a*C))*d^2)/2 - c*(A*
b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C
)*(2*b*c + a*d)))/2))/2 - b*(-1/2*(a*d*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2
+ (A*b - a*B - b*C)*d*(b*c - a*d))) + (((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(
3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2 - (b*(-(c*((-
3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (d^2
*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2)))*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) +
(2*Sqrt[b*c - a*d]*(-1/2*(a*b*(-(b*c) + a*d)*((-3*(A*b^2 - a*(b*B - a*C))*d
^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d - 2*a*A*(b*c - a*d)
 - (b*B - a*C)*(2*b*c + a*d)))/2)) + (a^2*b*(-(c*((-3*c*(A*b^2 - a*(b*B - a
*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*
c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 + b^2*(-1/2*(a*d*((-3*c*(A*b^2
 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (((b*d^2)/2 -
(c*(-(b*c) + a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c +
 a*d)))/2))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(S
qrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f)))/((-(b*c) + a*d)*(c^2 + d^2)) - (2*(-
(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d)))
+ (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/((-
(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/((a^2 + b^2)*(b*c - a
*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10087 vs. $2(411) = 822$.
time = 0.65, size = 10088, normalized size = 22.57

method	result	size
derivativedivides	Expression too large to display	10088
default	Expression too large to display	10088

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d*
tan(e + f*x))^(3/2)),x)
```

```
[Out] \text{Hanged}
```

3.122 $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=585

$$-\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}-\frac{(ia-b)^3(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+}}{\sqrt{c+id}}\right.}{(c+id)^{5/2}f}$$

```
[Out] -(a-I*b)^3*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d
)^(5/2)/f-(I*a-b)^3*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))
/(c+I*d)^(5/2)/f+2/3*b*(3*a*b*d*(8*c^4*C-2*B*c^3*d-c^2*(A-17*C)*d^2-8*B*c*d
^3+(5*A+3*C)*d^4)-b^2*(16*c^5*C-8*B*c^4*d+2*c^3*(A+15*C)*d^2-17*B*c^2*d^3+8
*c*(A+C)*d^4-3*B*d^5)+6*a^2*d^3*(2*c*(A-C)*d-B*(c^2-d^2)))*(c+d*tan(f*x+e))
^(1/2)/d^4/(c^2+d^2)^2/f+2/3*b^2*(b*(8*c^4*C-4*B*c^3*d+c^2*(A+15*C)*d^2-10*
B*c*d^3+(7*A+C)*d^4)+3*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(c+d*tan(f*x+e))^(1
/2)*tan(f*x+e)/d^3/(c^2+d^2)^2/f-2*(b*(2*A*d^4-B*c^3*d-3*B*c*d^3+2*C*c^4+4*
C*c^2*d^2)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^2/d^2/(c^2+d^2
)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^3/d/(
c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 2.01, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3726, 3718, 3711, 3620, 3618, 65, 214}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((a - I*b)^3*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(5/2)*f)) - ((I*a - b)^3*(A + I*B - C)*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^
2)*(a + b*Tan[e + f*x])^3)/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) -
 (2*(b*(2*c^4*C - B*c^3*d + 4*c^2*C*d^2 - 3*B*c*d^3 + 2*A*d^4) + a*d^2*(2*c
*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^2)/(d^2*(c^2 + d^2)^2*f*S
qrt[c + d*Tan[e + f*x]]) + (2*b*(3*a*b*d*(8*c^4*C - 2*B*c^3*d - c^2*(A - 17
*C)*d^2 - 8*B*c*d^3 + (5*A + 3*C)*d^4) - b^2*(16*c^5*C - 8*B*c^4*d + 2*c^3*
(A + 15*C)*d^2 - 17*B*c^2*d^3 + 8*c*(A + C)*d^4 - 3*B*d^5) + 6*a^2*d^3*(2*c
*(A - C)*d - B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(3*d^4*(c^2 + d^2)^2
*f) + (2*b^2*(b*(8*c^4*C - 4*B*c^3*d + c^2*(A + 15*C)*d^2 - 10*B*c*d^3 + (7
*A + C)*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Tan[e + f*x]*Sqrt[c
 + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{(a-ib)^3(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.}{(c-id)^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 6.59, size = 670, normalized size = 1.15

$$\frac{2C(a+b\tan(e+fx))^3}{3df(c+d\tan(e+fx))^{3/2}}+\frac{2\left(-\frac{3(2bcC-bBd-2aCd)(a+b\tan(e+fx))^2}{df(c+d\tan(e+fx))^{3/2}}+\frac{2\left(-\frac{3\left(b(Ab+aB-bC)d^2+4(bc-ad)(2bcC-bBd-2aCd)\right)(a+b\tan(e+fx))}{2df(c+d\tan(e+fx))^{3/2}}-\frac{3\left(-\frac{2\left(-16b^3c^3C+8b^3Bc^2d+48ab^2c^2Cd-2Ab^3cd^2-18ab^2Bcd^2-48a^2bcCd^2+2b^3cCd^2+9a^2bBd^3+b^3Bd^3+16a^3Cd^3\right)}{3d(c+d\tan(e+fx))^{3/2}}+\frac{2\left(\frac{\left(\frac{3}{2}c\left(a^3B-3ab^2B+3a^2b(A-C)-b^3(A-C)\right)d^4+\frac{3}{2}\left(3a^2bB-b^3B-a^3(A-C)+3ab^2(A-C)\right)d^5\right)\left(-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)}{3(ic+d)(c+d\tan(e+fx))^{3/2}}+\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)}{3(ic-d)(c+d\tan(e+fx))^{3/2}}\right)}{d}-\frac{3}{2}\left(a^3B-3ab^2B+3a^2b(A-C)-b^3(A-C)\right)d^3\left(-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)}{(ic+d)\sqrt{c+d\tan(e+fx)}}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)}{(ic-d)\sqrt{c+d\tan(e+fx)}}\right)\right)}{3d}\right)}{4df}\right)}{d}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*C*(a + b*Tan[e + f*x])^3)/(3*d*f*(c + d*Tan[e + f*x])^(3/2)) + (2*((-3*(
2*b*c*C - b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2)/(d*f*(c + d*Tan[e + f*x]
)^(3/2)) + (2*((-3*(b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - b*B*
d - 2*a*C*d))*(a + b*Tan[e + f*x]))/(2*d*f*(c + d*Tan[e + f*x])^(3/2)) - (3
*((-2*(-16*b^3*c^3*C + 8*b^3*B*c^2*d + 48*a*b^2*c^2*C*d - 2*A*b^3*c*d^2 - 1
8*a*b^2*B*c*d^2 - 48*a^2*b*c*C*d^2 + 2*b^3*c*C*d^2 + 9*a^2*b*B*d^3 + b^3*B*
d^3 + 16*a^3*C*d^3))/(3*d*(c + d*Tan[e + f*x])^(3/2)) + (2*((((3*c*(a^3*B -
 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C))*d^4)/2 + (3*(3*a^2*b*B - b^3*B
- a^3*(A - C) + 3*a*b^2*(A - C))*d^5)/2)*(-1/3*Hypergeometric2F1[-3/2, 1, -
1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*(c + d*Tan[e + f*x])^(3/2))
 + Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]/(3*(I*c
 - d)*(c + d*Tan[e + f*x])^(3/2))))/d - (3*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A
- C) - b^3*(A - C))*d^3*(-(Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f
*x])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e + f*x]])) + Hypergeometric2F1[-
1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/((I*c - d)*Sqrt[c + d*Tan[e +
f*x]])))/2))/(3*d)))/(4*d*f)))/d))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21767 vs. $2(550) = 1100$.
time = 0.65, size = 21768, normalized size = 37.21

method	result	size
derivativedivides	Expression too large to display	21768
default	Expression too large to display	21768

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.123 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=358

$$-\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}-\frac{(a+ib)^2(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e}}{\sqrt{c+ic}}\right.}{(c+id)^{5/2}f}$$

```
[Out] -(a-I*b)^2*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d
)^(5/2)/f-(a+I*b)^2*(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))/(c+I*d)^(5/2)/f+2/3*(-a*d+b*c)*(b*(4*c^4*C-B*c^3*d-2*c^2*(A-5*C)*d^2-7*B
*c*d^3+4*A*d^4)+3*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/d^3/(c^2+d^2)^2/f/(c+d*t
an(f*x+e))^(1/2)+2/3*b^2*(4*c^2*C-B*c*d+(A+3*C)*d^2)*(c+d*tan(f*x+e))^(1/2)
/d^3/(c^2+d^2)/f-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(
c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 1.04, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3726, 3716, 3711, 3620, 3618, 65, 214}

$$-\frac{2(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^2}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}+\frac{2(bc-ad)(3ad^2(2cd(A-C)-B(c^2-d^2))+b(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C))}{3d^3f(c^2+d^2)^2\sqrt{c+d\tan(e+fx)}}-\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}-\frac{(a+ib)^2(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{5/2}}+\frac{2b^2(d^2(A+3C)-Bcd+4c^2C)\sqrt{c+d\tan(e+fx)}}{3d^3f(c^2+d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((a - I*b)^2*(I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*
d]])/((c - I*d)^(5/2)*f)) - ((a + I*b)^2*(B - I*(A - C))*ArcTanh[Sqrt[c + d
*Tan[e + f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*
d^2)*(a + b*Tan[e + f*x])^2)/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
 + (2*(b*c - a*d)*(b*(4*c^4*C - B*c^3*d - 2*c^2*(A - 5*C)*d^2 - 7*B*c*d^3 +
 4*A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/(3*d^3*(c^2 + d^2)^2*
f*Sqrt[c + d*Tan[e + f*x]]) + (2*b^2*(4*c^2*C - B*c*d + (A + 3*C)*d^2)*Sqrt
[c + d*Tan[e + f*x]])/(3*d^3*(c^2 + d^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
```

```
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f\ldots}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/\ldots}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f\ldots}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/\ldots}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f\ldots}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/\ldots}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f\ldots}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/\ldots}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f\ldots}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/\ldots}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+f\ldots}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/\ldots}}$$

$$= -\frac{(a-ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c\ldots}}{\ldots}\right.}{(c-id)^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.68, size = 414, normalized size = 1.16

$$-\frac{-2(c-id)(c+id)\left(8a^2Cd^2+abd(-16cC+Bd)+b^2(8c^2C-2Bcd+(-A+C)d^2)\right)-d^2\left(-2ab(Ac-cC+Bd)-a^2(Bc+(-A+C)d)+b^2(Bc+(-A+C)d)\right)\left(i(c+id)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)-6(c-id)(c+id)d(4bcC-bBd-4aCd)(a+b\tan(e+fx))-6C(c-id)(c+id)d^2(a+b\tan(e+fx))^2-3(a^2B-b^2B+2ab(A-C))\,d^2\left(i(c+id)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)(c+d\tan(e+fx))}{3d^3\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/3*(-2*(c - I*d)*(c + I*d)*(8*a^2*C*d^2 + a*b*d*(-16*c*C + B*d) + b^2*(8*
c^2*C - 2*B*c*d + (-A + C)*d^2)) - d^2*(-2*a*b*(A*c - c*C + B*d) - a^2*(B*c
 + (-A + C)*d) + b^2*(B*c + (-A + C)*d))*(I*(c + I*d)*Hypergeometric2F1[-3/
2, 1, -1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1[-
3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]) - 6*(c - I*d)*(c + I*d)*d*(4
*b*c*C - b*B*d - 4*a*C*d)*(a + b*Tan[e + f*x]) - 6*C*(c - I*d)*(c + I*d)*d^
2*(a + b*Tan[e + f*x])^2 - 3*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2*(I*(c + I*
d)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c +
 d)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)])*(c + d
*Tan[e + f*x]))/(d^3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15608 vs. $2(325) = 650$.
time = 0.70, size = 15609, normalized size = 43.60

method	result	size
derivativedivides	Expression too large to display	15609
default	Expression too large to display	15609

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^2 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 116.90, size = 2500, normalized size = 6.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^(5/2),x)
```

```
[Out] atan((((c + d*tan(e + f*x))^(1/2)*(96*A^2*a^2*b^2*d^18*f^3 - 16*A^2*b^4*d^1
8*f^3 - 16*A^2*a^4*d^18*f^3 + 320*A^2*a^4*c^4*d^14*f^3 + 1024*A^2*a^4*c^6*d
^12*f^3 + 1440*A^2*a^4*c^8*d^10*f^3 + 1024*A^2*a^4*c^10*d^8*f^3 + 320*A^2*a
^4*c^12*d^6*f^3 - 16*A^2*a^4*c^16*d^2*f^3 + 320*A^2*b^4*c^4*d^14*f^3 + 1024
*A^2*b^4*c^6*d^12*f^3 + 1440*A^2*b^4*c^8*d^10*f^3 + 1024*A^2*b^4*c^10*d^8*f
^3 + 320*A^2*b^4*c^12*d^6*f^3 - 16*A^2*b^4*c^16*d^2*f^3 - 256*A^2*a*b^3*c*d
^17*f^3 + 256*A^2*a^3*b*c*d^17*f^3 - 1280*A^2*a*b^3*c^3*d^15*f^3 - 2304*A^2
*a*b^3*c^5*d^13*f^3 - 1280*A^2*a*b^3*c^7*d^11*f^3 + 1280*A^2*a*b^3*c^9*d^9*
f^3 + 2304*A^2*a*b^3*c^11*d^7*f^3 + 1280*A^2*a*b^3*c^13*d^5*f^3 + 256*A^2*a
```

*b^3*c^15*d^3*f^3 + 1280*A^2*a^3*b*c^3*d^15*f^3 + 2304*A^2*a^3*b*c^5*d^13*f^3 + 1280*A^2*a^3*b*c^7*d^11*f^3 - 1280*A^2*a^3*b*c^9*d^9*f^3 - 2304*A^2*a^3*b*c^11*d^7*f^3 - 1280*A^2*a^3*b*c^13*d^5*f^3 - 256*A^2*a^3*b*c^15*d^3*f^3 - 1920*A^2*a^2*b^2*c^4*d^14*f^3 - 6144*A^2*a^2*b^2*c^6*d^12*f^3 - 8640*A^2*a^2*b^2*c^8*d^10*f^3 - 6144*A^2*a^2*b^2*c^10*d^8*f^3 - 1920*A^2*a^2*b^2*c^12*d^6*f^3 + 96*A^2*a^2*b^2*c^16*d^2*f^3) + ((((8*A^2*a^4*c^5*f^2 + 8*A^2*b^4*c^5*f^2 - 48*A^2*a^2*b^2*c^5*f^2 - 80*A^2*a^4*c^3*d^2*f^2 - 80*A^2*b^4*c^3*d^2*f^2 - 32*A^2*a*b^3*d^5*f^2 + 32*A^2*a^3*b*d^5*f^2 + 40*A^2*a^4*c*d^4*f^2 + 40*A^2*b^4*c*d^4*f^2 - 160*A^2*a*b^3*c^4*d*f^2 + 160*A^2*a^3*b*c^4*d*f^2 + 320*A^2*a*b^3*c^2*d^3*f^2 - 240*A^2*a^2*b^2*c*d^4*f^2 - 320*A^2*a^3*b*c^2*d^3*f^2 + 480*A^2*a^2*b^2*c^3*d^2*f^2)^2/4 - (A^4*a^8 + A^4*b^8 + 4*A^4*a^2*b^6 + 6*A^4*a^4*b^4 + 4*A^4*a^6*b^2)*(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2) - 4*A^2*a^4*c^5*f^2 - 4*A^2*b^4*c^5*f^2 + 24*A^2*a^2*b^2*c^5*f^2 + 40*A^2*a^4*c^3*d^2*f^2 + 40*A^2*b^4*c^3*d^2*f^2 + 16*A^2*a*b^3*d^5*f^2 - 16*A^2*a^3*b*d^5*f^2 - 20*A^2*a^4*c*d^4*f^2 - 20*A^2*b^4*c*d^4*f^2 + 80*A^2*a*b^3*c^4*d*f^2 - 80*A^2*a^3*b*c^4*d*f^2 - 160*A^2*a*b^3*c^2*d^3*f^2 + 120*A^2*a^2*b^2*c*d^4*f^2 + 160*A^2*a^3*b*c^2*d^3*f^2 - 240*A^2*a^2*b^2*c^3*d^2*f^2)/(16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4)))^(1/2)*(32*A*b^2*d^21*f^4 - 32*A*a^2*d^21*f^4 - (c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^4*c^5*f^2 + 8*A^2*b^4*c^5*f^2 - 48*A^2*a^2*b^2*c^5*f^2 - 80*A^2*a^4*c^3*d^2*f^2 - 80*A^2*b^4*c^3*d^2*f^2 - 32*A^2*a*b^3*d^5*f^2 + 32*A^2*a^3*b*d^5*f^2 + 40*A^2*a^4*c*d^4*f^2 + 40*A^2*b^4*c*d^4*f^2 - 160*A^2*a*b^3*c^4*d*f^2 + 160*A^2*a^3*b*c^4*d*f^2 + 320*A^2*a*b^3*c^2*d^3*f^2 - 240*A^2*a^2*b^2*c*d^4*f^2 - 320*A^2*a^3*b*c^2*d^3*f^2 + 480*A^2*a^2*b^2*c^3*d^2*f^2)^2/4 - (A^4*a^8 + A^4*b^8 + 4*A^4*a^2*b^6 + 6*A^4*a^4*b^4 + 4*A^4*a^6*b^2)*(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2) - 4*A^2*a^4*c^5*f^2 - 4*A^2*b^4*c^5*f^2 + 24*A^2*a^2*b^2*c^5*f^2 + 40*A^2*a^4*c^3*d^2*f^2 + 40*A^2*b^4*c^3*d^2*f^2 + 16*A^2*a*b^3*d^5*f^2 - 16*A^2*a^3*b*d^5*f^2 - 20*A^2*a^4*c*d^4*f^2 - 20*A^2*b^4*c*d^4*f^2 + 80*A^2*a*b^3*c^4*d*f^2 - 80*A^2*a^3*b*c^4*d*f^2 - 160*A^2*a*b^3*c^2*d^3*f^2 + 120*A^2*a^2*b^2*c*d^4*f^2 + 160*A^2*a^3*b*c^2*d^3*f^2 - 240*A^2*a^2*b^2*c^3*d^2*f^2)/(16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4)))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5) - 160*A*a^2*c^2*d^19*f^4 - 128*A*a^2*c^4*d^17*f^4 + 896*A*a^2*c^6*d^15*f^4 + 3136*A*a^2*c^8*d^13*f^4 + 4928*A*a^2*c^10*d^11*f^4 + 4480*A*a^2*c^12*d^9*f^4 + 2432*A*a^2*c^14*d^7*f^4 + 736*A*a^2*c^16*d^5*f^4 + 96*A*a^2*c^18*d^3*f^4 + 160*A*b^2*c^2*d^19*f^4 + 128*A*b^2*c^4*d^17*f^4 - 896*A*b^2*c^6*d^15*f^4 - 3136*A*b^2*c^8*d^13*f^4 - 4928*A*b^2*c^10*d^11*f^4 - 4480*A*b^2*c^12*d^9*f^4 - 2432*A*b^2*c^14*d^7*f^4 - 736*A*b^2*c^16*d^5*f^4 - 96*A*b^2*c^18*d^3*f^4 + 192*A*a*b*c*d^20*f^4 + 1472*A*a*b*c^3*d^18*f^4 + 4864*A*a*b*c^5*d^16*f^4 + 8960*A*a*b*c^7*d^14*f^4 + 9856*A*a*b*c^9*d^12*f^4 + 6272*A*a*b*c^11*d^10*f^4 + 179

```
2*A*a*b*c^13*d^8*f^4 - 256*A*a*b*c^15*d^6*f^4 - 320*A*a*b*c^17*d^4*f^4 - 64
*A*a*b*c^19*d^2*f^4))*((((8*A^2*a^4*c^5*f^2 + 8*A^2*b^4*c^5*f^2 - 48*A^2*a^
2*b^2*c^5*f^2 - 80*A^2*a^4*c^3*d^2*f^2 - 80*A^2*b^4*c^3*d^2*f^2 - 32*A^2*a*
b^3*d^5*f^2 + 32*A^2*a^3*b*d^5*f^2 + 40*A^2*a^4*c*d^4*f^2 + 40*A^2*b^4*c*d^
4*f^2 - 160*A^2*a*b^3*c^4*d*f^2 + 160*A^2*a^3*b*c^4*d*f^2 + 320*A^2*a*b^3*c
^2*d^3*f^2 - 240*A^2*a^2*b^2*c*d^4*f^2 - 320*A^2*a^3*b*c^2*d^3*f^2 + 480*A^
2*a^2*b^2*c^3*d^2*f^2)^2/4 - (A^4*a^8 + A^4*b^8 + 4*A^4*a^2*b^6 + 6*A^4*a^4
*b^4 + 4*A^4*a^6*b^2)*(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4
*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2) - 4*A^2*a^4*c^5*f^2 - 4
*A^2*b^4*c^5*f^2 + 24*A^2*a^2*b^2*c^5*f^2 + 40*A^2*a^4*c^3*d^2*f^2 + 40*A^2
*b^4*c^3*d^2*f^2 + 16*A^2*a*b^3*d^5*f^2 - 16*A^2*a^3*b*d^5*f^2 - 20*A^2*a^4
*c*d^4*f^2 - 20*A^2*b^4*c*d^4*f^2 + 80*A^2*a*b^3*c^4*d*f^2 - 80*A^2*a^3*b*c
^4*d*f^2 - 160*A^2*a*b^3*c^2*d^3*f^2 + 120*A^2*...
```

3.124 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=273

$$-\frac{(a-ib)(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}+\frac{(ia-b)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+}}{\sqrt{c+id}}\right.}{(c+id)^{5/2}f}$$

```
[Out] -(a-I*b)*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^
(5/2)/f+(I*a-b)*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+
I*d)^(5/2)/f-2*(b*(c^4*C-c^2*(A-3*C)*d^2-2*B*c*d^3+A*d^4)+a*d^2*(2*c*(A-C)*
d-B*(c^2-d^2)))/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*(-a*d+b*c)*(A*
d^2-B*c*d+C*c^2)/d^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.54, antiderivative size = 271, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3716, 3709, 3620, 3618, 65, 214}

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{d^2f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{(b+ia)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}+\frac{(-b+ia)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*T
an[e + f*x])^(5/2),x]
```

```
[Out] -(((I*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/((c - I*d)^(5/2)*f)) + ((I*a - b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e +
 f*x]]/Sqrt[c + I*d]])/((c + I*d)^(5/2)*f) + (2*(b*c - a*d)*(c^2*C - B*c*d
+ A*d^2))/(3*d^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*(b*(c^4*C -
 c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d
^2))))/(d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3716

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Dist[1/(d*(c^2
 + d^2)), Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*(
c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*
Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n,
 -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} +$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} -$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} -$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} -$$

$$= \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}} -$$

$$= -\frac{(ia+b)(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+}}{}\right.}{(c-id)^{5/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.96, size = 300, normalized size = 1.10

$$-\frac{2(c-id)(c+id)(2bcC+bBd-2aCd)+d(Abc+aBc-bcC-aAd+bBd+aCd)\left(i(c+id)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)+6C(c-id)(c+id)d(a+b\tan(e+fx))-3(Ab+aB-bC)d\left(i(c+id)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)(c+d\tan(e+fx))}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/3*(2*(c - I*d)*(c + I*d)*(2*b*c*C + b*B*d - 2*a*C*d) + d*(A*b*c + a*B*c
- b*c*C - a*A*d + b*B*d + a*C*d)*(I*(c + I*d)*Hypergeometric2F1[-3/2, 1, -1
/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1[-3/2, 1,
-1/2, (c + d*Tan[e + f*x])/(c + I*d)]) + 6*C*(c - I*d)*(c + I*d)*d*(a + b*T
an[e + f*x]) - 3*(A*b + a*B - b*C)*d*(I*(c + I*d)*Hypergeometric2F1[-1/2, 1
, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1[-1/2,
1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)])*(c + d*Tan[e + f*x]))/(d^2*(c^2 +
d^2)*f*(c + d*Tan[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9671 vs. $2(246) = 492$.
time = 0.55, size = 9672, normalized size = 35.43

method	result	size
derivativedivides	Expression too large to display	9672
default	Expression too large to display	9672

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2)
,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan{(e + fx)})\,(A + B\tan{(e + fx)} + C\tan^2{(e + fx)})}{(c + d\tan{(e + fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)
)**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c +
 d*tan(e + f*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 88.47, size = 2500, normalized size = 9.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*t
an(e + f*x))^(5/2),x)
```

```
[Out] ((2*(C*b*c^3 + A*b*c*d^2 - B*b*c^2*d))/(3*(c^2 + d^2)) - (2*(c + d*tan(e +
f*x))*(A*b*d^4 + C*b*c^4 - 2*B*b*c*d^3 - A*b*c^2*d^2 + 3*C*b*c^2*d^2))/(c^2
 + d^2)^2)/(d^2*f*(c + d*tan(e + f*x))^(3/2)) - atan(-(((c + d*tan(e + f*x)
)^(1/2)*(16*A^2*b^2*d^18*f^3 - 16*B^2*b^2*d^18*f^3 + 16*C^2*b^2*d^18*f^3 -
320*A^2*b^2*c^4*d^14*f^3 - 1024*A^2*b^2*c^6*d^12*f^3 - 1440*A^2*b^2*c^8*d^1
0*f^3 - 1024*A^2*b^2*c^10*d^8*f^3 - 320*A^2*b^2*c^12*d^6*f^3 + 16*A^2*b^2*c
^16*d^2*f^3 + 320*B^2*b^2*c^4*d^14*f^3 + 1024*B^2*b^2*c^6*d^12*f^3 + 1440*B
^2*b^2*c^8*d^10*f^3 + 1024*B^2*b^2*c^10*d^8*f^3 + 320*B^2*b^2*c^12*d^6*f^3
- 16*B^2*b^2*c^16*d^2*f^3 - 320*C^2*b^2*c^4*d^14*f^3 - 1024*C^2*b^2*c^6*d^1
2*f^3 - 1440*C^2*b^2*c^8*d^10*f^3 - 1024*C^2*b^2*c^10*d^8*f^3 - 320*C^2*b^2
*c^12*d^6*f^3 + 16*C^2*b^2*c^16*d^2*f^3 - 32*A*C*b^2*d^18*f^3 - 128*A*B*b^2
*c*d^17*f^3 + 128*B*C*b^2*c*d^17*f^3 - 640*A*B*b^2*c^3*d^15*f^3 - 1152*A*B*
b^2*c^5*d^13*f^3 - 640*A*B*b^2*c^7*d^11*f^3 + 640*A*B*b^2*c^9*d^9*f^3 + 115
2*A*B*b^2*c^11*d^7*f^3 + 640*A*B*b^2*c^13*d^5*f^3 + 128*A*B*b^2*c^15*d^3*f^
3 + 640*A*C*b^2*c^4*d^14*f^3 + 2048*A*C*b^2*c^6*d^12*f^3 + 2880*A*C*b^2*c^8
*d^10*f^3 + 2048*A*C*b^2*c^10*d^8*f^3 + 640*A*C*b^2*c^12*d^6*f^3 - 32*A*C*b
^2*c^16*d^2*f^3 + 640*B*C*b^2*c^3*d^15*f^3 + 1152*B*C*b^2*c^5*d^13*f^3 + 64
0*B*C*b^2*c^7*d^11*f^3 - 640*B*C*b^2*c^9*d^9*f^3 - 1152*B*C*b^2*c^11*d^7*f^
3 - 640*B*C*b^2*c^13*d^5*f^3 - 128*B*C*b^2*c^15*d^3*f^3) + ((((8*A^2*b^2*c^
5*f^2 - 8*B^2*b^2*c^5*f^2 + 8*C^2*b^2*c^5*f^2 - 80*A^2*b^2*c^3*d^2*f^2 + 80
*B^2*b^2*c^3*d^2*f^2 - 80*C^2*b^2*c^3*d^2*f^2 + 16*A*B*b^2*d^5*f^2 - 16*A*C
*b^2*c^5*f^2 - 16*B*C*b^2*d^5*f^2 + 40*A^2*b^2*c*d^4*f^2 - 40*B^2*b^2*c*d^4
```

*f^2 + 40*C^2*b^2*c*d^4*f^2 + 80*A*B*b^2*c^4*d*f^2 - 80*A*C*b^2*c*d^4*f^2 - 80*B*C*b^2*c^4*d*f^2 - 160*A*B*b^2*c^2*d^3*f^2 + 160*A*C*b^2*c^3*d^2*f^2 + 160*B*C*b^2*c^2*d^3*f^2)^2/4 - (16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4)*(A^4*b^4 + B^4*b^4 + C^4*b^4 - 4*A*C^3*b^4 - 4*A^3*C*b^4 + 2*A^2*B^2*b^4 + 6*A^2*C^2*b^4 + 2*B^2*C^2*b^4 - 4*A*B^2*C*b^4))^(1/2) + 4*A^2*b^2*c^5*f^2 - 4*B^2*b^2*c^5*f^2 + 4*C^2*b^2*c^5*f^2 - 40*A^2*b^2*c^3*d^2*f^2 + 40*B^2*b^2*c^3*d^2*f^2 - 40*C^2*b^2*c^3*d^2*f^2 + 8*A*B*b^2*d^5*f^2 - 8*A*C*b^2*c^5*f^2 - 8*B*C*b^2*d^5*f^2 + 20*A^2*b^2*c*d^4*f^2 - 20*B^2*b^2*c*d^4*f^2 + 20*C^2*b^2*c*d^4*f^2 + 40*A*B*b^2*c^4*d*f^2 - 40*A*C*b^2*c*d^4*f^2 - 40*B*C*b^2*c^4*d*f^2 - 80*A*B*b^2*c^2*d^3*f^2 + 80*A*C*b^2*c^3*d^2*f^2 + 80*B*C*b^2*c^2*d^3*f^2)/(16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4)))^(1/2)*(128*A*b*c^15*d^6*f^4 - 32*B*b*d^21*f^4 - 736*A*b*c^3*d^18*f^4 - 2432*A*b*c^5*d^16*f^4 - 4480*A*b*c^7*d^14*f^4 - 4928*A*b*c^9*d^12*f^4 - 3136*A*b*c^11*d^10*f^4 - 896*A*b*c^13*d^8*f^4 - (c + d*tan(e + f*x))^(1/2)*((((8*A^2*b^2*c^5*f^2 - 8*B^2*b^2*c^5*f^2 + 8*C^2*b^2*c^5*f^2 - 80*A^2*b^2*c^3*d^2*f^2 + 80*B^2*b^2*c^3*d^2*f^2 - 80*C^2*b^2*c^3*d^2*f^2 + 16*A*B*b^2*d^5*f^2 - 16*A*C*b^2*c^5*f^2 - 16*B*C*b^2*d^5*f^2 + 40*A^2*b^2*c*d^4*f^2 - 40*B^2*b^2*c*d^4*f^2 + 40*C^2*b^2*c*d^4*f^2 + 80*A*B*b^2*c^4*d*f^2 - 80*A*C*b^2*c*d^4*f^2 - 80*B*C*b^2*c^4*d*f^2 - 160*A*B*b^2*c^2*d^3*f^2 + 160*A*C*b^2*c^3*d^2*f^2 + 160*B*C*b^2*c^2*d^3*f^2)^2/4 - (16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4 + 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4)*(A^4*b^4 + B^4*b^4 + C^4*b^4 - 4*A*C^3*b^4 - 4*A^3*C*b^4 + 2*A^2*B^2*b^4 + 6*A^2*C^2*b^4 + 2*B^2*C^2*b^4 - 4*A*B^2*C*b^4))^(1/2) + 4*A^2*b^2*c^5*f^2 - 4*B^2*b^2*c^5*f^2 + 4*C^2*b^2*c^5*f^2 - 40*A^2*b^2*c^3*d^2*f^2 + 40*B^2*b^2*c^3*d^2*f^2 - 40*C^2*b^2*c^3*d^2*f^2 + 8*A*B*b^2*d^5*f^2 - 8*A*C*b^2*c^5*f^2 - 8*B*C*b^2*d^5*f^2 + 20*A^2*b^2*c*d^4*f^2 - 20*B^2*b^2*c*d^4*f^2 + 20*C^2*b^2*c*d^4*f^2 + 40*A*B*b^2*c^4*d*f^2 - 40*A*C*b^2*c*d^4*f^2 - 40*B*C*b^2*c^4*d*f^2 - 80*A*B*b^2*c^2*d^3*f^2 + 80*A*C*b^2*c^3*d^2*f^2 + 80*B*C*b^2*c^2*d^3*f^2)/(16*(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4)))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5) + 160*A*b*c^17*d^4*f^4 + 32*A*b*c^19*d^2*f^4 - 160*B*b*c^2*d^19*f^4 - 128*B*b*c^4*d^17*f^4 + 896*B*b*c^6*d^15*f^4 + 3136*B*b*c^8*d^13*f^4 + 4928*B*b*c^10*d^11*f^4 + 4480*B*b*c^12*d^9*f^4 + 2432*B*b*c^14*d^7*f^4 + 736*B*b*c^16*d^5*f^4 + 96*B*b*c^18*d^3*f^4 + 736*C*b*c^3*d^18*f^4 + 2432*C*b*c^5*d^16*f^4 + 4480*C*b*c^7*d^14*f^4 + 4928*C*b*c^9*d^12*f^4 + 3136*C*b*c^11*d^10*f^4 + 896*C*b*c^13*d^8*f^4 - 128*C*b*c^15*d^6*f^4 - 160*C*b*c^17*d^4*f^4 - 32*C*b*c^19*d^2*f^4 - 96*A*b*c*d^20*f^4 + 96*C*b*c*d^20*f^4))*((((8*A^2*b^2*c^5*f^2 - 8*B^2*b^2*c^5*f^2 + 8*C^2*b^2*c^5*f^2 - 80*A^2*b^2*c^3*d^2*f^2 + 80*B^2*b^2*c^3*d^2*f^2 - 80*C^2*b^2*c^3*d^2*f^2 + 16*A*B*b^2*d^5*f^2 - 16*A*C*b^2*c^5*f^2 - 16*B*C*b^2*d^5*f^2 + 40*A^2*b^2*c*d^4*f^2 - 40*B^2*b^2*c*d^4*f^2 + 40*C^2*b^2*c*d^4*f^2 + 80*A*B*b^2*c^4*d*f^2 - 80*A*C*b^2*c*d^4*f^2 - 80*B*C*b^2*c^4*...

3.125 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f}-\frac{\ldots}{3d\,(c\ldots}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f-
(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(5/2)/f-2
*(2*c*(A-C)*d-B*(c^2-d^2))/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*d^2-
B*c*d+C*c^2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.32, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3709, 3610, 3620, 3618, 65, 214}

$$-\frac{2(Ad^2-Bcd+c^2C)}{3df\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}-\frac{2(2cd(A-C)-B(c^2-d^2))}{f\,(c^2+d^2)^2\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(c-id)^{5/2}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((c - I
*d)^(5/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c +
I*d]])/((c + I*d)^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^2))/(3*d*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])^(3/2)) - (2*(2*c*(A - C)*d - B*(c^2 - d^2)))/((c^2 +
d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3709

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{\int \frac{Ac-cC+Bd+(Bc-(A-}{(c+d\tan(e+f}}{c^2+d}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(2c(A-C)d-}{(c^2+d^2)^2 f\sqrt{c+d}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(2c(A-C)d-}{(c^2+d^2)^2 f\sqrt{c+d}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(2c(A-C)d-}{(c^2+d^2)^2 f\sqrt{c+d}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} - \frac{2(2c(A-C)d-}{(c^2+d^2)^2 f\sqrt{c+d}}$$

$$= -\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} - \frac{(B-i}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.65, size = 223, normalized size = 1.07

$$-\frac{2C(c^2+d^2)+(Bc+(-A+C)d)\left(i(c+id)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)-3B\left(i(c+id)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{c+d\tan(e+fx)}{c+id}\right)\right)(c+d\tan(e+fx))}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(5/2
),x]
```

```
[Out] -1/3*(2*C*(c^2 + d^2) + (B*c + (-A + C)*d)*(I*(c + I*d)*Hypergeometric2F1[-
3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1
[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]) - 3*B*(I*(c + I*d)*Hyperge
ometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hyperg
eometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)])*(c + d*Tan[e + f
*x]))/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4917 vs. 2(184) = 368.
time = 0.46, size = 4918, normalized size = 23.53

method	result	size
derivativedivides	Expression too large to display	4918
default	Expression too large to display	4918

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 2/f/d*(-1/3*(A*d^2-B*c*d+C*c^2)/(c^2+d^2)/(c+d*tan(f*x+e))^(3/2)-d*(2*A*c*d
-B*c^2+B*d^2-2*C*c*d)/(c^2+d^2)^2/(c+d*tan(f*x+e))^(1/2)+d/(c^2+d^2)^2*(1/4
/d/(5*c^4-10*c^2*d^2+d^4)/(c^2+d^2)^(3/2)*(1/2*(-10*C*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^4+10*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*
c)^(1/2)*c^2*d^6-5*A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^2-
A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^4+18*A*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d^2+10*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*c^4*d^4-10*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
c^2*d^6+C*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^4-18*C*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d^2-10*B*(c^2+d^2)^(1/2)*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*c^7*d+10*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*c^5*d^3+18*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^5-2*
B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^7+5*C*(c^2+d^2)^(3/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^2+12*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2
*d^7+3*C*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6-C*(c^2+d^2)^(3/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^6+15*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^8*
d+5*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^9-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^9
-5*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^9-22*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c
^4*d^5-20*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d^3+2*C*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*c^8+20*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^7*d^2-6*C
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d^4-3*A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)*c^6-28*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^6+3*C*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c*d^8+A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d
^6-2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^8-20*A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*c^7*d^2+6*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d^4+28*A*(2*
(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^6-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^8)
*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^
2+d^2)^(1/2))+2*(30*A*c^8*d^2-40*A*c^6*d^4-44*A*c^4*d^6+24*A*c^2*d^8-2*A*d^
10-10*B*c^9*d+40*B*c^7*d^3-12*B*c^5*d^5-56*B*c^3*d^7+6*B*c*d^9-30*C*c^8*d^2
+40*C*c^6*d^4+44*C*c^4*d^6-24*C*c^2*d^8+2*C*d^10+1/2*(-10*C*(c^2+d^2)^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^4+10*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*c^2*d^6-5*A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^
4*d^2-A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^4+18*A*(c^2+d^2
)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d^2+10*A*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c^4*d^4-10*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
```

```
(1/2)*c^2*d^6+C*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^4-18*C*
(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d^2-10*B*(c^2+d^2)^(1/2)*
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^7*d+10*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)
+2*c)^(1/2)*c^5*d^3+18*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*
d^5-2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^7+5*C*(c^2+d^2)^(
3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^2+12*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*c^2*d^7+3*C*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6-C*(c^2+d^2
)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*d^6+15*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)*c^8*d+5*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^9-B*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*d^9-5*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^9-22*B*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*c^4*d^5-20*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d^3+2*C*(c^2+d^2)^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^8+20*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^7*d
^2-6*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d^4-3*A*(c^2+d^2)^(3/2)*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c^6-28*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^6+3*C*(2*(
c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^8+A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)*d^6-2*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^8-20*A*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)*c^7*d^2+6*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^5*d^4+28
*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3*d^6-3*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
c*d^8)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(
(2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2)))+1/4/d/(5*c^4-10*c^2*d^2+d^4)/(c^2+d^2)^(3/2)*(1/2*(10*C*(c^2+
d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^4-10*C*(c^2+d^2)^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^6+5*A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c
)^(1/2)*c^4*d^2+A*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2*d^4-18*
A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d^2-10*A*(c^2+d^2)^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^4*d^4+10*A*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(
1/2)+2*c)^(1/2)*c^2*d^6-C*(c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2
*d^4+18*C*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^6*d^2+10*B*(c^2+d
^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^7*d-10*B*(c^2+d^2)^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)*c^5*d^3-18*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*c^3*d^5+2*B*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c*d^7-5*C*(
c^2+d^2)^(3/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^...
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan{(e + fx)} + C\tan^2{(e + fx)}}{(c + d\tan{(e + fx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c + d*tan(e + f*x))**(5/
2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 37.59, size = 2500, normalized size = 11.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^(5/2),x)
```

```
[Out] (log(96*A^3*c^3*d^13*f^2 - ((((((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 17
60*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) - 4*
```

A^2*c^5*f^2 + 40*A^2*c^3*d^2*f^2 - 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(((((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) - 4*A^2*c^5*f^2 + 40*A^2*c^3*d^2*f^2 - 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5))/4 - 32*A*d^21*f^4 - 160*A*c^2*d^19*f^4 - 128*A*c^4*d^17*f^4 + 896*A*c^6*d^15*f^4 + 3136*A*c^8*d^13*f^4 + 4928*A*c^10*d^11*f^4 + 4480*A*c^12*d^9*f^4 + 2432*A*c^14*d^7*f^4 + 736*A*c^16*d^5*f^4 + 96*A*c^18*d^3*f^4))/4 - (c + d*tan(e + f*x))^(1/2)*(320*A^2*c^4*d^14*f^3 - 16*A^2*d^18*f^3 + 1024*A^2*c^6*d^12*f^3 + 1440*A^2*c^8*d^10*f^3 + 1024*A^2*c^10*d^8*f^3 + 320*A^2*c^12*d^6*f^3 - 16*A^2*c^16*d^2*f^3))*(((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) - 4*A^2*c^5*f^2 + 40*A^2*c^3*d^2*f^2 - 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2))/4 + 240*A^3*c^5*d^11*f^2 + 320*A^3*c^7*d^9*f^2 + 240*A^3*c^9*d^7*f^2 + 96*A^3*c^11*d^5*f^2 + 16*A^3*c^13*d^3*f^2 + 16*A^3*c*d^15*f^2)*(((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) - 4*A^2*c^5*f^2 + 40*A^2*c^3*d^2*f^2 - 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2))/4 + (log(96*A^3*c^3*d^13*f^2 - ((((-((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) + 4*A^2*c^5*f^2 - 40*A^2*c^3*d^2*f^2 + 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(((-((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) + 4*A^2*c^5*f^2 - 40*A^2*c^3*d^2*f^2 + 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5))/4 - 32*A*d^21*f^4 - 160*A*c^2*d^19*f^4 - 128*A*c^4*d^17*f^4 + 896*A*c^6*d^15*f^4 + 3136*A*c^8*d^13*f^4 + 4928*A*c^10*d^11*f^4 + 4480*A*c^12*d^9*f^4 + 2432*A*c^14*d^7*f^4 + 736*A*c^16*d^5*f^4 + 96*A*c^18*d^3*f^4))/4 - (c + d*tan(e + f*x))^(1/2)*(320*A^2*c^4*d^14*f^3 - 16*A^2*d^18*f^3 + 1024*A^2*c^6*d^12*f^3 + 1440*A^2*c^8*d^10*f^3 + 1024*A^2*c^10*d^8*f^3 + 320*A^2*c^12*d^6*f^3 - 16*A^2*c^16*d^2*f^3))*(-((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) + 4*A^2*c^5*f^2 - 40*A^2*c^3*d^2*f^2 + 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2))/4 + 240*A^3*c^5*d^11*f^2 + 320*A^3*c^7*d^9*f^2 + 240*A^3*c^9*d^7*f^2 + 96*A^3*c^11*d^5*f^2 + 16*A^3*c^13*d^3*f^2 + 16*A^3*c*d^15*f^2)*(-((320*A^4

```
*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^
4 - 400*A^4*c^8*d^2*f^4)^(1/2) + 4*A^2*c^5*f^2 - 40*A^2*c^3*d^2*f^2 + 20*A^
2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6
*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2))/4 - log(96*A^3*c^3*d^13*f^2 - ((((320*A^4
*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^
4 - 400*A^4*c^8*d^2*f^4)^(1/2) - 4*A^2*c^5*f^2 + 40*A^2*c^3*d^2*f^2 - 20*A^
2*c*d^4*f^2)/(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4
+ 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2)*(896*A*c^6*d^15*f^4 - (((320*A^4
*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^
4 - 400*A^4*c^8*d^2*f^4)^(1/2) - 4*A^2*c^5*f^2 + 40*A^2*c^3*d^2*f^2 - 20*A^
2*c*d^4*f^2)/(16*c^10*f^4 + 16*d^10*f^4 + 80*c^2*d^8*f^4 + 160*c^4*d^6*f^4
+ 160*c^6*d^4*f^4 + 80*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(64*c
*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f^5 + 1344
0*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680*c^15*d^8*
f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5) - 160*A*c^2*d
^19*f^4 - 128*A*c^4*d^17*f^4 - 32*A*d^21*f^4 + 3136*A*c^8*d^13*f^4 + 4928*A
*c^10*d^11*f^4 + 4480*A*c^12*d^9*f^4 + 2432*A*c^14*d^7*f^4 + 736*A*c^16*d^5
*f^4 + 96*A*c^18*d^3*f^4) + (c + d*tan(e + f*x))^(1/2)*(320*A^2*c^4*d^14*f^
3 - 16*A^2*d^18*f^3 + 1024*A^2*c^6*d^12*f^3 + 1440*A^2*c^8*d^10*f^3 + 1024*
A^2*c^10*d^8*f^3 + 320*A^2*c^12*d^6*f^3 - 16*A^...
```

3.126 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=365

$$\frac{(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{5/2}f}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)(c+id)^{5/2}f}-\frac{2b^{3/2}(Ab}{}$$

```
[Out] (A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/(c-I*d)^(5/
2)/f+(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)/(c+I
*d)^(5/2)/f-2*b^(3/2)*(A*b^2-a*(B*b-C*a))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^
(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)/(-a*d+b*c)^(5/2)/f+2*(b*(c^4*C-2*B*c^3*d+
c^2*(3*A-C)*d^2+A*d^4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/(-a*d+b*c)^2/(c^2+d
^2)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*(A*d^2-B*c*d+C*c^2)/(-a*d+b*c)/(c^2+d^2)
/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [**A**]
time = 1.69, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3730, 3734, 3620, 3618, 65, 214, 3715}

$$-\frac{2b^{3/2}(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f(a^2+b^2)(bc-ad)^{5/2}}+\frac{2(Ad^2-Bcd+c^2C)}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2(b(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C)-ad^2(2cd(A-C)-B(c^2-d^2)))}{f(c^2+d^2)^2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}+\frac{(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f(b+ia)(c-id)^{5/2}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f(a+ib)(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*Ta
n[e + f*x])^(5/2)),x]
```

```
[Out] ((A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((I*a + b)*
(c - I*d)^(5/2)*f) + ((I*A - B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c + I*d]])/((a + I*b)*(c + I*d)^(5/2)*f) - (2*b^(3/2)*(A*b^2 - a*(b*B - a*
C))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^
2)*(b*c - a*d)^(5/2)*f) + (2*(c^2*C - B*c*d + A*d^2))/(3*(b*c - a*d)*(c^2 +
 d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + (2*(b*(c^4*C - 2*B*c^3*d + c^2*(3*A -
 C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/((b*c - a*d)^2*(
c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
```

```
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2(c^2C-Bcd+Ad^2)}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{-\frac{3}{2}(aA}{}}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(c^4C-}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(c^4C-}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(c^4C-}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(c^4C-}{}$$

$$= -\frac{2b^{3/2}(Ab^2-a(bB-aC))\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d\tan(e+}}{\sqrt{bc-ad}}\right.}{(a^2+b^2)\,(bc-ad)^{5/2}f}$$

$$= \frac{(A-iB-C)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{5/2}f} - \frac{(A+i}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1948 vs. $2(365) = 730$.
time = 6.19, size = 1948, normalized size = 5.34

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
+ d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (-2*(A*d^2 - c*(-(c*C) + B*d)))/(3*(-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[
e + f*x])^(3/2)) - (2*((-2*(((I*Sqrt[c - I*d]*((b*(-(b*c) + a*d)*((-3*c*(b*
```

```
c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*(c^2*C - B*c*d + A*d^2))/2 - (3*d*(a
*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2))/2 + a*((-3*((b*d^2)/2 - (c
*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - (a*d
*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2
))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - c*((3
*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2)))/
2) - I*((a*(-(b*c) + a*d)*((-3*c*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*
(c^2*C - B*c*d + A*d^2))/2 - (3*d*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d
^2)))/2))/2 - b*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C
 - B*d) - A*b*(c^2 + d^2)))/2 - (a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2
 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c*C
- B*d) - A*b*(c^2 + d^2)))/2 - c*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (
3*b*c*(c^2*C - B*c*d + A*d^2))/2)))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/S
qrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*(-(b*c) + a*d)*((-3*c*
(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*(c^2*C - B*c*d + A*d^2))/2 - (3*d
*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2))/2 + a*((-3*((b*d^2)/2 -
 (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - (
a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2)
)/2))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - c*
((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2)
))/2) + I*((a*(-(b*c) + a*d)*((-3*c*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b
*d*(c^2*C - B*c*d + A*d^2))/2 - (3*d*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2
+ d^2)))/2))/2 - b*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(
c*C - B*d) - A*b*(c^2 + d^2)))/2 - (a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d)
)/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c
*C - B*d) - A*b*(c^2 + d^2)))/2 - c*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2
- (3*b*c*(c^2*C - B*c*d + A*d^2))/2)))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]
]/Sqrt[c + I*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[b*c - a*d]*(-1/2*(a
*b*(-(b*c) + a*d)*((-3*c*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*(c^2*C -
 B*c*d + A*d^2))/2 - (3*d*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2)
) + (a^2*b*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - c*((
3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2)))
/2 + b^2*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C - B*d)
 - A*b*(c^2 + d^2)))/2 - (a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b
*c*(c^2*C - B*c*d + A*d^2))/2))/2))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x
]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f)))/((-(b*c) + a
*d)*(c^2 + d^2)) - (2*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)
))/2 - c*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A
*d^2))/2)))/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/(3*(-
(b*c) + a*d)*(c^2 + d^2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10188 vs. $2(328) = 656$.
time = 0.61, size = 10189, normalized size = 27.92

method	result	size
derivativedivides	Expression too large to display	10189
default	Expression too large to display	10189

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2)
,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*ta
n(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.127 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=679

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{5/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2(c+id)^{5/2}f}-\frac{b^{3/2}(}{}$$

```
[Out] -(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/(c-I*d
)^(5/2)/f-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)
^2/(c+I*d)^(5/2)/f-b^(3/2)*(7*a^3*b*B*d-5*a^4*C*d+b^4*(-5*A*d+2*B*c)+a*b^3*
(4*A*c+3*B*d-4*C*c)-a^2*b^2*(2*B*c+(9*A+C)*d))*arctanh(b^(1/2)*(c+d*tan(f*x
+e))^(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)^2/(-a*d+b*c)^(7/2)/f-d*(2*a^3*d^2*(B
*c^2-B*d^2+2*C*c*d)+2*b^3*c*(-3*B*c^2*d-B*d^3+2*C*c^3)-a*b^2*(B*c^4+3*B*d^4
-4*C*c*d^3)+a^2*b*(-6*B*c^3*d-2*B*c*d^3+5*C*c^4+2*C*c^2*d^2+C*d^4)-A*(4*a^3
*c*d^3+4*a*b^2*c*d^3-4*a^2*b*d^2*(2*c^2+d^2)-b^3*(c^4+10*c^2*d^2+5*d^4)))/(
a^2+b^2)/(-a*d+b*c)^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-1/3*d*(2*b^2*c*(
-B*d+C*c)-3*a*b*B*(c^2+d^2)+a^2*(-2*B*c*d+5*C*c^2+3*C*d^2)+A*(2*a^2*d^2+b^2
*(3*c^2+5*d^2)))/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)+
(-A*b^2+a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e
))^(3/2)
```

Rubi [A]
time = 3.47, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {3730, 3734, 3620, 3618, 65, 214, 3715}

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d*
Tan[e + f*x])^(5/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/((a - I
*b)^2*(c - I*d)^(5/2)*f)) - ((B - I*(A - C))*ArcTanh[Sqrt[c + d*Tan[e + f*x
]]/Sqrt[c + I*d]])/((a + I*b)^2*(c + I*d)^(5/2)*f) - (b^(3/2)*(7*a^3*b*B*d
- 5*a^4*C*d + b^4*(2*B*c - 5*A*d) + a*b^3*(4*A*c - 4*c*C + 3*B*d) - a^2*b^2
*(2*B*c + (9*A + C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c
 - a*d]])/((a^2 + b^2)^2*(b*c - a*d)^(7/2)*f) - (d*(2*a^2*A*d^2 + 2*b^2*c*(
c*C - B*d) - 3*a*b*B*(c^2 + d^2) + A*b^2*(3*c^2 + 5*d^2) + a^2*(5*c^2*C - 2
*B*c*d + 3*C*d^2)))/(3*(a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*Tan[e
 + f*x])^(3/2)) - (A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b
*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)) - (d*(2*a^3*d^2*(B*c^2 + 2*c*C*d
 - B*d^2) + 2*b^3*c*(2*c^3*C - 3*B*c^2*d - B*d^3) - a*b^2*(B*c^4 - 4*c*C*d^
```

```
3 + 3*B*d^4) + a^2*b*(5*c^4*C - 6*B*c^3*d + 2*c^2*C*d^2 - 2*B*c*d^3 + C*d^4
) - A*(4*a^3*c*d^3 + 4*a*b^2*c*d^3 - 4*a^2*b*d^2*(2*c^2 + d^2) - b^3*(c^4 +
 10*c^2*d^2 + 5*d^4))))/((a^2 + b^2)*(b*c - a*d)^3*(c^2 + d^2)^2*f*Sqrt[c +
 d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
```

```
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{Ab^2-a(bB-aC)}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-3abB(c^2+d^2)+Ab^2(3c^2}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-3abB(c^2+d^2)+Ab^2(3c^2}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-3abB(c^2+d^2)+Ab^2(3c^2}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-3abB(c^2+d^2)+Ab^2(3c^2}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+}$$

$$= -\frac{d(2a^2Ad^2+2b^2c(cC-Bd)-3abB(c^2+d^2)+Ab^2(3c^2}{3\,(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f(c+}$$

$$= -\frac{b^{3/2}(7a^3bBd-5a^4Cd+b^4(2Bc-5Ad)+ab^3(4Ac-4}{(a}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{5/2}f} - \frac{(B}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6052 vs. 2(679) = 1358.
time = 6.28, size = 6052, normalized size = 8.91

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(
c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 15714 vs. 2(639) = 1278.
time = 0.75, size = 15715, normalized size = 23.14

method	result	size
derivativedivides	Expression too large to display	15715
default	Expression too large to display	15715

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*x
+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e
))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d*
tan(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.128 $\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\,(A+B\,\mathrm{ta}$

Optimal. Leaf size=679

$$-\frac{(a-ib)^{5/2}(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{(a+ib)^{5/2}(B-i(A-C))}{}$$

```
[Out] -1/64*(5*a^4*C*d^4-20*a^3*b*d^3*(2*B*d+C*c)+30*a^2*b^2*d^2*(c^2*C-4*B*c*d-8
*(A-C)*d^2)-20*a*b^3*d*(c^3*C-2*B*c^2*d+8*c*(A-C)*d^2-16*B*d^3)+b^4*(5*c^4*
C-8*B*c^3*d+16*c^2*(A-C)*d^2+64*B*c*d^3+128*(A-C)*d^4))*arctanh(d^(1/2)*(a+
b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(3/2)/d^(7/2)/f-(a-I*
b)^(5/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(
1/2)/(c+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/f-(a+I*b)^(5/2)*(B-I*(A-C))*arct
anh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/
2))*(c+I*d)^(1/2)/f+1/64*(64*b*(a^2*B-b^2*B+2*a*b*(A-C))*d^3-(-a*d+b*c)*(16
*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*(-8*B*b*d-5*C*a*d+5*C*b*c)))*(a+b*tan(f*x+e
))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/d^3/f+1/32*(16*b*(A*b+B*a-C*b)*d^2+(-a*d+
b*c)*(-8*B*b*d-5*C*a*d+5*C*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3
/2)/d^3/f-1/24*(-8*B*b*d-5*C*a*d+5*C*b*c)*(a+b*tan(f*x+e))^(3/2)*(c+d*tan(f
*x+e))^(3/2)/d^2/f+1/4*C*(a+b*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 7.31, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - ((a
+ I*b)^(5/2)*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((5*a^4*C*d
^4 - 20*a^3*b*d^3*(c*C + 2*B*d) + 30*a^2*b^2*d^2*(c^2*C - 4*B*c*d - 8*(A -
C)*d^2) - 20*a*b^3*d*(c^3*C - 2*B*c^2*d + 8*c*(A - C)*d^2 - 16*B*d^3) + b^4
*(5*c^4*C - 8*B*c^3*d + 16*c^2*(A - C)*d^2 + 64*B*c*d^3 + 128*(A - C)*d^4))
*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x
]])])/(64*b^(3/2)*d^(7/2)*f) + ((64*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 -
 (b*c - a*d)*(16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d -
 5*a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(64*b*d^3*f)
 + ((16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d - 5*a*C*d)
```

```
)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(32*d^3*f) - ((5*b*c
*C - 8*b*B*d - 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/
2))/(24*d^2*f) + (C*(a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2))/
(4*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C(a+b\tan(e+\ldots}{\ldots}$$

$$= -\frac{(5bcC-8bBd\ldots}{\ldots}$$

$$= \frac{(16b(Ab+aB-\ldots}{\ldots}$$

$$= \frac{(64b(a^2B-b^2B\ldots}{\ldots}$$

$$= \frac{(64b(a^2B-b^2B\ldots}{\ldots}$$

$$= \frac{(64b(a^2B-b^2B\ldots}{\ldots}$$

$$= \frac{(64b(a^2B-b^2B\ldots}{\ldots}$$

$$= \frac{(64b(a^2B-b^2B\ldots}{\ldots}$$

$$= \frac{(64b(a^2B-b^2B\ldots}{\ldots}$$

$$= -\frac{(5a^4Cd^4-20a^3\ldots}{\ldots}$$

$$= -\frac{(a-ib)^{5/2}(iA+\ldots}{\ldots}$$

Mathematica [A]
time = 8.58, size = 1202, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e
+ f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2))/(4*d*f) + (((-5*b
*c*C + 8*b*B*d + 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(
3/2))/(6*d*f) + ((3*(16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*
b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(8*d
*f) + (((24*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 - (3*(b*c - a*d)*(16*b*(A
*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d - 5*a*C*d)))/8)*Sqrt[a
 + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((24*b*d^3*(b*(3*a^2*b
*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^
2*(B*c + (A - C)*d)) + Sqrt[-b^2]*(a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c
*C - B*d) - 3*a^2*b*(B*c + (A - C)*d) + b^3*(B*c + (A - C)*d)))*ArcTanh[(Sq
rt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*
Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)
/b]) - (24*b*d^3*(b*(3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^
3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d)) - Sqrt[-b^2]*(a^3*(A*c - c
*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) - 3*a^2*b*(B*c + (A - C)*d) + b^3*(B*
c + (A - C)*d)))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]
*Sqrt[c + (Sqrt[-b^2]*d)/b]) - (3*Sqrt[b]*Sqrt[c - (a*d)/b]*Sqrt[(c/(c - (a
*d)/b) - (a*d)/(b*(c - (a*d)/b)))^(-1)]*Sqrt[c/(c - (a*d)/b) - (a*d)/(b*(c
- (a*d)/b))]*(5*a^4*C*d^4 - 20*a^3*b*d^3*(c*C + 2*B*d) + 30*a^2*b^2*d^2*(c^
2*C - 4*B*c*d - 8*(A - C)*d^2) - 20*a*b^3*d*(c^3*C - 2*B*c^2*d + 8*c*(A - C
)*d^2 - 16*B*d^3) + b^4*(5*c^4*C - 8*B*c^3*d + 16*c^2*(A - C)*d^2 + 64*B*c*
d^3 + 128*(A - C)*d^4))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]
*Sqrt[c - (a*d)/b]*Sqrt[c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b))])]*Sqrt[(
c + d*Tan[e + f*x])/(c - (a*d)/b)])/(8*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(
b^2*f))/(2*d))/(3*d))/(4*d)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+d\tan\left(fx+e\right)}\,\left(a+b\tan\left(fx+e\right)\right)^{\frac{5}{2}}\left(A+B\tan\left(fx+e\right)+C\left(\tan^{2}\left(fx+e\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(5/2
)*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^{\frac{5}{2}}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(5/2)*sqrt(c + d*tan(e + f*x))*(A + B*tan(e
+ f*x) + C*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^{5/2}\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2),x)
[Out] int((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2), x)
```

3.129 $\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\,(A+B\tan$

Optimal. Leaf size=505

$$-\frac{(a-ib)^{3/2}(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{(a+ib)^{3/2}(iA-B-iC)\sqrt{c}}{}$$

```
[Out] -1/8*(a^3*C*d^3-3*a^2*b*d^2*(2*B*d+C*c)+3*a*b^2*d*(c^2*C-4*B*c*d-8*(A-C)*d^
2)-b^3*(c^3*C-2*B*c^2*d+8*c*(A-C)*d^2-16*B*d^3))*arctanh(d^(1/2)*(a+b*tan(f
*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(3/2)/d^(5/2)/f-(a-I*b)^(3/2
)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/f+(a+I*b)^(3/2)*(I*A-B-I*C)*arctanh((c+
I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+
I*d)^(1/2)/f+1/8*(8*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*(-2*B*b*d-C*a*d+C*b*c))*
(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/d^2/f-1/4*(-2*B*b*d-C*a*d+C
*b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/d^2/f+1/3*C*(a+b*tan(f*
x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 5.43, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{(a^3Cd^3-3a^2bd^2(2Bd+cC)+3ab^2d(-8d^2(A-C)-4Bcd+c^2C)-(b^3(8cd^2(A-C)-2Bc^2d-16Bd^3+c^3C)))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{8b^{3/2}d^{5/2}f}+\frac{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,(8bd^2(aB+Ab-bC)+(bc-ad)(-aCd-2bBd+bcC))}{8bd^2f}-\frac{(a-ib)^{3/2}\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{(a+ib)^{3/2}\sqrt{c+id}\,(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{(-aCd-2bBd+bcC)\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{4d^2f}+\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) + ((a
+ I*b)^(3/2)*(I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((a^3*C*d^3
 - 3*a^2*b*d^2*(c*C + 2*B*d) + 3*a*b^2*d*(c^2*C - 4*B*c*d - 8*(A - C)*d^2)
- b^3*(c^3*C - 2*B*c^2*d + 8*c*(A - C)*d^2 - 16*B*d^3))*ArcTanh[(Sqrt[d]*Sq
rt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*b^(3/2)*d^(
5/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*
d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*b*d^2*f) - ((b*c*
C - 2*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(
4*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2))/(3*d*f
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
```

```
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\,\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C(a+b\tan(e+ \cdots}{\cdots}$$

$$= -\frac{(bcC-2bBd \cdots}{\cdots}$$

$$= \frac{(8b(Ab+aB- \cdots}{\cdots}$$

$$= \frac{(8b(Ab+aB- \cdots}{\cdots}$$

$$= \frac{(8b(Ab+aB- \cdots}{\cdots}$$

$$= \frac{(8b(Ab+aB- \cdots}{\cdots}$$

$$= \frac{(8b(Ab+aB- \cdots}{\cdots}$$

$$= \frac{(8b(Ab+aB- \cdots}{\cdots}$$

$$= -\frac{(a^3Cd^3-3a^2 \cdots}{\cdots}$$

$$= -\frac{(a-ib)^{3/2}(iA \cdots}{\cdots}$$

Mathematica [A]
time = 7.89, size = 835, normalized size = 1.65

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df} + \frac{-\frac{3(bcC-2bBd-aCd)\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{4df} + \frac{3\left(8b(Ab+aB-bC)d^2+(bc-ad)(bcC-2bBd-aCd)\right)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bf} + \cdots}{3d}$$

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e
+ f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2))/(3*d*f) + ((-3*(b
*c*C - 2*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)
)/(4*d*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d -
 a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + ((6*b
*d^2*(Sqrt[-b^2]*(a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*
c + (A - C)*d)) + b*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*
(B*c + (A - C)*d)))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-a + Sqrt[
-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) + (6*b*d^2*(Sqrt[-b^2]*(a^2*(A*c - c*C
- B*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*c + (A - C)*d)) - b*(2*a*b*(A*c -
 c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d)))*ArcTanh[(Sqrt
[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt
[c + d*Tan[e + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) -
 (3*Sqrt[b]*Sqrt[c - (a*d)/b]*(a^3*C*d^3 - 3*a^2*b*d^2*(c*C + 2*B*d) + 3*a*
b^2*d*(c^2*C - 4*B*c*d - 8*(A - C)*d^2) - b^3*(c^3*C - 2*B*c^2*d + 8*c*(A -
 C)*d^2 - 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sq
rt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt[d]*Sq
rt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*d))/(3*d)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+d\tan(fx+e)}\,(a+b\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/2
)*sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan{(e+fx)})^{\frac{3}{2}}\sqrt{c+d\tan{(e+fx)}}\left(A+B\tan{(e+fx)}+C\tan^2{(e+fx)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*sqrt(c + d*tan(e + f*x))*(A + B*tan(e
+ f*x) + C*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\mathrm{tan}\left(e+f\,x\right)}\,\left(C\,{\mathrm{tan}\left(e+f\,x\right)}^2+B\,\mathrm{tan}\left(e+f\,x\right)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2), x)
```

3.130 $\int \sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,(A+B\tan$

Optimal. Leaf size=381

$$-\frac{\sqrt{a-ib}\,(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{\sqrt{a+ib}\,(B-i(A-C))\sqrt{c+}}{}$$

```
[Out] -1/4*(a^2*C*d^2-2*a*b*d*(2*B*d+C*c)+b^2*(c^2*C-4*B*c*d-8*(A-C)*d^2))*arctan
h(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(3/2)/d^
(3/2)/f-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1
/2)/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)*(c-I*d)^(1/2)/f-(B-I*(A-C))*arcta
nh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2
))*(a+I*b)^(1/2)*(c+I*d)^(1/2)/f-1/4*(-4*B*b*d-C*a*d+C*b*c)*(a+b*tan(f*x+e)
)^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/d/f+1/2*C*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(
f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 3.56, antiderivative size = 383, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{(a^2Cd^2-2abd(2Bd+cC)+b^2(-8d^2(A-C)-4Bcd+c^2C))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{4b^{3/2}d^{3/2}f}-\frac{\sqrt{a-ib}\,\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{\sqrt{a+ib}\,\sqrt{c+id}\,(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{(-aCd-4bBd+bcC)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4bdf}+\frac{C\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) + (Sqrt[
a + I*b]*(I*A - B - I*C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((a^2*C*d^2 - 2
*a*b*d*(c*C + 2*B*d) + b^2*(c^2*C - 4*B*c*d - 8*(A - C)*d^2))*ArcTanh[(Sqrt
[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(4*b^(3/
2)*d^(3/2)*f) - ((b*c*C - 4*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c
+ d*Tan[e + f*x]])/(4*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f
*x])^(3/2))/(2*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C\sqrt{a+b\tan(e+}}{}$$

$$= -\frac{(bcC-4bBd-}{}$$

$$= -\frac{(bcC-4bBd-}{}$$

$$= -\frac{(bcC-4bBd-}{}$$

$$= -\frac{(bcC-4bBd-}{}$$

$$= -\frac{(bcC-4bBd-}{}$$

$$= -\frac{(bcC-4bBd-}{}$$

$$= -\frac{(a^2Cd^2-2abd(}{}$$

$$= -\frac{\sqrt{a-ib}\,(iA+}{}$$

Mathematica [**A**]

time = 7.19, size = 619, normalized size = 1.62

$$\frac{C\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2df}+\frac{\frac{(-bcC+4bBd+aCd)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{2bf}+\frac{\frac{2bd\left(b(Abc+aBc-bcC+aAd-bBd-aCd)-\sqrt{-b^2}\,(bBc+b(A-C)d-a(Ac-cC-Bd))\right)\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}}-\frac{2bd\left(b(Abc+aBc-bcC+aAd-bBd-aCd)+\sqrt{-b^2}\,(bBc+b(A-C)d-a(Ac-cC-Bd))\right)\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}}-\frac{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}\,\left(a^2Cd^2-2abd(cC+2Bd)+b^2\left(c^2C-4Bcd-8(A-C)d^2\right)\right)\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{bc+bd\tan(e+fx)}{bc-ad}}}{2\sqrt{d}\,\sqrt{c+d\tan(e+fx)}}}{b^2f}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e +
f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*d*f) + (((-(b*c*
C) + 4*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(2
*b*f) + ((2*b*d*(b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) - Sqrt[-
b^2]*(b*B*c + b*(A - C)*d - a*(A*c - c*C - B*d)))*ArcTanh[(Sqrt[-c + (Sqrt[
-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan
[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (2*b*d*
(b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) + Sqrt[-b^2]*(b*B*c + b*
(A - C)*d - a*(A*c - c*C - B*d)))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqr
t[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) - (Sqrt[b]*Sqrt[c - (a*d)/b]*
(a^2*C*d^2 - 2*a*b*d*(c*C + 2*B*d) + b^2*(c^2*C - 4*B*c*d - 8*(A - C)*d^2))
*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sq
rt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(2*Sqrt[d]*Sqrt[c + d*Tan[e + f*x
]]))/(b^2*f))/(2*d)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan(fx+e)}\,\sqrt{c+d\tan(fx+e)}\,\left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)*
sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f
*x) + C*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.131 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=287

$$-\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f}-\frac{(B-i(A-C))\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{\sqrt{a+ib}}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/f/(a-I*b)^(1/2)-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I
*d)^(1/2)/f/(a+I*b)^(1/2)+(2*B*b*d-C*a*d+C*b*c)*arctanh(d^(1/2)*(a+b*tan(f*
x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(3/2)/f/d^(1/2)+C*(a+b*tan(f*
x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/f
```

Rubi [A]
time = 1.92, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}}-\frac{\sqrt{c+id}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}}+\frac{(-aCd+2bBd+bcC)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}\sqrt{d}\,f}+\frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt
[a + b*Tan[e + f*x]],x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) - ((B
- I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) + ((b*c*C + 2
*b*B*d - a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(b^(3/2)*Sqrt[d]*f) + (C*Sqrt[a + b*Tan[e + f*x]]*Sqrt
[c + d*Tan[e + f*x]])/(b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx = \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+\cdots}}{bf}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+\cdots}}{bf}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+\cdots}}{bf}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+\cdots}}{bf}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+\cdots}}{bf}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+\cdots}}{bf}$$

$$= \frac{(bcC+2bBd-aCd)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\cdots}}{\sqrt{b}\,\sqrt{\cdots}}\right.}{b^{3/2}\sqrt{d}\,f}$$

$$= -\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.}{\sqrt{a-ib\cdots}}$$

Mathematica [A]
time = 2.74, size = 441, normalized size = 1.54

$$\frac{\frac{b\left(bBc+b(A-C)d+\sqrt{-b^2}(Ac-cC-Bd)\right)\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}}+\frac{b\left(\sqrt{-b^2}(Ac-cC-Bd)-b(Bc+(A-C)d)\right)\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}}+bC\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}+\frac{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}\,(bcC+2bBd-aCd)\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{d}\,\sqrt{c+d\tan(e+fx)}}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((b*(b*B*c + b*(A - C)*d + Sqrt[-b^2]*(A*c - c*C - B*d))*ArcTanh[(Sqrt[-c +
 (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c
+ d*Tan[e + f*x]])]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) +
(b*(Sqrt[-b^2]*(A*c - c*C - B*d) - b*(B*c + (A - C)*d))*ArcTanh[(Sqrt[c + (
Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d
*Tan[e + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) + b*C*S
qrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]] + (Sqrt[b]*Sqrt[c - (a*d)/
b]*(b*c*C + 2*b*B*d - a*C*d)*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sq
rt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(Sqrt
[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan\left(fx+e\right)}\,\left(A+B\tan\left(fx+e\right)+C\left(\tan^{2}\left(fx+e\right)\right)\right)}{\sqrt{a+b\tan\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c)/
sqrt(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\tan{(e + fx)}}\,(A + B\tan{(e + fx)} + C\tan^2{(e + fx)})}{\sqrt{a + b\tan{(e + fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
sqrt(a + b*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.132 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}-\frac{(B-i(A-C))\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{a-}}\right.}{(a+ib)^{3/2}f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/(a-I*b)^(3/2)/f-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I
*d)^(1/2)/(a+I*b)^(3/2)/f+2*C*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2
)/(c+d*tan(f*x+e))^(1/2))*d^(1/2)/b^(3/2)/f-2*(A*b^2-a*(B*b-C*a))*(c+d*tan(
f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 2.88, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3726, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}-\frac{\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}}-\frac{\sqrt{c+id}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}}+\frac{2C\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (2*C*Sq
rt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e
+ f*x]])])/(b^(3/2)*f) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]
])/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{b\,(a^2+b^2)\,f\sqrt{a+b\tan(e+fx)}}$$

$$= \frac{2C\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+f}}{\sqrt{b}\,\sqrt{c+d\tan(e+f}}\right.}{b^{3/2}f}$$

$$= -\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{(a-ib)^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 33.57, size = 621058, normalized size = 2070.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [**F(-1)**] grade_annotation
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\tan{\left(e + fx \right)}}\left(A + B\tan{\left(e + fx \right)} + C\tan^{2}{\left(e + fx \right)}\right)}{\left(a + b\tan{\left(e + fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.133 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{(iA+B-iC)\sqrt{c-id}\ \tanh^{-1}\left(\frac{\sqrt{c-id}\ \sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\ \sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f}-\frac{(B-i(A-C))\sqrt{c+id}\ \tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{(a+ib)^{5/2}}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/(a-I*b)^(5/2)/f-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I
*d)^(1/2)/(a+I*b)^(5/2)/f-2/3*(2*a^3*b*B*d+a^4*C*d+b^4*(A*d+3*B*c)+2*a*b^3*
(3*A*c-2*B*d-3*C*c)-a^2*b^2*(5*A*d+3*B*c-7*C*d))*(c+d*tan(f*x+e))^(1/2)/b/(
a^2+b^2)^2/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a))*(c+d
*tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 1.34, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3726, 3730, 3697, 3696, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\,(a^4Cd+2a^3bBd-a^2b^2(5Ad+3Bc-7Cd)+2ab^3(3Ac-2Bd-3cC)+b^4(Ad+3Bc))}{3bf(a^2+b^2)^2(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}}-\frac{\sqrt{c+id}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(3*b*(a^2 + b^2)*f*(a + b*Tan
[e + f*x])^(3/2)) - (2*(2*a^3*b*B*d + a^4*C*d + b^4*(3*B*c + A*d) + 2*a*b^3
*(3*A*c - 3*c*C - 2*B*d) - a^2*b^2*(3*B*c + 5*A*d - 7*C*d))*Sqrt[c + d*Tan[
e + f*x]])/(3*b*(a^2 + b^2)^2*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{2(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{2(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{2(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{2(Ab^2-a(bB-aC))\,\sqrt{c+d\tan(e}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(a-ib)^{5/}}$$

Mathematica [A]
time = 6.15, size = 424, normalized size = 1.15

$$\frac{-\frac{3C\sqrt{c+d\tan(e+fx)}}{(a+b\tan(e+fx))^{3/2}}+\frac{(-2Ab^2+2abB+a^2C+3b^2C)\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(a+b\tan(e+fx))^{3/2}}+\frac{-3b\left(\frac{(a+ib)^2(iA+B-iC)\sqrt{-c+id}\,\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}}+\frac{(a-ib)^2(-iA+B+iC)\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}}\right)+\frac{2\left(2a^3bBd+a^4Cd+b^4(3Bc+Ad)+2ab^3(3Ac-3cC-2Bd)+a^2b^2(-3Bc-5Ad+7Cd)\right)\sqrt{c+d\tan(e+fx)}}{(-bc+ad)\sqrt{a+b\tan(e+fx)}}}{(a^2+b^2)^2}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-3*C*Sqrt[c + d*Tan[e + f*x]])/(a + b*Tan[e + f*x])^(3/2) + ((-2*A*b^2 +
2*a*b*B + a^2*C + 3*b^2*C)*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(a + b*Ta
n[e + f*x])^(3/2)) + (-3*b*(((a + I*b)^2*(I*A + B - I*C)*Sqrt[-c + I*d]*Arc
Tanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*T
an[e + f*x]])])/Sqrt[-a + I*b] + ((a - I*b)^2*((-I)*A + B + I*C)*Sqrt[c + I
*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c
```

```
+ d*Tan[e + f*x]])])/Sqrt[a + I*b]) + (2*(2*a^3*b*B*d + a^4*C*d + b^4*(3*B*
c + A*d) + 2*a*b^3*(3*A*c - 3*c*C - 2*B*d) + a^2*b^2*(-3*B*c - 5*A*d + 7*C*
d))*Sqrt[c + d*Tan[e + f*x]])/((-(b*c) + a*d)*Sqrt[a + b*Tan[e + f*x]]))/(a
^2 + b^2)^2)/(3*b*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan (fx+e)}\, (A+B\tan (fx+e)+C(\tan^2 (fx+e)))}{(a+b\tan (fx+e))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan (e+fx)}\, (A+B\tan (e+fx)+C\tan^2 (e+fx))}{(a+b\tan (e+fx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.134 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=597

$$-\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f}-\frac{(B-i(A-C))\sqrt{c+id}\,\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{a-}}\right.}{(a+ib)^{7/2}f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/(a-I*b)^(7/2)/f-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c+I
*d)^(1/2)/(a+I*b)^(7/2)/f+2/15*(8*a^5*b*B*d^2+2*a^6*C*d^2-a^4*b^2*d*(33*A*d
+25*B*c-39*C*d)-a^2*b^4*(45*A*c^2-29*A*d^2-90*B*c*d-45*C*c^2+23*C*d^2)+a^3*
b^3*(80*c*(A-C)*d+B*(15*c^2-49*d^2))-a*b^5*(40*c*(A-C)*d+B*(45*c^2-3*d^2))-
b^6*(5*c*(B*d+3*C*c)-A*(15*c^2+2*d^2)))*(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)^
3/(-a*d+b*c)^2/f/(a+b*tan(f*x+e))^(1/2)-2/5*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*
x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(5/2)-2/15*(4*a^3*b*B*d+a^4*C*d+
b^4*(A*d+5*B*c)+2*a*b^3*(5*A*c-3*B*d-5*C*c)-a^2*b^2*(9*A*d+5*B*c-11*C*d))*(
c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)^2/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 2.38, antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3726, 3730, 3697, 3696, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{5bf(a^2+b^2)(a+b\tan(e+fx))^{5/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\,(a^4Cd+4a^3bBd-a^2b^2(9Ad+5Bc-11Cd)+2ab^3(5Ac-3Bd-5cC)+b^4(Ad+5Bc))}{15bf(a^2+b^2)^2(bc-ad)(a+b\tan(e+fx))^{3/2}}+\frac{2\sqrt{c+d\tan(e+fx)}\,(2a^6Cd^2+8a^5bBd^2-a^4b^2d(33Ad+25Bc-39Cd)+a^3b^3(80cd(A-C)+B(15c^2-49d^2))-a^2b^4(45Ac^2-29Ad^2-90Bcd-45c^2C+23Cd^2)-ab^5(40cd(A-C)+B(45c^2-3d^2))-b^6(5c(Bd+3cC)-A(15c^2+2d^2)))}{15bf(a^2+b^2)^3(bc-ad)^2\sqrt{a+b\tan(e+fx)}}-\frac{\sqrt{c-id}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{7/2}}-\frac{\sqrt{c+id}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
 b*Tan[e + f*x])^(7/2),x]
```

```
[Out] -(((I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) - ((
B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(5*b*(a^2 + b^2)*f*(a + b*Tan
[e + f*x])^(5/2)) - (2*(4*a^3*b*B*d + a^4*C*d + b^4*(5*B*c + A*d) + 2*a*b^3
*(5*A*c - 5*c*C - 3*B*d) - a^2*b^2*(5*B*c + 9*A*d - 11*C*d))*Sqrt[c + d*Tan
[e + f*x]])/(15*b*(a^2 + b^2)^2*(b*c - a*d)*f*(a + b*Tan[e + f*x])^(3/2)) +
 (2*(8*a^5*b*B*d^2 + 2*a^6*C*d^2 - a^4*b^2*d*(25*B*c + 33*A*d - 39*C*d) - a
^2*b^4*(45*A*c^2 - 45*c^2*C - 90*B*c*d - 29*A*d^2 + 23*C*d^2) + a^3*b^3*(80
*c*(A - C)*d + B*(15*c^2 - 49*d^2)) - a*b^5*(40*c*(A - C)*d + B*(45*c^2 - 3
*d^2)) - b^6*(5*c*(3*c*C + B*d) - A*(15*c^2 + 2*d^2)))*Sqrt[c + d*Tan[e + f
*x]])/(15*b*(a^2 + b^2)^3*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{5b\,(a^2+b^2)\,f(a+b\tan(e+fx))^5}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{5b\,(a^2+b^2)\,f(a+b\tan(e+fx))^5}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{5b\,(a^2+b^2)\,f(a+b\tan(e+fx))^5}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{5b\,(a^2+b^2)\,f(a+b\tan(e+fx))^5}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{5b\,(a^2+b^2)\,f(a+b\tan(e+fx))^5}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+}}{5b\,(a^2+b^2)\,f(a+b\tan(e+fx))^5}$$

$$= -\frac{(iA+B-iC)\sqrt{c-id}\,\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{(a-ib)^{7/2}}$$

Mathematica [A]
time = 6.89, size = 1109, normalized size = 1.86

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] -1/2*(C*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[e + f*x])^(5/2)) - ((-2*(
(b^2*(-4*A*b*c + 5*b*c*C - a*C*d))/2 - a*(-2*b^2*(B*c + (A - C)*d) - (a*(b*
c*C - 4*b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/(5*(a^2 + b^2)*(b*c -
 a*d)*f*(a + b*Tan[e + f*x])^(5/2)) - (2*((-2*(b^2*(b*c - a*d)*(a^2*C*d + b
^2*(5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) - a*(a*(4*A*b^2 - 4*a*b*B - a
^2*C - 5*b^2*C)*d*(b*c - a*d) - 5*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C -
a*A*d - b*B*d + a*C*d)))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*
d)*f*(a + b*Tan[e + f*x])^(3/2)) - (2*((-15*b*(b*c - a*d)^2*(((I*a - b)^3*(
A - I*B - C)*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*b] - ((I*a + b)^
3*(A + I*B - C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b]))/(2*(a^2 + b^
2)*f) - (2*(b^2*((b*c - a*d)*(b^2*d - (3*a*(b*c - a*d))/2)*(a^2*C*d + b^2*(
5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) + ((-3*b*c)/2 + (a*d)/2)*(a*(4*A*
b^2 - 4*a*b*B - a^2*C - 5*b^2*C)*d*(b*c - a*d) - 5*b^2*(b*c - a*d)*(A*b*c -
 a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))) - a*((3*b*(b*c - a*d)*(b*(4*A*b^2
 - 4*a*b*B - a^2*C - 5*b^2*C)*d*(b*c - a*d) + 5*a*b*(b*c - a*d)*(A*b*c - a*
B*c - b*c*C - a*A*d - b*B*d + a*C*d) + b*(b*c - a*d)*(a^2*C*d + b^2*(5*B*c
+ A*d) + a*b*(5*A*c - 5*c*C - B*d))))/2 - a*d*(b^2*(b*c - a*d)*(a^2*C*d + b
^2*(5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) - a*(a*(4*A*b^2 - 4*a*b*B - a
^2*C - 5*b^2*C)*d*(b*c - a*d) - 5*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C -
a*A*d - b*B*d + a*C*d)))))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*
d)*f*Sqrt[a + b*Tan[e + f*x]])))/(3*(a^2 + b^2)*(b*c - a*d))))/(5*(a^2 + b^
2)*(b*c - a*d)))/(2*b)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan(fx+e)}\,\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{(a+b\tan(fx+e))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(7/2),x)
```

```
[Out] Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(a + b*tan(e + f*x))**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(7/2),x)
```

```
[Out] \text{Hanged}
```

3.135 $\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\,(A+B\tan$

Optimal. Leaf size=682

$$-\frac{(a-ib)^{3/2}(B+i(A-C))(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{(a+ib)^{3/2}(B-i(A-C}{}$$

```
[Out] -(a-I*b)^(3/2)*(B+I*(A-C))*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x
+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f-(a+I*b)^(3/2)*(B-I*(A-C)
)*(c+I*d)^(3/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/
(c+d*tan(f*x+e))^(1/2))/f+1/64*(3*a^4*C*d^4-4*a^3*b*d^3*(2*B*d+3*C*c)+6*a^2
*b^2*d^2*(3*c^2*C+12*B*c*d+8*(A-C)*d^2)-12*a*b^3*d*(c^3*C-6*B*c^2*d-24*c*(A
-C)*d^2+16*B*d^3)+b^4*(3*c^4*C-8*B*c^3*d+48*c^2*(A-C)*d^2-192*B*c*d^3-128*(
A-C)*d^4))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^
(1/2))/b^(5/2)/d^(5/2)/f+1/64*(64*b*(a^2*B-b^2*B+2*a*b*(A-C))*d^3+(-a*d+b*c
)*(48*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*(-8*B*b*d-3*C*a*d+3*C*b*c)))*(a+b*tan(
f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^2/d^2/f+1/96*(48*b*(A*b+B*a-C*b)*d^2
+(-a*d+b*c)*(-8*B*b*d-3*C*a*d+3*C*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x
+e))^(3/2)/b/d^2/f-1/24*(-8*B*b*d-3*C*a*d+3*C*b*c)*(a+b*tan(f*x+e))^(1/2)*(
c+d*tan(f*x+e))^(5/2)/d^2/f+1/4*C*(a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(
5/2)/d/f
```

Rubi [A]
time = 8.72, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*
x] + C*Tan[e + f*x]^2),x]
```

```
[Out] -(((a - I*b)^(3/2)*(B + I*(A - C))*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*S
qrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - ((
a + I*b)^(3/2)*(B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + ((3*a^4
*C*d^4 - 4*a^3*b*d^3*(3*c*C + 2*B*d) + 6*a^2*b^2*d^2*(3*c^2*C + 12*B*c*d +
8*(A - C)*d^2) - 12*a*b^3*d*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^
3) + b^4*(3*c^4*C - 8*B*c^3*d + 48*c^2*(A - C)*d^2 - 192*B*c*d^3 - 128*(A -
 C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(64*b^(5/2)*d^(5/2)*f) + ((64*b*(a^2*B - b^2*B + 2*a*b*(A -
C))*d^3 + (b*c - a*d)*(48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C -
8*b*B*d - 3*a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(64
```

```
*b^2*d^2*f) + ((48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*d
 - 3*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(96*b*d^2
*f) - ((3*b*c*C - 8*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e
+ f*x])^(5/2))/(24*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*
x])^(5/2))/(4*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{C(a+b\tan(e+}{}$$

$$=-\frac{(3bcC-8bBd}{}$$

$$=\frac{(48b(Ab+aB-}{}$$

$$=\frac{(64b(a^2B-b^2B}{}$$

$$=\frac{(64b(a^2B-b^2B}{}$$

$$=\frac{(64b(a^2B-b^2B}{}$$

$$=\frac{(64b(a^2B-b^2B}{}$$

$$=\frac{(64b(a^2B-b^2B}{}$$

$$=\frac{(64b(a^2B-b^2B}{}$$

$$=\frac{(3a^4Cd^4-4a^3b}{}$$

$$=-\frac{(a-ib)^{3/2}(B}{}$$

Mathematica [A]
time = 8.08, size = 1304, normalized size = 1.91

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[
e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2))/(4*d*f) + (((-3*b
*c*C + 8*b*B*d + 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/
2))/(6*d*f) + (((48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*
d - 3*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(8*b*f)
+ (((24*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 - (3*(-(b*c) + a*d)*(48*b*(A*
b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*d - 3*a*C*d)))/8)*Sqrt[a
+ b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((24*(-(b^4*Sqrt[-b^2]*
d^2*(a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d -
 C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))) - b^5*d^2
*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*
(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTanh[(Sqrt[-c + (Sq
rt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*
Tan[e + f*x]])])/(b^2*Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) -
(24*b^2*d^2*(Sqrt[-b^2]*(a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^
2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2
 - d^2))) - b*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(
A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTanh[(
Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*
Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b
]) + (3*Sqrt[b]*Sqrt[c - (a*d)/b]*Sqrt[(c/(c - (a*d)/b) - (a*d)/(b*(c - (a*
d)/b)))^(-1)]*Sqrt[c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b))]*(3*a^4*C*d^4
- 4*a^3*b*d^3*(3*c*C + 2*B*d) + 6*a^2*b^2*d^2*(3*c^2*C + 12*B*c*d + 8*(A -
C)*d^2) - 12*a*b^3*d*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^3) + b^
4*(3*c^4*C - 8*B*c^3*d + 48*c^2*(A - C)*d^2 - 192*B*c*d^3 - 128*(A - C)*d^4
))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b]*Sq
rt[c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b))])]*Sqrt[(c + d*Tan[e + f*x])/(
c - (a*d)/b)])/(8*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*b))/(3*d))
/(4*d)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(fx+e))^{\frac{3}{2}} (c+d\tan(fx+e))^{\frac{3}{2}} \left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/2
)*(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^{\frac{3}{2}} (c+d\tan(e+fx))^{\frac{3}{2}} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*(c + d*tan(e + f*x))**(3/2)*(A + B*tan
(e + f*x) + C*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^{3/2}\,(c+d\tan(e+f\,x))^{3/2}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2), x)
```

3.136 $\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}\,(A+B\,\text{ta}$

Optimal. Leaf size=508

$$-\frac{\sqrt{a-ib}\,(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{\sqrt{a+ib}\,(B-i(A-C))(c+}{}$$

```
[Out] 1/8*(a^3*C*d^3-a^2*b*d^2*(2*B*d+3*C*c)+a*b^2*d*(3*c^2*C+12*B*c*d+8*(A-C)*d^
2)-b^3*(c^3*C-6*B*c^2*d-24*c*(A-C)*d^2+16*B*d^3))*arctanh(d^(1/2)*(a+b*tan(
f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(5/2)/d^(3/2)/f-(I*A+B-I*C)
*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(
c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/f-(B-I*(A-C))*(c+I*d)^(3/2)*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a
+I*b)^(1/2)/f+1/8*(8*b*(A*b+B*a-C*b)*d^2-(-a*d+b*c)*(-6*B*b*d-C*a*d+C*b*c))
*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^2/d/f-1/12*(-6*B*b*d-C*a*d
+C*b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/b/d/f+1/3*C*(a+b*tan(
f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]

time = 5.34, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{(a^3Cd^3-a^2bd^2(2Bd+3cC)+ab^2d(8d^2(A-C)+12Bcd+3c^2C)-(b^3(-24cd^2(A-C)-6Bc^2d+16Bd^3+c^3C)))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{8b^{5/2}d^{3/2}f}+\frac{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,(8bd^2(aB+Ab-bC)-(bc-ad)(-aCd-6bBd+bcC))}{8b^2df}-\frac{\sqrt{a-ib}\,(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{\sqrt{a+ib}\,(c+id)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}-\frac{(-aCd-6bBd+bcC)\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{12bdf}+\frac{C\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) - (Sqr
t[a + I*b]*(B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f + ((a^3*C*d^3
 - a^2*b*d^2*(3*c*C + 2*B*d) + a*b^2*d*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2)
 - b^3*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^3))*ArcTanh[(Sqrt[d]*
Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*b^(5/2)*d
^(3/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 - (b*c - a*d)*(b*c*C - 6*b*B*d - a*
C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*b^2*d*f) - ((b*
c*C - 6*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))
/(12*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*d*
f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
```

```
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C\sqrt{a+b\tan(e}{}$$

$$= -\frac{(bcC-6bBd}{}$$

$$= \frac{(8b(Ab+aB-}{}$$

$$= \frac{(8b(Ab+aB-}{}$$

$$= \frac{(8b(Ab+aB-}{}$$

$$= \frac{(8b(Ab+aB-}{}$$

$$= \frac{(8b(Ab+aB-}{}$$

$$= \frac{(8b(Ab+aB-}{}$$

$$= \frac{(a^3Cd^3-a^2bd^2}{}$$

$$= -\frac{\sqrt{a-ib}\,(iA}{}$$

Mathematica [A]
time = 7.98, size = 867, normalized size = 1.71

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e
+ f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*d*f) + (((-(b*c*
C) + 6*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/
(4*b*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 - (b*c - a*d)*(b*c*C - 6*b*B*d - a
*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + ((6*b^2
*d*(b*(2*a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C
 + 2*B*c*d - C*d^2)) - Sqrt[-b^2]*(a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^
2)) + b*(2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)
/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x
]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (6*b^2*d*(b*(2*
a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C + 2*B*c*
d - C*d^2)) + Sqrt[-b^2]*(a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(
2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt
[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) + (3*Sqrt[b]*Sqrt[c - (a*d)/b]
*(a^3*C*d^3 - a^2*b*d^2*(3*c*C + 2*B*d) + a*b^2*d*(3*c^2*C + 12*B*c*d + 8*(
A - C)*d^2) - b^3*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^3))*ArcSin
h[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c
 + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b
^2*f))/(2*b))/(3*d)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan(fx+e)}\,(c+d\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)*
(d*tan(f*x + e) + c)^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan{(e+fx)}}\,(c+d\tan{(e+fx)})^{\frac{3}{2}}\,\left(A+B\tan{(e+fx)}+C\tan^{2}{(e+fx)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e
+ f*x) + C*tan(e + f*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\tan{(e+f\,x)}}\,\,{\left(c+d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}\,\left(C\,{\mathrm{tan}\left(e+f\,x\right)}^2+B\,\mathrm{tan}\left(e+f\,x\right)+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2), x)
```

3.137 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=384

$$-\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f}+\frac{(iA-B-iC)(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{a+ib}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(a-I*b)^(1/2)+(I*A-B-I*C)*(c+I*d)^(3/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/f/(a+I*b)^(1/2)+1/4*(3*a^2*C*d^2-2*a*b*d*(2*B*d+3*C*c)+b^2*(3*c^2
*C+12*B*c*d+8*(A-C)*d^2))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c
+d*tan(f*x+e))^(1/2))/b^(5/2)/f/d^(1/2)+1/4*(4*B*b*d-3*C*a*d+3*C*b*c)*(a+b*
tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^2/f+1/2*C*(a+b*tan(f*x+e))^(1/2)
*(c+d*tan(f*x+e))^(3/2)/b/f
```

Rubi [A]

time = 3.10, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{(3a^2Cd^2-2abd(2Bd+3cC)+b^2(8d^2(A-C)+12Bcd+3c^2C))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{4b^{5/2}\sqrt{d}\,f}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}}+\frac{(c+id)^{3/2}(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}}+\frac{(-3aCd+4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{4b^2f}+\frac{C\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[a + b*Tan[e + f*x]],x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) + ((
I*A - B - I*C)*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) + ((3*a^2
*C*d^2 - 2*a*b*d*(3*c*C + 2*B*d) + b^2*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2)
)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*
x]])])/(4*b^(5/2)*Sqrt[d]*f) + ((3*b*c*C + 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Ta
n[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b^2*f) + (C*Sqrt[a + b*Tan[e + f*x
]]*(c + d*Tan[e + f*x])^(3/2))/(2*b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
```

```
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx = \frac{C\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+}{2bf}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(}{4b^2f}$$

$$= \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(}{4b^2f}$$

$$= \frac{(3a^2Cd^2-2abd(3cC+2Bd)+b^2(3}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\Big(}{\sqrt{a-}}$$

Mathematica [A]
time = 6.55, size = 580, normalized size = 1.51

$$\frac{-\frac{4\left(-b(2c(A-C)d+B(c^2-d^2))+\sqrt{-b^2}\,(c^2C+2Bcd-Cd^2+A(-c^2+d^2))\right)\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}}-\frac{4\left(b(2c(A-C)d+B(c^2-d^2))+\sqrt{-b^2}\,(c^2C+2Bcd-Cd^2+A(-c^2+d^2))\right)\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}}+\frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{b}+2C\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}+\frac{\sqrt{c-\frac{ad}{b}}\,(3a^2Cd^2-2abd(3cC+2Bd)+b^2(3c^2C+12Bcd+8(A-C)d^2))\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{b^{3/2}\sqrt{d}\sqrt{c+d\tan(e+fx)}}}{4bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] ((-4*(-(b*(2*c*(A - C)*d + B*(c^2 - d^2))) + Sqrt[-b^2]*(c^2*C + 2*B*c*d -
C*d^2 + A*(-c^2 + d^2)))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-a +
Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (4*(b*(2*c*(A - C)*d + B*(c^2 -
d^2)) + Sqrt[-b^2]*(c^2*C + 2*B*c*d - C*d^2 + A*(-c^2 + d^2)))*ArcTanh[(Sqr
t[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqr
t[c + d*Tan[e + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b])
+ ((3*b*c*C + 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e
+ f*x]])/b + 2*C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2) + (Sqr
t[c - (a*d)/b]*(3*a^2*C*d^2 - 2*a*b*d*(3*c*C + 2*B*d) + b^2*(3*c^2*C + 12*B
*c*d + 8*(A - C)*d^2))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*
Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(b^(3/2)*Sq
rt[d]*Sqrt[c + d*Tan[e + f*x]]))/(4*b*f)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{\sqrt{a+b\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/2
)/sqrt(b*tan(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan{(e+fx)})^{\frac{3}{2}}\,(A+B\tan{(e+fx)}+C\tan^2{(e+fx)})}{\sqrt{a+b\tan{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/sqrt(a + b*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.138 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=382

$$-\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}-\frac{(B-i(A-C))(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\ }}{\sqrt{\ }}\right.}{(a+ib)^{3/}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/f-(B-I*(A-C))*(c+I*d)^(3/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(a+I*b)^(3/2)/f+(2*B*b*d-3*C*a*d+3*C*b*c)*arctanh(d^(1/2)*(a+b*ta
n(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))*d^(1/2)/b^(5/2)/f+(2*A*b^2-
2*B*a*b+3*C*a^2+C*b^2)*d*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^2/
(a^2+b^2)/f-2*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(3/2)/b/(a^2+b^2)/f/(a+b
*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 4.28, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3726, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}+\frac{d(3a^2C-2abB+2Ab^2+b^2C)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{b^2f(a^2+b^2)}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}}-\frac{(c+id)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}}+\frac{\sqrt{d}(-3aCd+2bBd+3bcC)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (Sq
rt[d]*(3*b*c*C + 2*b*B*d - 3*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(b^(5/2)*f) + ((2*A*b^2 - 2*a*b*B
+ 3*a^2*C + b^2*C)*d*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b^
2*(a^2 + b^2)*f) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(
b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
```

```
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e}{b\,(a^2+b^2)\,f\sqrt{a+b\tan(e+}$$

$$= \frac{(2Ab^2-2abB+3a^2C+b^2C)\,d\sqrt{a}}{b^2\,(a^2}$$

$$= \frac{(2Ab^2-2abB+3a^2C+b^2C)\,d\sqrt{a}}{b^2\,(a^2}$$

$$= \frac{(2Ab^2-2abB+3a^2C+b^2C)\,d\sqrt{a}}{b^2\,(a^2}$$

$$= \frac{(2Ab^2-2abB+3a^2C+b^2C)\,d\sqrt{a}}{b^2\,(a^2}$$

$$= \frac{(2Ab^2-2abB+3a^2C+b^2C)\,d\sqrt{a}}{b^2\,(a^2}$$

$$= \frac{(2Ab^2-2abB+3a^2C+b^2C)\,d\sqrt{a}}{b^2\,(a^2}$$

$$= \frac{\sqrt{d}\,(3bcC+2bBd-3aCd)\tanh^{-1}}{b^{5/2}f}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}}{(a-ib}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 36.19, size = 1073629, normalized size = 2810.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{3}{2}}\,(A+B\tan(fx+e)+C(\tan^2(fx+e)))}{(a+b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{3}{2}}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.139 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=402

$$-\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f}-\frac{(B-i(A-C))(c+id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(a+ib)^{5/}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/f-(B-I*(A-C))*(c+I*d)^(3/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(a+I*b)^(5/2)/f+2*C*d^(3/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2
)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(5/2)/f-2*(a^4*C*d+b^4*(A*d+B*c)+2*a*b^
3*(A*c-B*d-C*c)-a^2*b^2*(B*c+(A-3*C)*d))*(c+d*tan(f*x+e))^(1/2)/b^2/(a^2+b^
2)^2/f/(a+b*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(3/2
)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 5.14, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3726, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\,(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc))}{b^2f(a^2+b^2)^2\sqrt{a+b\tan(e+fx)}}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}}-\frac{(c+id)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}}+\frac{2Cd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) + (2*
C*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(b^(5/2)*f) - (2*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c -
 c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Sqrt[c + d*Tan[e + f*x]])/(b^2*(
a^2 + b^2)^2*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*b^2 - a*(b*B - a*C))*(c +
d*Tan[e + f*x])^(3/2))/(3*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
```

```
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e}{3b\,(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= -\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac}{b^2\,(a}$$

$$= -\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac}{b^2\,(a}$$

$$= -\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac}{b^2\,(a}$$

$$= -\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac}{b^2\,(a}$$

$$= -\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac}{b^2\,(a}$$

$$= -\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac}{b^2\,(a}$$

$$= \frac{2Cd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+}}{\sqrt{b}\,\sqrt{c+d\tan(e+}}\right.}{b^{5/2}f}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\right.}{(a-ib}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 36.90, size = 1347065, normalized size = 3350.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{(a+b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/2
)/(b*tan(f*x + e) + a)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{3}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.140 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=586

$$-\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f}-\frac{(B-i(A-C))(c+id)^{3/2}\tanh^{-1}\left(\cdots\right.}{(a+ib)^{7/}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(7/2)/f-(B-I*(A-C))*(c+I*d)^(3/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(a+I*b)^(7/2)/f-2/15*(2*a^5*b*B*d^2+3*a^6*C*d^2+a^4*b^2*d*(10*B*c
+(8*A+C)*d)+a^2*b^4*(45*A*c^2-49*A*d^2-90*B*c*d-45*C*c^2+58*C*d^2)-a^3*b^3*
(50*c*(A-C)*d+B*(15*c^2-39*d^2))+a*b^5*(70*c*(A-C)*d+B*(45*c^2-23*d^2))+b^6
*(5*c*(4*B*d+3*C*c)-3*A*(5*c^2-d^2)))*(c+d*tan(f*x+e))^(1/2)/b^2/(a^2+b^2)^
3/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)-2/15*(2*a^3*b*B*d+3*a^4*C*d+b^4*(3*A*
d+5*B*c)+2*a*b^3*(5*A*c-4*B*d-5*C*c)-a^2*b^2*(7*A*d+5*B*c-13*C*d))*(c+d*tan
(f*x+e))^(1/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))^(3/2)-2/5*(A*b^2-a*(B*b-C
*a))*(c+d*tan(f*x+e))^(3/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(5/2)
```

Rubi [A]

time = 2.46, antiderivative size = 586, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3726, 3730, 3697, 3696, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{5bf(a^2+b^2)(a+b\tan(e+fx))^{5/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\,(3a^4Cd+2a^3bBd-a^2b^2(7Ad+5Bc-13Cd)+2ab^3(5Ac-4Bd-5cC)+b^4(3Ad+5Bc))}{15b^2f(a^2+b^2)^2(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\,(3a^6Cd^2+2a^5bBd^2+a^4b^2d(d(8A+C)+10Bc)-a^3b^3(50cd(A-C)+B(15c^2-39d^2))+a^2b^4(45Ac^2-49Ad^2-90Bcd-45c^2C+58Cd^2)+ab^5(70cd(A-C)+B(45c^2-23d^2))+b^6(5c(4Bd+3cC)-3A(5c^2-d^2)))}{15b^2f(a^2+b^2)^3(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{(c-id)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{7/2}}-\frac{(c+id)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) -
((B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) - (2*
(2*a^3*b*B*d + 3*a^4*C*d + b^4*(5*B*c + 3*A*d) + 2*a*b^3*(5*A*c - 5*c*C - 4
*B*d) - a^2*b^2*(5*B*c + 7*A*d - 13*C*d))*Sqrt[c + d*Tan[e + f*x]])/(15*b^2
*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(2*a^5*b*B*d^2 + 3*a^6*C*
d^2 + a^4*b^2*d*(10*B*c + (8*A + C)*d) + a^2*b^4*(45*A*c^2 - 45*c^2*C - 90*
B*c*d - 49*A*d^2 + 58*C*d^2) - a^3*b^3*(50*c*(A - C)*d + B*(15*c^2 - 39*d^2
)) + a*b^5*(70*c*(A - C)*d + B*(45*c^2 - 23*d^2)) + b^6*(5*c*(3*c*C + 4*B*d
) - 3*A*(5*c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(15*b^2*(a^2 + b^2)^3*(b*
c - a*d)*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Ta
n[e + f*x])^(3/2))/(5*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e+}{5b\,(a^2+b^2)\,f(a+b\tan(e+fx)}$$

$$= -\frac{2(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad}{}$$

$$= -\frac{2(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad}{}$$

$$= -\frac{2(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad}{}$$

$$= -\frac{2(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad}{}$$

$$= -\frac{2(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{3/2}\tanh^{-1}\left(\right.}{(a-ib)^{7}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3134 vs. 2(586) = 1172.
time = 7.96, size = 3134, normalized size = 5.35

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] -((C*(c + d*Tan[e + f*x])^(3/2))/(b*f*(a + b*Tan[e + f*x])^(5/2))) - (-1/4*
((3*b*c*C - 2*b*B*d - 3*a*C*d)*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[e
+ f*x])^(5/2)) - ((-2*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B
*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - a*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) + (
b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c^2
- d^2))))*Sqrt[c + d*Tan[e + f*x]])/(5*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan
[e + f*x])^(5/2)) - (2*((-2*(b^2*(((2*b^2*d - (5*a*(b*c - a*d))/2)*(8*A*b^2
*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 + ((
-5*b*c)/2 + (a*d)/2)*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b
*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))) - a*(
(5*b*(b*c - a*d)*((b*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5
*b^2*c*(c*C + 2*B*d)))/4 - (b*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b
*c*C - 2*b*B*d - 3*a*C*d)))/4 - 2*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))/2
 - 2*a*d*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c
*(c*C + 2*B*d)))/4 - a*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3
*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2))))))*S
qrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^
(3/2)) - (2*((-15*b^2*(b*c - a*d)^2*(((3*a^2*A*b*c^2 - A*b^3*c^2 - a^3*B*c^
2 + 3*a*b^2*B*c^2 - 3*a^2*b*c^2*C + b^3*c^2*C - 2*a^3*A*c*d + 6*a*A*b^2*c*d
 - 6*a^2*b*B*c*d + 2*b^3*B*c*d + 2*a^3*c*C*d - 6*a*b^2*c*C*d - 3*a^2*A*b*d^
2 + A*b^3*d^2 + a^3*B*d^2 - 3*a*b^2*B*d^2 + 3*a^2*b*C*d^2 - b^3*C*d^2 + I*(
-(a^3*A*c^2) + 3*a*A*b^2*c^2 - 3*a^2*b*B*c^2 + b^3*B*c^2 + a^3*c^2*C - 3*a*
b^2*c^2*C - 6*a^2*A*b*c*d + 2*A*b^3*c*d + 2*a^3*B*c*d - 6*a*b^2*B*c*d + 6*a
^2*b*c*C*d - 2*b^3*c*C*d + a^3*A*d^2 - 3*a*A*b^2*d^2 + 3*a^2*b*B*d^2 - b^3*
B*d^2 - a^3*C*d^2 + 3*a*b^2*C*d^2))*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt
[-c + I*d]) + ((-3*a^2*A*b*c^2 + A*b^3*c^2 + a^3*B*c^2 - 3*a*b^2*B*c^2 + 3*
a^2*b*c^2*C - b^3*c^2*C + 2*a^3*A*c*d - 6*a*A*b^2*c*d + 6*a^2*b*B*c*d - 2*b
^3*B*c*d - 2*a^3*c*C*d + 6*a*b^2*c*C*d + 3*a^2*A*b*d^2 - A*b^3*d^2 - a^3*B*
d^2 + 3*a*b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3*C*d^2 + I*(-(a^3*A*c^2) + 3*a*A*b
^2*c^2 - 3*a^2*b*B*c^2 + b^3*B*c^2 + a^3*c^2*C - 3*a*b^2*c^2*C - 6*a^2*A*b*
c*d + 2*A*b^3*c*d + 2*a^3*B*c*d - 6*a*b^2*B*c*d + 6*a^2*b*c*C*d - 2*b^3*c*C
*d + a^3*A*d^2 - 3*a*A*b^2*d^2 + 3*a^2*b*B*d^2 - b^3*B*d^2 - a^3*C*d^2 + 3*
a*b^2*C*d^2))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*
b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d])))/(2*(a^2 + b^
2)*f) - (2*(b^2*((b^2*d - (3*a*(b*c - a*d))/2)*(((2*b^2*d - (5*a*(b*c - a*d
))/2)*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2
*B*d)))/4 + ((-5*b*c)/2 + (a*d)/2)*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) + (b
*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c^2 -
 d^2)))) + ((-3*b*c)/2 + (a*d)/2)*((5*b*(b*c - a*d)*((b*(8*A*b^2*c^2 + 3*a^
```

```
2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - (b*(8*b^2*d*(
B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 - 2*a*b^2*
(2*c*(A - C)*d + B*(c^2 - d^2))))/2 - 2*a*d*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^
2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - a*(-1/4*(a*(8*b^2*d
*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2
*c*(A - C)*d + B*(c^2 - d^2)))))) - a*((3*b*(b*c - a*d)*((-5*a*(b*c - a*d)*
((b*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B
*d)))/4 - (b*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d -
3*a*C*d)))/4 - 2*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))/2 - 2*b*d*((b^2*(8
*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/
4 - a*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d
- 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))) + b*(((2*b^2*d - (5*
a*(b*c - a*d))/2)*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^
2*c*(c*C + 2*B*d)))/4 + ((-5*b*c)/2 + (a*d)/2)*(-1/4*(a*(8*b^2*d*(B*c + (A
- C)*d) + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*
d + B*(c^2 - d^2))))))/2 - a*d*(b^2*(((2*b^2*d - (5*a*(b*c - a*d))/2)*(8*A*
b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 +
 ((-5*b*c)/2 + (a*d)/2)*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(
3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))) -
a*((5*b*(b*c - a*d)*((b*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d)
- 5*b^2*c*(c*C + 2*B*d)))/4 - (b*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(
3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 - 2*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))
)/2 - 2*a*d*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^
2*c*(c*C + 2*B*d)))/4 - a*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)
*(3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))))
)))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
 f*x]])))/(3*(a^2 + b^2)*(b*c - a*d))))/(5*(a^2...
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{(a+b\tan(fx+e))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\tan(e + fx))^{\frac{3}{2}} (A + B\tan(e + fx) + C\tan^2(e + fx))}{(a + b\tan(e + fx))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(7/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x))**(7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(7/2),x)
```

```
[Out] \text{Hanged}
```

3.141 $\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}\,(A+B\,\text{t}$

Optimal. Leaf size=697

$$-\frac{\sqrt{a-ib}\,(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f}+\frac{\sqrt{a+ib}\,(iA-B-iC)(c-}{}$$

```
[Out] -1/64*(5*a^4*C*d^4-4*a^3*b*d^3*(2*B*d+5*C*c)+2*a^2*b^2*d^2*(15*c^2*C+20*B*c
*d+8*(A-C)*d^2)-4*a*b^3*d*(5*c^3*C+30*B*c^2*d+40*c*(A-C)*d^2-16*B*d^3)+b^4*
(5*c^4*C-40*B*c^3*d-240*c^2*(A-C)*d^2+320*B*c*d^3+128*(A-C)*d^4))*arctanh(d
^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(7/2)/d^(3/
2)/f-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)
/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/f+(I*A-B-I*C)*(c+I*d)^
(5/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f
*x+e))^(1/2))*(a+I*b)^(1/2)/f+1/64*(64*b^2*d^2*(A*a*d+A*b*c+B*a*c-B*b*d-C*a
*d-C*b*c)+(-a*d+b*c)*(48*b*(A*b+B*a-C*b)*d^2-5*(-a*d+b*c)*(-8*B*b*d-C*a*d+C
*b*c)))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^3/d/f+1/96*(48*b*(A
*b+B*a-C*b)*d^2-5*(-a*d+b*c)*(-8*B*b*d-C*a*d+C*b*c))*(a+b*tan(f*x+e))^(1/2)
*(c+d*tan(f*x+e))^(3/2)/b^2/d/f-1/24*(-8*B*b*d-C*a*d+C*b*c)*(a+b*tan(f*x+e)
)^(1/2)*(c+d*tan(f*x+e))^(5/2)/b/d/f+1/4*C*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(
f*x+e))^(7/2)/d/f
```

Rubi [A]
time = 7.54, antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/f) + (Sqr
t[a + I*b]*(I*A - B - I*C)*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/f - ((5*a^4*C*d
^4 - 4*a^3*b*d^3*(5*c*C + 2*B*d) + 2*a^2*b^2*d^2*(15*c^2*C + 20*B*c*d + 8*(
A - C)*d^2) - 4*a*b^3*d*(5*c^3*C + 30*B*c^2*d + 40*c*(A - C)*d^2 - 16*B*d^3
) + b^4*(5*c^4*C - 40*B*c^3*d - 240*c^2*(A - C)*d^2 + 320*B*c*d^3 + 128*(A
- C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*T
an[e + f*x]])])/(64*b^(7/2)*d^(3/2)*f) + ((64*b^2*d^2*(A*b*c + a*B*c - b*c*
C + a*A*d - b*B*d - a*C*d) + (b*c - a*d)*(48*b*(A*b + a*B - b*C)*d^2 - 5*(b
*c - a*d)*(b*c*C - 8*b*B*d - a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*T
```

```
an[e + f*x]])/(64*b^3*d*f) + ((48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(
b*c*C - 8*b*B*d - a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/
2))/(96*b^2*d*f) - ((b*c*C - 8*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c +
 d*Tan[e + f*x])^(5/2))/(24*b*d*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan
[e + f*x])^(7/2))/(4*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
```

```
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C\sqrt{a+b\tan(e\ldots}}{\ldots}$$

$$= -\frac{(bcC-8bBd\ldots}{\ldots}$$

$$= \frac{(48b(Ab+aB-\ldots}{\ldots}$$

$$= \frac{(64b^2d^2(Abc+a\ldots}{\ldots}$$

$$= \frac{(64b^2d^2(Abc+a\ldots}{\ldots}$$

$$= \frac{(64b^2d^2(Abc+a\ldots}{\ldots}$$

$$= \frac{(64b^2d^2(Abc+a\ldots}{\ldots}$$

$$= \frac{(64b^2d^2(Abc+a\ldots}{\ldots}$$

$$= \frac{(64b^2d^2(Abc+a\ldots}{\ldots}$$

$$= -\frac{(5a^4Cd^4-4a^3\ldots}{\ldots}$$

$$= -\frac{\sqrt{a-ib}\,(iA+\ldots}{\ldots}$$

__**Mathematica**__ **[A]**

time = 8.41, size = 1261, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e
+ f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(7/2))/(4*d*f) + (((-(b*c*
C) + 8*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/
(6*b*f) + (((48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(b*c*C - 8*b*B*d -
a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(8*b*f) + (((2
4*b^2*d^2*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) - (3*(-(b*c) + a*
d)*(48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(b*c*C - 8*b*B*d - a*C*d)))/
8)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((-24*b^3*d*(
Sqrt[-b^2]*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^
3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3)) - b*(A*(b*c^3 + 3*a*c^2*d
 - 3*b*c*d^2 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^
3 - 3*c^2*C*d - 3*B*c*d^2 + C*d^3)))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*S
qrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])
/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (24*b^3*d*(Sqrt[-b^2
]*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^3*C - 3*B
*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3)) + b*(A*(b*c^3 + 3*a*c^2*d - 3*b*c*
d^2 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c^2
*C*d - 3*B*c*d^2 + C*d^3)))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*
Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a +
Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) - (3*Sqrt[b]*Sqrt[c - (a*d)/b]*Sqrt
[(c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b)))^(-1)]*Sqrt[c/(c - (a*d)/b) - (
a*d)/(b*(c - (a*d)/b))]*(5*a^4*C*d^4 - 4*a^3*b*d^3*(5*c*C + 2*B*d) + 2*a^2*
b^2*d^2*(15*c^2*C + 20*B*c*d + 8*(A - C)*d^2) - 4*a*b^3*d*(5*c^3*C + 30*B*c
^2*d + 40*c*(A - C)*d^2 - 16*B*d^3) + b^4*(5*c^4*C - 40*B*c^3*d - 240*c^2*(
A - C)*d^2 + 320*B*c*d^3 + 128*(A - C)*d^4))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b]*Sqrt[c/(c - (a*d)/b) - (a*d)/(b*(c
- (a*d)/b))])]*Sqrt[(c + d*Tan[e + f*x])/(c - (a*d)/b)])/(8*Sqrt[d]*Sqrt[c
+ d*Tan[e + f*x]]))/(b^2*f))/(2*b))/(3*b))/(4*d)
```

Maple [F]

time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\tan(fx+e)}\,(c+d\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x
+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)*
(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tan{(e + fx)}}\,(c + d \tan{(e + fx)})^{\frac{5}{2}} \left(A + B \tan{(e + fx)} + C \tan^{2}{(e + fx)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C
*tan(f*x+e)**2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2)*(A + B*tan(e
+ f*x) + C*tan(e + f*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*t
an(f*x+e)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\tan\left(e+fx\right)}\ \left(c+d\tan(e+fx)\right)^{5/2}\left(C\tan(e+fx)^2+B\tan(e+fx)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*
x) + C*tan(e + f*x)^2), x)
```

3.142 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

Optimal. Leaf size=505

$$-\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f}-\frac{(B-i(A-C))(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{a+ib}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(a-I*b)^(1/2)-(B-I*(A-C))*(c+I*d)^(5/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/f/(a+I*b)^(1/2)-1/8*(5*a^3*C*d^3-3*a^2*b*d^2*(2*B*d+5*C*c)+a*b^2*
d*(15*c^2*C+20*B*c*d+8*(A-C)*d^2)-b^3*(5*c^3*C+30*B*c^2*d+40*c*(A-C)*d^2-16
*B*d^3))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1
/2))/b^(7/2)/f/d^(1/2)+1/8*(8*b^2*d*(B*c+(A-C)*d)+(-a*d+b*c)*(6*B*b*d-5*C*a
*d+5*C*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^3/f+1/12*(6*B*
b*d-5*C*a*d+5*C*b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/b^2/f+1/
3*C*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)/b/f
```

Rubi [A]

time = 4.48, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{(5a^3Cd^3-3a^2bd^2(2Bd+5cC)+ab^2d(8d^2(A-C)+20Bcd+15c^2C)-(b^3(40cd^2(A-C)+30Bc^2d-16Bd^3+5c^3C)))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{8b^{7/2}\sqrt{d}\,f}+\frac{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,((bc-ad)(-5aCd+6bBd+5bcC)+8b^2d(d(A-C)+Bc))}{8b^3f}-\frac{(c-id)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}}-\frac{(c+id)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}}+\frac{(-5aCd+6bBd+5bcC)\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{12b^2f}+\frac{C\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[a + b*Tan[e + f*x]],x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f)) - ((
B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f) - ((5*a^3
*C*d^3 - 3*a^2*b*d^2*(5*c*C + 2*B*d) + a*b^2*d*(15*c^2*C + 20*B*c*d + 8*(A
- C)*d^2) - b^3*(5*c^3*C + 30*B*c^2*d + 40*c*(A - C)*d^2 - 16*B*d^3))*ArcTa
nh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/
(8*b^(7/2)*Sqrt[d]*f) + ((8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(5*b*c*C
+ 6*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8
*b^3*f) + ((5*b*c*C + 6*b*B*d - 5*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Ta
n[e + f*x])^(3/2))/(12*b^2*f) + (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e +
f*x])^(5/2))/(3*b*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx = \frac{C\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+}{3bf}$$

$$= \frac{(5bcC+6bBd-5aCd)\sqrt{a+b\tan(}{12b^2f}$$

$$= \frac{(8b^2d(Bc+(A-C)d)+(bc-ad)(5}{}$$

$$= \frac{(8b^2d(Bc+(A-C)d)+(bc-ad)(5}{}$$

$$= \frac{(8b^2d(Bc+(A-C)d)+(bc-ad)(5}{}$$

$$= \frac{(8b^2d(Bc+(A-C)d)+(bc-ad)(5}{}$$

$$= \frac{(8b^2d(Bc+(A-C)d)+(bc-ad)(5}{}$$

$$= \frac{(8b^2d(Bc+(A-C)d)+(bc-ad)(5}{}$$

$$= -\frac{(5a^3Cd^3-3a^2bd^2(5cC+2Bd)+a}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\Bigg(}{\sqrt{a-}}$$

Mathematica [A]
time = 7.97, size = 780, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/Sqrt[a + b*Tan[e + f*x]],x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*b*f) + (((5*b*c*
C + 6*b*B*d - 5*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))
/(4*b*f) + ((3*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(5*b*c*C + 6*b*B*d
- 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + ((
6*b^3*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) + Sqrt[-b^2]*(A*c^3
- c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3))*ArcTanh[(Sqrt[-c + (S
qrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d
*Tan[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (6*
b^3*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - Sqrt[-b^2]*(A*c^3 -
c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3))*ArcTanh[(Sqrt[c + (Sqrt
[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan
[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) - (3*Sqrt[b
]*Sqrt[c - (a*d)/b]*(5*a^3*C*d^3 - 3*a^2*b*d^2*(5*c*C + 2*B*d) + a*b^2*d*(1
5*c^2*C + 20*B*c*d + 8*(A - C)*d^2) - b^3*(5*c^3*C + 30*B*c^2*d + 40*c*(A -
 C)*d^2 - 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sq
rt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt[d]*Sq
rt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*b))/(3*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}}(A+B\tan(fx+e)+C(\tan^2(fx+e)))}{\sqrt{a+b\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(1/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan{(e+fx)})^{\frac{5}{2}}\,(A+B\tan{(e+fx)}+C\tan^{2}{(e+fx)})}{\sqrt{a+b\tan{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/sqrt(a + b*tan(e + f*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.143 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=535

$$-\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}-\frac{(B-i(A-C))(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(a+ib)^{3/}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/f-(B-I*(A-C))*(c+I*d)^(5/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(a+I*b)^(3/2)/f+1/4*(15*a^2*C*d^2-6*a*b*d*(2*B*d+5*C*c)+b^2*(15*c
^2*C+20*B*c*d+8*(A-C)*d^2))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/
(c+d*tan(f*x+e))^(1/2))*d^(1/2)/b^(7/2)/f-1/4*d*(15*a^3*C*d-8*A*b^2*(-a*d+b
*c)-3*a^2*b*(4*B*d+5*C*c)-b^3*(4*B*d+7*C*c)+a*b^2*(8*B*c+7*C*d))*(a+b*tan(f
*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)/f+1/2*(4*A*b^2-4*B*a*b+5*
C*a^2+C*b^2)*d*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/b^2/(a^2+b^2)/
f-2*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(5/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e
))^(1/2)
```

Rubi [A]
time = 6.18, antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3726, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}+\frac{d(5a^2C-4abB+4Ab^2+b^2C)\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}{2b^2f(a^2+b^2)}+\frac{\sqrt{d}\,(15a^2Cd^2-6abd(2Bd+5cC)+b^2(8d^2(A-C)+20Bcd+15c^2C))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{4b^{7/2}f}-\frac{d\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,(15a^3Cd-3a^2b(4Bd+5cC)-8Ab^2(bc-ad)+ab^2(8Bc+7Cd)-b^3(4Bd+7cC))}{4b^3f(a^2+b^2)}-\frac{(c-id)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}}-\frac{(c+id)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f) + (Sq
rt[d]*(15*a^2*C*d^2 - 6*a*b*d*(5*c*C + 2*B*d) + b^2*(15*c^2*C + 20*B*c*d +
8*(A - C)*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(4*b^(7/2)*f) - (d*(15*a^3*C*d - 8*A*b^2*(b*c - a*d) -
 3*a^2*b*(5*c*C + 4*B*d) - b^3*(7*c*C + 4*B*d) + a*b^2*(8*B*c + 7*C*d))*Sqr
t[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b^3*(a^2 + b^2)*f) + ((4
*A*b^2 - 4*a*b*B + 5*a^2*C + b^2*C)*d*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e
 + f*x])^(3/2))/(2*b^2*(a^2 + b^2)*f) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*T
an[e + f*x])^(5/2))/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e \ldots}{b\,(a^2+b^2)\,f\sqrt{a+b\tan(e+ \ldots}}$$

$$= \frac{(4Ab^2-4abB+5a^2C+b^2C)\,d\sqrt{a \ldots}}{2b^2\,(a^2 \ldots}$$

$$= -\frac{d(15a^3Cd-8Ab^2(bc-ad)-3a^2b \ldots}{\ldots}$$

$$= -\frac{d(15a^3Cd-8Ab^2(bc-ad)-3a^2b \ldots}{\ldots}$$

$$= -\frac{d(15a^3Cd-8Ab^2(bc-ad)-3a^2b \ldots}{\ldots}$$

$$= -\frac{d(15a^3Cd-8Ab^2(bc-ad)-3a^2b \ldots}{\ldots}$$

$$= -\frac{d(15a^3Cd-8Ab^2(bc-ad)-3a^2b \ldots}{\ldots}$$

$$= -\frac{d(15a^3Cd-8Ab^2(bc-ad)-3a^2b \ldots}{\ldots}$$

$$= \frac{\sqrt{d}\,(15a^2Cd^2-6abd(5cC+2Bd)+ \ldots}{\ldots}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\Big(\ldots}{(a-ib \ldots}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 39.16, size = 1654245, normalized size = 3092.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{(a+b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(3/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{5}{2}}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(3/2),x)
```

```
[Out] \text{Hanged}
```

3.144 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=545

$$-\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f}-\frac{(B-i(A-C))(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(a+ib)^{5/}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/f-(B-I*(A-C))*(c+I*d)^(5/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(a+I*b)^(5/2)/f+d^(3/2)*(2*B*b*d-5*C*a*d+5*C*b*c)*arctanh(d^(1/2)
*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(7/2)/f-d*(2*a^3*
b*B*d-5*a^4*C*d-2*a*b^3*(2*A*c-3*B*d-2*C*c)+2*a^2*b^2*(B*c-5*C*d)-b^4*(2*B*
c+(4*A+C)*d))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)^2
/f+2/3*(2*a^3*b*B*d-5*a^4*C*d-b^4*(5*A*d+3*B*c)-2*a*b^3*(3*A*c-4*B*d-3*C*c)
+a^2*b^2*(3*B*c+(A-11*C)*d))*(c+d*tan(f*x+e))^(3/2)/b^2/(a^2+b^2)^2/f/(a+b*
tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(5/2)/b/(a^2+b^2
)/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 8.26, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3726, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}+\frac{2(c+d\tan(e+fx))^{3/2}(-5a^4Cd+2a^3bBd+a^2b^2(d(A-11C)+3Bc)-2ab^3(3Ac-4Bd-3cC)-b^4(5Ad+3Bc))}{3b^2f(a^2+b^2)^2\sqrt{a+b\tan(e+fx)}}-\frac{d\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}(-5a^4Cd+2a^3bBd+2a^2b^2(Bc-5Cd)-2ab^3(2Ac-3Bd-2cC)-b^4(d(4A+C)+2Bc))}{b^3f(a^2+b^2)^2}-\frac{(c-id)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}}-\frac{(c+id)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}}+\frac{d^{3/2}(-5aCd+2bBd+5bcC)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*f) + (d^
(3/2)*(5*b*c*C + 2*b*B*d - 5*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(b^(7/2)*f) - (d*(2*a^3*b*B*d - 5*
a^4*C*d - 2*a*b^3*(2*A*c - 2*c*C - 3*B*d) + 2*a^2*b^2*(B*c - 5*C*d) - b^4*(
2*B*c + (4*A + C)*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b
^3*(a^2 + b^2)^2*f) + (2*(2*a^3*b*B*d - 5*a^4*C*d - b^4*(3*B*c + 5*A*d) - 2
*a*b^3*(3*A*c - 3*c*C - 4*B*d) + a^2*b^2*(3*B*c + (A - 11*C)*d))*(c + d*Tan
[e + f*x])^(3/2))/(3*b^2*(a^2 + b^2)^2*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(A*
b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(3*b*(a^2 + b^2)*f*(a + b*
Tan[e + f*x])^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e}{3b\,(a^2+b^2)\,f(a+b\tan(e+f}$$

$$= \frac{2(2a^3bBd-5a^4Cd-b^4(3Bc+5Ad}{}$$

$$= -\frac{d(2a^3bBd-5a^4Cd-2ab^3(2Ac-}{}$$

$$= -\frac{d(2a^3bBd-5a^4Cd-2ab^3(2Ac-}{}$$

$$= -\frac{d(2a^3bBd-5a^4Cd-2ab^3(2Ac-}{}$$

$$= -\frac{d(2a^3bBd-5a^4Cd-2ab^3(2Ac-}{}$$

$$= -\frac{d(2a^3bBd-5a^4Cd-2ab^3(2Ac-}{}$$

$$= -\frac{d(2a^3bBd-5a^4Cd-2ab^3(2Ac-}{}$$

$$= \frac{d^{3/2}(5bcC+2bBd-5aCd)\tanh^{-1}\Big(}{b^{7/2}f}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\Big(}{(a-ib}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 40.81, size = 2018669, normalized size = 3703.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}} (A+B\tan(fx+e)+C(\tan^2(fx+e)))}{(a+b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(e+fx))^{\frac{5}{2}} (A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(a + b*tan(e + f*x))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(5/2),x)
```

```
[Out] \text{Hanged}
```

3.145 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=590

$$-\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f}-\frac{(B-i(A-C))(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(a+ib)^{7/}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(7/2)/f-(B-I*(A-C))*(c+I*d)^(5/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(a+I*b)^(7/2)/f+2*C*d^(5/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2
)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(7/2)/f-2*(a^6*C*d^2+3*a^4*b^2*C*d^2-3*
a^2*b^4*(c^2*C+2*B*c*d-2*C*d^2-A*(c^2-d^2))+b^6*(c*(2*B*d+C*c)-A*(c^2-d^2))
-a^3*b^3*(2*c*(A-C)*d+B*(c^2-d^2))+3*a*b^5*(2*c*(A-C)*d+B*(c^2-d^2)))*(c+d*
tan(f*x+e))^(1/2)/b^3/(a^2+b^2)^3/f/(a+b*tan(f*x+e))^(1/2)-2/3*(a^4*C*d+b^4
*(A*d+B*c)+2*a*b^3*(A*c-B*d-C*c)-a^2*b^2*(B*c+(A-3*C)*d))*(c+d*tan(f*x+e))^
(3/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))^(3/2)-2/5*(A*b^2-a*(B*b-C*a))*(c+d
*tan(f*x+e))^(5/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 10.90, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3726, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{5bf(a^2+b^2)(a+b\tan(e+fx))^{5/2}}-\frac{2(c+d\tan(e+fx))^{3/2}(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc))}{3b^2f(a^2+b^2)^2(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\,(a^6Cd^2+3a^4b^2Cd^2-a^3b^3(2cd(A-C)+B(c^2-d^2))-3a^2b^4(-A(c^2-d^2)+2Bcd+c^2C-2Cd^2)+3ab^5(2cd(A-C)+B(c^2-d^2))+b^6(c(2Bd+cC)-A(c^2-d^2)))}{b^3f(a^2+b^2)^3\sqrt{a+b\tan(e+fx)}}-\frac{(c-id)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{7/2}}-\frac{(c+id)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{7/2}}+\frac{2Cd^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{b^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(7/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(7/2)*f) + (2*
C*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(b^(7/2)*f) - (2*(a^6*C*d^2 + 3*a^4*b^2*C*d^2 - 3*a^2*b^4*(c
^2*C + 2*B*c*d - 2*C*d^2 - A*(c^2 - d^2)) + b^6*(c*(c*C + 2*B*d) - A*(c^2 -
 d^2)) - a^3*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)) + 3*a*b^5*(2*c*(A - C)*d +
 B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/(b^3*(a^2 + b^2)^3*f*Sqrt[a + b*
Tan[e + f*x]]) - (2*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d)
- a^2*b^2*(B*c + (A - 3*C)*d))*(c + d*Tan[e + f*x])^(3/2))/(3*b^2*(a^2 + b^
2)^2*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[
e + f*x])^(5/2))/(5*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e}{5b\,(a^2+b^2)\,f(a+b\tan(e+fx}$$

$$= -\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac}{3b^2\,(a}$$

$$= -\frac{2(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C}{}$$

$$= -\frac{2(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C}{}$$

$$= -\frac{2(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C}{}$$

$$= -\frac{2(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C}{}$$

$$= -\frac{2(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C}{}$$

$$= -\frac{2(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C}{}$$

$$= \frac{2Cd^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+}}{\sqrt{b}\,\sqrt{c+d\tan(e+}}\right.}{b^{7/2}f}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\right.}{(a-ib}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 41.82, size = 2345519, normalized size = 3975.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(7/2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}} (A+B\tan(fx+e)+C(\tan^2(fx+e)))}{(a+b\tan(fx+e))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(7/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(7/2),x)
```

```
[Out] \text{Hanged}
```

3.146 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=946

$$-\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{9/2}f}-\frac{(B-i(A-C))(c+id)^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(a+ib)^{9/}}$$

```
[Out] -(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(9/2)/f-(B-I*(A-C))*(c+I*d)^(5/2
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(a+I*b)^(9/2)/f-2/105*(6*a^7*b*B*d^3+15*a^8*C*d^3+2*a^6*b^2*d^2*(
4*A*d+7*B*c+26*C*d)-2*a*b^7*(210*A*c^3-406*A*c*d^2-525*B*c^2*d+88*B*d^3-210
*C*c^3+406*C*c*d^2)-a^4*b^4*(525*A*c^2*d-311*A*d^3+105*B*c^3-749*B*c*d^2-52
5*C*c^2*d+221*C*d^3)+2*a^2*b^6*(875*A*c^2*d-261*A*d^3+315*B*c^3-812*B*c*d^2
-875*C*c^2*d+291*C*d^3)+2*a^5*b^3*d*(56*c*(A-C)*d+B*(35*c^2-12*d^2))-b^8*(5
*d*(49*A*c^2-3*A*d^2-49*C*c^2)+7*B*(15*c^3-23*c*d^2))-2*a^3*b^5*(210*c^3*C+
700*B*c^2*d-798*c*C*d^2-317*B*d^3-42*A*(5*c^3-19*c*d^2)))*(c+d*tan(f*x+e))^
(1/2)/b^3/(a^2+b^2)^4/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)-2/105*(6*a^5*b*B*
d^2+15*a^6*C*d^2+a^4*b^2*d*(8*A*d+14*B*c+37*C*d)+3*a^2*b^4*(35*A*c^2-39*A*d
^2-70*B*c*d-35*C*c^2+54*C*d^2)-a^3*b^3*(98*c*(A-C)*d+B*(35*c^2-75*d^2))+a*b
^5*(182*c*(A-C)*d+B*(105*c^2-71*d^2))+b^6*(7*c*(8*B*d+5*C*c)-5*A*(7*c^2-3*d
^2)))*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)^3/f/(a+b*tan(f*x+e))^(3/2)-2/35*
(2*a^3*b*B*d+5*a^4*C*d+b^4*(5*A*d+7*B*c)+2*a*b^3*(7*A*c-6*B*d-7*C*c)-a^2*b^
2*(9*A*d+7*B*c-19*C*d))*(c+d*tan(f*x+e))^(3/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f
*x+e))^(5/2)-2/7*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(5/2)/b/(a^2+b^2)/f/(
a+b*tan(f*x+e))^(7/2)
```

Rubi [A]
time = 4.32, antiderivative size = 946, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3726, 3730, 3697, 3696, 95, 214}

Antiderivative was successfully verified.

```
[In] Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
 + b*Tan[e + f*x])^(9/2),x]
```

```
[Out] -(((I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a - I*b)^(9/2)*f)) -
((B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a + I*b)^(9/2)*f) - (2*
(6*a^5*b*B*d^2 + 15*a^6*C*d^2 + a^4*b^2*d*(14*B*c + 8*A*d + 37*C*d) + 3*a^2
```

```
*b^4*(35*A*c^2 - 35*c^2*C - 70*B*c*d - 39*A*d^2 + 54*C*d^2) - a^3*b^3*(98*c
*(A - C)*d + B*(35*c^2 - 75*d^2)) + a*b^5*(182*c*(A - C)*d + B*(105*c^2 - 7
1*d^2)) + b^6*(7*c*(5*c*C + 8*B*d) - 5*A*(7*c^2 - 3*d^2)))*Sqrt[c + d*Tan[e
 + f*x]])/(105*b^3*(a^2 + b^2)^3*f*(a + b*Tan[e + f*x])^(3/2)) - (2*(6*a^7*
b*B*d^3 + 15*a^8*C*d^3 + 2*a^6*b^2*d^2*(7*B*c + 4*A*d + 26*C*d) - 2*a*b^7*(
210*A*c^3 - 210*c^3*C - 525*B*c^2*d - 406*A*c*d^2 + 406*c*C*d^2 + 88*B*d^3)
 - a^4*b^4*(105*B*c^3 + 525*A*c^2*d - 525*c^2*C*d - 749*B*c*d^2 - 311*A*d^3
 + 221*C*d^3) + 2*a^2*b^6*(315*B*c^3 + 875*A*c^2*d - 875*c^2*C*d - 812*B*c*
d^2 - 261*A*d^3 + 291*C*d^3) + 2*a^5*b^3*d*(56*c*(A - C)*d + B*(35*c^2 - 12
*d^2)) - b^8*(5*d*(49*A*c^2 - 49*c^2*C - 3*A*d^2) + 7*B*(15*c^3 - 23*c*d^2)
) - 2*a^3*b^5*(210*c^3*C + 700*B*c^2*d - 798*c*C*d^2 - 317*B*d^3 - 42*A*(5*
c^3 - 19*c*d^2)))*Sqrt[c + d*Tan[e + f*x]])/(105*b^3*(a^2 + b^2)^4*(b*c - a
*d)*f*Sqrt[a + b*Tan[e + f*x]]) - (2*(2*a^3*b*B*d + 5*a^4*C*d + b^4*(7*B*c
+ 5*A*d) + 2*a*b^3*(7*A*c - 7*c*C - 6*B*d) - a^2*b^2*(7*B*c + 9*A*d - 19*C*
d))*(c + d*Tan[e + f*x])^(3/2))/(35*b^2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]
)^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(7*b*(a^2
 + b^2)*f*(a + b*Tan[e + f*x])^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
```

```
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\left(c+d\tan(e\right.}{7b\left(a^2+b^2\right)f(a+b\tan(e+f}$$

$$= -\frac{2(2a^3bBd+5a^4Cd+b^4(7Bc+5A}{}$$

$$= -\frac{2(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14}{}$$

$$= -\frac{2(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14}{}$$

$$= -\frac{2(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14}{}$$

$$= -\frac{2(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14}{}$$

$$= -\frac{2(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14}{}$$

$$= -\frac{(iA+B-iC)(c-id)^{5/2}\tanh^{-1}\left(\right.}{(a-ib}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 46.37, size = 2719441, normalized size = 2874.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(a + b*Tan[e + f*x])^(9/2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{(a+b\tan(fx+e))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(9/2),x)
```

```
[Out] int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))
^(9/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan
(f*x+e))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f
*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a
 + b*tan(e + f*x))^(9/2),x)
```

```
[Out] \text{Hanged}
```

3.147 $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=505

$$-\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}-\frac{(a+ib)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{c+id}}$$

```
[Out] 1/8*(5*a^3*C*d^3-15*a^2*b*d^2*(-2*B*d+C*c)+5*a*b^2*d*(3*c^2*C-4*B*c*d+8*(A-
C)*d^2)-b^3*(5*c^3*C-6*B*c^2*d+8*c*(A-C)*d^2+16*B*d^3))*arctanh(d^(1/2)*(a+
b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(7/2)/f/b^(1/2)-(a-I*
b)^(5/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(
1/2)/(c+d*tan(f*x+e))^(1/2))/f/(c-I*d)^(1/2)-(a+I*b)^(5/2)*(B-I*(A-C))*arct
anh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/
2))/f/(c+I*d)^(1/2)+1/8*(8*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*(-6*B*b*d-5*C*a*d
+5*C*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^3/f-1/12*(-6*B*b
*d-5*C*a*d+5*C*b*c)*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)/d^2/f+1/3
*C*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 4.37, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{(5a^3Cd^3-15a^2bd^2(cC-2Bd)+5ab^2d(8d^2(A-C)-4Bcd+3c^2C)-(b^3(8cd^2(A-C)-6Bc^2d+16Bd^3+5c^3C)))\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{8\sqrt{b}\,d^{7/2}f}+\frac{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\,(8bd^2(aB+Ab-bC)+(bc-ad)(-5aCd-6bBd+5bcC))}{8d^3f}-\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c-id}}-\frac{(a+ib)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c+id}}-\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{12d^2f}+\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) - ((
a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) + ((5*a^3
*C*d^3 - 15*a^2*b*d^2*(c*C - 2*B*d) + 5*a*b^2*d*(3*c^2*C - 4*B*c*d + 8*(A -
 C)*d^2) - b^3*(5*c^3*C - 6*B*c^2*d + 8*c*(A - C)*d^2 + 16*B*d^3))*ArcTanh[
(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(8*
Sqrt[b]*d^(7/2)*f) + ((8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 6
*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(8*d^
3*f) - ((5*b*c*C - 6*b*B*d - 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d
*Tan[e + f*x]])/(12*d^2*f) + (C*(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e
 + f*x]])/(3*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(}{3df}$$

$$= -\frac{(5bcC-6bBd-5aCd)(a+b\tan(}{12d^2}$$

$$= \frac{(8b(Ab+aB-bC)d^2+(bc-ad)(5}{}$$

$$= \frac{(8b(Ab+aB-bC)d^2+(bc-ad)(5}{}$$

$$= \frac{(8b(Ab+aB-bC)d^2+(bc-ad)(5}{}$$

$$= \frac{(8b(Ab+aB-bC)d^2+(bc-ad)(5}{}$$

$$= \frac{(8b(Ab+aB-bC)d^2+(bc-ad)(5}{}$$

$$= \frac{(8b(Ab+aB-bC)d^2+(bc-ad)(5}{}$$

$$= \frac{(5a^3Cd^3-15a^2bd^2(cC-2Bd)+5a}{}$$

$$= -\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\Big(}{\sqrt{c-}}$$

Mathematica [A]

time = 7.64, size = 785, normalized size = 1.55

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) + (((-5*b*c
*C + 6*b*B*d + 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]
)/(4*d*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 6*b*B*d
 - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d*f) + (
(-6*(Sqrt[-b^2]*(3*a^2*b*B - b^3*B - a^3*(A - C) + 3*a*b^2*(A - C)) - b*(a^
3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C)))*d^3*ArcTanh[(Sqrt[-c + (S
qrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d
*Tan[e + f*x]])]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (6*
(Sqrt[-b^2]*(3*a^2*b*B - b^3*B - a^3*(A - C) + 3*a*b^2*(A - C)) + b*(a^3*B
- 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C)))*d^3*ArcTanh[(Sqrt[c + (Sqrt[-
b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e
 + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) + (3*Sqrt[b]*
Sqrt[c - (a*d)/b]*(5*a^3*C*d^3 - 15*a^2*b*d^2*(c*C - 2*B*d) + 5*a*b^2*d*(3*
c^2*C - 4*B*c*d + 8*(A - C)*d^2) - b^3*(5*c^3*C - 6*B*c^2*d + 8*c*(A - C)*d
^2 + 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
- (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt[d]*Sqrt[c
+ d*Tan[e + f*x]]))/(b*d*f))/(2*d))/(3*d)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{\sqrt{c+d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^{\frac{5}{2}}\left(A+B\tan(e+fx)+C\tan^{2}(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{5/2}\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{\sqrt{c+d\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(1/2), x)
```

3.148 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=383

$$-\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}+\frac{(a+ib)^{3/2}(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.}{\sqrt{c+id}}$$

```
[Out] 1/4*(3*a^2*C*d^2-6*a*b*d*(-2*B*d+C*c)+b^2*(3*c^2*C-4*B*c*d+8*(A-C)*d^2))*ar
ctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(5/2
)/f/b^(1/2)-(a-I*b)^(3/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e)
)^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(c-I*d)^(1/2)+(a+I*b)^(3/2)
*(I*A-B-I*C)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/f/(c+I*d)^(1/2)-1/4*(-4*B*b*d-3*C*a*d+3*C*b*c)*(a+b*ta
n(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^2/f+1/2*C*(c+d*tan(f*x+e))^(1/2)*(
a+b*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 3.03, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{(3a^2Cd^2-6abd(cC-2Bd)+b^2(8d^2(A-C)-4Bcd+3c^2C))\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4\sqrt{b}\,d^{5/2}f}-\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c-id}}+\frac{(a+ib)^{3/2}(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c+id}}-\frac{(-3aCd-4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4d^2f}+\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sq
rt[c + d*Tan[e + f*x]],x]
```

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) + ((
a + I*b)^(3/2)*(I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) + ((3*a^2
*C*d^2 - 6*a*b*d*(c*C - 2*B*d) + b^2*(3*c^2*C - 4*B*c*d + 8*(A - C)*d^2))*A
rcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]]
)])/(4*Sqrt[b]*d^(5/2)*f) - ((3*b*c*C - 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e
 + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d^2*f) + (C*(a + b*Tan[e + f*x])^(3/2
)*Sqrt[c + d*Tan[e + f*x]])/(2*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
```

```
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(}{2df}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan}{4d^2 j}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan}{4d^2 j}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan}{4d^2 j}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan}{4d^2 j}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan}{4d^2 j}$$

$$= -\frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan}{4d^2 j}$$

$$= \frac{(3a^2Cd^2-6abd(cC-2Bd)+b^2(3c^2}{}$$

$$= -\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\Big(}{\sqrt{c-}}$$

Mathematica [A]
time = 4.58, size = 582, normalized size = 1.52

$$\frac{\frac{4\left(b(a^2B-b^2B+2ab(A-C))-\sqrt{-b^2}\,(2abB+b^2(A-C)+a^2(-A+C))\right)d^2\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{b\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}}-\frac{4\left(b(a^2B-b^2B+2ab(A-C))+\sqrt{-b^2}\,(2abB+b^2(A-C)+a^2(-A+C))\right)d^2\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{b\sqrt{a+\sqrt{-b^2}}\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}}+(-3bcC+4bBd+3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}+2Cd(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}+\frac{\sqrt{c-\frac{ad}{b}}\,(3a^2Cd^2+6abd(-cC+2Bd)+b^2(3c^2C-4Bcd+8(A-C)d^2))\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{b}\sqrt{d}\sqrt{c+d\tan(e+fx)}}}{4d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] ((4*(b*(a^2*B - b^2*B + 2*a*b*(A - C)) - Sqrt[-b^2]*(2*a*b*B + b^2*(A - C)
+ a^2*(-A + C)))*d^2*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(b*Sqrt[-a + Sq
rt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (4*(b*(a^2*B - b^2*B + 2*a*b*(A -
C)) + Sqrt[-b^2]*(2*a*b*B + b^2*(A - C) + a^2*(-A + C)))*d^2*ArcTanh[(Sqrt[
c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[
c + d*Tan[e + f*x]])])/(b*Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b])
+ (-3*b*c*C + 4*b*B*d + 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e
+ f*x]] + 2*C*d*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]] + (Sqrt
[c - (a*d)/b]*(3*a^2*C*d^2 + 6*a*b*d*(-(c*C) + 2*B*d) + b^2*(3*c^2*C - 4*B*
c*d + 8*(A - C)*d^2))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*S
qrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)])/(Sqrt[b]*Sqr
t[d]*Sqrt[c + d*Tan[e + f*x]]))/(4*d^2*f)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{\sqrt{c+d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/2
)/sqrt(d*tan(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^{\frac{3}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{3/2}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{\sqrt{c+d\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(1/2),x)
```

```
[Out] int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(1/2), x)
```

3.149 $\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}\,f}+\frac{\sqrt{a+ib}\,(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+}}{\sqrt{a+}}\right.}{\sqrt{c+id}\,f}$$

```
[Out] -(-2*B*b*d-C*a*d+C*b*c)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d
*tan(f*x+e))^(1/2))/d^(3/2)/f/b^(1/2)-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+
b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/f/(
c-I*d)^(1/2)+(I*A-B-I*C)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*
b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I*b)^(1/2)/f/(c+I*d)^(1/2)+C*(a+b*tan(f
*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 1.95, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c-id}}+\frac{\sqrt{a+ib}\,(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{c+id}}-\frac{(-aCd-2bBd+bcC)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\,d^{3/2}f}+\frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt
[c + d*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f)) + (Sqr
t[a + I*b]*(I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f) - ((b*c*C - 2
*b*B*d - a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
+ d*Tan[e + f*x]])])/(Sqrt[b]*d^(3/2)*f) + (C*Sqrt[a + b*Tan[e + f*x]]*Sqrt
[c + d*Tan[e + f*x]])/(d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+}{df}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+}{df}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+}{df}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+}{df}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+}{df}$$

$$= \frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+}{df}$$

$$= -\frac{(bcC-2bBd-aCd)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{}}{\sqrt{b}\,\sqrt{}}\right.}{\sqrt{b}\,d^{3/2}f}$$

$$= -\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{\sqrt{c-id}\,f}$$

Mathematica [A]
time = 4.34, size = 450, normalized size = 1.55

$$\frac{C\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{df}-\frac{\frac{\left(-b(Ab+aB-bC)+\sqrt{-b^2}\,(bB+a(-A+C))\right)d\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}}+\frac{\left(b(Ab+aB-bC)+\sqrt{-b^2}\,(bB+a(-A+C))\right)d\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}}+\frac{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}\,(bcC-2bBd-aCd)\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{d}\,\sqrt{c+d\tan(e+fx)}}}{bdf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/Sqrt[c + d*Tan[e + f*x]],x]
```

```
[Out] (C*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f) - (((-(b*(A*b +
 a*B - b*C)) + Sqrt[-b^2]*(b*B + a*(-A + C)))*d*ArcTanh[(Sqrt[-c + (Sqrt[-b
^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e
 + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) + ((b*(A*b
+ a*B - b*C) + Sqrt[-b^2]*(b*B + a*(-A + C)))*d*ArcTanh[(Sqrt[c + (Sqrt[-b^
2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e +
 f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) + (Sqrt[b]*Sqrt
[c - (a*d)/b]*(b*c*C - 2*b*B*d - a*C*d)*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a
*d)])/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*d*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan \left(fx+e\right)}\,\left(A+B\tan \left(fx+e\right)+C(\tan^{2}\left(fx+e\right))\right)}{\sqrt{c+d\tan \left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)/
sqrt(d*tan(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan{(e+fx)}}\,(A+B\tan{(e+fx)}+C\tan^2{(e+fx)})}{\sqrt{c+d\tan{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
sqrt(c + d*tan(e + f*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(1/2),x)
```

```
[Out] \text{Hanged}
```

3.150 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=239

$$-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\sqrt{c-id}\,f}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}\,f}$$

```
[Out] -(B+I*(A-C))*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/f/(a-I*b)^(1/2)/(c-I*d)^(1/2)+(I*A-B-I*C)*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(a
+I*b)^(1/2)/(c+I*d)^(1/2)+2*C*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2
)/(c+d*tan(f*x+e))^(1/2))/f/b^(1/2)/d^(1/2)
```

Rubi [A]

time = 1.09, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\sqrt{c-id}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\sqrt{c+id}}+\frac{2C\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*Sqrt[
c + d*Tan[e + f*x]]),x]
```

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f)) + ((I*
A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b
]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d]*f) + (2*C*ArcTan
h[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(
Sqrt[b]*Sqrt[d]*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{C}{\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{A-C+Bx}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x}\right.\right.}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{A-C+Bx}{\sqrt{a+bx}\,\sqrt{c+dx}\,(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-B+i(A-C)}{2(i-x)\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{B+i(A-C)}{2(i+x)\sqrt{a+bx}\,\sqrt{c}}\right.\right.}{f}$$

$$= \frac{(-B+i(A-C))\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x\right.}{2f}$$

$$= \frac{2C\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\,\sqrt{d}\,f} + \frac{(-B+i(A-}{}$$

$$= -\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx}}\right.}{\sqrt{a-ib}\,\sqrt{c-id}\,f}$$

Mathematica [A]
time = 1.53, size = 362, normalized size = 1.51

$$\frac{\frac{\left(bB+\sqrt{-b^2}(A-C)\right)\tanh^{-1}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\,\sqrt{-c+\frac{\sqrt{-b^2}\,d}{b}}} - \frac{\left(bB+\sqrt{-b^2}(-A+C)\right)\tanh^{-1}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\,\sqrt{c+\frac{\sqrt{-b^2}\,d}{b}}} + \frac{2\sqrt{b}\,C\sqrt{c-\frac{ad}{b}}\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c-\frac{ad}{b}}}\right)\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}{\sqrt{d}\,\sqrt{c+d\tan(e+fx)}}}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]
*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (((b*B + Sqrt[-b^2]*(A - C))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-
a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - ((b*B + Sqrt[-b^2]*(-A + C))
```

```
*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sq
rt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[
-b^2]*d)/b]) + (2*Sqrt[b]*C*Sqrt[c - (a*d)/b]*ArcSinh[(Sqrt[d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b
*c - a*d)])/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*f)
```

Maple [**F(-1)**] grade_annotation
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(1/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/(sqrt(b*tan(f*x + e) + a)
*sqrt(d*tan(f*x + e) + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \tan{\left(e + fx \right)} + C \tan^{2}{\left(e + fx \right)}}{\sqrt{a + b \tan{\left(e + fx \right)}} \sqrt{c + d \tan{\left(e + fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*tan
(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(sqrt(a + b*tan(e + f*x))
*sqrt(c + d*tan(e + f*x))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(1/2)*(c
+ d*tan(e + f*x))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.151 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=251

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}\sqrt{c-id}\,f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}\sqrt{c+id}\,f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/f/(c-I*d)^(1/2)-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I
*b)^(3/2)/f/(c+I*d)^(1/2)-2*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(1/2)/(a^2
+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {3730, 3697, 3696, 95, 214}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}\sqrt{c-id}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*Sqr
t[c + d*Tan[e + f*x]]),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])]])/((a - I*b)^(3/2)*Sqrt[c - I*d]*f)) - ((
B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*Sqrt[c + I*d]*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sq
rt[a + b*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}} - \frac{2\int \frac{-\frac{1}{2}(b\ldots}{\ldots}}{\sqrt{\ldots}}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{(A-iB\ldots}{\ldots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{(A-iB\ldots}{\ldots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}} + \frac{(A-iB\ldots}{\ldots}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right.\ldots}{(a-ib)^{3/2}\sqrt{c-id}\,f}$$

Mathematica [**A**]
time = 1.72, size = 264, normalized size = 1.05

$$\frac{\frac{(a+ib)(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}} + \frac{(ia+b)(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}} + \frac{2(Ab^2+a(-bB+aC))\sqrt{c+d\tan(e+fx)}}{(-bc+ad)\sqrt{a+b\tan(e+fx)}}}{(a^2+b^2)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/
2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] (((a + I*b)*(I*A + B - I*C)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[-c + I*
d]) + ((I*a + b)*(A + I*B - C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d
]) + (2*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*Tan[e + f*x]])/((-(b*c) + a*d
)*Sqrt[a + b*Tan[e + f*x]]))/((a^2 + b^2)*f)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan(fx+e)+C(\tan^2(fx+e))}{\sqrt{c+d\tan(fx+e)}\,(a+b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)^(3/
2)*sqrt(d*tan(f*x + e) + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{\frac{3}{2}}\sqrt{c + d\tan(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(3
/2)*sqrt(c + d*tan(e + f*x))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(3/2)*(c
+ d*tan(e + f*x))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.152 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$

Optimal. Leaf size=375

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}\sqrt{c-id}\,f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}\sqrt{c+id}\,f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/f/(c-I*d)^(1/2)-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I
*b)^(5/2)/f/(c+I*d)^(1/2)-2/3*(5*a^3*b*B*d-2*a^4*C*d+b^4*(-2*A*d+3*B*c)+a*b
^3*(6*A*c-B*d-6*C*c)-a^2*b^2*(8*A*d+3*B*c-4*C*d))*(c+d*tan(f*x+e))^(1/2)/(a
^2+b^2)^2/(-a*d+b*c)^2/f/(a+b*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a))*(c+
d*tan(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 1.21, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {3730, 3697, 3696, 95, 214}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{3f\left(a^2+b^2\right)^2(bc-ad)^2\sqrt{a+b\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}\sqrt{c-id}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}\sqrt{c+id}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/2)*Sqr
t[c + d*Tan[e + f*x]]),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*Sqrt[c - I*d]*f)) - ((
B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*Sqrt[c + I*d]*f) - (2*(A*b
^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)*f*
(a + b*Tan[e + f*x])^(3/2)) - (2*(5*a^3*b*B*d - 2*a^4*C*d + b^4*(3*B*c - 2*
A*d) + a*b^3*(6*A*c - 6*c*C - B*d) - a^2*b^2*(3*B*c + 8*A*d - 4*C*d))*Sqrt[
c + d*Tan[e + f*x]])/(3*(a^2 + b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*
x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2\int \frac{1}{2}(\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2(5a^3\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2(5a^3\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2(5a^3\cdots}{\cdots}$$

$$= -\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2(5a^3\cdots}{\cdots}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx\cdots}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx\cdots}}\right.}{(a-ib)^{5/2}\sqrt{c-id}\,f}$$

<u>Mathematica</u> [A]

time = 4.11, size = 388, normalized size = 1.03

$$\frac{\frac{3(a+ib)^2(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}}+\frac{3i(a-ib)^2(A+iB-C)\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}}+\frac{2(a^2+b^2)(Ab^2+a(-bB+aC))\sqrt{c+d\tan(e+fx)}}{(-bc+ad)(a+b\tan(e+fx))^{3/2}}+\frac{2(-5a^3bBd+2a^4Cd+b^4(-3Bc+2Ad)+ab^3(-6Ac+6cC+Bd)+a^2b^2(3Bc+8Ad-4Cd))\sqrt{c+d\tan(e+fx)}}{(bc-ad)^2\sqrt{a+b\tan(e+fx)}}}{3(a^2+b^2)^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/
2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

```
[Out] ((3*(a + I*b)^2*(I*A + B - I*C)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-a + I*b]*Sqrt[-c
+ I*d]) + ((3*I)*(a - I*b)^2*(A + I*B - C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[a + I*b]*
Sqrt[c + I*d]) + (2*(a^2 + b^2)*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*Tan[e
 + f*x]])/((-(b*c) + a*d)*(a + b*Tan[e + f*x])^(3/2)) + (2*(-5*a^3*b*B*d +
2*a^4*C*d + b^4*(-3*B*c + 2*A*d) + a*b^3*(-6*A*c + 6*c*C + B*d) + a^2*b^2*(
3*B*c + 8*A*d - 4*C*d))*Sqrt[c + d*Tan[e + f*x]])/((b*c - a*d)^2*Sqrt[a + b
*Tan[e + f*x]]))/(3*(a^2 + b^2)^2*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan(fx+e) + C(\tan^2(fx+e))}{\sqrt{c + d\tan(fx+e)}\,(a + b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(a + b\tan(e+fx))^{\frac{5}{2}}\sqrt{c + d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(5
/2)*sqrt(c + d*tan(e + f*x))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(5/2)*(c
+ d*tan(e + f*x))^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.153 $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=528

$$-\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}-\frac{(a+ib)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(c+id)^{3/}}$$

```
[Out] -(a-I*b)^(5/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f-(a+I*b)^(5/2)*(B-I*(A-C)
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(c+I*d)^(3/2)/f+1/4*(15*a^2*C*d^2-10*a*b*d*(-2*B*d+3*C*c)+b^2*(15
*c^2*C-12*B*c*d+8*(A-C)*d^2))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2
)/(c+d*tan(f*x+e))^(1/2))*b^(1/2)/d^(7/2)/f-1/4*b*(3*(-a*d+b*c)*(5*c^2*C-4*
B*c*d+(4*A+C)*d^2)-4*d^2*((A-C)*(-a*d+b*c)+B*(a*c+b*d)))*(a+b*tan(f*x+e))^(
1/2)*(c+d*tan(f*x+e))^(1/2)/d^3/(c^2+d^2)/f+1/2*b*(5*c^2*C-4*B*c*d+(4*A+C)*
d^2)*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)/d^2/(c^2+d^2)/f-2*(A*d^2
-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(5/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 5.90, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3726, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{\sqrt{b}\left(15a^2Cd^2-10abd(3cC-2Bd)+b^2\left(8d^2(A-C)-12Bcd+15c^2C\right)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{4d^{7/2}f}+\frac{b\left(d^2(4A+C)-4Bcd+5c^2C\right)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2d^2f\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{b\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\left(3(bc-ad)\left(d^2(4A+C)-4Bcd+5c^2C\right)-4d^2((A-C)(bc-ad)+B(ac+bd))\right)}{4d^3f\left(c^2+d^2\right)}-\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}-\frac{(a+ib)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) -
((a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) + (Sq
rt[b]*(15*a^2*C*d^2 - 10*a*b*d*(3*c*C - 2*B*d) + b^2*(15*c^2*C - 12*B*c*d +
 8*(A - C)*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c
 + d*Tan[e + f*x]])])/(4*d^(7/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan
[e + f*x])^(5/2))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) - (b*(3*(b*c -
 a*d)*(5*c^2*C - 4*B*c*d + (4*A + C)*d^2) - 4*d^2*((A - C)*(b*c - a*d) + B*
(a*c + b*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d^3*(c^
2 + d^2)*f) + (b*(5*c^2*C - 4*B*c*d + (4*A + C)*d^2)*(a + b*Tan[e + f*x])^(
3/2)*Sqrt[c + d*Tan[e + f*x]])/(2*d^2*(c^2 + d^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
```

```
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+}$$

$$= \frac{\sqrt{b}\,(15a^2Cd^2-10abd(3cC-2Bd)}{}$$

$$= -\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\Big(}{(c-id}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 39.13, size = 1653959, normalized size = 3132.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan\left(fx+e\right)\right)^{\frac{5}{2}} \left(A+B\tan\left(fx+e\right)+C\left(\tan^{2}\left(fx+e\right)\right)\right)}{\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan{(e+fx)})^{\frac{5}{2}}\,(A+B\tan{(e+fx)}+C\tan^2{(e+fx)})}{(c+d\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{5/2}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{(c+d\tan{(e+f\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(3/2), x)
```

3.154 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=380

$$-\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}-\frac{(a+ib)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(c+id)^{3/}}$$

```
[Out] -(a-I*b)^(3/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f-(a+I*b)^(3/2)*(B-I*(A-C)
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(c+I*d)^(3/2)/f-(-2*B*b*d-3*C*a*d+3*C*b*c)*arctanh(d^(1/2)*(a+b*t
an(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))*b^(1/2)/d^(5/2)/f+b*(3*c^2
*C-2*B*c*d+(2*A+C)*d^2)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^2/(
c^2+d^2)/f-2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(3/2)/d/(c^2+d^2)/f/(c+d*
tan(f*x+e))^(1/2)
```

Rubi [A]

time = 4.12, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3726, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}{d^2f\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}-\frac{(a+ib)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{3/2}}-\frac{\sqrt{b}\,(-3aCd-2bBd+3bcC)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) -
((a + I*b)^(3/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) - (Sq
rt[b]*(3*b*c*C - 2*b*B*d - 3*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(5/2)*f) - (2*(c^2*C - B*c*d +
A*d^2)*(a + b*Tan[e + f*x])^(3/2))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]
]) + (b*(3*c^2*C - 2*B*c*d + (2*A + C)*d^2)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c
 + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
```

```
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\left(a+b\tan(e\right.}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\left(a+b\tan(e\right.}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\left(a+b\tan(e\right.}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\left(a+b\tan(e\right.}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\left(a+b\tan(e\right.}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\left(a+b\tan(e\right.}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\left(a+b\tan(e\right.}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+}}$$

$$= -\frac{\sqrt{b}\,(3bcC-2bBd-3aCd)\tanh^{-1}}{d^{5/2}f}$$

$$= -\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\right.}{(c-id}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 36.19, size = 1073499, normalized size = 2825.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^{\frac{3}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{3/2}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{(c+d\tan\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(3/2), x)
```

3.155 $\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}-\frac{\sqrt{a+ib}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{a-}}\right.}{(c+id)^{3/2}f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/(c-I*d)^(3/2)/f-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I
*b)^(1/2)/(c+I*d)^(3/2)/f+2*C*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2
)/(c+d*tan(f*x+e))^(1/2))*b^(1/2)/d^(3/2)/f-2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(
f*x+e))^(1/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 2.48, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$,
Rules used = {3726, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{3/2}}-\frac{\sqrt{a+ib}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{3/2}}+\frac{2\sqrt{b}\,C\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
 d*Tan[e + f*x])^(3/2),x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(3/2)*f)) - (S
qrt[a + I*b]*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f) + (2*Sqrt
[b]*C*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e
+ f*x]])])/(d^(3/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]
])/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\,\sqrt{a+b\tan(e+}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,\sqrt{a+b\tan(e+}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,\sqrt{a+b\tan(e+}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,\sqrt{a+b\tan(e+}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,\sqrt{a+b\tan(e+}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+fx)}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,\sqrt{a+b\tan(e+}{d\,(c^2+d^2)\,f\,\sqrt{c+d\tan(e+fx)}}$$

$$= \frac{2\sqrt{b}\,C\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+f}{\sqrt{b}\,\sqrt{c+d\tan(e+f}}\right.}{d^{3/2}f}$$

$$= -\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c}{\sqrt{a}}\right.}{(c-id)^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 33.56, size = 621084, normalized size = 2077.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(c + d*Tan[e + f*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan\left(fx+e\right)}\,\left(A+B\tan\left(fx+e\right)+C(\tan^{2}\left(fx+e\right))\right)}{\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan\left(e+fx\right)}\left(A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)\right)}{\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\tan\left(e+f\,x\right)}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{\left(c+d\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(3/2),x)
```

```
[Out] int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(3/2), x)
```

3.156 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,(c-id)^{3/2}f}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,(c+id)^{3/2}f}$$

```
[Out] -(B+I*(A-C))*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f/(a-I*b)^(1/2)+(I*A-B-I*C)*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I
*d)^(3/2)/f/(a+I*b)^(1/2)+2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(1/2)/(-a*
d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {3730, 3697, 3696, 95, 214}

$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\,(c-id)^{3/2}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\,(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c +
d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*(c - I*d)^(3/2)*f)) + ((
I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*(c + I*d)^(3/2)*f) + (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f*Sq
rt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}} + \frac{2\int \frac{\frac{1}{2}(bc-a}{\sqrt{a}}}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}} + \frac{(A-iB}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}} + \frac{(A-iB}{}$$

$$= \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}} + \frac{(A-iB}{}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx}}\right.}{\sqrt{a-ib}\,(c-id)^{3/2}f}$$

Mathematica [A]
time = 2.20, size = 275, normalized size = 1.10

$$-\frac{(bc-ad)\left(\frac{(iA+B-iC)(c+id)\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\,\sqrt{-c+id}}+\frac{(A+iB-C)(ic+d)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,\sqrt{c+id}}\right)+\frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}}{(-bc+ad)(c^2+d^2)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]
*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((b*c - a*d)*(((I*A + B - I*C)*(c + I*d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a +
 b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*
b]*Sqrt[-c + I*d]) + ((A + I*B - C)*(I*c + d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*
b]*Sqrt[c + I*d])) + (2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/S
qrt[c + d*Tan[e + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan(fx+e)+C(\tan^2(fx+e))}{\sqrt{a+b\tan(fx+e)}\,(c+d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/(sqrt(b*tan(f*x + e) + a)
*(d*tan(f*x + e) + c)^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan{(e + fx)} + C\tan^2{(e + fx)}}{\sqrt{a + b\tan{(e + fx)}}\,(c + d\tan{(e + fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(sqrt(a + b*tan(e + f*x))
*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(1/2)*(c
+ d*tan(e + f*x))^(3/2)),x)
```

```
[Out] \text{Hanged}
```

3.157 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=383

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{3/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}(c+id)^{3/2}f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/(c-I*d)^(3/2)/f-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I
*b)^(3/2)/(c+I*d)^(3/2)/f-2*(A*b^2-a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b
*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2)-2*d*(b^2*c*(-B*d+C*c)-a*b*B*(c^2+
d^2)+a^2*(-B*c*d+2*C*c^2+C*d^2)+A*(a^2*d^2+b^2*(c^2+2*d^2)))*(a+b*tan(f*x+e
))^(1/2)/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 1.28, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {3730, 3697, 3696, 95, 214}

$$-\frac{2d\sqrt{a+b\tan(e+fx)}\,(a^2Ad^2+a^2(-Bcd+2c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+2d^2)+b^2c(cC-Bd))}{f\,(a^2+b^2)\,(c^2+d^2)\,(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{2(Ab^2-a(bB-aC))}{f\,(a^2+b^2)\,(bc-ad)\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}(c-id)^{3/2}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*(c
+ d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*(c - I*d)^(3/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*(c + I*d)^(3/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]
]*Sqrt[c + d*Tan[e + f*x]]) - (2*d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - a*b*B*(
c^2 + d^2) + A*b^2*(c^2 + 2*d^2) + a^2*(2*c^2*C - B*c*d + C*d^2))*Sqrt[a +
b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e
+ f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(}}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right.}{(a-ib)^{3/2}(c-id)^{3/2}f}$$

Mathematica [A]
time = 6.50, size = 484, normalized size = 1.26

$$-\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}-\frac{2\left(\frac{(bc-ad)^2\left(\frac{(a+ib)(iA+B-iC)(c+id)\tanh^{-1}\left(\frac{\sqrt{-c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\,\sqrt{-c+id}}+\frac{(ia+b)(A+iB-C)(c-id)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,\sqrt{c+id}}\right)}{2(-bc+ad)(c^2+d^2)f}-\frac{2\left(-c\left(-c(Ab^2-a(bB-aC))d+\frac{1}{2}(Ab-aB-bC)d(bc-ad)\right)+\frac{1}{2}d^2(2Ab^2d-aA(bc-ad)-(bB-aC)(bc+ad))\right)\sqrt{a+b\tan(e+fx)}}{(-bc+ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}\right)}{(a^2+b^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/
2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (2*(((b*c - a*d)^2*(((a + I*b)*(I*A + B -
 I*C)*(c + I*d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a
+ I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[-c + I*d]) + ((I*a
+ b)*(A + I*B - C)*(c - I*d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d])
))/(2*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(-(c*(-(c*(A*b^2 - a*(b*B - a*C))*
d) + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)) + (d^2*(2*A*b^2*d - a*A*(b*c - a
*d) - (b*B - a*C)*(b*c + a*d)))/2)*Sqrt[a + b*Tan[e + f*x]])/((-(b*c) + a*d
)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/((a^2 + b^2)*(b*c - a*d))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan\left(fx+e\right)+C\left(\tan^{2}\left(fx+e\right)\right)}{\left(a+b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)}{\left(a+b\tan\left(e+fx\right)\right)^{\frac{3}{2}}\left(c+d\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(3/2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(3
/2)*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(3/2)*(c
+ d*tan(e + f*x))^(3/2)),x)
```

```
[Out] \text{Hanged}
```

3.158 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=598

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}(c-id)^{3/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(}}{\sqrt{a+ib}\,\sqrt{c+d\tan(}}\right.}{(a+ib)^{5/2}(c+id)^{3/2}f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/(c-I*d)^(3/2)/f-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I
*b)^(5/2)/(c+I*d)^(3/2)/f-2/3*(7*a^3*b*B*d-4*a^4*C*d+b^4*(-4*A*d+3*B*c)+a*b
^3*(6*A*c+B*d-6*C*c)-a^2*b^2*(3*B*c+2*(5*A-C)*d))/(a^2+b^2)^2/(-a*d+b*c)^2/
f/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2)-2/3*d*(8*a^3*b*B*d*(c^2+d^2
)+2*a*b^3*(3*A*c+B*d-3*C*c)*(c^2+d^2)-a^4*d*(8*c^2*C-3*B*c*d+(3*A+5*C)*d^2)
-a^2*b^2*(11*A*c^2*d+17*A*d^3+3*B*c^3-3*B*c*d^2+5*C*c^2*d-C*d^3)-b^4*(d*(5*
A*c^2+8*A*d^2+3*C*c^2)-3*B*(c^3+2*c*d^2)))*(a+b*tan(f*x+e))^(1/2)/(a^2+b^2)
^2/(-a*d+b*c)^3/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a))/
(a^2+b^2)/(-a*d+b*c)/f/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 2.36, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {3730, 3697, 3696, 95, 214}

$$-\frac{2(Ab^2-a(bB-aC))}{3f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}-\frac{2d\sqrt{a+b\tan(e+fx)}\,(a^4(-d)(d^2(3A+5C)-3Bcd+8c^2C)+8a^3bBd(c^2+d^2)-a^2b^2(11Ac^2d+17Ad^3+3Bc^3-3Bcd^2+5c^2Cd-Cd^3)+2ab^3(c^2+d^2)(3Ac+Bd-3cC)-b^4(d(5Ac^2+8Ad^2+3c^2C)-3B(c^3+2cd^2)))}{3f(a^2+b^2)^2(c^2+d^2)(bc-ad)^3\sqrt{c+d\tan(e+fx)}}-\frac{2(-4a^4Cd+7a^3bBd-a^2b^2(2d(5A-C)+3Bc)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad))}{3f(a^2+b^2)^2(bc-ad)^2\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{5/2}(c-id)^{3/2}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{5/2}(c+id)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/2)*(c
+ d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(5/2)*(c - I*d)^(3/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(5/2)*(c + I*d)^(3/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])^
(3/2)*Sqrt[c + d*Tan[e + f*x]]) - (2*(7*a^3*b*B*d - 4*a^4*C*d + b^4*(3*B*c
- 4*A*d) + a*b^3*(6*A*c - 6*c*C + B*d) - a^2*b^2*(3*B*c + 2*(5*A - C)*d)))/
(3*(a^2 + b^2)^2*(b*c - a*d)^2*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e
+ f*x]]) - (2*d*(8*a^3*b*B*d*(c^2 + d^2) + 2*a*b^3*(3*A*c - 3*c*C + B*d)*(c
^2 + d^2) - a^4*d*(8*c^2*C - 3*B*c*d + (3*A + 5*C)*d^2) - a^2*b^2*(3*B*c^3
+ 11*A*c^2*d + 5*c^2*C*d - 3*B*c*d^2 + 17*A*d^3 - C*d^3) - b^4*(d*(5*A*c^2
+ 3*c^2*C + 8*A*d^2) - 3*B*(c^3 + 2*c*d^2)))*Sqrt[a + b*Tan[e + f*x]])/(3*(
a^2 + b^2)^2*(b*c - a*d)^3*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

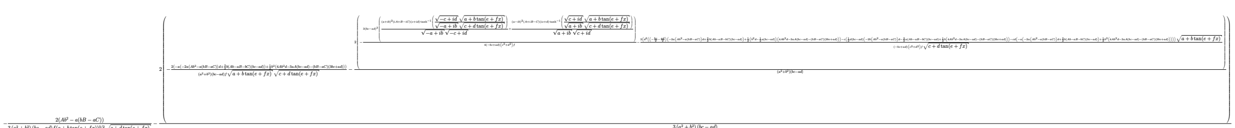
$$= -\frac{2(Ab^2-a(bB-aC))}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d}}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+f}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+f}}\right.}{(a-ib)^{5/2}(c-id)^{3/2}f}$$

Mathematica [A]
time = 6.62, size = 902, normalized size = 1.51



Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/
2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C)))/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*
x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]) - (2*((-2*(-(a*(-2*a*(A*b^2 - a*(b*B -
a*C))*d + (3*b*(A*b - a*B - b*C)*(b*c - a*d))/2)) + (b^2*(4*A*b^2*d - 3*a*A
*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2))/((a^2 + b^2)*(b*c - a*d)*f*S
qrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (2*((-3*(b*c - a*d)^3*(
((a + I*b)^2*(I*A + B - I*C)*(c + I*d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*T
```

```
an[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*S
qrt[-c + I*d]) + ((a - I*b)^2*(A + I*B - C)*(I*c + d)*ArcTanh[(Sqrt[c + I*d
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqr
t[a + I*b]*Sqrt[c + I*d])))/(4*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(d^2*((-1
/2*(b*c) - (a*d)/2)*(-2*a*(A*b^2 - a*(b*B - a*C))*d + (3*b*(A*b - a*B - b*C
)*(b*c - a*d))/2) + ((b^2*d - (a*(b*c - a*d))/2)*(4*A*b^2*d - 3*a*A*(b*c -
a*d) - (b*B - a*C)*(3*b*c + a*d)))/2) - c*((d*(b*c - a*d)*(-2*b*(A*b^2 - a*
(b*B - a*C))*d - (3*a*(A*b - a*B - b*C)*(b*c - a*d))/2 + (b*(4*A*b^2*d - 3*
a*A*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2))/2 - c*d*(-(a*(-2*a*(A*b^2
 - a*(b*B - a*C))*d + (3*b*(A*b - a*B - b*C)*(b*c - a*d))/2)) + (b^2*(4*A*b
^2*d - 3*a*A*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2)))*Sqrt[a + b*Tan[
e + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/((a^2
+ b^2)*(b*c - a*d))))/(3*(a^2 + b^2)*(b*c - a*d))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan \left(fx+e\right)+C(\tan^2 \left(fx+e\right))}{\left(a+b\tan \left(fx+e\right)\right)^{\frac{5}{2}} \left(c+d\tan \left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))
^(3/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="fricas")
```

[Out] Timed out

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{\frac{5}{2}}(c + d\tan(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(5/2)/(c+d*tan
(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(5
/2)*(c + d*tan(e + f*x))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f
*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(5/2)*(c
+ d*tan(e + f*x))^(3/2)),x)
```

```
[Out] \text{Hanged}
```

3.159 $\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=549

$$-\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}-\frac{(a+ib)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(c+id)^{5/}}$$

```
[Out] -(a-I*b)^(5/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f-(a+I*b)^(5/2)*(B-I*(A-C)
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(c+I*d)^(5/2)/f-b^(3/2)*(-2*B*b*d-5*C*a*d+5*C*b*c)*arctanh(d^(1/2
)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(7/2)/f+b*(b*(5*
c^4*C-2*B*c^3*d+10*c^2*C*d^2-6*B*c*d^3+(4*A+C)*d^4)+2*a*d^2*(2*c*(A-C)*d-B*
(c^2-d^2)))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^3/(c^2+d^2)^2/f
-2/3*(b*(5*c^4*C-2*B*c^3*d-c^2*(A-11*C)*d^2-8*B*c*d^3+5*A*d^4)+3*a*d^2*(2*c
*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(3/2)/d^2/(c^2+d^2)^2/f/(c+d*tan(f*
x+e))^(1/2)-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(5/2)/d/(c^2+d^2)/f/(c
+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 7.96, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3726, 3728, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{b\sqrt{a+b\tan(e+fx)}\,\sqrt{c+d\tan(e+fx)}\left(2ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(d^4(4A+C)-2Bc^3d-6Bcd^3+5c^4C+10c^2Cd^2\right)\right)}{d^3f\left(c^2+d^2\right)^2}-\frac{2(a+b\tan(e+fx))^{3/2}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-11C)+5Ad^4-2Bc^3d-8Bcd^3+5c^4C\right)\right)}{3d^2f\left(c^2+d^2\right)^2\sqrt{c+d\tan(e+fx)}}-\frac{(a-ib)^{5/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{5/2}}-\frac{(a+ib)^{5/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{5/2}}-\frac{b^{3/2}(-5aCd-2bBd+5bcC)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((a - I*b)^(5/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) -
((a + I*b)^(5/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (b^
(3/2)*(5*b*c*C - 2*b*B*d - 5*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(d^(7/2)*f) - (2*(c^2*C - B*c*d +
A*d^2)*(a + b*Tan[e + f*x])^(5/2))/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^
(3/2)) - (2*(b*(5*c^4*C - 2*B*c^3*d - c^2*(A - 11*C)*d^2 - 8*B*c*d^3 + 5*A*
d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f*x])^(3/2))
/(3*d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]]) + (b*(b*(5*c^4*C - 2*B*c^
3*d + 10*c^2*C*d^2 - 6*B*c*d^3 + (4*A + C)*d^4) + 2*a*d^2*(2*c*(A - C)*d -
B*(c^2 - d^2)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d^3*(c^
2 + d^2)^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e\text{-}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e\text{-}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e\text{-}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e\text{-}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e\text{-}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e\text{-}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e\text{-}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e\text{-}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{b^{3/2}(5bcC-2bBd-5aCd)\tanh^{-1}}{d^{7/2}f}$$

$$= -\frac{(a-ib)^{5/2}(B+i(A-C))\tanh^{-1}}{(c-ic}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 40.93, size = 2018643, normalized size = 3676.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(fx+e))^{\frac{5}{2}} \left(A + B\tan(fx+e) + C(\tan^2(fx+e))\right)}{(c + d\tan(fx+e))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\tan(e+fx))^{\frac{5}{2}} \left(A + B\tan(e+fx) + C\tan^2(e+fx)\right)}{(c + d\tan(e+fx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{5/2}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{(c+d\tan\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(5/2), x)
```

3.160 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=407

$$-\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}-\frac{(a+ib)^{3/2}(B-i(A-C))\tanh^{-1}\left(\cdots\right)}{(c+id)^{5/\cdots}}$$

```
[Out] -(a-I*b)^(3/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-
I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f-(a+I*b)^(3/2)*(B-I*(A-C)
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/(c+I*d)^(5/2)/f+2*b^(3/2)*C*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2
)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(5/2)/f-2*(b*(c^4*C-c^2*(A-3*C)*d^2-2*B
*c*d^3+A*d^4)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(1/2)/d^2/(
c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e)
)^(3/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 5.47, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3726, 3736, 6857, 65, 223, 212, 95, 214}

$$-\frac{2(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{3/2}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}-\frac{2\sqrt{a+b\tan(e+fx)}\,(ad^2(2cd(A-C)-B(c^2-d^2))+b(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C))}{d^2f(c^2+d^2)^2\sqrt{c+d\tan(e+fx)}}-\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{5/2}}-\frac{(a+ib)^{3/2}(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{5/2}}+\frac{2b^{3/2}C\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\,\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
 + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -(((a - I*b)^(3/2)*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) -
((a + I*b)^(3/2)*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) + (2*
b^(3/2)*C*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])])/(d^(5/2)*f) - (2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x]
)^(3/2))/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*(b*(c^4*C - c^
2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)
))*Sqrt[a + b*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
```

```
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e-}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e-}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e-}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e-}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e-}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e-}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e-}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx}$$

$$= \frac{2b^{3/2}C\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+b\tan(e+}}{\sqrt{b}\,\sqrt{c+d\tan(e+}}\right.}{d^{5/2}f}$$

$$= -\frac{(a-ib)^{3/2}(iA+B-iC)\tanh^{-1}\left(\right.}{(c-id}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 37.12, size = 1347117, normalized size = 3309.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2))/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C(\tan^2(fx+e))\right)}{(c+d\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/2
)/(d*tan(f*x + e) + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^{\frac{3}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)**
2)/(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^{3/2}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{(c+d\tan\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(5/2), x)
```

3.161 $\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=373

$$-\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}-\frac{\sqrt{a+ib}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-\ldots}}{\sqrt{a-\ldots}}\right)}{(c+id)^{5/2}f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/(c-I*d)^(5/2)/f-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I
*b)^(1/2)/(c+I*d)^(5/2)/f+2/3*(b*(c^4*C+2*B*c^3*d-c^2*(5*A-7*C)*d^2-4*B*c*d
^3+A*d^4)+3*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(1/2)/d/(-a*d
+b*c)/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan
(f*x+e))^(1/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 1.30, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3726, 3730, 3697, 3696, 95, 214}

$$-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}+\frac{2\sqrt{a+b\tan(e+fx)}\,(3ad^2(2cd(A-C)-B(c^2-d^2))+b(-c^2d^2(5A-7C)+Ad^4+2Bc^3d-4Bcd^3+c^4C))}{3df(c^2+d^2)^2(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c-id)^{5/2}}-\frac{\sqrt{a+ib}\,(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
 d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -((Sqrt[a - I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c - I*d)^(5/2)*f)) - (S
qrt[a + I*b]*(B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(5/2)*f) - (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*d*(c^2 + d^2)*f*(c + d*Tan
[e + f*x])^(3/2)) + (2*(b*(c^4*C + 2*B*c^3*d - c^2*(5*A - 7*C)*d^2 - 4*B*c*
d^3 + A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[a + b*Tan[e +
f*x]])/(3*d*(b*c - a*d)*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3726

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Dis
t[1/(d*(n + 1)*(c^2 + d^2)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
+ f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*(
n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b*
(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
```

```
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/}}$$

$$= -\frac{\sqrt{a-ib}\,(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{(c-id)^{5/2}}$$

Mathematica [A]
time = 6.11, size = 434, normalized size = 1.16

$$\frac{-\frac{3C\sqrt{a+b\tan(e+fx)}}{(c+d\tan(e+fx))^{3/2}}+\frac{(c^2C+2Bcd+(-2A+3C)d^2)\sqrt{a+b\tan(e+fx)}}{(c^2+d^2)(c+d\tan(e+fx))^{3/2}}+\frac{-3d(bc-ad)\left(\frac{\sqrt{-a+ib}\,(iA+B-iC)(c+id)^2\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-c+id}}+\frac{\sqrt{a+ib}\,(-iA+B+iC)(c-id)^2\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}}\right)+\frac{2\left(b\left(c^4C+2Bc^3d+c^2(-5A+7C)d^2-4Bcd^3+Ad^4\right)+3ad^2\left(2c(A-C)d+B\left(-c^2+d^2\right)\right)\right)\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}}{(bc-ad)(c^2+d^2)^2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
)/(c + d*Tan[e + f*x])^(5/2),x]
```

```
[Out] ((-3*C*Sqrt[a + b*Tan[e + f*x]])/(c + d*Tan[e + f*x])^(3/2) + ((c^2*C + 2*B
*c*d + (-2*A + 3*C)*d^2)*Sqrt[a + b*Tan[e + f*x]])/((c^2 + d^2)*(c + d*Tan[
e + f*x])^(3/2)) + (-3*d*(b*c - a*d)*((Sqrt[-a + I*b]*(I*A + B - I*C)*(c +
I*d)^2*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sq
rt[c + d*Tan[e + f*x]])])/Sqrt[-c + I*d] + (Sqrt[a + I*b]*((-I)*A + B + I*C
)*(c - I*d)^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*
```

```
b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c + I*d]) + (2*(b*(c^4*C + 2*B*c^3*d +
c^2*(-5*A + 7*C)*d^2 - 4*B*c*d^3 + A*d^4) + 3*a*d^2*(2*c*(A - C)*d + B*(-c^
2 + d^2)))*Sqrt[a + b*Tan[e + f*x]])/Sqrt[c + d*Tan[e + f*x]])/((b*c - a*d)
*(c^2 + d^2)^2))/(3*d*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan\left(fx+e\right)}\,\left(A+B\tan\left(fx+e\right)+C\left(\tan^{2}\left(fx+e\right)\right)\right)}{\left(c+d\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assume?`
 for mo
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\tan\left(e+fx\right)}\left(A+B\tan\left(e+fx\right)+C\tan^{2}\left(e+fx\right)\right)}{\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\tan\left(e+f\,x\right)}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{\left(c+d\tan\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(5/2),x)
```

```
[Out] int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c
 + d*tan(e + f*x))^(5/2), x)
```

3.162 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=379

$$-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,(c-id)^{5/2}f}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,(c+id)^{5/2}f}$$

```
[Out] -(B+I*(A-C))*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f/(a-I*b)^(1/2)+(I*A-B-I*C)*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c+I
*d)^(5/2)/f/(a+I*b)^(1/2)+2/3*(b*(2*c^4*C-5*B*c^3*d+4*c^2*(2*A-C)*d^2+B*c*d
^3+2*A*d^4)-3*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(1/2)/(-a*d
+b*c)^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*(A*d^2-B*c*d+C*c^2)*(a+b*t
an(f*x+e))^(1/2)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]

time = 1.24, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {3730, 3697, 3696, 95, 214}

$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2\sqrt{a+b\tan(e+fx)}\,(b(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C)-3ad^2(2cd(A-C)-B(c^2-d^2)))}{3f(c^2+d^2)^2(bc-ad)^2\sqrt{c+d\tan(e+fx)}}-\frac{(B+i(A-C))\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\,(c-id)^{5/2}}+\frac{(iA-B-iC)\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\,(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c +
d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*(c - I*d)^(5/2)*f)) + ((
I*A - B - I*C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*(c + I*d)^(5/2)*f) + (2*(c^2
*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*(b*c - a*d)*(c^2 + d^2)*f*
(c + d*Tan[e + f*x])^(3/2)) + (2*(b*(2*c^4*C - 5*B*c^3*d + 4*c^2*(2*A - C)*
d^2 + B*c*d^3 + 2*A*d^4) - 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[a
+ b*Tan[e + f*x]])/(3*(b*c - a*d)^2*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]
])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2\int \frac{1}{2}(2A \cdots}{\cdots}$$

$$= \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(2c^4C \cdots}{\cdots}$$

$$= \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(2c^4C \cdots}{\cdots}$$

$$= \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(2c^4C \cdots}{\cdots}$$

$$= \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(2c^4C \cdots}{\cdots}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx \cdots}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx \cdots}}\right.}{\sqrt{a-ib}\,(c-id)^{5/2}f} \cdots$$

Mathematica [A]
time = 3.72, size = 403, normalized size = 1.06

$$\frac{3(bc-ad)^2\left(\frac{(iA+B-iC)(c+id)^2\tanh^{-1}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}}+\frac{i(A+iB-C)(c-id)^2\tanh^{-1}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}}\right)+\frac{2(bc-ad)(c^2+d^2)(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{(c+d\tan(e+fx))^{3/2}}+\frac{2(b(2c^4C-5Bc^3d+4c^2(2A-C)d^2+Bcd^3+2Ad^4)+3ad^2(2c(-A+C)d+B(c^2-d^2)))\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}}{3(bc-ad)^2(c^2+d^2)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]
*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (3*(b*c - a*d)^2*(((I*A + B - I*C)*(c + I*d)^2*ArcTanh[(Sqrt[-c + I*d]*Sqrt
[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a
+ I*b]*Sqrt[-c + I*d]) + (I*(A + I*B - C)*(c - I*d)^2*ArcTanh[(Sqrt[c + I*d
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqr
t[a + I*b]*Sqrt[c + I*d])) + (2*(b*c - a*d)*(c^2 + d^2)*(c^2*C - B*c*d + A*
d^2)*Sqrt[a + b*Tan[e + f*x]])/(c + d*Tan[e + f*x])^(3/2) + (2*(b*(2*c^4*C
- 5*B*c^3*d + 4*c^2*(2*A - C)*d^2 + B*c*d^3 + 2*A*d^4) + 3*a*d^2*(2*c*(-A +
 C)*d + B*(c^2 - d^2)))*Sqrt[a + b*Tan[e + f*x]])/Sqrt[c + d*Tan[e + f*x]])
/(3*(b*c - a*d)^2*(c^2 + d^2)^2*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan(fx+e) + C(\tan^2(fx+e))}{\sqrt{a+b\tan(fx+e)}\,(c+d\tan(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(sqrt(a + b*tan(e + f*x))
*(c + d*tan(e + f*x))**(5/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(1/2)*(c
+ d*tan(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.163 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=651

$$-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{5/2}f}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e}{\sqrt{a+ib}\,\sqrt{c+d\tan(e}\right.}{(a+ib)^{3/2}(c+id)^{5/2}f}$$

```
[Out] -(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/(c-I*d)^(5/2)/f-(B-I*(A-C))*arctanh((c+I
*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a+I
*b)^(3/2)/(c+I*d)^(5/2)/f-2/3*d*(b^3*c*(-8*B*c^2*d-2*B*d^3+5*C*c^3-C*c*d^2)
+a^2*b*(-8*B*c^3*d-2*B*c*d^3+8*C*c^4+5*C*c^2*d^2+3*C*d^4)+3*a^3*d^2*(2*c*C*
d+B*(c^2-d^2))+3*a*b^2*(2*c*C*d^3-B*(c^4+c^2*d^2+2*d^4))-A*(6*a^3*c*d^3+6*a
*b^2*c*d^3-a^2*b*d^2*(11*c^2+5*d^2)-b^3*(3*c^4+17*c^2*d^2+8*d^4)))*(a+b*tan
(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-
2*(A*b^2-a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)/(c+d*ta
n(f*x+e))^(3/2)-2/3*d*(b^2*c*(-B*d+C*c)-3*a*b*B*(c^2+d^2)+a^2*(-B*c*d+4*C*c
^2+3*C*d^2)+A*(a^2*d^2+b^2*(3*c^2+4*d^2)))*(a+b*tan(f*x+e))^(1/2)/(a^2+b^2)
/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 2.40, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {3730, 3697, 3696, 95, 214}

$$-\frac{2d\sqrt{a+b\tan(e+fx)}\,(a^2Ad^2+a^2(-Bcd+4c^2C+3Cd^2)-3abB(c^2+d^2)+Ab^2(3c^2+4d^2)+b^2c(cC-Bd))}{3f(a^2+b^2)(c^2+d^2)(bc-ad)^2(c+d\tan(e+fx))^{3/2}}-\frac{2(Ab^2-a(bB-aC))}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e+fx))^{3/2}}-\frac{2d\sqrt{a+b\tan(e+fx)}\,(3a^3d^2(B(c^2-d^2)+2cCd)+a^2b(-8Bc^3d-2Bcd^3+8c^4C+5c^2Cd^2+3Cd^4)-A(6a^3cd^3-a^2bd^2(11c^2+5d^2)+6ab^2cd^3-(b^3(3c^4+17c^2d^2+8d^4)))+3ab^2(2cCd^3-B(c^4+c^2d^2+2d^4))+b^3c(-8Bc^2d-2Bd^3+5c^3C-cCd^2))}{3f(a^2+b^2)(c^2+d^2)^2(bc-ad)^3\sqrt{c+d\tan(e+fx)}}-\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a-ib)^{3/2}(c-id)^{5/2}}-\frac{(B-i(A-C))\tanh^{-1}\left(\frac{\sqrt{c+id}\,\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\,\sqrt{c+d\tan(e+fx)}}\right)}{f(a+ib)^{3/2}(c+id)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*(c
+ d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -(((I*A + B - I*C)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a - I*b)^(3/2)*(c - I*d)^(5/2)*f)) -
((B - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*(c + I*d)^(5/2)*f) - (2*
(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]
]*(c + d*Tan[e + f*x])^(3/2)) - (2*d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - 3*a*b
*B*(c^2 + d^2) + A*b^2*(3*c^2 + 4*d^2) + a^2*(4*c^2*C - B*c*d + 3*C*d^2))*S
qrt[a + b*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)^2*(c^2 + d^2)*f*(c + d*
Tan[e + f*x])^(3/2)) - (2*d*(b^3*c*(5*c^3*C - 8*B*c^2*d - c*C*d^2 - 2*B*d^3
) + a^2*b*(8*c^4*C - 8*B*c^3*d + 5*c^2*C*d^2 - 2*B*c*d^3 + 3*C*d^4) + 3*a^3
*d^2*(2*c*C*d + B*(c^2 - d^2)) + 3*a*b^2*(2*c*C*d^3 - B*(c^4 + c^2*d^2 + 2*
d^4)) - A*(6*a^3*c*d^3 + 6*a*b^2*c*d^3 - a^2*b*d^2*(11*c^2 + 5*d^2) - b^3*(
```

```
3*c^4 + 17*c^2*d^2 + 8*d^4)))*Sqrt[a + b*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c
 - a*d)^3*(c^2 + d^2)^2*f*Sqrt[c + d*Tan[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3696

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[A^2/f, Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

Rule 3697

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A + I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*Ta
n[e + f*x]), x], x] + Dist[(A - I*B)/2, Int[(a + b*Tan[e + f*x])^m*(c + d*T
an[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2, 0]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e}$$

$$= -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)\,(bc-ad)f\sqrt{a+b\tan(e+fx)}\,(c+d\tan(e}$$

$$= -\frac{(iA+B-iC)\tanh^{-1}\left(\frac{\sqrt{c-id}\,\sqrt{a+b\tan(e+fx}}{\sqrt{a-ib}\,\sqrt{c+d\tan(e+fx}}\right.}{(a-ib)^{3/2}(c-id)^{5/2}f}$$

Mathematica [A]
time = 6.65, size = 903, normalized size = 1.39

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/
2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
f*x]]*(c + d*Tan[e + f*x])^(3/2)) - (2*((-2*(-(c*(-2*c*(A*b^2 - a*(b*B - a*
C))*d + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)) + (d^2*(4*A*b^2*d - a*A*(b*c
- a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2)*Sqrt[a + b*Tan[e + f*x]])/(3*(-(b*c
```

```
) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*((3*(b*c - a*d)^3*(
((a + I*b)*(I*A + B - I*C)*(c + I*d)^2*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*S
qrt[-c + I*d]) + ((I*a + b)*(A + I*B - C)*(c - I*d)^2*ArcTanh[(Sqrt[c + I*d
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqr
t[a + I*b]*Sqrt[c + I*d])))/(4*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(d^2*(((b
*c)/2 - (3*a*d)/2)*(-2*c*(A*b^2 - a*(b*B - a*C))*d + ((A*b - a*B - b*C)*d*(
b*c - a*d))/2) + ((b*d^2 - (3*c*(-(b*c) + a*d))/2)*(4*A*b^2*d - a*A*(b*c -
a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2) - c*((3*d*(-(b*c) + a*d)*(-2*(A*b^2 -
 a*(b*B - a*C))*d^2 - (c*(A*b - a*B - b*C)*(b*c - a*d))/2 + (d*(4*A*b^2*d -
 a*A*(b*c - a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2))/2 - b*c*(-(c*(-2*c*(A*b^
2 - a*(b*B - a*C))*d + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)) + (d^2*(4*A*b^
2*d - a*A*(b*c - a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2)))*Sqrt[a + b*Tan[e +
 f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/(3*(-(b*c
) + a*d)*(c^2 + d^2))))/((a^2 + b^2)*(b*c - a*d))
```

Maple [**F(-1)**] grade_annotation
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))
^(5/2),x)
```

```
[Out] int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))
^(5/2),x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{\frac{3}{2}}(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(3/2)/(c+d*tan
(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(3
/2)*(c + d*tan(e + f*x))**(5/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f
*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(3/2)*(c
+ d*tan(e + f*x))^(5/2)),x)
```

```
[Out] \text{Hanged}
```

3.164 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e\right.$

Optimal. Leaf size=376

$$-\frac{(B+i(A-C))F_1\left(1+m;-n,1;2+m;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}(c+d\tan(}{2(a-ib)f(1+m)}$$

```
[Out] -1/2*(B+I*(A-C))*AppellF1(1+m,1,-n,2+m,(a+b*tan(f*x+e))/(a-I*b),-d*(a+b*tan
(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+e))^(1+m)*(c+d*tan(f*x+e))^n/(a-I*b)/f/(1
+m)/((b*(c+d*tan(f*x+e))/(-a*d+b*c))^n)-1/2*(A+I*B-C)*AppellF1(1+m,1,-n,2+m
,(a+b*tan(f*x+e))/(a+I*b),-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+e))^
(1+m)*(c+d*tan(f*x+e))^n/(I*a-b)/f/(1+m)/((b*(c+d*tan(f*x+e))/(-a*d+b*c))^n
)+C*hypergeom([-n, 1+m],[2+m],-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+
e))^(1+m)*(c+d*tan(f*x+e))^n/b/f/(1+m)/((b*(c+d*tan(f*x+e))/(-a*d+b*c))^n)
```

Rubi [A]
time = 0.62, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3736, 6857, 72, 71, 142, 141}

$$-\frac{(B+i(A-C))(a+b\tan(e+fx))^{m+1}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(a-ib)}-\frac{(A+iB-C)(a+b\tan(e+fx))^{m+1}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)}+\frac{C(a+b\tan(e+fx))^{m+1}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{bf(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] -1/2*((B + I*(A - C))*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*Tan[e + f*x
]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 +
 m)*(c + d*Tan[e + f*x])^n)/((a - I*b)*f*(1 + m)*((b*(c + d*Tan[e + f*x]))/
(b*c - a*d))^n) - ((A + I*B - C)*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*
Tan[e + f*x]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e +
 f*x])^(1 + m)*(c + d*Tan[e + f*x])^n)/(2*(I*a - b)*f*(1 + m)*((b*(c + d*Ta
n[e + f*x]))/(b*c - a*d))^n) + (C*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*
(a + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan
[e + f*x])^n)/(b*f*(1 + m)*((b*(c + d*Tan[e + f*x]))/(b*c - a*d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 3736

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[ff/f, Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^2
)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f,
 A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
d^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^m(\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{\text{Subst}\left(\int \left(C(a+\cdots\right.\right.}{\cdots}$$

$$= \frac{(-B+i(A-C)\cdots}{\cdots}$$

$$= \frac{\left(\left(-B+i(A-C\cdots\right.\right.}{\cdots}$$

$$= -\frac{(B+i(A-C)\cdots}{\cdots}$$

Mathematica [F]
time = 16.61, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
[Out] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2), x]
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int (a+b\tan(fx+e))^m\,(c+d\tan(fx+e))^n\left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(d
*tan(f*x + e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(d*
tan(f*x + e) + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**n*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(d
*tan(f*x + e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))^n\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^n*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^n*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2), x)
```

3.165 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e\right.$

Optimal. Leaf size=560

$$\frac{(bc(2+m)\left(b^2d(Bc+(A-C)d)(3+m)(4+m)-2(bc-ad)(3aCd-b(3cC+Bd(4+m)))\right)+d(b^3(2c(A-}{}$$

```
[Out] (b*c*(2+m)*(b^2*d*(B*c+(A-C)*d)*(3+m)*(4+m)-2*(-a*d+b*c)*(3*a*C*d-b*(3*c*C+
B*d*(4+m))))+d*(b^3*(2*c*(A-C)*d+B*(c^2-d^2))*(2+m)*(3+m)*(4+m)-a*(b^2*d*(B
*c+(A-C)*d)*(3+m)*(4+m)-2*(-a*d+b*c)*(3*a*C*d-b*(3*c*C+B*d*(4+m))))))*(a+b*
tan(f*x+e))^(1+m)/b^4/f/(1+m)/(2+m)/(3+m)/(4+m)+1/2*(A-I*B-C)*(c-I*d)^3*hyp
ergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a
+b)/f/(1+m)-1/2*(A+I*B-C)*(c+I*d)^3*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e
))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/f/(1+m)+d*(b^2*d*(B*c+(A-C)*d)*(
3+m)*(4+m)-2*(-a*d+b*c)*(3*a*C*d-b*(3*c*C+B*d*(4+m))))*tan(f*x+e)*(a+b*tan(
f*x+e))^(1+m)/b^3/f/(2+m)/(3+m)/(4+m)-(3*a*C*d-b*(3*c*C+B*d*(4+m)))*(a+b*ta
n(f*x+e))^(1+m)*(c+d*tan(f*x+e))^2/b^2/f/(3+m)/(4+m)+C*(a+b*tan(f*x+e))^(1+
m)*(c+d*tan(f*x+e))^3/b/f/(4+m)
```

Rubi [A]
time = 1.55, antiderivative size = 551, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3711, 3620, 3618, 70}

$$\frac{d\tan(e+fx)(a+b\tan(e+fx))^{m+1}\left(2(bc-ad)(-3aCd+bBd(m+4)+3bcC)+b^2d(m+3)(m+4)(d(A-C)+Bc)\right)}{b^3f(m+2)(m+3)(m+4)}+\frac{(a+b\tan(e+fx))^{m+1}\left(bc(m+2)\left(2(bc-ad)(-3aCd+bBd(m+4)+3bcC)+b^2d(m+3)(m+4)(d(A-C)+Bc)\right)+d\left(b^3(m+2)(m+3)(m+4)\left(2cd(A-C)+B(c^2-d^2)\right)-a\left(2(bc-ad)(-3aCd+bBd(m+4)+3bcC)+b^2d(m+3)(m+4)(d(A-C)+Bc)\right)\right)\right)}{b^4f(m+1)(m+2)(m+3)(m+4)}+\frac{(c-id)^3(A-iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}-\frac{(c+id)^3(A+iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)}+\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{b^2f(m+3)(m+4)}+\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] ((b*c*(2 + m)*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b
*c*C - 3*a*C*d + b*B*d*(4 + m))) + d*(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(
2 + m)*(3 + m)*(4 + m) - a*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*
c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))))*(a + b*Tan[e + f*x])^(1 + m
))/(b^4*f*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((A - I*B - C)*(c - I*d)^3*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[
e + f*x])^(1 + m))/(2*(I*a + b)*f*(1 + m)) - ((A + I*B - C)*(c + I*d)^3*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[
e + f*x])^(1 + m))/(2*(I*a - b)*f*(1 + m)) + (d*(b^2*d*(B*c + (A - C)*d)*(3
 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))*Tan[e +
f*x]*(a + b*Tan[e + f*x])^(1 + m))/(b^3*f*(2 + m)*(3 + m)*(4 + m)) + ((3*b*
c*C - 3*a*C*d + b*B*d*(4 + m))*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e +
f*x])^2)/(b^2*f*(3 + m)*(4 + m)) + (C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*T
an[e + f*x])^3)/(b*f*(4 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
```

```
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{C(a+b\tan(e+f\cdots}{b\cdots}$$

$$=\frac{(3bcC-3aCd+b\cdots}{\cdots}$$

$$=\frac{d(b^2d(Bc+(A-\cdots}{\cdots}$$

$$=\frac{(bc(2+m)\left(b^2d(B\cdots\right.}{\cdots}$$

$$=\frac{(bc(2+m)\left(b^2d(B\cdots\right.}{\cdots}$$

$$=\frac{(bc(2+m)\left(b^2d(B\cdots\right.}{\cdots}$$

$$=\frac{(bc(2+m)\left(b^2d(B\cdots\right.}{\cdots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1390 vs. 2(560) = 1120.
time = 6.30, size = 1390, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*x])^3)/(b*f*(4 + m)) + (((
3*b*c*C - 3*a*C*d + b*B*d*(4 + m))*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[
e + f*x])^2)/(b*f*(3 + m)) + ((d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) +
 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))*Tan[e + f*x]*(a + b*Tan
```

```
[e + f*x])^(1 + m))/(b*f*(2 + m)) - (((-(b*c*(2 + m)*(b^2*d*(B*c + (A - C)*
d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))) +
d*(-(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(2 + m)*(3 + m)*(4 + m)) + a*(b^2*
d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*
B*d*(4 + m)))))*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((I/2)*(a*d*(
b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d
+ b*B*d*(4 + m))) + b*c*(2 + m)*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) +
2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) - b*c*(2 + m)*(-((2*a*d
+ b*c*(1 + m))*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + b*c*(3 + m)*(A*b*c*(4
 + m) - C*(3*a*d + b*c*(1 + m)))) - d*(-(b^3*(2*c*(A - C)*d + B*(c^2 - d^2)
)*(2 + m)*(3 + m)*(4 + m)) + a*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2
*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))) - I*b*(2 + m)*(b^2*c*(2*
c*(A - C)*d + B*(c^2 - d^2))*(3 + m)*(4 + m) - d*(b^2*d*(B*c + (A - C)*d)*(
3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + d*(-(
(2*a*d + b*c*(1 + m))*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + b*c*(3 + m)*(A
*b*c*(4 + m) - C*(3*a*d + b*c*(1 + m))))))*Hypergeometric2F1[1, 1 + m, 2 +
m, ((-I)*a - I*b*Tan[e + f*x])/((-I)*a + b)]*(a + b*Tan[e + f*x])^(1 + m))/
((a + I*b)*f*(1 + m)) - ((I/2)*(a*d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m
) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + b*c*(2 + m)*(b^2*d
*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B
*d*(4 + m))) - b*c*(2 + m)*(-((2*a*d + b*c*(1 + m))*(3*b*c*C - 3*a*C*d + b*
B*d*(4 + m))) + b*c*(3 + m)*(A*b*c*(4 + m) - C*(3*a*d + b*c*(1 + m)))) - d*
(-(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(2 + m)*(3 + m)*(4 + m)) + a*(b^2*d*
(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*
d*(4 + m)))) + I*b*(2 + m)*(b^2*c*(2*c*(A - C)*d + B*(c^2 - d^2))*(3 + m)*(
4 + m) - d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*
C - 3*a*C*d + b*B*d*(4 + m))) + d*(-((2*a*d + b*c*(1 + m))*(3*b*c*C - 3*a*C
*d + b*B*d*(4 + m))) + b*c*(3 + m)*(A*b*c*(4 + m) - C*(3*a*d + b*c*(1 + m))
))))*Hypergeometric2F1[1, 1 + m, 2 + m, -((I*a + I*b*Tan[e + f*x])/((-I)*a
- b))]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)))/(b*(2 + m)))/(b
*(3 + m)))/(b*(4 + m))
```

Maple [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int (a+b\tan (fx+e))^m (c+d\tan (fx+e))^3 \left(A+B\tan (fx+e)+C\left(\tan^2 (fx+e)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)

[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] integral((C*d^3*tan(f*x + e)^5 + (3*C*c*d^2 + B*d^3)*tan(f*x + e)^4 + A*c^3
 + (3*C*c^2*d + 3*B*c*d^2 + A*d^3)*tan(f*x + e)^3 + (C*c^3 + 3*B*c^2*d + 3*
A*c*d^2)*tan(f*x + e)^2 + (B*c^3 + 3*A*c^2*d)*tan(f*x + e))*(b*tan(f*x + e)
 + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^m (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))**3*(A + B*tan(e + f*x
) + C*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^3*(b
*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))^3\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2), x)
```

3.166 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2 (A+B\tan(e$

Optimal. Leaf size=363

$$\frac{(2a^2Cd^2 - abd(2cC+Bd)(3+m) + b^2(2+m)(2c^2C + 2Bcd(3+m) + (A-C)d^2(3+m)))(a+b\tan(e+}{b^3f(1+m)(2+m)(3+m)}$$

```
[Out] (2*a^2*C*d^2-a*b*d*(B*d+2*C*c)*(3+m)+b^2*(2+m)*(2*c^2*C+2*B*c*d*(3+m)+(A-C)
*d^2*(3+m)))*(a+b*tan(f*x+e))^(1+m)/b^3/f/(1+m)/(2+m)/(3+m)+1/2*(A-I*B-C)*(
c-I*d)^2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e)
)^(1+m)/(I*a+b)/f/(1+m)+1/2*(I*A-B-I*C)*(c+I*d)^2*hypergeom([1, 1+m],[2+m],
(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(a+I*b)/f/(1+m)-d*(2*a*C*d
-b*(2*c*C+B*d*(3+m)))*tan(f*x+e)*(a+b*tan(f*x+e))^(1+m)/b^2/f/(2+m)/(3+m)+C
*(a+b*tan(f*x+e))^(1+m)*(c+d*tan(f*x+e))^2/b/f/(3+m)
```

Rubi [A]
time = 0.77, antiderivative size = 360, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3728, 3718, 3711, 3620, 3618, 70}

$$\frac{(a+b\tan(e+fx))^{m+1}(2a^2Cd^2 - abd(m+3)(Bd+2cC) + b^2(m+2)(d^2(m+3)(A-C) + 2Bcd(m+3) + 2c^2C))}{b^3f(m+1)(m+2)(m+3)} + \frac{(c-id)^2(A-iB-C)(a+b\tan(e+fx))^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)} + \frac{(c+id)^2(iA-B-iC)(a+b\tan(e+fx))^{m+1}{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)} + \frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{b^2f(m+2)(m+3)} + \frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*T
an[e + f*x]^2),x]
```

```
[Out] ((2*a^2*C*d^2 - a*b*d*(2*c*C + B*d)*(3 + m) + b^2*(2 + m)*(2*c^2*C + 2*B*c*
d*(3 + m) + (A - C)*d^2*(3 + m)))*(a + b*Tan[e + f*x])^(1 + m))/(b^3*f*(1 +
 m)*(2 + m)*(3 + m)) + ((A - I*B - C)*(c - I*d)^2*Hypergeometric2F1[1, 1 +
m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*
(I*a + b)*f*(1 + m)) + ((I*A - B - I*C)*(c + I*d)^2*Hypergeometric2F1[1, 1
+ m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(
2*(a + I*b)*f*(1 + m)) + (d*(2*b*c*C - 2*a*C*d + b*B*d*(3 + m))*Tan[e + f*x
]*(a + b*Tan[e + f*x])^(1 + m))/(b^2*f*(2 + m)*(3 + m)) + (C*(a + b*Tan[e +
 f*x])^(1 + m)*(c + d*Tan[e + f*x])^2)/(b*f*(3 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
```

```
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C(a+b\tan(e+f\ldots}{b\ldots}$$

$$= \frac{d(2bcC-2aCd+\ldots}{\ldots}$$

$$= \frac{(2a^2Cd^2-abd(2c\ldots}{\ldots}$$

$$= \frac{(2a^2Cd^2-abd(2c\ldots}{\ldots}$$

$$= \frac{(2a^2Cd^2-abd(2c\ldots}{\ldots}$$

$$= \frac{(2a^2Cd^2-abd(2c\ldots}{\ldots}$$

Mathematica [A]
time = 4.84, size = 308, normalized size = 0.85

$$\frac{(a+b\tan(e+fx))^{1+m}\left(\frac{2bc(2+m)(2bcC-2aCd+bBd(3+m))+2d(b^2(Bc+(A-C)d)(2+m)(3+m)-a(2bcC-2aCd+bBd(3+m)))-\frac{ib^3(A-iB-C)(c-id)^2(6+5m+m^2)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{ib^3(A+iB-C)(c+id)^2(6+5m+m^2)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}}{b^2(1+m)(2+m)}+\frac{2d(2bcC-2aCd+bBd(3+m))\tan(e+fx)}{b(2+m)}+2C(c+d\tan(e+fx))^2\right)}{2bf(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x]
 + C*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b*Tan[e + f*x])^(1 + m)*((2*b*c*(2 + m)*(2*b*c*C - 2*a*C*d + b*B*d*(3
 + m)) + 2*d*(b^2*(B*c + (A - C)*d)*(2 + m)*(3 + m) - a*(2*b*c*C - 2*a*C*d
+ b*B*d*(3 + m))) - (I*b^3*(A - I*B - C)*(c - I*d)^2*(6 + 5*m + m^2)*Hyperg
eometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)])/(a - I*b) + (
I*b^3*(A + I*B - C)*(c + I*d)^2*(6 + 5*m + m^2)*Hypergeometric2F1[1, 1 + m,
 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b))/(b^2*(1 + m)*(2 + m)) +
 (2*d*(2*b*c*C - 2*a*C*d + b*B*d*(3 + m))*Tan[e + f*x])/(b*(2 + m)) + 2*C*(
c + d*Tan[e + f*x])^2))/(2*b*f*(3 + m))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(a+b\tan\left(fx+e\right)\right)^{m}\left(c+d\tan\left(fx+e\right)\right)^{2}\left(A+B\tan\left(fx+e\right)+C\left(\tan^{2}\left(fx+e\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^2*(b
*tan(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="fricas")
```

```
[Out] integral((C*d^2*tan(f*x + e)^4 + (2*C*c*d + B*d^2)*tan(f*x + e)^3 + A*c^2 +
 (C*c^2 + 2*B*c*d + A*d^2)*tan(f*x + e)^2 + (B*c^2 + 2*A*c*d)*tan(f*x + e))
*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^m (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x
+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))**2*(A + B*tan(e + f*x
) + C*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e
)^2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^2*(b
*tan(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))^2\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*t
an(e + f*x)^2), x)
```

3.167 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx)) \left(A+B\tan(e\right.$

Optimal. Leaf size=247

$$-\frac{(aCd-b(cC+Bd)(2+m))(a+b\tan(e+fx))^{1+m}}{b^2f(1+m)(2+m)}+\frac{(A-iB-C)(c-id)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e}{a-ib}\right.}{2(ia+b)f(1+m)}$$

```
[Out] -(a*C*d-b*(B*d+C*c)*(2+m))*(a+b*tan(f*x+e))^(1+m)/b^2/f/(1+m)/(2+m)+1/2*(A-
I*B-C)*(c-I*d)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(
f*x+e))^(1+m)/(I*a+b)/f/(1+m)-1/2*(A+I*B-C)*(c+I*d)*hypergeom([1, 1+m],[2+m
],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/f/(1+m)+C*d*tan(
f*x+e)*(a+b*tan(f*x+e))^(1+m)/b/f/(2+m)
```

Rubi [A]
time = 0.36, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3718, 3711, 3620, 3618, 70}

$$\frac{(c-id)(A-iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}-\frac{(c+id)(A+iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)}-\frac{(aCd-b(m+2)(Bd+cC))(a+b\tan(e+fx))^{m+1}}{b^2f(m+1)(m+2)}+\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan
[e + f*x]^2),x]
```

```
[Out] -(((a*C*d - b*(c*C + B*d)*(2 + m))*(a + b*Tan[e + f*x])^(1 + m))/(b^2*f*(1
+ m)*(2 + m))) + ((A - I*B - C)*(c - I*d)*Hypergeometric2F1[1, 1 + m, 2 + m
, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b
)*f*(1 + m)) - ((A + I*B - C)*(c + I*d)*Hypergeometric2F1[1, 1 + m, 2 + m,
(a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a - b)*
f*(1 + m)) + (C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rule 3718

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Dist[1/(d*(n + 2)), Int[(c + d*Tan[e + f*x])^n*Sim
p[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*d
*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{Cd\tan(e+fx)(a+}{bf(2}$$

$$=-\frac{(aCd-b(cC+B}{b}$$

$$=-\frac{(aCd-b(cC+B}{b}$$

$$=-\frac{(aCd-b(cC+B}{b}$$

$$=-\frac{(aCd-b(cC+B}{b}$$

Mathematica [**A**]

time = 1.99, size = 202, normalized size = 0.82

$$\frac{(a+b\tan(e+fx))^{1+m}\left(\frac{-2aCd+2b(cC+Bd)(2+m)}{b(1+m)}-\frac{ib(A-iB-C)(c-id)(2+m)\,_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{(a-ib)(1+m)}+\frac{ib(A+iB-C)(c+id)(2+m)\,_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{(a+ib)(1+m)}+2Cd\tan(e+fx)\right)}{2bf(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] +
 C*Tan[e + f*x]^2),x]
```

```
[Out] ((a + b*Tan[e + f*x])^(1 + m)*((-2*a*C*d + 2*b*(c*C + B*d)*(2 + m))/(b*(1 +
 m)) - (I*b*(A - I*B - C)*(c - I*d)*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 +
 m, (a + b*Tan[e + f*x])/(a - I*b)])/((a - I*b)*(1 + m)) + (I*b*(A + I*B -
C)*(c + I*d)*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x]
)/(a + I*b)])/((a + I*b)*(1 + m)) + 2*C*d*Tan[e + f*x]))/(2*b*f*(2 + m))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (a+b\tan(fx+e))^m (c+d\tan(fx+e)) \left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)*(b*t
an(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="fricas")
```

```
[Out] integral((C*d*tan(f*x + e)^3 + (C*c + B*d)*tan(f*x + e)^2 + A*c + (B*c + A*
d)*tan(f*x + e))*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\tan(e+fx))^m (c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) +
 C*tan(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^
2),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)*(b*t
an(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan
(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan
(e + f*x)^2), x)
```

3.168 $\int (a+b\tan(e+fx))^m (A+B\tan(e+fx)+C\tan^2(e$

Optimal. Leaf size=178

$$\frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)}+\frac{(A-iB-C)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)f(1+m)}+\frac{(iA-$$

```
[Out] C*(a+b*tan(f*x+e))^(1+m)/b/f/(1+m)+1/2*(A-I*B-C)*hypergeom([1, 1+m],[2+m],(
a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a+b)/f/(1+m)+1/2*(I*A-B-
I*C)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1
+m)/(a+I*b)/f/(1+m)
```

Rubi [A]
time = 0.13, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3711, 3620, 3618, 70}

$$\frac{(A-iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)}+\frac{(iA-B-iC)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)}+\frac{C(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((A - I*B - C)*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x]
)^(1 + m))/(2*(I*a + b)*f*(1 + m)) + ((I*A - B - I*C)*Hypergeometric2F1[1,
1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))
/(2*(a + I*b)*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
```

```
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx &= \frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)}+\int (a+b\,\mathrm{ta}\\ &= \frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)}+\frac{1}{2}(A-iB\\ &= \frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)}+\frac{(iA+B-}{}\\ &= \frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)}-\frac{(iA+B-}{}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 135, normalized size = 0.76

$$\frac{\left(\frac{2C}{b}-\frac{i(A-iB-C)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{i(A+iB-C)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
[Out] (((2*C)/b - (I*(A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[
e + f*x])/(a - I*b)])/(a - I*b) + (I*(A + I*B - C)*Hypergeometric2F1[1, 1 +
 m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b))*(a + b*Tan[e + f*x])
^(1 + m))/(2*f*(1 + m))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (a+b\tan(fx+e))^m \left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tan \left(e + f x\right)\right)^{m} \left(A + B \tan \left(e + f x\right) + C \tan^{2}\left(e + f x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(A + B*tan(e + f*x) + C*tan(e + f*x)**2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\tan(e+f\,x))^m\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

```
[Out] int((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2), x)
```

3.169 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$

Optimal. Leaf size=258

$$-\frac{(iA+B-iC)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a-ib)(c-id)f(1+m)}-\frac{(A+iB-C)\,{}_2F_1\left(1,1+m;2\right.}{2(ia-}$$

```
[Out] -1/2*(I*A+B-I*C)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*ta
n(f*x+e))^(1+m)/(a-I*b)/(c-I*d)/f/(1+m)-1/2*(A+I*B-C)*hypergeom([1, 1+m],[2
+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/(c+I*d)/f/(1+m
)+(A*d^2-B*c*d+C*c^2)*hypergeom([1, 1+m],[2+m],-d*(a+b*tan(f*x+e))/(-a*d+b*
c))*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)/(c^2+d^2)/f/(1+m)
```

Rubi [A]

time = 0.34, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3734, 3620, 3618, 70, 3715}

$$\frac{(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{f(m+1)(c^2+d^2)(bc-ad)}-\frac{(iA+B-iC)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(a-ib)(c-id)}-\frac{(A+iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(-b+ia)(c+id)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x]),x]
```

```
[Out] -1/2*((I*A + B - I*C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x
])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*(c - I*d)*f*(1 + m))
 - ((A + I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(
a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a - b)*(c + I*d)*f*(1 + m)) +
 ((c^2*C - B*c*d + A*d^2)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Ta
n[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2
+ d^2)*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = \frac{\int (a+b\tan(e+fx))^m(Ac-cC+Bd}{c^2+}$$

$$= \frac{(A-iB-C)\int (1+i\tan(e+fx))(a-}{2(c-id)}$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)\,{}_2F_1\Big(1,1+m;2+}{(bc-ad)\,(c}$$

$$= -\frac{(iA+B-iC)\,{}_2F_1\Big(1,1+m;2+m;}{2(a-ib)(c-}$$

Mathematica [A]
time = 0.75, size = 204, normalized size = 0.79

$$\frac{\left(\frac{(A-iB-C)(-ic+d)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{(A+iB-C)(ic+d)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}+\frac{2\left(c^2C-Bcd+Ad^2\right)\,{}_2F_1\left(1,1+m;2+m;\frac{d(a+b\tan(e+fx))}{-bc+ad}\right)}{bc-ad}\right)(a+b\tan(e+fx))^{1+m}}{2\left(c^2+d^2\right)f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x]),x]
```

```
[Out] ((((A - I*B - C)*((-I)*c + d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan
[e + f*x])/(a - I*b)])/(a - I*b) + ((A + I*B - C)*(I*c + d)*Hypergeometric2
F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b) + (2*(c^2*C
- B*c*d + A*d^2)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*Tan[e + f*x])
)/(-(b*c) + a*d)])/(b*c - a*d))*(a + b*Tan[e + f*x])^(1 + m))/(2*(c^2 + d^2
)*f*(1 + m))
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tan\left(fx+e\right)\right)^{m}\left(A+B\tan\left(fx+e\right)+C\left(\tan^{2}\left(fx+e\right)\right)\right)}{c+d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

```
[Out] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d*
tan(f*x + e) + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^m (A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e)),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
)),x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^m \left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{c+d\tan(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x)),x)
```

```
[Out] int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x)), x)
```

3.170 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

Optimal. Leaf size=403

$$\frac{(A-iB-C)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)^2f(1+m)}+\frac{(iA-B-iC)\,{}_2F_1\left(1,1+m;2+\right.}{2(a+ib)}$$

```
[Out] 1/2*(A-I*B-C)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f
*x+e))^(1+m)/(I*a+b)/(c-I*d)^2/f/(1+m)+1/2*(I*A-B-I*C)*hypergeom([1, 1+m],[
2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(a+I*b)/(c+I*d)^2/f/(
1+m)-(a*d^2*(2*c*(A-C)*d-B*(c^2-d^2))-b*(A*d^2*(c^2*(2-m)-d^2*m)-B*c*d*(c^2
*(1-m)-d^2*(1+m))-c^2*C*(c^2*m+d^2*(2+m))))*hypergeom([1, 1+m],[2+m],-d*(a+
b*tan(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)^2/(c^2+d^2)^2/f
/(1+m)+(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)/(c^2+d^2)/f/(c
+d*tan(f*x+e))
```

Rubi [A]

time = 0.84, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3730, 3734, 3620, 3618, 70, 3715}

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}-\frac{(a+b\tan(e+fx))^{m+1}\left(ad^2(2cd(A-C)-B(c^2-d^2))-b(Ac^2d^2(2-m)-Ad^4m-B(c^3d(1-m)-cd^3(m+1))+c^4(-C)m-c^2Cd^2(m+2))\right)\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{f(m+1)\left(c^2+d^2\right)^2(bc-ad)^2}+\frac{(A-iB-C)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(b+ia)(c-id)^2}+\frac{(iA-B-iC)(a+b\tan(e+fx))^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)(c+id)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^2,x]
```

```
[Out] ((A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a -
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)^2*f*(1 + m)) +
((I*A - B - I*C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a
 + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a + I*b)*(c + I*d)^2*f*(1 + m))
- ((a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)) - b*(A*c^2*d^2*(2 - m) - c^4*C*m
- A*d^4*m - c^2*C*d^2*(2 + m) - B*(c^3*d*(1 - m) - c*d^3*(1 + m))))*Hyperge
ometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b
*Tan[e + f*x])^(1 + m))/((b*c - a*d)^2*(c^2 + d^2)^2*f*(1 + m)) + ((c^2*C -
 B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2 + d^2)*f*(c
 + d*Tan[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx = \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+}$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+}$$

$$= \frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+}$$

$$= -\frac{\left(ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)-b}{}$$

$$= -\frac{(iA+B-iC)\,{}_2F_1\Big(1,1+m;2+m}{2(a-ib)(c-}$$

Mathematica [A]
time = 5.70, size = 360, normalized size = 0.89

$$\frac{(a+b\tan(e+fx))^{1+m}\left(-\frac{i\left(\frac{(A-iB-C)(c+id)^2(-bc+ad)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{(A+iB-C)(c-id)^2(bc-ad)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}\right)}{(c^2+d^2)(1+m)}+\frac{2\left(ad^2\left(2c(A-C)d+B\left(-c^2+d^2\right)\right)+b\left(Ac^2d^2(-2+m)+c^4Cm+Ad^4m+c^2Cd^2(2+m)-B\left(c^3d(-1+m)+cd^3(1+m)\right)\right)\right)\,{}_2F_1\left(1,1+m;2+m;\frac{d(a+b\tan(e+fx))}{-bc+ad}\right)}{(bc-ad)(c^2+d^2)(1+m)}-\frac{2\left(c^2C-Bcd+Ad^2\right)}{c+d\tan(e+fx)}\right)}{2(-bc+ad)\left(c^2+d^2\right)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^2,x]
```

```
[Out] ((a + b*Tan[e + f*x])^(1 + m)*(((-I)*(((A - I*B - C)*(c + I*d)^2*(-(b*c) +
a*d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)])/(a
 - I*b) + ((A + I*B - C)*(c - I*d)^2*(b*c - a*d)*Hypergeometric2F1[1, 1 + m
, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b)))/((c^2 + d^2)*(1 + m))
 + (2*(a*d^2*(2*c*(A - C)*d + B*(-c^2 + d^2)) + b*(A*c^2*d^2*(-2 + m) + c^4
*C*m + A*d^4*m + c^2*C*d^2*(2 + m) - B*(c^3*d*(-1 + m) + c*d^3*(1 + m))))*H
ypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*Tan[e + f*x]))/(-(b*c) + a*d)])
/((b*c - a*d)*(c^2 + d^2)*(1 + m)) - (2*(c^2*C - B*c*d + A*d^2))/(c + d*Tan
[e + f*x])))/(2*(-(b*c) + a*d)*(c^2 + d^2)*f)
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^m\left(A+B\tan(fx+e)+C\left(\tan^2(fx+e)\right)\right)}{(c+d\tan(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x
)
```

```
[Out] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d^
2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^2,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^m\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{\left(c+d\tan\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^2,x)
```

```
[Out] int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^2, x)
```

3.171 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

Optimal. Leaf size=702

$$\frac{(A-iB-C)\,{}_2F_1\left(1,1+m;2+m;\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)^3f(1+m)}+\frac{(A+iB-C)\,{}_2F_1\left(1,1+m;2+m\right.}{2(a+ib)(i}$$

```
[Out] 1/2*(A-I*B-C)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f
*x+e))^(1+m)/(I*a+b)/(c-I*d)^3/f/(1+m)+1/2*(A+I*B-C)*hypergeom([1, 1+m],[2+
m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(a+I*b)/(I*c-d)^3/f/(1+
m)+1/2*(2*a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2))-2*a*b*d^2*(B*(6*c^2
*d^2-c^4*(2-m)-d^4*m)+2*c*(A-C)*d*(c^2*(3-m)-d^2*(1+m)))-b^2*(A*d^2*(d^4*(1
-m)*m+2*c^2*d^2*(-m^2+3*m+1)-c^4*(m^2-5*m+6))+B*c*d*(d^4*m*(1+m)-2*c^2*d^2*
(-m^2+m+3)+c^4*(m^2-3*m+2))+c^2*C*(c^4*(1-m)*m+2*c^2*d^2*(-m^2-m+3)-d^4*(m^
2+3*m+2))))*hypergeom([1, 1+m],[2+m],-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(a+b*t
an(f*x+e))^(1+m)/(-a*d+b*c)^3/(c^2+d^2)^3/f/(1+m)+1/2*(A*d^2-B*c*d+C*c^2)*(
a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^2-1/2*(2*a*d^
2*(2*c*(A-C)*d-B*(c^2-d^2))-b*(c^4*C*(1-m)+A*d^4*(1-m)-B*c^3*d*(3-m)+B*c*d^
3*(1+m)+c^2*d^2*(A*(5-m)-C*(3+m))))*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)^2/(c^
2+d^2)^2/f/(c+d*tan(f*x+e))
```

Rubi [A]
time = 2.11, antiderivative size = 702, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3730, 3734, 3620, 3618, 70, 3715}

Antiderivative was successfully verified.

```
[In] Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d
*Tan[e + f*x])^3,x]
```

```
[Out] ((A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a -
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(I*a + b)*(c - I*d)^3*f*(1 + m)) +
((A + I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a +
 I*b)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(a + I*b)*(I*c - d)^3*f*(1 + m)) +
((2*a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*d^2*(B*(6
*c^2*d^2 - c^4*(2 - m) - d^4*m) + 2*c*(A - C)*d*(c^2*(3 - m) - d^2*(1 + m))
) - b^2*(A*d^2*(d^4*(1 - m)*m + 2*c^2*d^2*(1 + 3*m - m^2) - c^4*(6 - 5*m +
m^2)) + B*(c*d^5*m*(1 + m) - 2*c^3*d^3*(3 + m - m^2) + c^5*d*(2 - 3*m + m^2
)) + c^2*C*(c^4*(1 - m)*m + 2*c^2*d^2*(3 - m - m^2) - d^4*(2 + 3*m + m^2)))
)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d)
)]*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c - a*d)^3*(c^2 + d^2)^3*f*(1 + m))
+ ((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c - a*d)*(c^
```

```
2 + d^2)*f*(c + d*Tan[e + f*x])^2) - ((2*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^
2)) - b*(c^4*C*(1 - m) + A*d^4*(1 - m) - B*c^3*d*(3 - m) + B*c*d^3*(1 + m)
+ c^2*d^2*(A*(5 - m) - C*(3 + m))))*(a + b*Tan[e + f*x])^(1 + m))/(2*(b*c -
 a*d)^2*(c^2 + d^2)^2*f*(c + d*Tan[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3618

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[c*(d/f), Subst[Int[(a + (b/d)*x)^m/(d^2 + c
*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Rule 3620

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Dist[(c + I*d)/2, Int[(a + b*Tan[e + f*x])^m*(1
 - I*Tan[e + f*x]), x], x] + Dist[(c - I*d)/2, Int[(a + b*Tan[e + f*x])^m*(
1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Rule 3715

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Dist[A/f, Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /; F
reeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
```

```
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3734

```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n
*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Dist[(
A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(c + d*Tan[e + f*x])^n*((1 + Tan[e
+ f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C
, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&
 !GtQ[n, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^m\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx = \frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx)}{2(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+f}$$

$$= \frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx)}{2(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+f}$$

$$= \frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx)}{2(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+f}$$

$$= \frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx)}{2(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+f}$$

$$= \frac{(2a^2d^3((A-C)d(3c^2-d^2)-B(c^3-3}{}$$

$$= -\frac{(A-iB-C)\,{}_2F_1\Big(1,1+m;2+m;}{2(a-ib)(ic+}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2238 vs. 2(702) = 1404.
time = 6.17, size = 2238, normalized size = 3.19

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/
(c + d*Tan[e + f*x])^3,x]
```

```
[Out] -1/2*((A*d^2 - c*(-(c*C) + B*d))*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) + a
*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (-(((-(c*(2*d*(b*c - a*d)*(B*c
- (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a
*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))))*(a + b*Tan[e + f
*x])^(1 + m))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))) - (-(((-
(c*d*(-(b*c) + a*d)*(-2*c*(b*c - a*d)*(B*c - (A - C)*d) - b*d*(c^2*C - B*c*
d + A*d^2)*(1 - m) + d*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(
2*a*d - b*c*(1 + m))))) - b*c^2*m*(-(c*(2*d*(b*c - a*d)*(B*c - (A - C)*d) -
 b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a*d) + b*d^2*(1
 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))) + d^2*((2*d*(b*c - a*d)*(B*c -
 (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) + b*c*(1 + m)) +
 (-(c*(-(b*c) + a*d)) - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*
C - B*d)*(2*a*d - b*c*(1 + m)))))*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a
+ b*Tan[e + f*x]))/(-(b*c) + a*d)]*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) +
 a*d)*(c^2 + d^2)*f*(1 + m))) + (((I/2)*(d*(-(b*c) + a*d)*(-2*c*(b*c - a*d)
*(B*c - (A - C)*d) - b*d*(c^2*C - B*c*d + A*d^2)*(1 - m) + d*(A*(2*c*(b*c -
 a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))) + c*((2*d*(b*c
 - a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) +
b*c*(1 + m)) + (-(c*(-(b*c) + a*d)) - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*
(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))) + b*m*(-(c*(2*d*(b*c - a*d)*(
B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c
 - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))))) + I*(c*(-(b
*c) + a*d)*(-2*c*(b*c - a*d)*(B*c - (A - C)*d) - b*d*(c^2*C - B*c*d + A*d^2
)*(1 - m) + d*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b
*c*(1 + m)))) - d*((2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d
+ A*d^2)*(1 - m))*(-(a*d) + b*c*(1 + m)) + (-(c*(-(b*c) + a*d)) - b*d^2*m)*
(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))) +
 b*m*(-(c*(2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*
(1 - m))) + d^2*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d -
 b*c*(1 + m)))))))*Hypergeometric2F1[1, 1 + m, 2 + m, ((-I)*a - I*b*Tan[e +
 f*x])/((-I)*a + b)]*(a + b*Tan[e + f*x])^(1 + m))/((a + I*b)*f*(1 + m)) -
((I/2)*(d*(-(b*c) + a*d)*(-2*c*(b*c - a*d)*(B*c - (A - C)*d) - b*d*(c^2*C -
 B*c*d + A*d^2)*(1 - m) + d*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B
*d)*(2*a*d - b*c*(1 + m)))) + c*((2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(
c^2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) + b*c*(1 + m)) + (-(c*(-(b*c) + a*d
)) - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b
*c*(1 + m))) + b*m*(-(c*(2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B
*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C -
 B*d)*(2*a*d - b*c*(1 + m))))) - I*(c*(-(b*c) + a*d)*(-2*c*(b*c - a*d)*(B*c
 - (A - C)*d) - b*d*(c^2*C - B*c*d + A*d^2)*(1 - m) + d*(A*(2*c*(b*c - a*d)
 + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))) - d*((2*d*(b*c - a*
d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) + b*c*(
1 + m)) + (-(c*(-(b*c) + a*d)) - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*(1 -
m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))) + b*m*(-(c*(2*d*(b*c - a*d)*(B*c -
 (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a*
```

```
d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))))))*Hypergeometric
2F1[1, 1 + m, 2 + m, -((I*a + I*b*Tan[e + f*x])/((-I)*a - b))]*(a + b*Tan[e
 + f*x])^(1 + m))/((a - I*b)*f*(1 + m)))/(c^2 + d^2))/((-(b*c) + a*d)*(c^2
+ d^2)))/(2*(-(b*c) + a*d)*(c^2 + d^2))
```

Maple [F]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(fx+e))^{m}\left(A+B\tan(fx+e)+C(\tan^{2}(fx+e))\right)}{(c+d\tan(fx+e))^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x
)
```

```
[Out] int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="maxima")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="fricas")
```

```
[Out] integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d^
3*tan(f*x + e)^3 + 3*c*d^2*tan(f*x + e)^2 + 3*c^2*d*tan(f*x + e) + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tan(e+fx))^{m}\left(A+B\tan(e+fx)+C\tan^{2}(e+fx)\right)}{(c+d\tan(e+fx))^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x
+e))**3,x)
```

```
[Out] Integral((a + b*tan(e + f*x))**m*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(
c + d*tan(e + f*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e
))^3,x, algorithm="giac")
```

```
[Out] integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\tan(e+f\,x))^m\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{(c+d\tan\left(e+f\,x\right))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^3,x)
```

```
[Out] int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d
*tan(e + f*x))^3, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section 1120
4.2 Listing of Grading functions 1120

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```